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Electromagnetic analogue of a first-type leaky surface elastic wave for the single interface between transparent dielectric media

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Contents

Abstract. Under conditions of total internal reflection of a TM- (TE-) type of plane volume electromagnetic wave from the surface of a semi-infinite transparent anisotropic dielectric medium, a special type of fast improper surface wave can be formed (an exceptional surface wave). For these types of waves, the instantaneous flow of energy through the interface is zero. In this case, the reflection of a quasi-plane (or quasi-monochromatic) wave of the corresponding polarization leads to the excitation of the leaky surface wave and to the maximum of the resonant amplification of the Goos – Hänchen effect (or the Wigner delay effect).

Keywords: evanescent wave, total internal reflection, bianisotropic medium, antiferromagnet, Goos–Hänchen shift, Wigner delay, leaky surface wave, multilayered structure

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1. Introduction

Despite unabating active studies of the wave dynamics of layered media over several decades, the physics of leaky surface waves remains an actively investigated field [1–6]. This is not least due to the ever-expanding range of practical applications of waves of this type (primarily in antenna technology [7]), as well as due to the possibility of explaining a variety of physical phenomena [8].

According to the classification proposed in Ref. [2], among the possible wave field types (surface, proper complex, leaky) corresponding to the Fresnel pole of the reflection coefficient for a plane electromagnetic wave incident on the plane surface of a dissipation-free layered structure, only the improper complex (leaky) wave attenuates in its propagation along the layered structure due to the radiation of a volume electromagnetic wave into the medium adjacent to the layered structure (radiative decay).

In acoustics [4], in the case of a single media interface, it was suggested to class leaky surface waves into two types. For type-I waves, the generation of the volume wave responsible for radiative decay occurs in the medium adjacent to the medium in which the propagating leaky surface wave is localized. In the case of type-II leaky surface waves, the volume wave generation occurs in the same medium in which the initial propagating surface wave is localized. In both cases, the leaky wave in its propagation along the media interface is localized in some domain near the radiation source due to radiative decay. As a consequence, at a given frequency, this wave is characterized by complex values of the longitudinal (along the direction of propagation) wavenumber $h = h' + \mathrm{i}h''$. When $|h'| \gg |h''|$, the localization domain near the source turns out to be rather large (a slowly leaking wave). In the same wave configuration, the inverse effect is not only of academic but also of purely practical interest: the resonance enhancement of the amplitude of a slowly leaking wave (surface wave resonance). This is possible when the interface is irradiated by a volume wave with the same frequency and polarization as the volume wave responsible for the radiative decay of the propagating leaky surface wave (of type I or type II) and the projection of the wave vector of the wave incident on the media interface is equal to the longitudinal wavenumber of the excited leaky wave (see, for instance, Refs [9–11]).

As is well known, under the conditions of total internal reflection (TIR) of an incident plane volume wave from a single media interface, an evanescent wave (EW) may exist in the adjacent medium even in the absence of absorption (in particular, in an optically transparent medium) [12, 13]. In the Russian scientific literature, an analogue of the term 'evanescent' ('vanishing') wave is the term 'inhomogeneous' wave [12]. Recent years have seen a surge of interest in the formation conditions and properties of electromagnetic (EM) excitations of this type. This is not least due to the rapid development of the physics of metamaterials and nanooptics (in particular, of photon scanning tunnel microscopy), since the use of EWs makes it possible to go beyond the diffraction limit [13]. In this connection, one of the central issues is to analyze the optimum conditions for maximizing the intensity of these propagating EWs.

To resonantly excite and enhance the amplitude of an EW on spatially uniform surfaces, traditionally investigated and used are multilayer configurations (in particular, Otto and Kretschmann configurations [13, 14]). As for the single interface of optically transparent dielectrics, in monographic Ref. [13] it is noted that, for optically isotropic nonabsorbing media, the greatest (four-fold) intensity enhancement of TM-type evanescent waves relative to the volume outside a p-wave is achieved when the incidence angle ϑ_{p} is equal to the limiting TIR angle ϑ_{pc}.

2. Single interface of two media. Reflection of a plane wave

2.1 Fast improper exceptional volume wave

Consider the interface $\zeta = 0$ of two half-spaces with the normal $\mathbf{q}$ and assume, for definiteness, that the upper half-space ($\zeta > 0$) is occupied by a nonmagnetic, optically isotropic medium with material relations of the form [12, 13]

$$\tilde{\mathbf{B}}_i = \tilde{\mathbf{H}}_i\,, \qquad \tilde{\mathbf{D}}_i = \tilde{\varepsilon}\tilde{\mathbf{E}}_i\,, \qquad i = x, y, z\,, \tag{1}$$

where $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{D}}$ are the magnetic and electric induction vectors, $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{E}}$ are the magnetic and electric fields, $\tilde{\varepsilon}$ is the permittivity of the medium, the tilde marks the quantities relating to this medium, and ζ is the current coordinate along the $\mathbf{q}$ direction. We also assume that the selected sagittal plane (characterized by the normal vector $\mathbf{a}$ ($\mathbf{a} \perp \mathbf{q}$)) is such that independent propagation of EM waves with given values of frequency ω, longitudinal wavenumber h, and polarization $\alpha = \mathrm{p}, \mathrm{s}$ is possible in both contacting media ($\alpha = \mathrm{p}$ corresponds to a transverse magnetic (TM) wave ($\mathbf{H} \parallel \mathbf{a}$, $\mathbf{E} \perp \mathbf{a}$) and $\alpha = \mathrm{s}$ to a transverse electric (TE) wave ($\mathbf{E} \parallel \mathbf{a}$, $\mathbf{H} \perp \mathbf{a}$)). In this case, using Maxwellian electromagnetic boundary conditions [12, 15]

$$\begin{gathered} \tilde{\mathbf{E}}\mathbf{b} = \mathbf{E}\mathbf{b}\,, \quad \tilde{\mathbf{H}}\mathbf{a} = \mathbf{H}\mathbf{a}\,, \quad \tilde{\mathbf{H}}\mathbf{b} = \mathbf{H}\mathbf{b}\,, \quad \tilde{\mathbf{E}}\mathbf{a} = \mathbf{E}\mathbf{a}\,, \\ \mathbf{a} = [\mathbf{b}\mathbf{q}]\,, \quad \zeta = 0\,, \end{gathered} \tag{2}$$

it is possible, following Ref. [16], to determine the Fresnel transmission coefficients for a plane EM TM (T_{p}) wave as the ratio of magnetic field components and for a plane TE (T_{s}) wave as the ratio of electric field components of the waves transmitted through and incident on the interface. As a result, under TIR conditions, for the case of a single interface between two optically transparent isotropic dielectrics observed in [13], we obtain

$$\begin{gathered} T_\alpha = \frac{2\tilde{Z}_\alpha}{\tilde{Z}_\alpha + \mathrm{i}Z_\alpha}\,, \qquad \alpha = \mathrm{p}, \mathrm{s}\,, \\ \tilde{Z}_{\mathrm{p}} = \frac{\sqrt{\tilde{\varepsilon}k_0^2 - h^2}}{\tilde{\varepsilon}k_0}\,, \quad \tilde{Z}_{\mathrm{s}} = \frac{\sqrt{\tilde{\varepsilon}k_0^2 - h^2}}{k_0}\,, \\ Z_{\mathrm{p}} = \frac{\sqrt{\varepsilon k_0^2 - h^2}}{\varepsilon k_0}\,, \quad Z_{\mathrm{s}} = \frac{\sqrt{\varepsilon k_0^2 - h^2}}{k_0}\,, \end{gathered} \tag{3}$$

$k_0 \equiv \omega/c$, and c is the speed of light. To characterize the wave properties of contacting media with the relations [16]

$$\begin{gathered} \tilde{\mathbf{E}}\mathbf{b} \equiv \tilde{Z}_{\mathrm{p}}\tilde{\mathbf{H}}\mathbf{a}\,, \quad \tilde{\mathbf{H}}\mathbf{b} \equiv -\tilde{Z}_{\mathrm{s}}\tilde{\mathbf{E}}\mathbf{a}\,, \quad \mathbf{E}\mathbf{b} \equiv Z_{\mathrm{p}}\mathbf{H}\mathbf{a}\,, \\ \mathbf{H}\mathbf{b} \equiv -Z_{\mathrm{s}}\mathbf{E}\mathbf{a}\,, \quad \zeta = 0\,, \end{gathered} \tag{4}$$

we introduced the surface wave impedance Z_{p} (for the case of a TM wave) and the surface wave conductance Z_{s} (for a TE wave). Vector $\mathbf{b}$ lies along the intersection line of the plane of the medium interface and the sagittal plane ($\mathbf{a} = [\mathbf{b}\mathbf{q}]$) [17]. In this case, in the TIR domain in the dissipationless approximation, in Eqns (3),

$$\operatorname{Im}\tilde{Z}_\alpha = 0\,, \quad \operatorname{Re}Z_\alpha \neq 0\,, \quad \alpha = \mathrm{p}, \mathrm{s}\,. \tag{5}$$

The conclusion of Ref. [13] is that the maximum (four-fold) TM EW intensity enhancement at the single interface of optically isotropic nonabsorbing and nonmagnetic media relative to the plane volume p wave incident from the outside ($|T_{\mathrm{p}}|^2 = 4$) is reached for the incidence angle ϑ_{p} equal to the limiting TIR angle $\vartheta_{\mathrm{pc}}(\omega)$. This conclusion relies on the fact that

$$Z_{\mathrm{p}}(\omega, h) = 0 \tag{6}$$

in Eqn (3) for $\vartheta = \vartheta_{\mathrm{pc}}$. In this case, due to Eqn (6), the instantaneous energy flux [12, 13, 17] across the medium interface,

$$\mathbf{S}\mathbf{q} = 0\,, \quad \mathbf{S} = \frac{c}{4\pi}[\mathbf{E}\mathbf{H}]\,, \tag{7}$$

is zero at any point in time. However, in view of Eqn (3), the fulfillment of equality $Z_{\mathrm{p}}(\omega, h) = 0$ for $\vartheta = \vartheta_{\mathrm{pc}}$ corresponds to the most intense excitation not of the evanescent wave but of a plane volume homogeneous monochromatic single-partial TM wave in the optically less dense isotropic medium. Earlier, in Ref. [18], by analogy with homogeneous plane elastic waves, which have been actively studied in crystal acoustics for several decades [19], a homogeneous volume plane monochromatic single-partial TM EM wave propagating along the interface between a semi-infinite isotropic dielectric and an ideal metal has received the name exceptional bulk wave (EBW). It is noteworthy that, according to the classification of possible types of waves propagating along open uniform waveguides proposed in Ref. [20], the EBW under discussion may be termed a fast improper EBW, because it exists for $h^2 < k_0^2\tilde{\varepsilon}$ and is not localized in the optically denser medium near the single interface with the optically less dense medium. At the same time, in accordance with its definition, this EBW is not localized near the medium interface in the optically less dense medium, either.

As a result, until recently, under TIR conditions it was considered impossible to achieve the maximum EW amplitude enhancement due to the formation of a leaky surface type-I EM wave with $\alpha = \mathrm{p}$ as well as with $\alpha = \mathrm{s}$ at the single interface of optically transparent dielectrics (see, for instance, Ref. [6]).

At the same time, in the acoustic TIR domain in crystal acoustics, the formation conditions have been adequately studied not only for exceptional bulk waves but also for type-I leaky surface elastic waves, both including and disregarding dissipation, for the case of a single acoustically continuous interface between dielectrics [9–11]. We use a specific example to study the conditions required for the formation of a type-I leaky surface EM wave at the single interface between optically transparent dielectric media. To this end, in the dissipation-free approximation, we consider the reflection of a shear plane bulk wave incident, under acoustic TIR conditions, on a plane interface with rigid gluing [21] between a semi-infinite elastic isotropic dielectric and a piezo-crystalline medium [22].

2.2 Fast improper surface acoustic wave

We assume that the boundary conditions [22, 23]

$$\bar{\bar{\tilde{\sigma}}}\mathbf{q} = \bar{\bar{\sigma}}\mathbf{q}\,, \quad \tilde{\mathbf{u}} = \mathbf{u}\,, \quad \mathbf{D}\mathbf{q} = \tilde{\mathbf{D}}\mathbf{q}\,, \quad [\mathbf{E}\mathbf{q}] = [\tilde{\mathbf{E}}\mathbf{q}]\,, \quad \zeta = 0\,, \tag{8}$$

which are standard for the physics of piezo crystals, are fulfilled at the acoustically continuous interface between an elastic isotropic medium, $\zeta > 0$, and the piezo-crystalline one, $\zeta < 0$. Here, $\mathbf{u}$ is the elastic displacement vector and $\bar{\bar{\sigma}}$ is the elastic stress tensor; the quantities relating to the non-piezo-active dielectric are marked with a tilde. The equations of elastic dynamics and the material relations for the non-piezo-active and piezo-active media may, according to Ref. [24], be respectively represented as

$$\begin{aligned}
&\tilde{\rho}\,\frac{\partial^2\tilde{\mathbf{u}}}{\partial t^2} = \operatorname{div}\bar{\bar{\tilde{\sigma}}}^{\mathrm{T}}\,, \quad \bar{\bar{\tilde{\sigma}}} = \bar{\bar{\tilde{\sigma}}}(\bar{\bar{\tilde{u}}})\,, \quad \tilde{\mathbf{D}} = \tilde{\mathbf{D}}(\tilde{\mathbf{E}})\,,\\
&\rho\,\frac{\partial^2\mathbf{u}}{\partial t^2} = \operatorname{div}\bar{\bar{\sigma}}^{\mathrm{T}}\,, \quad \operatorname{div}\mathbf{D} = 0\,, \quad \operatorname{rot}\mathbf{E} = 0\,,\\
&\bar{\bar{\sigma}} = \bar{\bar{\sigma}}(\bar{\bar{u}}, \mathbf{E})\,, \quad \mathbf{D} = \mathbf{D}(\bar{\bar{u}}, \mathbf{E})\,,
\end{aligned} \tag{9}$$

where $\tilde{\rho}\,(\rho)$ and $\bar{\bar{\tilde{u}}}\,(\bar{\bar{u}})$ are the density and the elastic deformation tensor of the non-piezo-active (piezo-active) dielectric and the superscript T corresponds to transposition. In accordance with Refs [22, 23], we define the amplitude reflection coefficient V_{SH} of a plane shear bulk wave (an SH-type wave, $\mathbf{u}\|\tilde{\mathbf{u}}\|\mathbf{a}$) as the ratio of the amplitudes of reflected and incident waves for the component of elastic displacements orthogonal to the plane of incidence. For a plane bulk wave with frequency ω and longitudinal wavenumber h incident on a medium interface (8) from the acoustically less dense medium (9) under acoustic TIR conditions, in the Coulomb limit ($k_0 \to 0$), V_{SH} has the following structure [22, 23]:

$$V_{\mathrm{SH}} = \frac{\tilde{Z}_{\mathrm{SH}} - \mathrm{i}Z_{\mathrm{SH}}}{\tilde{Z}_{\mathrm{SH}} + \mathrm{i}Z_{\mathrm{SH}}}\,,$$

$$\tilde{Z}_{\mathrm{SH}}(\omega, h) \equiv \frac{a_i\tilde{\sigma}_{ik}q_k}{\tilde{\mathbf{u}}\mathbf{a}}\,, \quad Z_{\mathrm{SH}}(\omega, h) \equiv \left.\frac{a_i\sigma_{ik}q_k}{\mathbf{u}\mathbf{a}}\right|_{\mathbf{D}\mathbf{q}+\tilde{\varepsilon}\psi=0}\,,$$

$$\operatorname{Im}\tilde{Z}_{\mathrm{SH}}(\omega, h) = 0\,, \quad \operatorname{Re}Z_{\mathrm{SH}}(\omega, h) \neq 0\,, \quad \mathbf{E} = -\nabla\psi\,, \tag{10}$$

where $\tilde{Z}_{\mathrm{SH}} = \tilde{\rho}\tilde{s}_{\mathrm{t}}^2\sqrt{\omega^2/\tilde{s}_{\mathrm{t}}^2 - h^2}$ is the surface wave impedance for a shear wave in the acoustically less dense medium ($\zeta > 0$), $\tilde{s}_{\mathrm{t}}$ is the phase velocity of an SH wave for an infinite medium, $Z_{\mathrm{SH}}(\omega, h)$ is the surface wave impedance for a shear wave with frequency ω and longitudinal wavenumber h in the acoustically denser piezoelectric medium [22, 23], and ψ is the electrostatic potential. According to Ref. [23], under acoustic TIR conditions ($\omega/s_{\mathrm{t}} < h < \omega/\tilde{s}_{\mathrm{t}}$), the amplitude transmission coefficient of a shear bulk SH wave incident from the acoustically less dense medium on the medium interface in the sagittal plane with the normal aligned with $\mathbf{a}$, $T_{\mathrm{SH}} = 1 + V_{\mathrm{SH}}$, in view of expressions (10) takes on the form

$$T_{\mathrm{SH}} = \frac{2\tilde{Z}_{\mathrm{SH}}}{\tilde{Z}_{\mathrm{SH}} + \mathrm{i}Z_{\mathrm{SH}}}\,, \quad \operatorname{Im}\tilde{Z}_{\mathrm{SH}} = \operatorname{Im}Z_{\mathrm{SH}} = 0\,. \tag{11}$$

When the lower medium is also non-piezo-active, elastic, and isotropic, in Eqns (10) and (11), $Z_{\mathrm{SH}} = \rho s_{\mathrm{t}}^2\sqrt{h^2 - \omega^2/s_{\mathrm{t}}^2}$ [23], where s_{t} is the velocity of a shear elastic wave in an isotropic dielectric. As a result, at the boundary of the acoustic TIR domain, i.e., for $\omega^2/\tilde{s}_{\mathrm{t}}^2 > h^2 = \omega^2/s_{\mathrm{t}}^2$, the fulfillment of $Z_{\mathrm{SH}}(\omega, h = \omega/s_{\mathrm{t}}) = 0$ in Eqns (10) and (11) corresponds to the excitation of a plane shear homogeneous bulk wave, which slides along the mechanically free surface of an elastic half-space (a single-partial elastic EBW) [19, 23]. For it, in Eqn (11), $T_{\mathrm{SH}}(\omega, h = \omega/s_{\mathrm{t}}) = 2$, and hence the amplitude of the shear SH-type EBW, as in the case of its electromagnetic analogue in Ref. [13], is two times higher than the amplitude of the plane bulk SH outside wave incident at the limiting TIR angle [23]. When $Z_{\mathrm{SH}} = 0$ in expression (8) ($T_{\mathrm{SH}} = 2$ in Eqn (11)), the surface is referred to as acoustically soft, and when $Z_{\mathrm{SH}} = \infty$ in expression (8) ($T_{\mathrm{SH}} = 0$ in Eqn (11)), it is termed acoustically hard.

Interest in studying the conditions for the formation and propagation of shear EBWs in acoustics is largely due to the extremely high sensitivity of localization conditions of the wave excitations of this class to the character of the boundary conditions (see, for instance, Ref. [4]). We assume that the lower half-space in the acoustically continuous structure under consideration is occupied by a class-6 piezoelectric. Calculations for this piezoelectric medium show that material relations for a shear wave with $\mathbf{u}\,\|\,z$ and the incidence plane $\mathbf{k} \in xy$ may be represented as [22]

$$\begin{cases}
\sigma_{zx} = c_{44}\,\dfrac{\partial u_z}{\partial x} - e_{15}E_x\,, \quad D_x = \varepsilon E_x + 4\pi e_{15}\,\dfrac{\partial u_z}{\partial x}\,,\\[2ex]
\sigma_{zy} = c_{44}\,\dfrac{\partial u_z}{\partial y} - e_{15}E_y\,, \quad D_y = \varepsilon E_y + 4\pi e_{15}\,\dfrac{\partial u_z}{\partial y}\,,
\end{cases} \tag{12}$$

where e_{15} is the piezoelectric modulus.

When $\mathbf{q}\,\|\,y$, for the acoustically continuous interface between a semi-infinite isotropic dielectric (9) and a semi-infinite piezoelectric medium (12), the boundary conditions (8) in the electrostatic approximation are expressed as follows:

$$\begin{aligned}
&\sigma_{zx} = \tilde{\sigma}_{zx}\,, \quad u_z = \tilde{u}_z\,, \quad y = 0\,, \quad u_z(y \to -\infty) \to 0\,,\\
&D_y = -h\psi\,, \quad y = 0\,, \quad \psi(y \to -\infty) \to 0\,.
\end{aligned} \tag{13}$$

According to calculations, in view of expressions (12) and (13), under conditions of acoustic TIR, the amplitude transmission coefficient T_{SH} for an SH wave incident in the sagittal plane with the normal along **a** from an acoustically less dense medium (9) on the surface of a piezoelectric medium (12) also coincides structurally in this case with expressions (10) and (11). However, the elastic surface wave impedance for a shear wave with frequency ω and longitudinal wavenumber h in the piezoelectric now is of the form

$$Z_{\mathrm{SH}}(\omega, h) \equiv c'_{44}\eta_{\mathrm{SH}} - 4\pi\,\frac{e_{15}^2\tilde{\varepsilon}}{\varepsilon+\tilde{\varepsilon}}\,,$$
$$\eta_{\mathrm{SH}} \equiv \sqrt{h^2 - \frac{\rho\omega^2}{c'_{44}}}\,,\quad c'_{44} \equiv c_{44}\left(1 + \frac{4\pi e_{15}^2}{c_{44}\varepsilon}\right), \tag{14}$$

where c'_{44} is the effective elastic modulus. As a result, under the conditions of acoustic TIR ($\eta_{\mathrm{SH}}^2 > 0$), for the acoustically continuous interface between the isotropic dielectric (9) and the piezo crystal (12), the fulfillment of relation

$$Z_{\mathrm{SH}}(\omega, h) = 0 \tag{15}$$

corresponds to the maximum intensity of excitation of the elastic evanescent ($\eta_{\mathrm{SH}}^2 > 0$) SH ($\mathbf{u}\,\|\,\mathbf{a}$) wave in the piezocrystalline medium (12) by an outside plane shear bulk wave: $T_{\mathrm{SH}}(\omega, h) = 2$. In this case, relations (14) and (15) define the spectrum of the Gulyaev–Blustein shear surface acoustic wave (SAW), which propagates along the mechanically free and electrically open (2) surface of the medium (1) and elastic semi-infinite piezo space (12) [22]:

$$h^2 = h_{\mathrm{SAW}}^2(\omega)\,,\quad h_{\mathrm{SAW}}^2(\omega) \equiv \frac{\rho\omega^2}{c'_{44}}\left[1 - \left(4\pi\,\frac{e_{15}^2\tilde{\varepsilon}}{\varepsilon+\tilde{\varepsilon}}\right)^2\right]^{-1}. \tag{16}$$

We note that the fulfillment of Eqns (14) and (15) under TIR conditions signifies that instantaneous elastic (but not electric!) energy flux across the interface between media (9) and (12) is strictly zero at any point in time for the corresponding plane evanescent wave with $\mathbf{u}\,\|\,\mathbf{a}$. As a result, in this case, in piezoelectric medium (12) a SAW characterized by the vanishing of the surface wave impedance, $Z_{\mathrm{SH}}(\omega, h = h_{\mathrm{SAW}}(\omega)) = 0$, forms for boundary problem (13). If in this case, too, we resort to the 'acoustic' analogue of the classification of Ref. [20], the Gulyaev–Blustein SAW (16) formed in the layered structure (8), (9), (12), and (13) under consideration may be termed a fast improper SAW when $\rho\omega^2/c'_{44} < h^2 < \omega^2\tilde{s}_{\mathrm{t}}^{-2}$.

However, a remark is in order. The hybridization of two partial evanescent waves (elastic and electromagnetic) is not the necessary condition for the formation of a fast improper SAW and fulfillment of Eqns (14) and (15). By way of example, mention can made of the case when the lower half-space ($\zeta < 0$) in the acoustically continuous structure under consideration, which comprises a semi-infinite ($\zeta > 0$) elastic isotropic dielectric (9), is occupied by a ferromagnet (FM). Ignoring the finiteness of the EM wave propagation velocity, the magnetoelastic dynamics of the acoustically less dense FM medium is described by the closed system of equations consisting of the Landau–Lifshitz equations for the unit-volume magnetization vector **M**, the main equation of continuum mechanics, and the equations of magnetostatics [25].

By way of example, we consider the single-sublattice easy-axis (EA) FM, whose magnetoelastic and elastic properties will hereinafter be assumed to be isotropic for simplicity and ease of calculations (μ is the shear modulus of the FM medium). We restrict ourselves to the case when the z axis is the light FM axis and $\mathbf{M}\,\|\,\mathbf{H}_0\,\|\,z$ is in the equilibrium state. For a shear wave with $\mathbf{u}\,\|\,z$ and $\mathbf{k} \in xy$, the material relations for the FM model under consideration with the inclusion of magnetoelastic and magnetostriction interactions in the exchangeless approximation may be represented as [26]

$$\begin{cases} \sigma_{zx} = c_\perp\,\dfrac{\partial u_z}{\partial x} + \mathrm{i}c_*\,\dfrac{\partial u_z}{\partial y} + \beta_{15}H_x - \mathrm{i}\beta_* H_y\,, \\[2mm] \sigma_{zy} = c_\perp\,\dfrac{\partial u_z}{\partial y} - \mathrm{i}c_*\,\dfrac{\partial u_z}{\partial x} + \beta_{15}H_y + \mathrm{i}\beta_* H_x\,, \end{cases}$$
$$\begin{cases} B_x = \mu_\perp H_x - \mathrm{i}\mu_* H_y - 4\pi\beta_{15}\,\dfrac{\partial u_z}{\partial x} + 4\pi\mathrm{i}\beta_*\,\dfrac{\partial u_z}{\partial y}\,, \\[2mm] B_y = \mu_\perp H_y + \mathrm{i}\mu_* H_x - 4\pi\beta_{15}\,\dfrac{\partial u_z}{\partial y} - 4\pi\mathrm{i}\beta_*\,\dfrac{\partial u_z}{\partial x}\,, \end{cases} \tag{17}$$

where $\mathbf{H} = -\nabla\varphi$ and φ is the magnetostatic potential.

Let $\mathbf{q}\,\|\,y$ as before, but, instead of boundary conditions (13), the following system of boundary conditions applies to the surface of the semi-infinite EA-FM of the magnetoacoustic configuration under consideration:

$$\sigma_{zx} = \tilde{\sigma}_{zx}\,,\quad u_z = \tilde{u}_z\,,\quad y = 0\,,\quad u_z(y \to -\infty) \to 0\,,$$
$$B_x = -h\varphi\,,\quad y = 0\,,\quad \varphi(y \to -\infty) \to 0\,, \tag{18}$$

which corresponds to the acoustically continuous interface between nonmagnetic (9) and magnetic (17) media. In this case, in accordance with the general propositions of the theory of wave processes in layered media [5, 22], in view of Eqns (9), (17), and (18), it may be shown that the amplitude transmission coefficient of a bulk shear wave with $\mathbf{u}\,\|\,\mathbf{H}_0\,\|\,z$ incident from the depth of an elastically isotropic nonmagnetic medium (9) coincides structurally with expression (11). Now, however, it is not expression (14), but

$$Z_{\mathrm{SH}}(\omega, h) \equiv c'_{44}\eta_{\mathrm{SH}} - c_*\sigma + \left[\frac{4\pi}{\mu_\perp}\,\beta_{15}^2 - \frac{4\pi(\beta_{15} + \beta_*\sigma)^2}{\mu_\perp + \mu_*\sigma + 1}\right]\frac{1}{\mu}\,,$$
$$c'_{44} \equiv \frac{c_\perp}{\mu} + \frac{4\pi\beta_{15}^2}{\mu\mu_\perp}\,,\quad \sigma \equiv \operatorname{sign} h\,. \tag{19}$$

In this case, the effect of a two-fold increase in the amplitude of the excited elastic evanescent SH wave in comparison with the amplitude of a plane shear wave, which is incident from a nonmagnetic medium (9), persists, provided expressions (15) and (19) are fulfilled, even when ignoring the quasistatic electromagnetic field (which is formally in line with the passage to the limit $4\pi \to 0$ in relations (17)–(19)).

Here, a note is in order. In view of relations (19), for a plane shear bulk SH wave incident on an acoustically continuous (18) single interface between media (9) and (17) from an acoustically less dense medium, the reflection for $Z_{\mathrm{SH}}(\omega, h = \omega/\tilde{s}_{\mathrm{t}}) = 0$, as in the case (13), (15), is completely analogous to the reflection from a mechanically free surface (an analogue of the incidence at the Rayleigh angle [9]).

We emphasize, in the case of acoustically continuous interface (8) between elastic isotropic dielectric (9) and piezo-crystalline medium (12) (or ferromagnetic medium (17)), that the instantaneous elastic energy flux across the interface is not strictly zero at any point in time when a bulk shear plane wave is incident from dielectric (9) on the surface of semi-infinite piezoelectric (12) (or ferromagnet (17)) under the limiting acoustic TIR angle.

Considering the above acoustic examples (8)–(19) and the structure of the Poynting vector for a plane EM wave [12, 13] propagating along the normal to the medium interface, it can be assumed that the condition for the maximum intensity enhancement of TM- or TE-type evanescent EM waves ($T_\alpha = 2$, $\alpha = \mathrm{p, s}$) at the single interface between optically transparent dielectrics within the TIR domain is the vanishing of the surface wave impedance Z_p (for the TM wave) or of the surface wave resistance Z_s (for the TE wave), i.e.,

$$\mathbf{Eb} = 0\,, \quad \mathbf{Ha} = \tilde{\mathbf{H}}\mathbf{a}\,, \quad \alpha = \mathrm{p}\,, \quad \zeta = 0\,, \tag{20}$$

$$\mathbf{Hb} = 0\,, \quad \mathbf{Ea} = \tilde{\mathbf{E}}\mathbf{a}\,, \quad \alpha = \mathrm{s}\,, \quad \zeta = 0\,. \tag{21}$$

As follows from Refs [27–29], for a dielectric–ideal metal interface, a constant external electric $\mathbf{E}_0$ or magnetic $\mathbf{H}_0$ field (or their combination) may give rise to a single-partial plane surface electromagnetic TM wave propagating in the dielectric. However, the possibility of maximizing, with the use of a constant external magnetic or electric field (or their combination), the enhancement of a TM or TE evanescent wave as well as of attendant dynamic effects under TIR conditions has never been considered for a single interface between two transparent dielectric media (the formation of a type I surface leaky EM wave).

We determine the conditions whereby the incidence of a plane bulk TM or TE EM wave on the single interface between optically transparent, spatially uniform dielectric media maximizes, under TIR conditions, the excitation intensity of the electromagnetic EW with the corresponding polarization [30].

2.3 Fast improper exceptional surface wave

By way of example, we consider the plane interface between two transparent semi-infinite dielectric media with the interface normal $\mathbf{q}$ and assume, as in (1)–(3), that the upper half-space ($\zeta > 0$) is occupied by the denser nonmagnetic medium, which is isotropic in electromagnetic properties (1). As for the lower half-space, we assume that it is occupied by a bianisotropic (BA) medium, whose material relations, according to Ref. [17], may be represented in the form

$$\mathbf{B} = \bar{\bar{\mu}}\mathbf{H} + \bar{\bar{A}}^{*}\mathbf{E}\,, \quad \mathbf{D} = \bar{\bar{\varepsilon}}\mathbf{E} + \bar{\bar{A}}^{\mathrm{T}}\mathbf{H}\,, \tag{22}$$

where $\bar{\bar{\mu}}$ and $\bar{\bar{\varepsilon}}$ are the magnetic permeability and permittivity tensors, $\bar{\bar{A}}$ is the magnetoelectric tensor, and superscript $*$ corresponds to complex conjugation.

When discussing in what follows only the case of independent TM or TE wave propagation in the selected sagittal plane with the normal along $\mathbf{a}$, as relating to the tensor coefficients in Eqns (22), we assume that, disregarding dissipation, in the dyad representation [17] they have the following structure ($\mathbf{b} = [\mathbf{qa}]$):

$$\begin{aligned}
\bar{\bar{\varepsilon}} &= \varepsilon_1 \mathbf{b}\otimes\mathbf{b} + \varepsilon_2 \mathbf{q}\otimes\mathbf{q} + \varepsilon_3 \mathbf{b}\otimes\mathbf{q} + \varepsilon_3^* \mathbf{q}\otimes\mathbf{b} + \varepsilon_4 \mathbf{a}\otimes\mathbf{a}\,,\\
\bar{\bar{\mu}} &= \mu_1 \mathbf{b}\otimes\mathbf{b} + \mu_2 \mathbf{q}\otimes\mathbf{q} + \mu_3 \mathbf{b}\otimes\mathbf{q} + \mu_3^* \mathbf{q}\otimes\mathbf{b} + \mu_4 \mathbf{a}\otimes\mathbf{a}\,,\\
\bar{\bar{A}} &= A_1 \mathbf{b}\otimes\mathbf{a} + A_2 \mathbf{q}\otimes\mathbf{a} + A_3 \mathbf{a}\otimes\mathbf{b} + A_4 \mathbf{a}\otimes\mathbf{q}\,.
\end{aligned} \tag{23}$$

Furthermore, in the subsequent discussion, we restrict ourselves to the case when, ignoring dissipation, the diagonal tensor components appearing in Eqns (22) and (23) are real and the off-diagonal ones are complex. Tretyakov et al. [31] proposed a classification of BA media (22) based on the structure analysis of magnetoelectric interaction tensor $\bar{\bar{A}}$. According to this classification, tensor $\bar{\bar{A}}$, which appears in Eqns (23), corresponds to a pseudo-Tellegen medium for $\mathrm{Re}\,A_{ik} = \mathrm{Re}\,A_{ki}$ and to a moving medium for $\mathrm{Re}\,A_{ik} = -\mathrm{Re}\,A_{ki}$. When $\mathrm{Im}\,A_{ik} = \mathrm{Im}\,A_{ki}$ in Eqns (23), tensor $\bar{\bar{A}}$, which appears in Eqns (22), according to Ref. [31], corresponds to a pseudo-chiral medium and for $\mathrm{Im}\,A_{ik} = -\mathrm{Im}\,A_{ki}$, to an omega medium.

In accordance with Eqns (22) and (23), independent propagation of a plane TM wave ($\mathbf{Ha} \propto \exp(\mathrm{i}\mathbf{k}_\mathrm{p}\mathbf{r} - \mathrm{i}\omega t)$) or a TE wave ($\mathbf{Ea} \propto \exp(\mathrm{i}\mathbf{k}_\mathrm{s}\mathbf{r} - \mathrm{i}\omega t)$) is possible in the plane with the normal along $\mathbf{a}$, with $\mathbf{r} \perp \mathbf{a}$. When the wave vector $\mathbf{k}$ may be represented as $\mathbf{k} = h\mathbf{b} + k_{\parallel\alpha}\mathbf{q}$ ($\alpha = \mathrm{p, s}$), depending on the TM or TE wave polarization, the dispersion equation for the spectrum of TM or TE normal polaritons propagating in unlimited medium (22), (23) corresponds to the roots of the following equation:

$$k_{\parallel\alpha}^2 + N_{1\alpha}k_{\parallel\alpha} + N_{2\alpha} = 0\,, \tag{24}$$

$$\begin{cases}
N_{1\mathrm{p}} = \dfrac{2}{\varepsilon_2}\left[h\,\mathrm{Re}\,\varepsilon_3 + \mathrm{Re}\,(\varepsilon_2 A_3) - \mathrm{Re}\,(A_4\varepsilon_3)\right],\\
\qquad \Delta\varepsilon \equiv \varepsilon_1\varepsilon_2 - \varepsilon_3\varepsilon_3^*\,,\\
\varepsilon_2 N_{2\mathrm{p}} = h^2\varepsilon_1 + 2h\left[-\mathrm{Re}\,(\varepsilon_1 A_4) + \mathrm{Re}\,(\varepsilon_3 A_3^*)\right]\\
\qquad + \mu_4\Delta\varepsilon + \varepsilon_2 A_3 A_3^* + \varepsilon_1 A_4 A_4^* - 2\,\mathrm{Re}\,(\varepsilon_3 A_3^* A_4^*)\,,
\end{cases}$$

$$\begin{cases}
N_{1\mathrm{s}} = \dfrac{2}{\mu_2}\left[h\,\mathrm{Re}\,\mu_3 - 2\,\mathrm{Re}\,(\mu_2 A_1) + 2\,\mathrm{Re}\,(\mu_3 A_2^*)\right],\\
\qquad \Delta\mu \equiv \mu_1\mu_2 - \mu_3\mu_3^*\,,\\
\mu_2 N_{2\mathrm{s}} = \mu_1 h^2 + 2h\left[\mu_1\,\mathrm{Re}\,A_2 - \mathrm{Re}\,(\mu_3 A_1)\right]\\
\qquad + \varepsilon_4\Delta\mu + \mu_2 A_1 A_1^* + \mu_1 A_2 A_2^* - 2\,\mathrm{Re}\,(\mu_3 A_1 A_2^*)\,.
\end{cases}$$

We consider a plane EM wave with polarization $\alpha = \mathrm{p, s}$. Calculations suggest that under TIR conditions (i.e., when $N_{1\alpha}^2 < 4N_{2\alpha}^2$ in Eqns (24)) the required wave vector component $\eta_\alpha(\omega, h)$ ($k_{\parallel\alpha}^2 = \eta_\alpha^2 < 0$) normal to the surface of BA medium (22), (23), which occupies the lower half-space, according to Eqns (24) is of the form

$$\eta_\alpha = \eta_\alpha' - \mathrm{i}\eta_\alpha''\,, \quad \mathrm{Im}\,\eta_\alpha' = \mathrm{Im}\,\eta_\alpha'' = 0\,, \quad \alpha = \mathrm{p, s}\,. \tag{25}$$

Consequently, the amplitude of the EW with polarization α may not merely decay exponentially with distance from the interface in the lower half-space with exponent $\eta_\alpha'' \neq 0$, but may also experience spatial oscillations (with a period $2\pi/\eta_\alpha'$), provided that $\eta_\alpha' < \eta_\alpha''$. It is also noteworthy that such spatial oscillations (with a period $2\pi/\eta_\alpha'$) of TM (TE) wave amplitude with distance from the interface in the BA medium (22), (23) under consideration may also persist for $\eta_\alpha'' = 0$ (the case of surface irradiation by an outside TM (TE) electromagnetic wave at the limiting TIR angle $\vartheta = \vartheta_{\mathrm{c}\alpha}$ for a wave of given polarization).

In view of expressions (4), (22)–(25), for $\eta_\alpha^2 > 0$, the relations for Z_α, which appear in formulas (3), may be represented in the form

$$\begin{aligned}
Z_\mathrm{s}(\omega, h) &\equiv \frac{1}{\Delta\mu}\left[\mu_2\eta_\mathrm{s}'' - h\,\mathrm{Im}\,\mu_3 + \mu_2\,\mathrm{Im}\,A_1^* - \mathrm{Im}\,(\mu_3 A_2^*)\right],\\
Z_\mathrm{p}(\omega, h) &\equiv \frac{1}{\Delta\varepsilon}\left[\varepsilon_2\eta_\mathrm{p}'' - h\,\mathrm{Im}\,\varepsilon_3 - \varepsilon_2\,\mathrm{Im}\,A_3 + \mathrm{Im}\,(\varepsilon_3 A_4)\right].
\end{aligned} \tag{26}$$

Therefore, the maximum (four-fold) increase in an evanescent EM wave of the corresponding polarization $\alpha = \mathrm{p}, \mathrm{s}$ ($|T_\alpha|^2 = 4$) is possible at the single interface between optically transparent dielectrics inside the TIR domain. This is so provided the condition

$$Z_\alpha(\omega, h) = 0\,, \quad \alpha = \mathrm{p}, \mathrm{s} \tag{27}$$

is the result of the fulfillment of relation (20) or (21) in the $\omega - h$ external parameter plane for the given EM wave polarization, $\alpha = \mathrm{p}$ or $\alpha = \mathrm{s}$, in view of expressions (24)–(26) inside the TIR domain (for $(\eta''_\alpha)^2 > 0$) (see also Ref. [31]). A combined analysis of expressions (3)–(5) and (22)–(26) in the BA medium case suggests that the possibility of the simultaneous formation also of a single-partial electromagnetic EBW of the corresponding polarization (i.e., for $\eta''_\alpha(\omega, h) = 0$) corresponds to the fulfillment of

$$Z_\alpha(\vartheta = \vartheta_{\mathrm{c}\alpha}) = 0\,, \quad |T_\alpha|^2 = 4\,, \quad \alpha = \mathrm{p}, \mathrm{s}\,, \tag{28}$$

depending on the polarization of the outside plane bulk EM wave. As follows from expressions (22)–(27), for $\alpha = \mathrm{p}$, relation (27), in combination with expressions (24) and (25), defines, in the $\omega - h$ external parameter plane, the spectrum of a TM surface wave propagating in the selected sagittal plane along the interface between nonmagnetic dielectric (23) and a perfect electric conductor. When $\alpha = \mathrm{s}$ in expressions (24)–(27), we are dealing with those $\omega - h$ combinations which correspond to the dispersion law of the surface TE polariton with $\mathbf{k} \in xy$, which propagates along the interface between nonmagnetic dielectric (23) and a perfect magnetic conductor (formally, $\tilde{Z}_\mathrm{s} = 0$) in the optical configuration under consideration. On the strength of expression (27), for this class of traveling plane EW localized near the interface inside the TIR domain the group velocity is strictly parallel to the media interface at any point in time (the instantaneous energy flux across the interface between the media is zero at any point in time, $\mathbf{S}_\alpha \mathbf{q} = 0$). Therefore, following the analogy to exceptional bulk TM waves [18], the surface polariton excitations with $\alpha = \mathrm{p}$ or $\alpha = \mathrm{s}$, which are formed under TIR conditions and are defined by expressions (26) and (27), may be termed TM or TE exceptional surface waves (ESWs), respectively.

Depending on the polarization of an ESW, the domain of its possible existence in the $\omega - h$ external parameter domain, in view of expression (26), is defined by the following relations:

$$\begin{aligned} &\mu_2\left[-h \operatorname{Im}\mu_3 + \mu_2 \operatorname{Im} A_1^* - \operatorname{Im}(\mu_3 A_2^*)\right] < 0\,, \quad \alpha = \mathrm{s}\,, \\ &\varepsilon_2\left[-h \operatorname{Im}\varepsilon_3 - \varepsilon_2 \operatorname{Im} A_3 + \operatorname{Im}(\varepsilon_3 A_4)\right] < 0\,, \quad \alpha = \mathrm{p}\,. \end{aligned} \tag{29}$$

When the classifications of gyrotropic media [17] and BA media [31] are simultaneously taken into account, proceeding from the combined analysis of expressions (23), (26), (27), and (29), it is possible to point out the following independent formation mechanisms for the ESWs under consideration:

(i) spontaneous or induced gyrotropy (the axis of gyrotropy is orthogonal to the plane of incidence), $\operatorname{Im}\varepsilon_3 \neq 0$, $\operatorname{Im}\mu_3 \neq 0$, $\overline{\overline{A}} = \overline{\overline{0}}$;

(ii) an omega medium or a pseudo-chiral medium, $\operatorname{Im} A_1 \neq 0$, $\operatorname{Im} A_3 \neq 0$, $\operatorname{Im}\varepsilon_3 = 0$, $\operatorname{Im}\mu_3 = 0$, $\operatorname{Re} A_2 = 0$, $\operatorname{Re} A_4 = 0$;

(iii) hybridization of spontaneous or induced gyrotropy and the asymmetric linear ME effect (a moving or pseudo-Tellegen medium), $\operatorname{Im} A_1 = \operatorname{Im} A_3 = 0$, $\operatorname{Im}\varepsilon_3 \neq 0$, $\operatorname{Im}\mu_3 \neq 0$, $\operatorname{Re} A_2 \neq 0$, $\operatorname{Re} A_4 \neq 0$.

Furthermore, as follows from an analysis of expressions (23), (24), (26), (27), and (29), there may be several independent mechanisms of the nonreciprocity of the spectra of TM or TE ESWs relative to the inversion of the direction of propagation along the media interface ($\omega(h) \neq \omega(-h)$). In particular, the nonreciprocity of the ESW spectrum may be due to spontaneous or induced gyrotropy (the gyrotropy axis is orthogonal to the plane of incidence) of the BA medium when $\overline{\overline{A}} = \overline{\overline{0}}$, then $\operatorname{Im}\varepsilon_3 \neq 0$ for $\alpha = \mathrm{p}$ and $\operatorname{Im}\mu_3 \neq 0$ for $\alpha = \mathrm{s}$.

When simultaneously

$$\operatorname{Re}(\varepsilon_1 A_4 - \varepsilon_3 A_3^*) \neq 0 \ \text{ and } \ \operatorname{Im}\varepsilon_3 \operatorname{Im} A_3 = 0 \ \text{ for } \ \alpha = \mathrm{p}\,,$$

the ESW spectrum nonreciprocity mechanism is induced by the asymmetric linear ME effect (a moving or pseudo-Tellegen medium). For $\alpha = \mathrm{s}$, the corresponding relations are of the form $\operatorname{Re}(\mu_1 A_2 - \mu_3 A_1) \neq 0$, $\operatorname{Im}\mu_3 \operatorname{Im} A_1 = 0$.

When simultaneously

$$\operatorname{Re}(\varepsilon_1 A_4 - \varepsilon_3 A_3^*) \neq 0 \ \text{ and } \ \operatorname{Im}\varepsilon_3 \operatorname{Im} A_3 \neq 0 \ \text{ for } \ \alpha = \mathrm{p}\,,$$

the ESW spectrum nonreciprocity mechanism is induced by the hybridization of spontaneous or induced gyrotropy and the pseudo-chiral interaction in the BA medium. For $\alpha = \mathrm{s}$, the corresponding relations are of the form $\operatorname{Re}(\mu_1 A_2 - \mu_3 A_1) \neq 0$, $\operatorname{Im}\mu_3 \operatorname{Im} A_1 \neq 0$.

Considering the magnitude and sign of the coefficients appearing in the constraint equations (22), (23) of the BA medium under discussion, the TM or TE ESW defined by Eqns (24)–(27), (29) may be either a direct wave ($h\,\partial\omega/\partial h > 0$) or a backward one ($h\,\partial\omega/\partial h < 0$). Under TIR conditions, this corresponds to the direction of the energy flux carried by this ESW in the BA medium relative to the direction of its propagation along the surface (2). As a result, for the formation of the TM or TE backward ESW (24)–(27), (29), it is necessary, in view of expression (24), that

$$\operatorname{Im}\left(\eta''_\alpha \frac{\partial Z_\alpha}{\partial h}\right) < 0\,, \quad \alpha = \mathrm{p}, \mathrm{s} \tag{30}$$

under TIR conditions (see also Ref. [32]). The opposite sign in inequality (30) corresponds to the TM or TE direct ESW (26), (27), (29). The vanishing of the left side of inequality (30) defines those $\omega - h$ combinations for which a maximum or a minimum forms in the ESW dispersion curve (24)–(27), (29) in the $\omega - h$ plane.

3. Reflection of a plane wave from the single interface between media in the resonance excitation of an exceptional surface wave

So far, we have discussed only the conditions whereby inside the TIR domain the amplitude of a TM or TE EW near the surface of an optically less dense semi-infinite transparent medium attains its maximum.

Let us now consider how a plane bulk wave in an optically denser medium (1) reflected from a single interface of transparent dielectrics changes when the wave frequency and incidence angle simultaneously obey the dispersion law of the above ESW (24)–(27), (29) (EM is an analogue of the Rayleigh angle [9]).

Let φ_α be the phase shift of a bulk wave with polarization α reflected under TIR conditions from the surface of the BA dielectric medium (23) under consideration into the optically denser upper medium (1). Then, in view of the notation assumed above,

$$\tan\frac{\varphi_\alpha}{2} = -\frac{Z_\alpha}{\tilde{Z}_\alpha}\,, \quad \alpha = \mathrm{s}, \mathrm{p}\,. \tag{31}$$

Therefore, in the case (24)–(27), (29), for a wave with a given polarization $\alpha = \mathrm{p}$ ($\alpha = \mathrm{s}$) in the TIR domain ($(\eta''_\alpha)^2 > 0$), also fulfilled at the interface between media (1) and (22), (23) are the following conditions:

$$R_\alpha = 1\,, \quad \varphi_\alpha = 0\,, \quad Z_\alpha = 0\,, \quad \mathbf{S}_\alpha\mathbf{q} = 0\,, \quad \zeta = 0\,, \quad \alpha = \mathrm{s}, \mathrm{p}\,. \tag{32}$$

This signifies that, in the excitation in a BA medium (22), (23) of a fast improper TM ESW propagating in an optically denser adjacent medium (1), the total magnetic field amplitude near the medium interface is twice the amplitude of the magnetic field in the TM wave incident on the interface. This effect is typical for the reflection of a plane TM EM wave from the surface of an ideal metal [33] (a perfect electric conductor [34]). In the framework of the model under discussion, for a plane bulk monochromatic single-partial TM wave, which is incident from the outside on the surface of an optically transparent dielectric and whose frequency and incidence angle correspond to the TM ESW spectrum (24)–(27), (29), (32), the reflection will therefore be the same as from a perfect electric conductor.

Similarly, from expressions (1), (31), and (32), it follows that an outside plane bulk monochromatic single-partial TE wave which is incident on the surface of an optically transparent dielectric within the TIR domain and whose frequency and incidence angle simultaneously correspond to the spectrum of a TE ESW (24)–(27), (29) will be reflected, in the context of the model under consideration, in the same way as from the surface of a perfect magnetic conductor [34].

Here, a remark is in order. The above effect of reflection of an outside plane bulk TM EM wave which is incident on the surface of an optically transparent dielectric, like that from an ideal metal, is, in a sense, the reverse effect relative to the effect, known in optics, of the reflection of a plane bulk EM wave from a metal, like that from a dielectric. The latter is possible for the Drude model when the frequency of the outside plane bulk TM wave incident on the metal surface is greater than the plasma frequency of the metal (see, for instance, Ref. [35]).

The appearance of frequency–longitudinal wavenumber combinations in the frequency–longitudinal wavenumber plane corresponding to expressions (24)–(27), (29) may be treated, following Ref. [20], as the formation, under TIR conditions, of a TM-type (for $R_\mathrm{p} = 1$) or TE-type (for $R_\mathrm{s} = 1$) fast improper ESW at the interface between an isotropic dielectric (1) and an optically less dense BA medium (22)–(24).

It is significant that the instantaneous energy flux across the single media interface is zero at any point in time for all of the above fast improper ESWs. This signifies that the TM or TE ESW (24)–(27), (29) under consideration is not a leaky wave and exists in the optically less dense medium (22), (23) only in the presence of a plane bulk monochromatic TM (for $R_\mathrm{p} = 1$) or TE (for $R_\mathrm{s} = 1$) wave standing along the normal to the media interface in the adjacent optically denser medium (1) with the same ω and h values (see also Ref. [16]). This is a consequence of the circumstance pointed out in Ref. [3] that a leaky wave cannot be excited by an outside plane bulk wave.

We recall the case studied in Refs [36, 37]: in the reflection of an elliptically polarized plane bulk EM wave from the single interface between optically transparent dielectric media under TIR conditions, the instantaneous energy flux across the interface is also strictly zero at any point in time. However, the necessary condition for the realization of this effect was, according to Ref. [36], the formation of a circularly polarized evanescent EM wave traveling along the media interface in the optically less dense medium.

Under TIR conditions, at a single media interface the formation of a TM or TE exceptional surface wave in the optically denser adjacent medium (prism) is attended by a simple [10] (pure [38]) reflection of a plane bulk wave of the corresponding polarization (both the incident and reflected waves belong to the same surface of refraction). However, although the instantaneous energy flux across the medium interface is strictly zero in this case, the ESW under discussion cannot be treated as the locus of 'pure' spectrum points (as a dissipationless localized state against the continuum background [10, 39]), since the amplitudes of the ESW and plane waves participating in the simple reflection due to boundary conditions are not independent, unlike those in Refs [10, 39]. This also applies to the formation of a circularly polarized exceptional two-partial evanescent wave at the interface of isotropic optically transparent media under TIR conditions, which was considered in Ref. [36].

So far, everywhere above we have considered under TIR conditions only the possibility of the enhancement of TM or TE EWs at the single interface of optically transparent media due to the incidence of a plane bulk wave of the corresponding polarization. At the same time, as is well known (see, for instance, Ref. [40]), a plane wave is no more than a physical idealization, just because the finiteness of the real source of excitation is ignored. Correct inclusion of this circumstance in the framework of a more realistic model may qualitatively alter the character of wave reflection in layered media (see, for instance, Refs [3, 5, 9–11, 41–43]). In particular, when a bulk EM wave incident on the media interface at the limiting TIR angle is not plane, the fast improper EBW formed in the optically less dense medium becomes the source of a side wave. Note that the side wave in seismic physics is more often referred to as a refracted or head wave [5].

4. Single interface between two media. Resonance reflection of a quasi-plane or quasi-monochromatic wave

As is well known, in view of the relations for surface wave impedance (for $\alpha = \mathrm{p}$) and surface wave conductance (for $\alpha = \mathrm{s}$) (4) introduced in Section 2, for all magnetooptical configurations discussed above the expression for the Fresnel reflection coefficient of a plane bulk monochromatic TM or TE wave under TIR conditions may be represented as [12, 13]

$$\tilde{\Psi}_\alpha = \left[\exp(-\mathrm{i}k_\alpha\zeta) + R_\alpha \exp(\mathrm{i}k_\alpha\zeta)\right]\exp(-\mathrm{i}\omega t + \mathrm{i}h\tau)\,, \quad \zeta > 0\,,$$

$$R_\alpha = \frac{\tilde{Z}_\alpha - \mathrm{i}Z_\alpha}{\tilde{Z}_\alpha + \mathrm{i}Z_\alpha}\,, \quad \mathrm{Im}\,\tilde{Z}_\alpha = 0\,, \quad \mathrm{Im}\,Z_\alpha = 0\,, \tag{33}$$

$$\tilde{\Psi}_\mathrm{p} \equiv \tilde{\mathbf{H}}\mathbf{a}\,, \quad \tilde{\Psi}_\mathrm{s} \equiv \tilde{\mathbf{E}}\mathbf{a}\,, \quad \alpha = \mathrm{s}, \mathrm{p}$$

(τ is the current coordinate along the **b** direction). Therefore, in the immediate vicinity of the point corresponding, for a given frequency in the frequency–longitudinal wavenumber external parameter plane, to the spectrum of a fast ESW (24)–(27), (29), (32), the surface wave impedance (for a TM wave) (or the surface wave conductance for a TE wave) may be expressed as a power series in small deviations of the longitudinal wavenumber around $h'_\alpha(\omega)$ $(Z_\alpha(\omega, h = h'_\alpha(\omega)) = 0)$. As a result, in the immediate vicinity of (24)–(27), (29), (32), the structure of the Fresnel reflection coefficient of the TM or TE wave with a fixed frequency ω may be represented in the form [2, 3, 5]

$$R_\alpha(\omega) \approx \frac{h - h'_\alpha(\omega) + \mathrm{i}h''_\alpha(\omega)}{h - h'_\alpha(\omega) - \mathrm{i}h''_\alpha(\omega)}\,, \qquad (34)$$
$$Z_\alpha\big(\omega, h'_\alpha(\omega)\big) = 0\,, \quad h''_\alpha \approx \frac{\tilde{Z}_\alpha}{\partial Z_\alpha/\partial h}\bigg|_{h=h'_\alpha}\,, \quad \alpha = \mathrm{p}, \mathrm{s}\,.$$

If it is assumed that the monochromatic field source of frequency ω is infinitely far from the impedance surface and possesses an angular spectrum $F_\alpha(\omega, h)$ in the case of a TM or TE wave, the field structure of the reflected non-plane TM or TE wave in the optically denser medium may be represented, instead of by expressions (33), as

$$\tilde{\Psi}_\alpha(\zeta, \tau) = \tilde{\Psi}_{i\alpha}(\zeta, \tau) + \tilde{\Psi}_{r\alpha}(\zeta, \tau)\,,$$
$$\tilde{\Psi}_{i\alpha}(\zeta, \tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp\left[-\mathrm{i}k_{\parallel\alpha}(\omega, h)\zeta + \mathrm{i}h\tau\right] F_\alpha(\omega, h)\,\mathrm{d}h\,, \qquad (35)$$
$$\tilde{\Psi}_{r\alpha}(\zeta, \tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_\alpha(\omega, h) \exp\left[\mathrm{i}k_{\parallel\alpha}(\omega, h)\zeta + \mathrm{i}h\tau\right] F_\alpha(\omega, h)\,\mathrm{d}h\,.$$

As a result, in accordance with the general propositions of the wave theory in layered media [3, 5, 9–11, 41–43], the following conclusions are drawn from expressions (34), (35). The fast improper ESW, which is formed in the TIR domain in the incidence of an outside quasi-plane bulk TM or TE wave, propagates along the interface (24)–(27), (29), (32) and transforms into a leaky ESW with the same polarization. As is well known [43], in the incidence of a plane wave, its angular spectrum is defined by the delta function. However, considering the finite angular width of an incident beam, the following versions become possible, which permit making approximate analytical calculations, in view of expressions (34), of the integral on the right side of expressions (35):

$$\left|h - h'_\alpha(\omega)\right| \ll \left|h''_\alpha(\omega)\right|\,, \qquad (36)$$
$$\left|h - h'_\alpha(\omega)\right| \gg \left|h''_\alpha(\omega)\right|\,. \qquad (37)$$

When the finite curvature of the front of a real quasi-plane bulk wave (the finiteness of the size of the wave source) is taken into account, at the interface the condition $\mathbf{S}_\alpha\mathbf{q} = 0$, according to calculations, is no longer fulfilled for the majority of plane waves forming a highly directional wave beam for $\alpha = \mathrm{p}$ as well as for $\alpha = \mathrm{s}$. However, for the axial vector of a quasi-plane TM or TE wave with a given frequency ω, the relation $\operatorname{Re} Z_\alpha(\omega, h) = 0$ may be realized, as before, under TIR conditions, provided expressions (24)–(26), (28), (29), (32) hold true.

When inequality (36) holds, it is possible to expand in a series the phase of the reflection coefficient. Taking into account only the zero and first orders of the expansion in small deviations of h from the axial beam direction under TIR conditions gives only a shift of the reflected beam as a whole relative to the beam trajectory determined proceeding from geometrical optics (i.e., the reflection of a perfectly plane wave) [3, 5, 9–11, 41–43].

When inequality (37) holds, according to Refs [3, 5, 9, 41], a qualitative change in the shape and trajectory of the reflected beam relative to the shape and trajectory defined by approximation (36) becomes possible.

The above effect of excitation of a fast improper TM(TE) ESW (like that of a TM(TE) EBW or a shear SAW), which does nor radiate in its propagation, by an outside quasi-plane bulk wave under TIR conditions is consistent with the assertion of Refs [3, 5, 42] that a leaky wave (like the side one) may be excited by a wave with a finite angular spectrum. When expressions (24)–(27), (29) hold, this gives grounds to expect in the TIR domain ($R_\alpha = \exp(\mathrm{i}\varphi_\alpha)$) a local enhancement of the Goos–Hänchen effect $\varDelta_\alpha = -\partial\varphi_\alpha/\partial h$ (see, for instance, Ref. [5]) in the incidence of a quasi-plane TM(TE) wave, provided its incidence angle and frequency correspond to inequalities (36) or (37).

Although over 70 years have passed since the discovery of the Goos–Hänchen effect [44], interest in the analysis of the spatial evolution of the beam of bulk electromagnetic waves incident on the interface of optically transparent media under TIR conditions has not weakened [45, 46].

As is well known, the Goos–Hänchen effect consists of a beam of bulk waves incident under TIR conditions on the interface from an optically denser medium undergoing on reflection a longitudinal shift along the line of intersection of the sagittal plane and the media interface. The magnitude of the Goos–Hänchen effect increases significantly when leaky surface polaritons participate in energy transfer. This was experimentally demonstrated with different optical configurations, which permitted exciting surface polaritons in the framework of the frustrated TIR technique (cases in point, in particular, are the Otto and Kretschmann configurations [12–14]). However, in all of these studies, the Goos–Hänchen shift enhancement did not result from the excitation of a surface EM wave directly at the interface with the optically denser medium. The necessary element of these optical configurations was the inevitable presence of an intermediate layer (structure) on the surface of the optically lower-density medium. This is due to the fact that the formation of a surface polariton at the interface between two media requires that (i) one of the media be surface-active for the given values of ω and h; (ii) the same ω and h values simultaneously satisfy the TIR domain for both contacting media.

We will show that the enhancement of the Goos–Hänchen effect for a quasi-plane TM(TE) wave under TIR conditions may be realized even for a single interface between transparent media (1)–(6), (22), (23) when there is resonance excitation of a leaky electromagnetic ESW with the same polarization as the incident wave. When the relation for the Fresnel reflection coefficient R_α in expression (35) under TIR conditions is represented as $R_\alpha = \exp(\mathrm{i}\varphi_\alpha)$, in view of expression (31) we find [5, 41, 44] that the longitudinal shift of the beam of bulk TM(TE)-polarized waves away from the limiting TIR angle $\vartheta = \vartheta_{\mathrm{c}\alpha}$ assumes the form

$$\varDelta_\alpha = \frac{2\,\mathrm{sign}\,h}{1 + (Z_\alpha/\tilde{Z}_\alpha)^2}\,\frac{\partial}{\partial h}\left(\frac{Z_\alpha}{\tilde{Z}_\alpha}\right), \quad \alpha = \mathrm{p}, \mathrm{s}\,. \qquad (38)$$

Since the Artmann relation [47] is valid, as is well known [5, 41, 43], only for incidence angles distant from grazing incidence, in the subsequent discussion we restrict ourselves to the condition $\tilde{Z}_\alpha\,\partial Z_\alpha/\partial h \gg Z_\alpha\,\partial\tilde{Z}_\alpha/\partial h$ to obtain from expression (38)

$$\Delta_\alpha \cong \frac{2\,\mathrm{sign}\,h}{1+(Z_\alpha/\tilde{Z}_\alpha)^2}\,\frac{\partial Z_\alpha}{\tilde{Z}_\alpha\,\partial h} = \frac{\mathrm{sign}\,h}{2\tilde{Z}_\alpha}|T_\alpha|^2\,\frac{\partial Z_\alpha}{\partial h}\,, \quad \alpha = \mathrm{p,s}\,. \tag{39}$$

Calculations show that $\partial Z_\alpha/\partial h$ defines the ratio of the cycle-averaged energy flux carried by a TM(TE) evanescent wave along the surface of an optically lower-density medium and the reciprocal of the penetration depth of this nonuniform wave into the optically lower-density medium.

Although relation (39) is inapplicable in the immediate vicinity of the limiting TIR angle for an incident wave of this type ($\vartheta_{\mathrm{c}\alpha}$), it shows the possibility of an increase in the Goos–Hänchen shift for $\vartheta \to \vartheta_{\mathrm{c}\alpha}$, both when $\partial Z_\alpha/\partial h$ increases and when Z_α tends to zero.

Since evanescent wave formation inside the TIR domain will limit the increase in $\partial Z_\alpha/\partial h$, one might expect that the realization of condition (27) will result in enhancement of the Goos–Hänchen effect even in the case of a single interface between transparent media possessing optical contrast (i.e., without introducing 'additional interfaces' employed in traditional optical configurations [12–14]).

In particular, for a high-directivity beam of plane bulk TM(TE) waves in the existence domain of a leaky TM(TE) ESW (24)–(27), (29), the magnitude of the shift, according to expressions (38) and (39), in view of expression (34) is of the form [2, 5, 41, 48]

$$\Delta_\alpha(h) = -\frac{\partial R_\alpha}{\mathrm{i}R_\alpha\,\partial h} \cong \frac{2h''_\alpha}{(h-h'_\alpha)^2+(h''_\alpha)^2}\,, \quad \alpha = \mathrm{p,s}\,. \tag{40}$$

As a result, in the resonance excitation of a slowly leaking (i.e., for $|h''_\alpha| \ll |h'_\alpha|$) TM(TE) ESW (24)–(27), (29) at the single interface between optically transparent dielectrics, in expression (40), $h'_\alpha\Delta_\alpha(h=h'_\alpha) \gg 1$.

As a consequence, in the ray representation, much as it takes place in the case of a side wave [5], in this case of ESW excitation at a single interface under TIR conditions, the ray which connects in the optically denser medium the radiation source point and the observation point characterizes, in view of expression (40), the nonlocal interaction of the incident bulk EM wave with the interface. In particular, correct inclusion of this circumstance, according to Ref. [5], has the following consequence. When a beam of plane bulk TM(TE) waves is incident on the interface at an angle close to the angle corresponding to the electromagnetic ESW of the corresponding polarization ($h = h'_\alpha$ determined by expression (27)), for $h > h'_\alpha$ the effect of focusing of the highly directional reflected wave beam becomes basically possible, the envelope of these rays corresponding to a caustic. Calculations in this case do not basically differ from those outlined in Ref. [5]. The only difference is that the straight line, from which the distance to the caustic is found in Ref. [5], is now determined not by the limiting TIR angle (as in the case of side wave excitation considered in Ref. [5]) but by the angle calculated for the given frequency ω from the ESW spectrum (27).

When it is not a quasi-plane but a quasi-monochromatic outside TM(TE) wave that is incident on the surface of the BA medium (22), (23) under consideration, in the vicinity of the existence domain of the leaky ESW (24)–(27), (29) under discussion, similarly to expression (36), for a fixed incidence angle [49, 50]

$$R_\alpha(h) \approx \frac{\omega-\omega'_\alpha(h)-\mathrm{i}\omega''_\alpha(h)}{\omega-\omega'_\alpha(h)+\mathrm{i}\omega''_\alpha(h)}\,, \quad Z_\alpha\big(\omega'_\alpha(h),h\big)=0\,,$$
$$\omega''_\alpha(h) \approx \frac{\tilde{Z}_\alpha}{\partial Z_\alpha/\partial\omega}\bigg|_{\omega=\omega'_\alpha(h)}\,, \tag{41}$$

where, according to expressions (24)–(27), (29), $Z_\alpha(\omega=\omega'_\alpha(h),h)=0$. Here, $2\pi/\omega''_\alpha$ characterizes the lifetime of the quasistationary surface electromagnetic TM(TE) state with $\omega=\omega'_\alpha(h)$, whose finite value is due to the emission of a bulk wave (reflected quasi-monochromatic wave packet) propagating from the medium interface into the upper medium. As is well known [12], the term surface electromagnetic state (SES) is used in reference to a surface electromagnetic wave (SEW) whose dispersion law satisfies boundary conditions and the energy flux in the plane of the media interface is strictly zero.

According to calculations similar to those made in Ref. [49], for a reflected quasi-monochromatic wave with $\alpha = \mathrm{p,s}$ and a narrow frequency spectrum, $\omega = \omega_\alpha(h)+\Delta\omega_\alpha$, $|\Delta\omega_\alpha/\omega_\alpha| \ll 1$, under TIR conditions and ignoring signal shape deformation,

$$\Delta t_\alpha(h,\omega) = \frac{\partial\varphi_\alpha(\omega,h)}{\partial\omega}\bigg|_{\omega=\omega_\alpha(h)}\,, \quad \alpha=\mathrm{p,s} \tag{42}$$

defines the group delay of the arrival of the envelope of the reflected wave packet at a given point of observation relative to the incident one (an analogue of the Wigner formula for particle delay in the interaction region in elastic scattering [51]). As a result, we take into account expression (41) to find that the group delay of the pulse of a bulk TM(TE)-polarized wave reflected from the medium interface for $\tilde{Z}_\alpha\,\partial Z_\alpha/\partial\omega \gg Z_\alpha\,\partial\tilde{Z}_\alpha/\partial\omega$ assumes the form

$$\Delta t_\alpha(\omega) \cong \frac{2\omega''_\alpha}{(\omega-\omega'_\alpha)^2+(\omega''_\alpha)^2}\,, \quad \alpha=\mathrm{p,s}\,. \tag{43}$$

Therefore, the condition $\omega'_\alpha\Delta t_\alpha \gg 1$ may be fulfilled for the reflected pulse only when there is a long-lived ($|\omega''_\alpha/\omega'_\alpha| \ll 1$) quasistationary SES (or SEW) whose frequency is equal to the carrier frequency of the wave packet with $\omega=\omega'_\alpha$ incident from the outside. The maximum group delay of a reflected pulse for a TM(TE) wave is determined by the lifetime of the resonantly excited quasistationary SES of the corresponding polarization ($|\Delta t_\alpha| \cong 2/\omega''_\alpha$). By analogy with the surface electronic resonance considered in Ref. [52], it is valid to say that relations (24)–(27), (29), (41)–(43) correspond to the resonance scattering from a quasistationary ($|\omega''_\alpha/\omega'_\alpha| \ll 1$) non-Tamm SES (or SEW) for a plane quasi-monochromatic bulk TM(TE) wave incident on a single medium interface (surface polariton resonance) [48].

Although the first investigations involving SESs and SEWs were performed about 40 years ago, interest in localized excitations of this type and attending dynamic effects in the electrodynamics of layered media (in particular, in surface polariton resonance) is constantly growing. As this takes place, the overwhelming majority of work in this area is traditionally related to studies of surface polariton excitations (electromagnetic analogues of Tamm states). For their existence, it is necessary that at least one of the contacting media possess additional translation symmetry along the normal to the interface plane [53, 54].

5. Exceptional surface waves and interference mechanisms of evanescent wave intensity enhancement in layered structures

As shown in Sections 2–4, in the domain of TIR at the single interface between transparent dielectric media, the necessary condition for the maximal intensity enhancement of electromagnetic TM(TE) EWs in an optically lower-density medium in the incidence of an outside plane bulk wave with the corresponding polarization is the strict equality to zero of the instantaneous energy flux across the interface at any point in time (24)–(27). We consider how, under TIR conditions, the fulfillment of these relations on the outer surface of a reflecting multilayer dielectric structure affects the intensity enhancement of electromagnetic TM(TE) EWs at the inner interlayer interfaces.

Consider a vacuum–transparent dielectric layered structure, and let the optically lower-density BA medium (22), (23) occupy not the entire lower half-space ($\zeta < 0$) but only the layer $0 \leqslant \zeta \leqslant d_C$, whose lower surface ($\zeta = 0$) is coated with a perfect metal (for a TM incident outside wave) or a perfect magnetic conductor (for a TE incident wave), i.e., an analogue of the Gires–Tournois etalon [55]. We assume that the magnetooptical configuration, which allows the existence of conditions (24)–(27), (29) on the surface of the BA medium (22), (23), has remained invariable relative to that studied in Section 4. As before, the system of Maxwell boundary conditions (2) applies to the upper layer surface (for $\zeta = d_C$) adjacent to medium (1), while on the lower surface of the layer, depending on the polarization of the plane EM wave incident from the vacuum,

$$\begin{aligned} &\mathbf{Eb} = 0\,, \quad \mathbf{Ea} = 0\,, \quad \zeta = 0\,, \quad \alpha = \mathrm{p}\,, \\ &\mathbf{Hb} = 0\,, \quad \mathbf{Ha} = 0\,, \quad \zeta = 0\,, \quad \alpha = \mathrm{s}\,. \end{aligned} \tag{44}$$

As a result, for a TM(TE) wave, the transition matrix for a layer of thickness d_C of the BA medium (22), (23) under TIR conditions takes on the following form:

$$\begin{aligned} &\begin{pmatrix} \tilde{\mathbf{H}}\mathbf{a} \\ \tilde{\mathbf{E}}\mathbf{b} \end{pmatrix} = \begin{pmatrix} C_{11}^{\mathrm{p}} & C_{12}^{\mathrm{p}} \\ C_{21}^{\mathrm{p}} & C_{22}^{\mathrm{p}} \end{pmatrix} \begin{pmatrix} \mathbf{Ha} \\ \mathbf{Eb} \end{pmatrix}, \\ &\begin{pmatrix} \tilde{\mathbf{E}}\mathbf{a} \\ \tilde{\mathbf{H}}\mathbf{b} \end{pmatrix} = \begin{pmatrix} C_{11}^{\mathrm{s}} & C_{12}^{\mathrm{s}} \\ C_{21}^{\mathrm{s}} & C_{22}^{\mathrm{s}} \end{pmatrix} \begin{pmatrix} \mathbf{Ea} \\ \mathbf{Hb} \end{pmatrix}, \end{aligned} \tag{45}$$

$$\begin{aligned} &C_{21}^{\alpha} = -\frac{2Z_{\alpha}^{+}Z_{\alpha}^{-}}{Z_{\alpha}^{+} - Z_{\alpha}^{-}} \sinh(\eta_{\alpha}'' d)\,, \\ &C_{11}^{\alpha} = \cosh(\eta_{\alpha}'' d) + \frac{Z_{\alpha}^{+} + Z_{\alpha}^{-}}{Z_{\alpha}^{+} - Z_{\alpha}^{-}} \sinh(\eta_{\alpha}'' d)\,, \\ &C_{12}^{\alpha} = \frac{2}{Z_{\alpha}^{+} - Z_{\alpha}^{-}} \sinh(\eta_{\alpha}'' d)\,, \\ &C_{22}^{\alpha} = \cosh(\eta_{\alpha}'' d) - \frac{Z_{\alpha}^{+} + Z_{\alpha}^{-}}{Z_{\alpha}^{+} - Z_{\alpha}^{-}} \sinh(\eta_{\alpha}'' d)\,, \\ &Z_{\mathrm{s}}^{\pm}(\omega, h) \equiv \frac{1}{\Delta\mu}\left[\pm\mu_2\eta_{\mathrm{s}}'' - h \operatorname{Im}\mu_3 + \mu_2 \operatorname{Im} A_1^{*} - \operatorname{Im}(\mu_3 A_2^{*})\right], \\ &\Delta\mu \equiv \mu_1\mu_2 - \mu_3\mu_3^{*}\,, \\ &Z_{\mathrm{p}}^{\pm}(\omega, h) \equiv \frac{1}{\Delta\varepsilon}\left[\pm\varepsilon_2\eta_{\mathrm{p}}'' - h \operatorname{Im}\varepsilon_3 - \varepsilon_2 \operatorname{Im} A_3 + \operatorname{Im}(\varepsilon_3 A_4)\right], \\ &\Delta\varepsilon \equiv \varepsilon_1\varepsilon_2 - \varepsilon_3\varepsilon_3^{*}\,. \end{aligned} \tag{46}$$

Therefore, for $\alpha = \mathrm{p}$ (or $\alpha = \mathrm{s}$), the ratio of the amplitude of the transmitted plane wave to the amplitude of the outside incident wave on the lower surface $W_{\alpha}(\zeta = 0)$ of the layer of a BA medium (22), (23) is expressed, in view of expressions (44)–(46), as

$$W_{\alpha}(\zeta = 0) = \frac{1}{C_{11}^{\alpha}} \frac{2\tilde{Z}_{\alpha}}{\tilde{Z}_{\alpha} + \mathrm{i}Z_{\mathrm{in}\,\alpha}}\,, \quad Z_{\mathrm{in}\,\alpha}(\zeta = d_C) = \frac{C_{21}^{\alpha}}{C_{11}^{\alpha}}\,. \tag{47}$$

As a result, when the frequency and incidence angle of the plane bulk TM(TE) wave are simultaneously such that $Z_{\mathrm{in}\,\alpha}(\zeta = d_C) = 0$ in expressions (22), (23), and (47), then in the excitation, in the layer of the BA medium, of a TM(TE) ESW corresponding to $Z_{\alpha}^{+}(\omega, h) = 0$ in expressions (45)–(47), $W_{\alpha}(\zeta = 0) = \exp(\eta_{\alpha}'' d_C)$, while for $Z_{\alpha}^{-}(\omega, h) = 0$ in expressions (46), (47), $W_{\alpha}(\zeta = 0) = \exp(-\eta_{\alpha}'' d_C)$. At the same time, both for $Z_{\alpha}^{+}(\omega, h) = 0$ and for $Z_{\alpha}^{-}(\omega, h) = 0$, $W_{\alpha}(\zeta = d_C) = 2$. Therefore, in the incidence of an outside plane bulk wave, the instantaneous energy flux is simultaneously zero at any point in time at both interfaces of the structure under consideration.

Now consider the bilayer Kretschmann configuration with a layer of BA medium (45), (46) (medium C) [14]. To this end, we assume that there is an optically transparent one-dimensional reflective layered structure, which comprises two optically isotropic semi-infinite media not equivalent to each other: A ($\zeta > d_B + d_C$) and D ($\zeta < 0$). Their surfaces are connected by a bilayer sandwich structure consisting of a layer of optically isotropic medium B of thickness d_B with the transition matrix

$$\begin{aligned} &\begin{pmatrix} \mathbf{H}_B\mathbf{a} \\ \mathbf{E}_B\mathbf{b} \end{pmatrix}_{\zeta = d_B} = \begin{pmatrix} B_{11}^{\mathrm{p}} & B_{12}^{\mathrm{p}} \\ B_{21}^{\mathrm{p}} & B_{22}^{\mathrm{p}} \end{pmatrix} \begin{pmatrix} \mathbf{H}_B\mathbf{a} \\ \mathbf{E}_B\mathbf{b} \end{pmatrix}_{\zeta = 0}, \\ &\begin{pmatrix} \mathbf{E}_B\mathbf{a} \\ \mathbf{H}_B\mathbf{b} \end{pmatrix}_{\zeta = d_B} = \begin{pmatrix} B_{11}^{\mathrm{s}} & B_{12}^{\mathrm{s}} \\ B_{21}^{\mathrm{s}} & B_{22}^{\mathrm{s}} \end{pmatrix} \begin{pmatrix} \mathbf{E}_B\mathbf{a} \\ \mathbf{H}_B\mathbf{b} \end{pmatrix}_{\zeta = 0}, \\ &B_{21}^{\alpha} = Z_{\alpha B} \sinh(\eta_{\alpha B} d_B)\,, \quad B_{11}^{\alpha} = \cosh(\eta_{\alpha B} d_B)\,, \\ &B_{12}^{\alpha} = \frac{1}{Z_{\alpha B}} \sinh(\eta_{\alpha B} d_B)\,, \quad B_{22}^{\alpha} = \cosh(\eta_{\alpha B} d_B)\,, \\ &Z_{\mathrm{pB}} = \frac{\sqrt{h^2 - \varepsilon_B k_0^2}}{\varepsilon_B k_0}\,, \quad Z_{\mathrm{sB}} = \frac{\sqrt{h^2 - \varepsilon_B k_0^2}}{k_0}\,, \end{aligned} \tag{48}$$

and a layer of BA medium (22), (23) of thickness d_C with transition matrix (45), (46). Let Maxwellian boundary conditions (2) be applicable to all layer interfaces of the structure under consideration. For a plane bulk TM(TE) wave incident from medium A, Fresnel reflection $V_{\alpha}(\zeta = d_B + d_C)$ and transmission $W_{\alpha}(\zeta)$ coefficients for one of the inner layer boundaries (i.e., for $\zeta = d_B + d_C$, $\zeta = d_C$, or $\zeta = 0$) of the reflective layered structure under discussion are related by the following relations:

$$\begin{aligned} &\begin{pmatrix} 1 + V_{\alpha} \\ \mathrm{i}Z_{\alpha A}(-1 + V_{\alpha}) \end{pmatrix}_{\zeta = d_B + d_C} = \overline{\overline{Q}}^{\alpha}(\zeta) \begin{pmatrix} W_{\alpha} \\ Z_{\mathrm{in}\,\alpha}(\zeta) W_{\alpha} \end{pmatrix}_{\zeta}, \\ &\zeta = d_B + d_C, d_C, 0\,, \\ &\overline{\overline{Q^{\alpha}}}(\zeta = d_B + d_C) \equiv \overline{\overline{I}}\,, \quad \overline{\overline{Q^{\alpha}}}(\zeta = d_C) \equiv \overline{\overline{B^{\alpha}}}(d_B)\,, \\ &\overline{\overline{Q^{\alpha}}}(\zeta = 0) \equiv \overline{\overline{B^{\alpha}}}(d_B)\overline{\overline{C^{\alpha}}}(d_C)\,, \end{aligned} \tag{49}$$

where $\overline{\overline{I}}$ is the identity matrix.

Hereinafter, our analysis is restricted to the case when, simultaneously,

$$k_{\mathrm{A}}^2 = \varepsilon_{\mathrm{A}} k_0^2 - h^2 > 0\,, \quad \eta_{\mathrm{B}}^2 = h^2 - \varepsilon_{\mathrm{B}} k_0^2 > 0\,,$$
$$\eta_{\mathrm{D}}^2 = h^2 - \varepsilon_{\mathrm{D}} k_0^2 > 0\,, \tag{50}$$
$$Z_{\mathrm{pD}} = \frac{\sqrt{h^2 - \varepsilon_{\mathrm{D}} k_0^2}}{\varepsilon_{\mathrm{D}} k_0}\,, \quad Z_{\mathrm{sD}} = \frac{\sqrt{h^2 - \varepsilon_{\mathrm{D}} k_0^2}}{k_0}\,, \quad \alpha = \mathrm{p}, \mathrm{s}\,,$$

which corresponds to a plane bulk TM(TE) wave incident from medium A on the upper surface $\zeta = d_{\mathrm{B}} + d_{\mathrm{C}}$ of reflective structure (49). Calculations suggest that, under TIR conditions on the outer surface of reflective structure (49), (50), i.e., for $\zeta = d_{\mathrm{B}} + d_{\mathrm{C}}$, the maximal enhancement of the amplitude of the excited evanescent wave with the corresponding polarization $\alpha = \mathrm{p}, \mathrm{s}$

$$W_\alpha(\zeta = d_{\mathrm{B}} + d_{\mathrm{C}}) = 2\,, \tag{51}$$

when

$$Z_{\mathrm{in}\,\alpha}(\zeta = d_{\mathrm{B}} + d_{\mathrm{C}}) = \frac{Q_{21}^\alpha + Q_{22}^\alpha Z_{\mathrm{D}\alpha}}{Q_{11}^\alpha + Q_{12}^\alpha Z_{\mathrm{D}\alpha}} = 0\,, \quad \alpha = \mathrm{p}, \mathrm{s}\,. \tag{52}$$

Consequently, in this case, on the surface $\zeta = d_{\mathrm{B}} + d_{\mathrm{C}}$ of reflective structure (49), $\mathbf{S}_\alpha \mathbf{q} = 0$ ($\alpha = \mathrm{p}, \mathrm{s}$). This signifies that condition (52), in combination with expressions (50) for $\alpha = \mathrm{p}$, defines the dispersion law of an interference exceptional surface TM EM wave propagating along the interface between a perfect electric conductor and a multilayer structure (49), (50). Accordingly, for $\alpha = \mathrm{s}$, the fulfillment of relation (52) under conditions (50) corresponds to the dispersion law of an interference exceptional surface TE EM wave propagating along the interface between the perfect magnetic conductor and the multilayer structure (49), (50).

Therefore, in the case of a multilayer reflective structure, too, in the incidence of an outside plane bulk TM(TE) wave on its surface, the greatest enhancement of the excitation intensity of an evanescent wave with the corresponding TM(TE) polarization stems from the realization of the ESW mode with $\alpha = \mathrm{p}$ or $\alpha = \mathrm{s}$ on the outer surface of the multilayer structure. However, equal to zero at any point in time is the instantaneous energy flux only across the interface between the reflective layered structure (49) and the semi-infinite medium, from which a plane bulk wave with polarization $\alpha = \mathrm{p}, \mathrm{s}$ falls, i.e., for $\zeta = d_{\mathrm{B}} + d_{\mathrm{C}}$. Simultaneously, equal to zero at the remaining interfaces (for $\zeta = d_{\mathrm{C}}$ and $\zeta = 0$) is only the cycle-averaged energy flux across the interface.

According to calculations, when Eqn (52) holds, simultaneously,

$$W_\alpha(\zeta = d_{\mathrm{C}}) = 2B_{22}^\alpha\,,$$
$$Z_{\mathrm{in}\,\alpha}(\zeta = d_{\mathrm{C}}) = \frac{C_{21}^\alpha + C_{22}^\alpha Z_{\mathrm{D}\alpha}}{C_{11}^\alpha + C_{12}^\alpha Z_{\mathrm{D}\alpha}}\,, \quad \alpha = \mathrm{p}, \mathrm{s}\,, \tag{53}$$

$$W_\alpha(\zeta = 0) = 2Q_{22}^\alpha\,, \quad Z_{\mathrm{in}\,\alpha}(\zeta = 0) = Z_{\mathrm{D}\alpha}\,, \quad \alpha = \mathrm{p}, \mathrm{s} \tag{54}$$

(see also Refs [56, 57]).

In this case, for reflective structure (49), (50) under consideration, relation (53) under the fulfillment of Eqn (52) defines in the dissipative-free limit the greatest amplitude enhancement of the TM(TE) evanescent wave excited in the Otto configuration [13, 14]. In this case, medium A plays the role of a prism, the intermediate surface-inactive layer is the layer of thickness d_{B} of medium B, and the surface-active medium is the layer of thickness d_{C} of medium C, which is related to semi-infinite medium D by electrodynamic boundary conditions (2).

As for relation (54), for reflective structure (49), (50) under consideration, it defines, under the fulfillment of Eqn (52) and disregarding dissipation, the greatest amplitude enhancement of a TM(TE) evanescent wave excited in the Kretschmann configuration [13, 14]. In this case, medium A plays the role of a prism, the surface-active layer is the bilayer structure of thickness $d_{\mathrm{B}} + d_{\mathrm{C}}$ formed by the layers of media B and C, respectively, and the surface-inactive medium is medium D.

When the one-dimensional reflective multilayer structure is a two-component semi-infinite photonic crystal ($\zeta < 0$) with elementary period $D = d_{\mathrm{B}} + d_{\mathrm{C}}$ and transition matrix $\overline{\overline{Q^\alpha}} \equiv \overline{\overline{B^\alpha}}(d_{\mathrm{B}})\overline{\overline{C^\alpha}}(d_{\mathrm{C}})$ (49) for TM(TE) waves, under TIR conditions ($Q_{11}^\alpha + Q_{22}^\alpha > 2$), the greatest intensity enhancement ($|W_\alpha(\zeta = 0)|^2 = 4$) of a TM(TE) evanescent wave on its outer surface is realized when [58]

$$Z_{\mathrm{in}\,\alpha}(\zeta = 0) = 0\,,$$
$$Z_{\mathrm{in}\,\alpha}(\zeta = 0) \equiv \frac{Q_{21}^\alpha(\zeta = -D)}{\exp(-q_\alpha D) - Q_{11}^\alpha(\zeta = -D)}\,, \quad \alpha = \mathrm{p}, \mathrm{s}\,, \tag{55}$$
$$\cosh(q_\alpha D) = \frac{1}{2}(Q_{11}^\alpha + Q_{22}^\alpha)\,, \quad q_\alpha^2 > 0\,.$$

Relation (55) defines, for this layered structure, the law of dispersion of Tamm-type ESWs with $\alpha = \mathrm{p}, \mathrm{s}$ (see also Ref. [59]). When relation (55) is fulfilled, the maxima of TM(TE) evanescent wave intensity enhancement are also realized in the depth of the one-dimensional superlattice under consideration, when $\zeta = -\nu D$, $\nu = 1, 2, \ldots$. As a result,

$$W_\alpha(\zeta = 0) = 2\,, \quad W_\alpha(\zeta = -\nu D) = \frac{2}{Q_{11}^\alpha U_{\nu-1}(x) - U_{\nu-2}(x)}\,,$$
$$x = \operatorname{arcosh}\left[\frac{1}{2}\left(Q_{11}^\alpha + \frac{1}{Q_{11}^\alpha}\right)\right]\,, \quad \alpha = \mathrm{p}, \mathrm{s}\,, \tag{56}$$

where $U_\nu(x)$ is the Chebyshev polynomial of the second kind of degree ν and for $\zeta = 0, -\nu D$, $\nu = 1, 2, \ldots$ the instantaneous energy flux across such an interlayer interface is zero ($\mathbf{S}_\alpha \mathbf{q} = 0$) at any point in time. Simultaneously, when relation (55) is fulfilled, $\langle \mathbf{S}_\alpha \mathbf{q} \rangle = 0$ for the interlayer interface inside each of the elementary periods of the semi-infinite one-dimensional photonic crystal under discussion (i.e., for $\zeta = -d_{\mathrm{B}} - \nu D$, $\nu = 0, 1, 2, \ldots$), while $\mathbf{S}_\alpha \mathbf{q} \neq 0$.

We emphasize that, in the case of relation (53), as well as in the case of relation (54), the greatest intensity enhancement of the TM(TE) evanescent wave excitation by an outside plane bulk wave of the corresponding polarization in the Otto configuration (53) and in the Kretschmann configuration (54) is defined by expression (52) rather than by the transverse resonance condition [2] at the interface between the surface-active and surface-inactive transparent semi-infinite media. Therefore, in both specified configurations, the maximum enhancement of TM(TE) evanescent wave excitation corresponds to the formation of an ESW of the corresponding polarization $\alpha = \mathrm{p}, \mathrm{s}$ in this interference-type reflective structure.

6. Concrete example of exceptional surface wave formation in a bianisotropic medium

As an example of a BA medium we consider the two-sublattice ($\mathbf{M}_{1,2}$ are the magnetizations of the sublattices, $|\mathbf{M}_1| = |\mathbf{M}_2| = M_0$) model of an exchange-collinear uniaxial (z-axis) antiferromagnet (AFM) [60, 61]. Following Ref. [62], for a mechanism providing in the AFM the interaction between the spin subsystem and constant external electric field $\mathbf{E}_0$, we consider quadratic magnetooptical interaction (QMOI). In this case, in terms of ferromagnetism ($\mathbf{m} = (\mathbf{M}_1 + \mathbf{M}_2)/(2M_0)$) and antiferromagnetism ($\mathbf{l} = (\mathbf{M}_1 - \mathbf{M}_2)/(2M_0)$) vectors, the thermodynamic potential density of the AFM under consideration takes on the form

$$F = M_0^2\left(\frac{\delta}{2}\,\mathbf{m}^2 - \frac{b}{2}\,l_z^2 + \frac{b_1}{2}\,l_x^2 l_y^2 - 2\mathbf{mh} - \frac{r_m}{2}(\mathbf{mP})^2 - \frac{r_l}{2}(\mathbf{lP})^2 - \frac{s_m}{2}\,\mathbf{m}^2\mathbf{P}^2 - \frac{s_l}{2}\,\mathbf{l}^2\mathbf{P}^2\right) + \left(\frac{P_x^2 + P_y^2}{2\kappa_\perp} + \frac{P_z^2}{2\kappa_\parallel} - \mathbf{PE}\right), \tag{57}$$

where δ and b, b_1 are the respective constants of uniform exchange and magnetic anisotropy, $\mathbf{h}$ is the renormalized magnetic field, $\mathbf{E}$ and $\mathbf{P}$ are the electric field and polarization vectors, respectively, $\kappa_\parallel$ and $\kappa_\perp$ are the longitudinal and transverse dielectric susceptibilities, r_m, r_l, s_m, and s_l are the QMOI coefficients.

It is traditionally assumed (see, for instance, Ref. [63]) that all coefficients in expression (57) are functions of temperature and pressure. When $b < 0$, which corresponds to the collinear phase with hard magnetic axis z, for $b_1 > 0$ in a constant magnetic field $\mathbf{H}_0 \parallel x$ in the ground state,

$$\mathbf{m}_0 \parallel x\,, \quad \mathbf{l}_0 \parallel y\,, \tag{58}$$

where $\mathbf{m}_0$ and $\mathbf{l}_0$ are the equilibrium ferromagnetism and antiferromagnetism vectors.

According to calculations, in the framework of the AFM model (57) under consideration, the additional application, orthogonally to $\mathbf{H}_0 \parallel x$, of a constant external electric field $\mathbf{E}_0$, both orthogonally and parallel to the easy magnetic axis, does not change the ground state (58) relative to that for $|\mathbf{E}_0| = 0$. As a result, for the model of a magnetic medium for $\mathbf{E}_0 \parallel y$, in the linear approximation in the amplitude of small oscillations, the constraint equations have a structure similar to that of expressions (22), (23) and may be represented in the form (see Ref. [61])

$$\begin{aligned}
\mathbf{m} &= \begin{pmatrix} \chi_{xx}(\omega) & 0 & 0 \\ 0 & \chi_{yy}(\omega) & -\mathrm{i}\chi_*(\omega) \\ 0 & \mathrm{i}\chi_*(\omega) & \chi_{zz}(\omega) \end{pmatrix}\mathbf{H} + \begin{pmatrix} 0 & \beta_4(\omega) & -\mathrm{i}\beta_1(\omega) \\ \beta_3(\omega) & 0 & 0 \\ \mathrm{i}\beta_2(\omega) & 0 & 0 \end{pmatrix}\mathbf{E}\,, \\
\mathbf{p} &= \begin{pmatrix} \alpha_{xx}(\omega) & 0 & 0 \\ 0 & \alpha_{yy}(\omega) & -\mathrm{i}\alpha_*(\omega) \\ 0 & \mathrm{i}\alpha_*(\omega) & \alpha_{zz}(\omega) \end{pmatrix}\mathbf{E} + \begin{pmatrix} 0 & \beta_3(\omega) & -\mathrm{i}\beta_2(\omega) \\ \beta_4(\omega) & 0 & 0 \\ \mathrm{i}\beta_1(\omega) & 0 & 0 \end{pmatrix}\mathbf{H}\,,
\end{aligned} \tag{59}$$

where

$$\begin{aligned}
&\chi_{xx} = T_x\,\frac{\omega_{\mathrm{AF}}^2}{\Delta_{\mathrm{AF}}}\,, \quad \chi_{yy} = T_y\,\frac{\omega_{\mathrm{F}}^2}{\Delta_{\mathrm{F}}}\,, \quad \chi_{zz} = T_z\,\frac{\omega_{\mathrm{F}}^2}{\Delta_{\mathrm{F}}}\,, \\
&\chi_* = \sqrt{T_y T_z}\,\frac{\omega_{\mathrm{F}}\omega}{\Delta_{\mathrm{F}}}\,, \quad \alpha_{xx} = \alpha_{x0} + R_x\,\frac{\omega_{\mathrm{F}}^2}{\Delta_{\mathrm{F}}}\,, \\
&\alpha_{yy} = \alpha_{y0} + R_y\,\frac{\omega_{\mathrm{AF}}^2}{\Delta_{\mathrm{AF}}}\,, \quad \alpha_* = \sqrt{R_y R_z}\,\frac{\omega_{\mathrm{AF}}\omega}{\Delta_{\mathrm{AF}}}\,, \\
&\alpha_{zz} = \alpha_{z0} + R_z\,\frac{\omega_{\mathrm{AF}}^2}{\Delta_{\mathrm{AF}}}\,, \quad \beta_1 = \sqrt{R_z T_x}\,\frac{\omega_{\mathrm{AF}}\omega}{\Delta_{\mathrm{AF}}}\,, \\
&\beta_2 = \sqrt{R_x T_z}\,\frac{\omega_{\mathrm{F}}\omega}{\Delta_{\mathrm{F}}}\,, \quad \beta_3 = \sqrt{R_x T_y}\,\frac{\omega_{\mathrm{F}}^2}{\Delta_{\mathrm{F}}}\,, \quad \beta_4 = \sqrt{R_y T_x}\,\frac{\omega_{\mathrm{AF}}^2}{\Delta_{\mathrm{AF}}}\,, \\
&\Delta_{\mathrm{F}} = \omega_{\mathrm{F}}^2 - \omega^2\,, \quad \Delta_{\mathrm{AF}} = \omega_{\mathrm{AF}}^2 - \omega^2\,.
\end{aligned} \tag{60}$$

Here, T_i is the static susceptibility, $i = x, y, z$, $\alpha_{i0} + R_i$ is the static dielectric susceptibility with the inclusion of the effect of the magnetic subsystem (i.e., with the inclusion of quadratic magnetoelectric interaction [62]), and $\omega_{\mathrm{AF,\,F}}$ are the frequencies of uniform AFM resonance for quasi-antiferromagnetic and quasi-ferromagnetic modes of the spin wave spectrum of unlimited AFM [61, 64].

Therefore, from the viewpoint of electromagnetic properties, the uncompensated single-phase exchange-collinear AFM dielectric possesses simultaneously gyrotropy, the linear antisymmetric magnetoelectric effect, and pseudo-chirality, i.e., a concrete example of a BA medium (22), (23).

We assume that the semi-infinite AFM (57)–(60) occupies the lower semi-space ($y < 0$) relative to the nonmagnetic dielectric (1) and that the standard system of boundary electromagnetic conditions (2) applies to the interface between the magnetic and optically isotropic nonmagnetic media. As suggested by calculations in this case, for $\mathbf{k} \in yz$ under refraction conditions, in the description of an outside TM(TE) wave incident on the surface of the uncompensated AFM with ground state (58) under consideration, the relations for the amplitude transmission coefficient T_α for the TM(TE) wave coincide structurally with relations (3) as before. However, expressions (26) for Z_{p} and Z_{s}, depending on the polarization ($\alpha = \mathrm{p, s}$) of an evanescent TM(TE) wave excited in a magnet under TIR conditions, assume the following form ($\mathbf{q} \parallel y$, $\mathbf{k} \in yz$) for the magnetooptical configuration under consideration:

$$Z_{\mathrm{p}} = \frac{\varepsilon_{yy}}{k_0(\varepsilon_{yy}\varepsilon_{zz} - \varepsilon_*^2)}\left(-\eta_{\mathrm{p}}'' + \frac{\varepsilon_*}{\varepsilon_{yy}}\,h + k_0\,\frac{\varepsilon_*\bar{\beta}_4 - \varepsilon_{yy}\bar{\beta}_1}{\varepsilon_{yy}}\right), \tag{61}$$

$$\eta_{\mathrm{p}}'' = \left[\frac{\varepsilon_{zz}}{\varepsilon_{yy}}\left(h + k_0\,\frac{\varepsilon_{zz}\bar{\beta}_4 - \varepsilon_*\bar{\beta}_1}{\varepsilon_{zz}}\right)^2 - k_0^2\,\frac{(\varepsilon_{yy}\varepsilon_{zz} - \varepsilon_*^2)(\mu_{xx}\varepsilon_{zz} - \bar{\beta}_1^2)}{\varepsilon_{yy}\varepsilon_{zz}}\right]^{1/2},$$

where $\bar{\beta}_j = 4\pi\beta_j$, $j = 1, 2, 3, 4$,

$$Z_{\mathrm{s}} = \frac{\mu_{yy}}{k_0(\mu_{yy}\,\mu_{zz} - \mu_*^2)}\left(\eta_{\mathrm{s}}'' - \frac{\mu_*}{\mu_{yy}}\,h + k_0\,\frac{\mu_*\bar{\beta}_3 - \mu_{yy}\bar{\beta}_2}{\mu_{yy}}\right), \tag{62}$$

$$\eta_{\mathrm{s}}'' = \left[\frac{\mu_{zz}}{\mu_{yy}}\left(h - k_0\,\frac{\mu_{zz}\bar{\beta}_3 - \mu_*\bar{\beta}_2}{\mu_{zz}}\right)^2 - k_0^2\,\frac{(\mu_{yy}\,\mu_{zz} - \mu_*^2)(\varepsilon_{xx}\,\mu_{zz} - \bar{\beta}_2^2)}{\mu_{yy}\,\mu_{zz}}\right]^{1/2}.$$

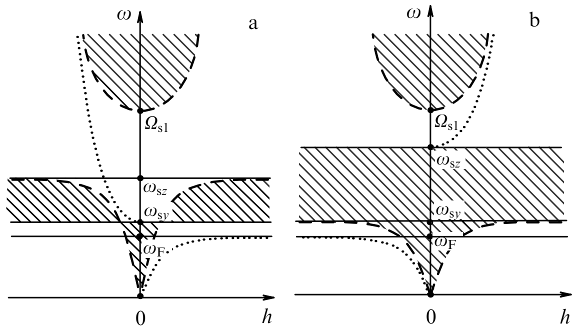

Figure 1. Domains of bulk (hatched) and evanescent (not hatched) TE wave formation in semi-infinite AFM (59), (60) for $\mathbf{E}_0 = 0$, $\mathbf{m}_0 \parallel x$, $\mathbf{l}_0 \parallel y$, $\mathbf{k} \in yz$: (a) $\mathbf{q} \parallel y$; the dotted line — ESW spectrum (63) [61]; (b) $\mathbf{q} \parallel z$; characteristic frequencies are determined from relations $\mu_{yy}(\omega_{sy}) = 0$, $\mu_{zz}(\omega_{sz}) = 0$, $\varepsilon_{xx}(\omega_{sx}) = 0$, $\mu_{yy}(\Omega_{s1})\mu_{zz}(\Omega_{s1}) - \mu_*^2(\Omega_{s1}) = 0$. Dotted line — ESW spectrum $h^2 = k_0^2 \varepsilon_{xx}\mu_{zz}$.

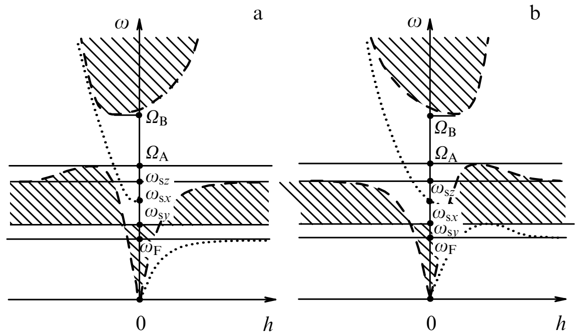

Figure 2. Domains of bulk (hatched) and evanescent (not hatched) TE wave formation in semi-infinite AFM (59), (60) for $\mathbf{E}_0 \parallel \mathbf{q} \parallel \mathbf{l}_0 \parallel y$, $\mathbf{m}_0 \parallel x$, $\mathbf{k} \in yz$, for (a) $\mathbf{E}_0\mathbf{q} > 0$, (b) $\mathbf{E}_0\mathbf{q} < 0$. Additional, relative to Fig. 1, characteristic frequencies are defined as $\Omega_A = \min\{\Omega_{s1}, \Omega_{s2}\}$, $\varepsilon_{xx}(\omega_{sx}) = 0$, $\varepsilon_{xx}(\Omega_{s2})\mu_{zz}(\Omega_{s2}) - \bar{\beta}_2^2(\Omega_{s2}) = 0$, $\Omega_B = \max\{\Omega_{s1}, \Omega_{s2}\}$. Dotted line — spectrum of exceptional surface TE wave (64).

For $\mathbf{E}_0 = 0$, simultaneously $\beta_{1-4} \equiv 0$ and $R_{x,y,z} \equiv 0$ in constraint equations (59), (60), i.e., both the linear magnetoelectric effect ($\beta_{3,4} \equiv 0$) and the pseudo-chiral effect ($\beta_{1,2} \equiv 0$) are absent in the system. In this case, in the $\omega - h$ plane, the domains of evanescent TM(TE) waves with $\mathbf{k} \in yz$ for a given propagation geometry are determined from expressions (59)–(62) by the condition $(\eta''_\alpha)^2 > 0$. The presence of gyrotropy has the result that the condition $Z_s = 0$ may be met for $\mathbf{q} \parallel y$ in the TIR domain for the model of a centrally symmetric uncompensated AFM under discussion. In accordance with expressions (24)–(27) and (29), this signifies the possibility of the formation of a single-partial TE ESW (similar to that considered in Ref. [5]), which corresponds to the interface between an uncompensated AFM and a perfect magnet. For the $\omega - h$ combinations defined by this condition, the instantaneous energy flux across the magnetic–nonmagnetic medium interface will be zero at any point in time. As a result, for the magnetooptical configuration ($\mathbf{k} \in yz$, $\mathbf{q} \parallel \mathbf{l}_0 \parallel y$) under consideration, the dispersion equation for a TE ESW assumes the form

$$h^2 = \frac{\omega^2}{c^2}\,\varepsilon_{xx}\,\mu_{yy}\,, \quad \frac{h\mu_*}{\mu_{yy}} > 0\,, \quad \alpha = \mathrm{s}\,, \tag{63}$$

i.e., the spectrum of this ESW with $\alpha = \mathrm{s}$ is unidirectional (Fig. 1).

In the case of a TM wave, for $\mathbf{E}_0 = 0$, the above effects are nonexistent at the boundary of the semi-infinite AFM model under consideration.

For $\mathbf{E}_0 \neq 0$, in constraint equations (59), (60), $\beta_{1-4} \neq 0$ and $R_{x,y,z} \neq 0$. As a result, resonance (four-fold) enhancement of the excitation intensity of not only a TE-type but also of a TM-type evanescent wave near the surface of a semi-infinite AFM medium becomes possible under TIR conditions. For $\mathbf{E}_0 \parallel \mathbf{q} \parallel \mathbf{l} \parallel y$ and $\mathbf{k} \in yz$, the analytical expressions of these curves, in view of expressions (21), (26), (27) and (59), (60), are of the form

$$(h - k_0\bar{\beta}_3)^2 = k_0^2 \varepsilon_{xx}\,\mu_{yy}\,, \qquad \frac{\mu_*}{\mu_{yy}}\,h - k_0\,\frac{\mu_*\bar{\beta}_3 - \mu_{yy}\bar{\beta}_2}{\mu_{yy}} > 0\,, \quad \alpha = \mathrm{s}\,, \tag{64}$$

$$(h + k_0\bar{\beta}_4)^2 = k_0^2 \mu_{xx}\varepsilon_{yy}\,, \qquad \frac{\varepsilon_*}{\varepsilon_{yy}}\,h + k_0\,\frac{\varepsilon_*\bar{\beta}_4 - \varepsilon_{yy}\bar{\beta}_1}{\varepsilon_{yy}} > 0\,, \quad \alpha = \mathrm{p}\,. \tag{65}$$

Therefore, the spectra of the ESW (both TE and TM) under consideration not only are characterized by unidirectionality relative to inversion of the propagation direction ($h \to -h$) but may correspond, depending on the magnitude of h, to forward ($h\,\partial\omega/\partial h > 0$) or backward ($h\,\partial\omega/\partial h < 0$) waves, and may have extremum points for $h \neq 0$. As follows from expressions (64), (65), the frequency range of the existence of ESWs (64), (65) and therefore the condition of attaining the maximum of evanescent TM(TE) wave intensity ($T_\alpha = 2$) in Eqns (1) depend on whether $\mathbf{E}_0$, $\mathbf{B}_0$, and $\mathbf{h}$ make up a right-hand or left-hand triple of vectors as well as on their relative magnitude. Some possible versions of the spectrum of exceptional surface TE waves are shown in Fig. 2. In particular, as is clear from Fig. 2, there are frequency ranges in which the ESW can change the sign of the group velocity under variations of the magnitude of h without changing the sign of the phase velocity.

Furthermore, for $\mathbf{E}_0 \parallel \mathbf{q} \parallel \mathbf{l} \parallel y$ and $\mathbf{k} \in yz$, the formation of a TM(TE) ESW near the surface of a semi-infinite AFM medium (57)–(62) is possible even for $|\mathbf{H}_0| = 0$, i.e., when simultaneously $|\mathbf{m}_0| = 0$ and $\mathbf{l}_0 \parallel y$ in the equilibrium state. In this case, the relations for the TM(TE) ESW spectrum take on a form which also corresponds to expressions (24)–(27), (29):

$$h^2 = k_0^2 \varepsilon_{xx}\,\mu_{yy}\,, \quad \bar{\beta}_2 > 0\,, \quad \alpha = \mathrm{s}\,, \tag{66}$$

$$h^2 = k_0^2 \mu_{xx}\,\varepsilon_{yy}\,, \quad -\bar{\beta}_1 > 0\,, \quad \alpha = \mathrm{p}\,. \tag{67}$$

In this case, relations (66), (67) correspond to only waves of the forward type without extremum points. However, the frequency ranges of the existence of exceptional surface TM(TE) waves depend on the sign of the quantity $\mathbf{E}_0\mathbf{q}$ (Fig. 3).

Until now, we have considered only gyrotropic or pseudo-chiral interactions for the mechanisms allowing under TIR conditions the formation of TM(TE) ESWs on the surface of an optically transparent semi-infinite dielectric. However, other mechanisms of formation of waves of this type on the single interface of optically transparent dielectrics are also possible under TIR conditions. In particular, they may be the nonlinear properties of an optically lower-density semi-infinite dielectric medium. By way of illustration, Section 7 exemplifies the realization of this mechanism when an outside plane bulk wave with $\alpha = \mathrm{s}$ is incident on the surface of a semi-infinite optically transparent dielectric with a Kerr nonlinearity from an optically transparent isotropic dielectric.

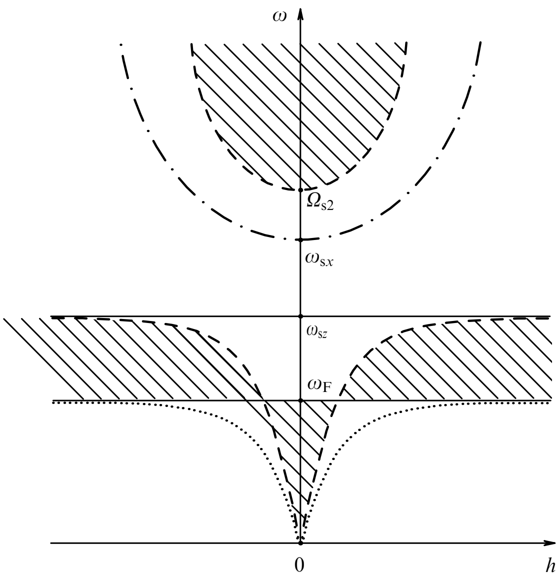

Figure. 3. Existence domains of bulk and evanescent TE waves in an AFM (59), (60) for $|\mathbf{m}_0| = 0$, $\mathbf{E}_0 \parallel \mathbf{q} \parallel \mathbf{l}_0 \parallel y$, $\mathbf{k} \in yz$. The dashed-dotted and dotted curves are the TE ESW spectrum (66) for $\mathbf{E}_0\mathbf{q} > 0$ and $\mathbf{E}_0\mathbf{q} < 0$, respectively.

7. Exceptional surface wave in an optically transparent nonlinear dielectric

As an example, consider the case when the upper half-space is, as before, occupied by optically isotropic dielectric (1), while the optically lower-density semi-infinite medium is an optically transparent nonmagnetic dielectric with a positive Kerr nonlinearity and constraint equations of the form [65, 66]

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + a|\mathbf{E}|^2 \mathbf{E}\,, \quad \mathbf{B} = \mathbf{H}\,, \quad a > 0\,. \tag{68}$$

In this case, under TIR conditions, the spatial structure of the electric field of the evanescent TE wave with $\mathbf{k} \in yz$ propagating with $\mathbf{q} \parallel z$ along the surface of optically lower-density medium (68) may be, according to Refs [65, 66], represented in the form

$$\begin{gathered} E_x = \sqrt{\frac{2}{a}} \frac{\eta_s}{\cosh\left[\eta_s (z - z_0)\right]} \exp(\mathrm{i}hy - \mathrm{i}\omega t)\,, \\ \eta_s^2 = h^2 - k_0^2 \varepsilon_0\,. \end{gathered} \tag{69}$$

When standard Maxwellian boundary conditions apply to the interface between the nonmagnetic dielectrics under consideration, on the strength of formulas (69), the surface wave conductance is of the form $Z_s = \eta_s \tanh(\eta_s z_0)$ in this case. Therefore, an outside plane bulk TE EM wave incident on the surface of such a nonmagnetic dielectric will reflect in the same way as from the surface of a perfect magnetic conductor when $z_0 = 0$. To do this, the following condition must be met:

$$h^2 = k_0^2 (\varepsilon_0 + 2a\tilde{A}^2)\,. \tag{70}$$

In other words, for given values of the frequency, incidence angle, and nonlinearity parameter, the formation of a TE ESW is possible only for a certain amplitude $\tilde{A}$ of a plane bulk TE wave incident from the outside. Therefore, this ESW is a nonlinear wave, and formula (70) defines the angle at which the plane bulk TE wave should be incident on the surface of the semi-infinite nonlinear dielectric from the optically isotropic linear dielectric and which corresponds, according to Ref. [65], to the limiting angle of nonlinear TIR. As also suggested by calculations, not only is the cycle-averaged energy flux across the medium interface equal to zero when formula (70) holds (i.e., when $H_y(z = 0) = 0$) (as shown in Ref. [65]), but also the instantaneous energy flux across the surface of the semi-infinite nonlinear dielectric is zero at any point in time.

It is noteworthy that there is another way to fulfill the relation $\mathbf{Hb} = 0$ ($T_s = 2$) under TIR conditions in the same optical configuration. This version, as suggested by calculations, corresponds to the case when an outside plane bulk TE wave with an amplitude $\tilde{A}$, which is incident on the surface of a semi-infinite nonlinear dielectric under consideration, excites a bulk nonlinear TE wave with a constant amplitude $2\tilde{A}$ in the nonlinear dielectric (68). The spatial field structure and the dispersion law of this wave are, respectively, defined by the following relations:

$$E_x = 2\tilde{A} \exp(\mathrm{i}hy - \mathrm{i}\omega t)\,, \quad h^2 = k_0^2 (\varepsilon_0 + 4a\tilde{A}^2)\,. \tag{71}$$

Unlike relations (69), (70), this version may be treated as the excitation of a nonlinear TE EBW in optically lower-density dielectric (68) by the outside plane bulk TE wave. Note that, according to Ref. [65], for given values of the frequency and amplitude $\tilde{A}$ of an incident plane bulk wave, the interval of incidence angles $k_0^2(\varepsilon_0 + 2a\tilde{A}^2) \leqslant h^2 \leqslant k_0^2(\varepsilon_0 + 4a\tilde{A}^2)$ corresponds to the hysteresis domain in the reflection of a plane light TE wave from the surface of an optically transparent nonlinear medium.

8. Conclusions

So, for a plane bulk TM(TE) wave incident on the surface of an optically transparent semi-infinite anisotropic dielectric under TIR conditions, such combinations of frequency and incidence angle are basically possible whereby inside the TIR domain the instantaneous energy flux across the interface is zero at any point in time (in an optically lower-density dielectric they correspond to a fast improper undamped exceptional surface wave of the TM or TE type, respectively). In this case, the imaginary part of the surface wave impedance (for the TM wave) or of the surface wave conductance (for the TE wave) turns to zero. As a consequence, the amplitude of an excited evanescent TM(TE) wave will be twice the amplitude of the plane bulk wave with the same values of the frequency and longitudinal wavenumber with the corresponding polarization incident on the interface from the optically denser medium. As a result, an outside plane bulk TM(TE) EM wave incident on the surface of a transparent dielectric, whose frequency and incidence angle, under TIR conditions, simultaneously satisfy the ESW spectrum (24)–(26), (28), (29) of the corresponding polarization, will be reflected in the same way as from a perfect electromagnetic metasurface of the corresponding type (the TM wave reflects as if from a perfect electric conductor and the TE wave as if from a perfect magnetic conductor), which corresponds to the EM analogue of the Rayleigh angle [9].

Consider the case when the finiteness of the angular spectrum of an outside TM(TE) wave is taken into account. According to calculations, even for a single interface between transparent media, under TIR conditions, a local lengthening of the longitudinal shift of the TM(TE) wave beam along the interface is possible due to the resonance excitation of a leaky ESW of the corresponding polarization. This nonlocal interaction of the outside incident EM wave with a single interface makes basically possible the focusing of the highly directional reflected beam and the formation of a caustic for the envelope of reflected waves (in perfect analogy with the well-known similar effect induced by a side wave). Accordingly, in the incidence of a wave pulse with a plane front, even in the case of normal incidence, in the reflection under TIR conditions a sharp increase in its delay time becomes possible when the incident wave frequency corresponds to the dispersion law of a fast ESW with a zero longitudinal wavenumber.

This signifies that a single interface between transparent, optically isotropic, and bianisotropic dielectric media may, under certain conditions, be treated as a special class of structures capable of maintaining the mode of a leaky surface EM wave (leaky wave structures). Traditionally, such systems in optics were systems with more than one interface of a platelet type or multilayer structures, which border on an optically denser semi-infinite medium.

Therefore, an ESW may be treated, considering the nonlocal nature of its interaction with a single media interface, an additional (with respect to those mentioned in Ref. [67]) type of diffraction wave.

When an outside plane monochromatic bulk TM(TE) wave is incident on the surface of a multilayer transparent reflective plane-layered structure, the condition for attaining the highest excitation intensity of an EW of the corresponding polarization now turns out to be related to the excitation in this layered structure of an interference exceptional surface wave of the corresponding polarization. However, for this fast improper EM wave inside the TIR domain, the instantaneous energy flux across the interface is zero at any point in time only on the outer surface of the corresponding layered structure (unless it is a one-dimensional photonic crystal). As a result, at this interface, the amplitude of an excited evanescent TM(TE) wave will be twice the amplitude of a plane bulk wave with the corresponding polarization incident from an optically denser medium on the surface of the layered structure. Simultaneously, at other (inner) interfaces of this reflective layered structure, the intensity enhancement of an evanescent wave with the same polarization (TM or TE) is also possible, and it may far exceed the four-fold one.

Note that this result correlates closely with the earlier investigations of why, even in the dissipationless limit, both in the Otto and in the Kretschmann configurations, the criterion for maximizing the excitation intensity of a surface TM(TE) wave is not the fulfillment of transverse resonance condition [2] at the interface between semi-infinite surface-active and surface-inactive media (see, for instance, Refs [68–72]).

Here, a note is in order. The definition of an ESW includes the condition that the instantaneous energy flux across the plane interface with an optically denser medium (prism), from which a plane bulk wave of the same polarization is incident under TIR conditions, be strictly zero, making the existence of this ESW highly hypothetical. However, the frequencies, the longitudinal wavenumber, and the polarization determined from the dispersion equation of this ESW correspond to the maximum amplitude enhancement of the corresponding EW excited in this case in the optically lower-density layered reflective structure adjacent to the prism. This permits pointing out a certain analogy between the notion of an ESW considered above and the notion of a praphase [73], which was proposed in Refs [74, 75]. A praphase is a hypothetical high-temperature phase which establishes a symmetry relation to lower-symmetry phases symmetry-unrelated to each other, which are realized in a real substance due to a phase transition. In this case, the phase itself is not observable in reality. The properties of such a praphase are determined analytically by analyzing the properties of low-symmetry phases realized below the point of this phase transition [73].

All the aforesaid effects have analogues not only in the acoustics of reflective multilayer structures (for instance, those similar studied in Refs [9–11, 76]), but also in the spin-wave dynamics and electrodynamics [77, 78] of layered media.

This study was carried out in the framework of a state assignment.

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