

Ultrapерipheral nuclear interactions

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Abstract. Long-distance ultraperipheral collisions of two relativistic ions are considered. Clouds of photons surrounding the ions are responsible for their distant electromagnetic interaction. The perturbative approach and the method of equivalent photons are described. It is shown that the total cross section of these collisions rapidly increases with an energy increase and is especially large for heavy ions. Some experimental data and their comparison with theoretical approaches are described. Further proposals are discussed.

Keywords: proton, nucleus, ultraperipheral interaction, cross section, form factor, impact parameter

1. Introduction

Ultrapерipheral nuclear collisions are distinguished from others by the nature of the interacting fields. They occur when the ions do not come close enough to interact strongly. Then the electromagnetic fields surrounding the ions enter the game. We here concentrate on pure ultraperipheral collisions, where the photons from the two electromagnetic clouds surrounding both ions collide.¹ Interest in them is related to the fact that electromagnetic fields become extremely compressed in the longitudinal direction and very strong at high velocities of the ions. The cross section of these processes

increases at high energies even faster than the strong interaction cross section. This opens the way to studying strong electromagnetic fields and their possible nonlinear effects.

The peripherality of these interactions is characterized by the transverse distance between the trajectories of the centers of two colliding ions, called the impact parameter b . For ultraperipheral collisions, it must be larger than the sum of the radii of the ions, $b > R_1 + R_2$. Otherwise, the ions interact strongly. The total cross sections of strong hadronic interactions at present energies are very large. Ion collisions with small impact parameters are studied, e.g., to search for some effects due to the production of quark–gluon plasma. The mean multiplicities of particles created by strong interactions are very high. Therefore, particles produced by ultraperipheral processes would be lost in the huge background from strong interactions. The special selection criteria dictated by the kinematics of ultraperipheral processes must be imposed to separate them. At the impact parameters slightly exceeding the sum of the radii, the exchanged photon can excite one of the ions by interacting directly with quarks and producing some bosons. The strong interactions are then partly involved, and the theoretical treatment becomes more complicated. That is why we do not consider such processes, known as photoproduction (or photonuclear) reactions.

The present review is rather brief. It is aimed at those who have just started to acquaint themselves with this problem. Its main purpose is to be a guide to papers where the discussed problems are expounded in greater detail. Therefore, we concentrate on some particular aspects of ultraperipheral collisions related to studies in colliders. We deal mostly with work done during the last decade, with some references to previous stages. To shorten the review, no figures or graphs abundantly shown in many papers are demonstrated, but multiple references to them are given with a short summary of the conclusions obtained.

We start with a brief reminder on the early history of the problem of ultraperipheral nuclear collisions. The perturba-

¹ Sometimes, they are called two-photon processes. More photons can be involved in the interaction (radiative corrections to the two-photon diagrams).

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tive approach to its solution is described. The asymptotic energy behavior of the cross section of electron–positron pair production in ultraperipheral collisions is demonstrated. Higher-order corrections are discussed in connection with the preasymptotic energy dependence of the cross section. The method of equivalent photons is formulated and applied to the calculation of the cross sections. Special features of the bound–free processes, where the produced electron becomes bound to one of the ions, are considered. Nuclear form factors and the suppression mechanism are discussed. Some experimental data are compared with theoretical predictions; the comparison motivates further analysis of the main assumptions used in the theoretical approaches. The search for new physics in ultraperipheral processes is described.

2. Early history

Almost a century ago, in 1924, Fermi [1, 2] considered the problem of interaction of charged objects with matter: “Let’s calculate, first of all, the spectral distributions corresponding to those of the electric field created by a particle with electric charge, e , passing with velocity, v , at a minimum distance, b , from a point, P .” He obtained a formula for the electromagnetic field strength created in this process. It was used in 1934 by Weizsäcker [3] and Williams [4] for their formulation of the method of equivalent photons, as discussed below.

The same year, Landau and Lifshitz [5], impressed by the prediction of positrons in the Dirac-sea theory, used the Dirac equation and calculated the asymptotic behavior of the cross section of electron–positron pair production in the electromagnetic fields of colliding relativistic nuclei. It turned out to increase very rapidly at high energies E as $\ln^3 \gamma$, where $\gamma = E/M$ is the Lorentz factor of the colliding ions of mass M . That was a test for the newly born Dirac theory of the positron. It is remarkable that paper [5] was published almost immediately after the discovery of positrons in cosmic ray interactions in 1932 (published in 1933 [6]).

Three years later, Racah [7] obtained an expression for the cross section in the lowest order of the perturbation theory (Born approximation), which contained some preasymptotic terms increasing more slowly than $\ln^3 \gamma$.

These papers were the start for more detailed theoretical studies of such processes. Experimental research became extremely intensive after high-energy colliders came into operation.

A very careful and detailed review of theoretical predictions and some early experimental data was given in 1975 by the Novosibirsk group [8]. We also mention some later review papers [9–14].

3. Electron–positron pair production in ultraperipheral collisions according to the perturbation theory

As stated above, the process of electron–positron pair production in ultraperipheral interactions of ions was the first one described theoretically. In these collisions, the two colliding protons or nuclei interact electromagnetically but not hadronically. They effectively miss each other with no change to their states. They interact only by photon clouds, which create electron–positron pairs. No nuclear transitions occur at small transferred momenta. The large spatial extension of electromagnetic fields and their high strength at increased velocities lead to a strong energy increase (propor-

tional to $\ln^3 \gamma$) in the cross section of these processes. The high density of the photon clouds surrounding heavy ions leads to large coefficients in this expression, proportional to the squares of their electric charges $Z_1 e$ and $Z_2 e$. It is Z^4 times less for proton (the hydrogen atom nucleus) collisions. These fields act only for a short time, and the perturbation theory is applicable. The famous Racah formula [7] for the total cross section of ultraperipheral production of electron–positron pairs in collisions of fast nuclei derived in the Born approximation is

$$\sigma_{Z_1 Z_2 \rightarrow Z_1 Z_2 e^+ e^-} = \frac{28(Z_1 Z_2 \alpha^2)^2}{27\pi m_e^2} \times (l^3 - 6.36l^2 + 15.7l - 13.8), \quad (1)$$

where

$$l = \ln \frac{2p_1 p_2}{M_1 M_2} = \ln \frac{s_{nn}}{m^2} = \ln (4\gamma_c^2), \quad (2)$$

m_e is the electron mass, m is the nucleon mass, p_i are the 4-momenta of colliding ions with masses M_i (considered equal in the right-hand side), γ_c is their Lorentz factor in the center-of-mass system, and s_{nn} is the squared total energy per colliding nucleon pair. The formula contains the preasymptotic terms proportional to l^2 and l , increasing more slowly with energy. The small mass of the electron in the denominator favors large values of the cross section. The ultraperipheral production cross sections for heavy-lepton pairs ($\mu^+ \mu^-$ or $\tau^+ \tau^-$) can be obtained at relativistic energies from Racah formula (1), which does not take the form factors of the colliding objects into account, by inserting their masses in place of the electron mass.² The cross sections are proportional to the inverse squares of the lepton masses and therefore become much smaller than those for the electron pairs.

In terms of the energy per pair of colliding nucleons, $\sqrt{s_{nn}}$, Racah formula (1) can be rewritten as

$$\sigma_{Z_1 Z_2 \rightarrow Z_1 Z_2 e^+ e^-} = \frac{28(Z_1 Z_2 \alpha^2)^2}{27\pi m_e^2} \times \left(\ln^3 \frac{s_{nn}}{8.3m^2} + 2.2 \ln \frac{s_{nn}}{8.3m^2} + 0.4 \right). \quad (3)$$

This formula absorbs the strongest correction terms l^2 in Eqn (1) into the leading term due to the numerical factor in the argument of the logarithms. Therefore, this formula can be directly applied for studies of the preasymptotic behavior of the cross section. This is important in view of the newly constructed NICA and FAIR facilities with energies $\sqrt{s_{nn}}$ of about 10 GeV. Surely, the leading term dominates in the RHIC and LHC colliders with available energies of hundreds and thousands of GeV.

All terms in Eqn (3) are positive at $\sqrt{s_{nn}} > 3$ GeV, and the leading term dominates at $\sqrt{s_{nn}} > 6$ GeV. These energies are below those in NICA and FAIR. Thus, the Racah formula predicts a quite noticeable effect already at energies of about 10 GeV. In particular, the values of the cross section for PbPb collisions are 1.4 kb at $\sqrt{s_{nn}} = 10$ GeV, 22.8 kb at 100 GeV, and 97.5 kb at 1 TeV. The small mass of electrons gives rise to such large values of the cross sections.

² The form factors are usually accounted for in the framework of the equivalent-photon approximation.

This formula was also confirmed by considering Feynman diagrams with two photons emitted by colliding ions and producing an electron–positron pair. That is why ultraperipheral collisions are often called two-photon processes. Correspondingly, the higher-order corrections due to the additional photons emitted by ions were evaluated. The graphs are reproduced in many publications on this subject. The estimated part of the Coulomb correction to the Racah formula proportional to l^2 is negative [15–18],

$$\sigma_C = -\frac{56}{9\pi} \frac{Z^4 \alpha^4}{m_e^2} f(Z) l^2, \quad (4)$$

where

$$f(Z) = (Z\alpha)^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2 + (Z\alpha)^2)}. \quad (5)$$

It is negligibly small for protons but becomes substantial for heavy ions. Taking it into account for Pb ions leads to the replacement of the factor 8.3 in Eqn (3) by approximately 16. The preasymptotic behavior of the cross section changes. The above estimates at different energies should be corrected accordingly. At 10 GeV, the cross section becomes less than one half. The estimates are less reliable because the l^3 term no longer dominates. Even at the LHC energies, the cross section becomes smaller by about 13%. Energies of NICA and FAIR are close to the threshold.

The unitarity corrections accounting for light-by-light scattering loops in Feynman diagrams are small for the ultraperipheral graphs of electron–positron production. In contrast, the Coulomb correction becomes much less for muon pair production, while the role of unitarity corrections increases [19]. At the same time, these conclusions and quantitative estimates of the cross section values according to Eqns (3) and (4) can change when taking the nuclear form factors into account [20]. These problems are crucial in connection with the so-called ultraperipherality parameter introduced below. It is related to the numerical factors discussed above.

The multiple pair production was also estimated by different theoretical methods (see, e.g., [12, 15, 16]). At small impact parameters, multiple pair production happens to be even more active than the creation of a single pair. However, the total cross section is not very sensitive to small impact parameters. The main contribution comes from large impact parameters; therefore, the single pairs dominate.

3.1 Differential distributions

The precise differential distributions of the electron–positron pairs produced in two-photon collisions are rather complicated. They contain 20 independent helicity amplitudes [21]. In the perturbative approach, matrix elements squared become strongly intermixed in differential distributions. Some simplified expressions are written in review [8]. As an example, we show the leading term of the distribution of the mass $^* W$ of the produced e^+e^- system (Eqn (5.27) in [8]):

$$\frac{d\sigma}{dW^2} = \frac{2(Z_1 Z_2 \alpha)^2 \sigma_{\gamma\gamma \rightarrow e^+e^-}(W^2)}{3\pi^2 W^2} \ln^3 \frac{p_1 p_2}{M_1 M_2}. \quad (6)$$

* The pairs produced in peripheral process turn out to have especially low masses. This can explain the abundance of soft electrons and positrons registered both in collider experiments and in astrophysical observations [48]. (Author's note to the proofs.)

Detailed studies of the characteristics of dilepton pair production are still at the very initial stage. Their analysis shows that the leading contribution to the total cross section at relativistic energies is provided by the region of production of electrons at small angles, small transverse momenta, and small pseudorapidities of the pair.³ Therefore, the photons with small squared 4-momenta (virtualities) are most important. They can be considered to be almost real (massless). Then the differential cross section can be approximated by a product of the total cross section of the $\gamma\gamma$ transition into an electron–positron pair and the differential fluxes of photons, which appeared already in Fermi's papers [1, 2]. From here, the equivalent photon approximation (see Section 4) follows [3, 4]. The maximum photon energy $\omega_{\max} = \sqrt{s_{nn}}/(mb)$ increases at higher collision energies and becomes smaller at large impact parameters. Formulas (1) and (6) are valid for a point-like source of electromagnetic radiation. The charge distribution inside the colliding protons and heavy nuclei must be taken into account. The strength of the photon fields depends on the transverse distance between the centers of the colliding nuclei (impact parameter b). Therefore, their radii R_i start playing a major role due to the requirement $b > R_1 + R_2$. These problems are considered within the equivalent-photon approximation.

3.2 Bound-free processes

Before delving into these problems, we mention the so-called bound-free effect induced by the production of electron–positron pairs. This name is used when the produced electron is captured by one of the ions while the positron flies away. This is an important source of beam ion loss. The charge-to-mass ratio Z/A changes, and new ions do not follow the former trajectory. This loss puts some limits on luminosity. Such ions can damage the accelerator magnets, chamber walls, and even external safety walls at distances of hundreds of meters. They deposit their energy in a localized region of the chamber walls and heat them, which is of practical importance for the operation of accelerators in heavy-ion modes. The capture cross section is higher for heavier ions. However, the cross section of these processes increases with energy only logarithmically [9, 23], i.e., more slowly than the main process cross section, which increases as the cube of the logarithm. The total cross section of the ultraperipheral collisions of lead nuclei can be as large as 200 kb in the LHC, while the capture cross section is about 200 b. Pair production with capture will become comparable with the production of free pairs at the lower energies of NICA and FAIR**.

4. Equivalent-photon approximation

The essence of the equivalent-photon approximation is already demonstrated by Eqn (6). The Feynman diagrams of all processes with two-photon interactions contain a box describing the transformation of these photons into some final states (e.g., e^+e^- considered above). Thus, the box can be seen as the cross sections of these processes. The missing element of the whole picture is the photon fluxes between the

³ The problem of the widening of these distributions compared to their expressions in the perturbative approach is considered in the recent paper [22].

** At much lower energies of the NICA collider near 10 GeV, the cross section of electron capture by an Au ion was estimated in recent paper [86] to be in the range 10–70 b. (Author's note to the proofs.)

colliding charged objects, which were the main purpose of Fermi's research, as clearly stated in the quotation at the beginning of this review. The photons carrying small fractions x of the nucleon energy dominate in these fluxes. The distribution of equivalent photons generated by a moving (point-like!) nucleus with the charge Ze and carrying a small fraction x of the nucleon energy integrated over the transverse momentum up to some value (see, e.g., [24]) leads, according to the method of equivalent photons, to the flux

$$\frac{dn}{dx} = \frac{2Z^2\alpha}{\pi x} \ln \frac{u(Z)}{x}. \quad (7)$$

The ultraperipherality parameter $u(Z)$ depends on the nature of colliding objects and created states. Its physical meaning is the ratio of the maximum adoptable transverse momentum to the nucleon mass, which gives the only massless parameter in the problem. In the perturbative approach, it should incorporate the radiative corrections that change the pre-asymptotic dependence of the cross section (see the comments after Eqn (4)). This parameter differs numerically in various approaches [8, 25–34]. It depends on considered processes as well as on the charges $Z_i e$, the sizes, and the impact parameters of colliding objects (form factors and absorptive factors).

The impact parameters cannot be measured, but should certainly exceed the sum of the radii of colliding ions. Otherwise, the strong (QCD) and photonuclear interactions enter the picture. This requirement can be restated as a bound on the exchanged transverse momenta, such that the objects are not destroyed but slightly deflected by the collision and no excitations or nuclear transitions occur. The bound depends on their internal structure, i.e., on forces inside them. These forces are stronger for a proton than for heavy nuclei. Therefore, protons allow larger transverse momenta. Quantitative estimates of the parameter for different processes are obtained from comparison with experimental data and confronted with theoretical approaches described in more detail in the next section.

The equivalent-photon approximation allows a clear separation into a purely kinematical effect of photon fluxes and the dynamical cross sections of their interactions. Besides the electron–positron pairs considered theoretically in Refs [5, 7] and observed, e.g., in Refs [35, 36], other pairs of oppositely charged particles with even C -parity can be created in two-photon collisions. For example, pairs of muons produced in ultraperipheral collisions are observed at the LHC [37–41]. Light-by-light scattering described theoretically by a loop of charged particles is also detected at the LHC [42–44]. Some neutral C -even bosons composed of quark–antiquark pairs can be produced in two-photon interactions. This process is especially suitable for a brief theoretical demonstration [33] of the $\ln^3 \gamma$ law.

The exclusive cross section of the production of a resonance R in two-photon collisions of nuclei A can be written as

$$\sigma_{AA}(R) = \int dx_1 dx_2 \frac{dn}{dx_1} \frac{dn}{dx_2} \sigma_{\gamma\gamma}(R), \quad (8)$$

where the fluxes dn/dx_i for the colliding objects 1 and 2 are given by Equation (7) and (see Ref. [8])

$$\sigma_{\gamma\gamma}(R) = \frac{8\pi^2 \Gamma_{\text{tot}}(R)}{m_R} \text{Br}(R \rightarrow \gamma\gamma) \text{Br}_d(R) \delta(x_1 x_2 s_{mm} - m_R^2). \quad (9)$$

Here, m_R is the mass of R , $\Gamma_{\text{tot}}(R)$ is its total width, $\text{Br}_d(R)$ denotes the branching ratio to a considered channel of its decay, $s_{mm} = (2m\gamma)^2$, and m is the nucleon mass. The δ -function approximation is used for resonances with small widths compared to their masses. The resonance is registered according to the peak in the distribution of the effective mass of the decay products $\sqrt{x_1 x_2 s_{mm}}$. As can be seen, the perturbative matrix element approach is replaced in the equivalent-photon approximation with a semiclassical probabilistic scheme accounting for the structure of Feynman diagrams.

The integrals in Eqn (8) can be easily calculated, yielding the analytic formula

$$\sigma_{AA}(R) = \frac{128}{3} Z^4 \alpha^2 \text{Br}(R \rightarrow \gamma\gamma) \text{Br}_d(R) \frac{\Gamma_{\text{tot}}(R)}{m_R^3} \ln^3 \frac{2um\gamma}{m_R}. \quad (10)$$

The asymptotic $\ln^3 \gamma$ behavior is valid again. The factor $2mu/m_R = 1/\gamma_0$ defines the preasymptotic behavior of the ultraperipheral cross section of production of the resonance R . The structure of this formula is similar to that used for e^+e^- production (3). Variations in the parameter u can account for the subleading terms proportional to $\ln^2 \gamma$. The asymptotic limit is reached at

$$\gamma \gg \frac{m_R}{2um}, \quad (11)$$

where the terms increasing more slowly than $\ln^3 \gamma$ can be ignored.

The parameter u can be found from Eqn (10) if the exclusive cross sections of the ultraperipheral production of π^0 mesons or a parapositronium are measured. The analogous formulas are obtained in [45, 46] for the creation of C -odd states like ρ^0 mesons or the orthopositronium.

5. Preasymptotic behavior of cross sections

The fast asymptotic increase in the total cross section of ultraperipheral collisions as $\ln^3 \gamma$ raises a question about its comparison with the total cross section of purely hadronic interactions, which cannot increase faster than $\ln^2 \gamma$ according to the Froissart theorem [47], following from general theoretical principles. According to experimental data on proton–proton collisions, their increase is even slower at present energies.

The cross section for single neutral pion production of two protons in ultraperipheral collisions, according to (10), is compared in Ref. [48] with experimental data on the corresponding production channel in strong interactions at TeV energies. This cross section is about 0.6 nb, while single π^0 are produced in strong interactions with cross sections of the order of 0.3 mb.

It has been shown that the additional $\ln \gamma$ factor is not large enough and absolutely insufficient for such ultraperipheral processes to dominate in proton–proton collisions over strong forces at any realistic energies. The background for π^0 production due to strong interactions, given small impact parameters, must be enormously large. Some special cutoffs should be imposed to separate the ultraperipheral events. The specific kinematics of ultraperipheral processes can be used for such cutoffs, as shown in Section 7.

Similar estimates for collisions of heavy nuclei are more complicated due to the lack of information about the definite reaction channels. One can just state that the large numerical

factor $Z^4/A^{2/3} \approx 10^6$ in the ratio of the ultraperipheral to purely nuclear (strong) interactions would favor heavy nuclei over protons.

Another preasymptotic problem is related to the energy behavior of ultraperipheral cross sections in the lower-energy region. The Racah formula written as (3) clearly demonstrates the $\ln^3 \gamma$ asymptotic behavior and shows that the numerical factor 8.3 determines the preasymptotic behavior of the ultraperipheral cross section. Moreover, it is modified by radiative corrections. This numerical factor transforms in the equivalent-photon approximation into the ultraperipherality parameter $u(Z)$, which has the meaning of the ratio of the maximum adoptable transferred momentum to the nucleon mass. This parameter incorporates the form factors and the impact parameter suppression. It influences the estimates of the cross sections, especially at lower energies. This becomes important, for example, at energies of NICA and FAIR. The electron–positron pair (as well as para- and ortho-positronium) production seems to be feasible there [48] if the optimistic results from studies at the LHC [34, 50, 51] are taken into account and extrapolated. The π^0 -production processes turn out to be close to those of NICA and are very sensitive to estimates of the parameter u shown in Section 7.

6. Theoretical analysis of exclusive dilepton production

The perturbative matrix element approach in Eqns (1), (4) clearly demonstrates that higher-order corrections can change the preasymptotic values of the calculated cross section by changing the numerical factor in the argument of the \ln^3 term. Its predictive power also suffers from considering colliding objects in Feynman diagrams as structureless point-like particles. This deficiency can be partially overcome within the equivalent-photon approximation (8) by the insertion and the interpretation of the ultraperipherality parameter $u(Z)$.

To take the size and the electric charge distribution inside the colliding hadrons and ions into account, we must somewhat generalize Eqn (8).

The photon flux density depends on the structure of colliding objects and on their impact parameter.⁴ In terms of the form factors F of colliding objects, it has the form [10, 34, 53]

$$\frac{d^3n}{d^2b dx} = \frac{Z\alpha}{\pi^2 x} \left[\int dq_\perp q_\perp^2 J_1(bq_\perp) \frac{F(q_\perp^2 + m^2 x^2)}{q_\perp^2 + m^2 x^2} \right]^2. \quad (12)$$

The electromagnetic fields of relativistic ions look like narrow pancakes perpendicular to their trajectories and moving together with ions. Their interaction can be approximated by the δ -function in this plane at small $x < 0.1/(mb)$ [27]. It is strongest when the ions come close to one another. However, the impact parameter between them should be larger than the sum of their radii. Otherwise, they are involved in strong interactions, and a huge background to the products of ultraperipheral interactions appears. To exclude such collisions, the cutoff factor $P(|\mathbf{b}_1 - \mathbf{b}_2|)$ is introduced. It is determined by soft physics with low transferred momenta. Therefore, it is nonperturbative and phenomenological. If the colliding ions are considered to be black disks, this factor

completely forbids impact parameters smaller than the sum of ion radii and is written as

$$P = \theta(|\mathbf{b}_1 - \mathbf{b}_2| - R_1 - R_2), \quad (13)$$

where R_i are their radii.

Equation (8) can be generalized⁵ as

$$\sigma_{AA \rightarrow AAX} = \int dx_1 dx_2 d^2b_1 d^2b_2 \frac{d^3n}{d^2b_1 dx_1} \times \frac{d^3n}{d^2b_2 dx_2} \sigma_{\gamma\gamma \rightarrow X} P(|\mathbf{b}_1 - \mathbf{b}_2|). \quad (14)$$

The choice of the cutoff factor determines the ultraperipherality parameter $u(Z)$. If the heavy ions stay intact after the collision, then Eqn (13) should be used. At smaller impact parameters, they do not survive. The hope that the form factors in Eqn (12) satisfy this requirement automatically is hardly realistic. The photon flux computed in Ref. [50] for PbPb interactions at the energy of 5.02 GeV per nucleon pair (see Fig. 3a in Ref. [50]) becomes much smaller if the additional cutoff according to (13) is imposed on it even for a realistic form factor. For processes with initial protons, the elastic scattering with small impact parameters can be taken into account. The spatial distribution of their inelastic profile depends on the collision energy [54]. The cutoff factor can be generalized either by accounting for the proton opacity, as proposed in [55], or by the Glauber modification of (13), as proposed in [50]. The suppression factor S^2 has been used for the quantitative estimate of the cutoff factor effect on the cross section of ultraperipheral processes:

$$S^2 = \frac{\int \int d^2b_1 d^2b_2 (d^3n/d^2b_1 dx_1)(d^3n/d^2b_2 dx_2) P(|\mathbf{b}_1 - \mathbf{b}_2|)}{\int_{b_1>0} \int_{b_2>0} d^2b_1 d^2b_2 (d^3n/d^2b_1 dx_1)(d^3n/d^2b_2 dx_2)}. \quad (15)$$

Its evaluation depends on the choice of the lower limits of the impact parameters in the numerator (e.g., compare Eqn (7) in [53] and Appendix 3 in [34]). This is the main root of disagreement among the different choices of the ultraperipherality parameter mentioned above [8, 25–34].

We note, however, that the formulas for the total cross sections considered above are useful for understanding the energy behavior of ultraperipheral processes and some general estimates, but they are not very practical in direct applications to experimental results. The experimentally measured phase space volume is usually much smaller than the total one. The detector structure and possible backgrounds reduce it. The so-called fiducial cross sections taking these ‘caveats’ and the ‘ultraperipherality’ requirement into account are measured. Monte Carlo generators, e.g., STARlight [56] or SuperChic [57], are often used both for the selection of the most favorable and admissible conditions and for further comparison with experimental results. The corresponding cutoffs are imposed on the matrix elements or on the formulas derived according to the equivalent-photon approximation, for the computation of the fiducial cross sections, i.e., those that account for experimental cutoffs. The advantage of the equivalent-photon approximation over perturbative calculations or the diagram approach is that similar calculations can be done analytically (up to comput-

⁴ The flux in Eqn (7) is integrated over the transverse momentum and the impact parameter.

⁵ The symbol R must be replaced with $X = l^+l^-$ for dileptons. The cross section $\sigma_{\gamma\gamma \rightarrow l^+l^-}$ is given by the Breit–Wheeler formula [52].

ing some simple integrals). Thus, the possibility arises of comparing different approaches and parameterizations with experimental data and controlling the accuracy of the equivalent-photon approximation. These experimental cutoffs are taken into account in the papers considered below [34, 50, 51].

A more general problem is related to the spatio-temporal inhomogeneities of the considered electromagnetic fields. They can play a prominent role in the production of secondary particles. In particular, it is shown in [58] that they can result in an increased number of soft photons, i.e., in higher photon fluxes. Nonlinear effects of strong-field QED are related to the string problem [59] and can become important in heavy-ion collisions.

7. Comparison with experimental data

Predicting large cross sections of ultraperipheral heavy-ion collisions stimulated their experimental studies at the RHIC [35] and LHC [36–41]. There are special signatures of these processes. The dilepton pairs in the final state have a very small transverse momentum. There are two rapidity gaps that separate the intact very forward ions from the dilepton pair. From the theoretical side, the main problem is to properly estimate the photon fluxes, i.e., evaluate the parameter $u(Z)$.

Production of e^+e^- pairs in heavy-ion collisions was first studied at RHIC by the STAR Collaboration [35] and then at the LHC by the ALICE Collaboration [36]. The obtained rapidity and invariant mass distributions for exclusive e^+e^- production by $\gamma\gamma$ interactions in PbPb collisions at $\sqrt{s} = 2.76$ TeV were compared in Refs [50, 51] with theoretical results using the above formulas. Besides the rigid absorption factor of the black disks (13), its Glauber model-type modification was considered. As regards the form factors, three of them were used: point-like, monopole, and realistic, which corresponds to the Fourier transform of the Wood–Saxon charge density distribution of the nucleus. The general behavior of both distributions is well reproduced by theoretical results, except in the region of small invariant masses below 2.3 GeV (see Fig. 5 in [50]). The experimental cutoffs were imposed on the computed distributions. It was concluded that the modification of the absorption factor is unimportant. Both precise and monopole form factors fit experimental results at high masses rather well within error bars, while the point-like one deviates from them. The experimental distribution is higher than the theoretical one at small masses less than 2.3 GeV in [50] but deviates slightly only at that single point in [51] (see Fig. 3 there). The small masses become most important at the lower energies of NICA and FAIR. This region must be carefully studied.

The production of $\mu^+\mu^-$ pairs in pp collisions was first observed in 1990 in CERN’s Intersecting Storage Rings (ISR) [60]. However, detailed experimental studies [37, 39–41] and a comparison with theory [34, 50, 51] became possible only recently.

The ATLAS data presented in Ref. [37] matched the theoretical results in Refs [50, 51] in the same manner as done above for e^+e^- data. Rapidity and invariant mass distributions for the exclusive $\mu^+\mu^-$ production by $\gamma\gamma$ interactions in PbPb collisions at $\sqrt{s} = 5.02$ TeV have been plotted (see Fig. 4 in [50] and Fig. 4 in [51]). The agreement with theory is less satisfactory within the precision of experimental data than for electron–positron pairs, especially in the case of point-like form factors, as expected.

A very detailed comparison with experiment is done in [34]. It helped to glean more definite information on the parameter $u(Z)$. The experimental results from pp collisions at 13 TeV [39] and PbPb collisions at 5.02 TeV per nucleon pair [40] were considered.

The total cross section of the ultraperipheral production of muon pairs in the equivalent-photon approximation is

$$\sigma(ZZ(\gamma\gamma) \rightarrow ZZ(\mu^+\mu^-)) = \frac{28}{27} \frac{Z^4 \alpha^4}{\pi m_\mu^2} \ln^3 \frac{u^2 s_{\text{m}}}{4m_\mu^2}. \quad (16)$$

We note that the energy dependence in Eqns (10) and (16) is the same for $m_R = 2m_\mu$, as expected. The preasymptotic behavior is determined by the factor u .

The cutoffs on the invariant mass of the $\mu^+\mu^-$ pair (on the fraction x in Eqn (7)), on the muon transverse momentum (on the differential p_T distribution of $\gamma\gamma$ processes), and on the pseudorapidity (required by the detector geometry) were imposed in Eqn (14) both for pp collisions at 13 TeV ($Z=1$) and for PbPb collisions at 5.02 TeV per nucleon pair. The corresponding integrals are easily computed. These cutoffs drastically reduce the cross section values.

For example, for pp processes, the value 0.22 μb according to Eqn (16) is reduced to 3.35 pb. If corrected for absorptive effects [53], it becomes 3.06 ± 0.05 pb. The chosen cutoffs coincide with those imposed in studies of the ATLAS collaboration [39], which measured the value 3.12 ± 0.07 (stat.) ± 0.10 (syst.) pb. The SuperChic Monte Carlo program [57], which, in principle, incorporates both ordinary and ultraperipheral processes, predicts 3.45 ± 0.06 pb. Theoretical results are in agreement with experimental data and show that ultraperipheral processes dominate over other sources in this fiducial volume. Analogous conclusions were obtained for PbPb collisions [34]. Here, due to the Z^4 enhancement, the measured fiducial cross sections are on the μb scale compared to pb’s for pp-collisions.

The ultraperipherality parameter $u(Z)$ is the least precisely determined element of the equivalent photon approximation. As described above, its evaluation crucially depends on two main factors accounting for the impact parameter suppression $P(b)$ and for the charge distribution inside ions (form factors $F(q)$). The careful treatment of form factors of protons and nuclei taking the photon virtuality (see also Refs [25, 27], where the problem was treated in more detail) and the suppression factors [34] into account lead to the values $u_{\text{pp}} \approx 0.2$ for pp and $u_{\text{PbPb}} \approx 0.02$ for PbPb collisions within factors of about 1.5, which depend on the particular shape of the form factors. At the very beginning, these values of $u(Z)$ were qualitatively estimated from general physics arguments and then confirmed by a successful comparison of theoretical predictions with experimental data.

It is remarkable that these values of the ultraperipherality parameter u agree quite well with those obtained in Ref. [46] for the ultraperipheral cross sections of π^0 production according to Eqn (10) at the RHIC and LHC energies. Their values (28 mb at the LHC) shown in the Table in Ref. [46] easily lead to $u_{\text{PbPb}} \approx 0.013$. This value agrees up to the factor of 1.5 with that shown above.

Knowledge of the ultraperipherality parameters helps to estimate possible effects at the lower energies of NICA and FAIR. They favor the ultraperipheral processes of e^+e^- and positronia production [48] there, while π^0 production is questionable because the argument of the logarithm in Eqn (10) is close to 1 and therefore nonleading terms similar

to those in Eqn (3) must be taken into account (available near the threshold in the estimates in [34] and unavailable in the estimates in [46]).⁶

8. Search for new physics

The clean channel of $\gamma\gamma$ interactions in ultraperipheral collisions is often discussed in connection with the search for new physics. Before the discovery of the Higgs boson, it was actively debated as one of the possible sources for its production (see, e.g., [61, 62]). Nowadays, the main topics include the search for supersymmetric particles [61, 63–70], magnetic monopoles [71–73], gravitons and possible extra spatial dimensions of the Kaluza–Klein theory with large compact dimensions, in addition to the 4 dimensions of Minkowski spacetime [62, 74], axion-like pseudoscalar particles [75–81], which would induce anomalous light-by-light scattering, radions [82], unparticles [83], and the impact of supersymmetry on the properties of the observed particles (e.g., virtual sparticles in Feynman diagrams for the anomalous magnetic moment of the tau lepton [84]). There are no experimental signatures of these effects yet. Some lower limits, e.g., on the masses of supersymmetric particles [85] or on the masses of spin 0, 1/2, or 1 Dirac monopoles [72], have been established.

A very detailed proposal to search for supersymmetric particles in ultraperipheral proton–proton collisions at the LHC was presented recently in Ref. [70]. It is proposed to search for charginos⁷ with masses of about 100 GeV. Even though the LHC results exclude production with masses below 1 TeV within a large interval of theoretical parameters there according to [85], this conclusion is not valid when the masses of the lightest chargino and lightest neutralino are approximately equal. Just this rather exotic possibility with the lightest chargino somewhat heavier than the lightest neutralino is studied. The leading-order Feynman diagrams for chargino pair production in ultraperipheral collisions look similar to those for the production of electron–positron pairs considered above. Estimates of the total cross sections give the values of 2.84 fb for pp collisions at 13 TeV and 21.2 pb for PbPb collisions at 5.02 TeV per nucleon. If both protons are required to be registered in forward detectors⁸ and experimental cutoffs similar to the cutoffs described above for muon pairs data are imposed, the fiducial cross section in proton–proton processes is reduced to 0.72 fb. The background due to muon pairs and pile-up is estimated. It is shown that the peaks from charginos and muons are well separated and the chargino peak is clearly visible over the pile-up if a special cutoff on the longitudinal momentum of the final state system is used (see Fig. 6 in Ref. [70]).

9. Conclusions

Ultraperipheral processes provide very important information about strong *electromagnetic* fields. Experimental data on dilepton production are successfully described by the equivalent-photon approximation. The interaction of high-

energy photons in the dense electromagnetic clouds surrounding relativistic protons and heavy ions opens ways to study new physics in the processes of production of new objects. The theoretical methods of their description are well developed and prove their applicability when compared with experimental data.

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⁶ The earlier estimates [33] based on extremely strong cutoffs of the impact parameters imposed in [32] produced a much higher threshold for such processes.

⁷ Two charginos and four neutralinos are the supersymmetric partners of electroweak bosons.

⁸ It allows strong background suppression and complete reconstruction of the event kinematics.

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