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# Mechanisms and kinetics of gravity separation of granular materials

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## Contents



Abstract. **Particle separation mechanisms and kinetics in granular materials in a gravity chute and under vibrational impact are analyzed. Special attention is given to separation and kinetics mechanisms operating under conditions of high heterogeneity of structural and kinematic characteristics of granular flows. An analysis of alternative expressions for the separation driving force arising from local and spatial heterogeneity of granular media is based on results of experimental and analytical studies. Predictive properties of mathematical models are discussed with reference to the dynamics of separation of granular media according to the size and density of their constituent particles due to shear flow heterogeneity.**

**Keywords:** granular medium, rapid shear gravity flow, granular temperature, separation, segregation, migration

## 1. Introduction

Granular matter (GM) [1] is the most representative object encountered in everyday life. These materials are of importance in various fields of professional practice, including the chemical and food industries, mining, and agriculture. Moreover, they are widely involved in geological processes, such as landslides and erosion, to say nothing of large-scale tectonic events determining major morphological features of Earth. Despite the apparent simplicity of granular materials as large assemblies of individual macroscopic particles, their properties give reason to consider them to be an additional form of matter. The peculiarities of granular materials are such that they can be regarded as solids, gases, or liquids with unusual characteristics, depending on the dynamic conditions of their existence [1]. In other words, GM properties are of a mesoscopic character.

The science of granular media has a more than three century history [1]; it owes its status and achievements to the efforts and seminal work of many great researchers, like Coulomb [2], who suggested the idea of static friction in contact interactions between particles, Faraday [3], who was the first to observe convection inhomogeneity in powder vibration, and Reynolds [4], who introduced the term dilatancy to describe the effect of GM volume enlargement (dilation) under the action of shear deformation. Knowledge of granular matter is relevant for many branches of industry, agriculture, exploitation of natural resources, and environmental management.

GM physics is currently considered one of key aspects of soft condensed matter research [5]. The main study subjects are materials composed of macroscopic particles in which atoms are organized into mesoscopic-scale structures bigger than individual atoms but much smaller than the total volume of the material. Such materials include solid particles, liquid droplets in emulsions, and gas bubbles in foams [1, 6, 7]. These particles are larger than their colloidal counterparts; therefore, they do not undergo thermal motion in the general case. Particle-to-particle interactions are largely determined by contact forces but can be just as well initiated by an interparticle fluid and electrostatic forces. There is an obvious relationship between this and other branches of physics, such as colloid physics, mechanics, and rheology. Moreover, an analogy is discernible [5] between the behavior of granular

[1] The term granular matter is commonly used in the English-language literature to define a certain kind of particulate solids consisting of large enough particles adhesive forces between which are weaker than inertial forces. An identical term widely used in the Russian-language literature is grained material. The authors preferred the former term as the internationally accepted definition even though it is often used to describe loose materials containing particles produced by such forming technology as granulation.

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materials and that of active living matter, such as large gatherings of people and their traffic. The difference among the behaviors of such different entities is due to social relations among complicated constituent elements of living matter.

GMs are characterized by the following typical features [5]: disorder and heterogeneity, combined with a tendency toward structuring, marked dissipative properties, a highly nonequilibrium state, and a well apparent nonlinearity of the response to external force action. Taken together, these properties largely account for the lack of definite knowledge of what particle assemblage can be regarded as adequate for the granular material in terms of major properties; nor is there adequate statistical mechanics to describe such materials [5].

Approaches to the investigation of crystalline materials, fluids, and the majority of other forms of soft condensed matter are inapplicable to the study of GM, whose unusual behavior has frequently nothing in common with that of other materials, especially as far as friction is concerned, the role of which is still poorly understood despite its importance for both the statics and dynamics of granular media. Apart from certain idealized situations, it is thus far unknown how energy dissipation and friction influence the reaction of a material to external impacts. In this connection, in many cases, it is difficult to understand the physical nature of the behavior of seemingly simple objects composed of particle aggregates, such as sand, and to control their state.

Analysis [1] demonstrates that the unique properties of GMs are largely due to their two inherent features, viz. dissipative effects of static friction and inelasticity of collisions, as well as the absence of an appreciable temperature influence. The latter property hampers GM analysis in terms of classical thermodynamics. A vivid illustration of the relevance of such a conclusion is provided by the effects of particle separation by size in rotating and vibrating containers. The accompanying concentration of uniform particles in a particular part of the container in the absence of attractive forces between them suggests a deviation of the system from the entropy principle. Because particles intermix in the container in accordance with this principle, the observed separation gives evidence of the predominance of dynamic effects in GM responsible for the system deviation from thermodynamic equilibrium.

The striking instances of the manifestation of nonordinary GM properties include [1] the absence of a linear dependence of bottom pressure in a high container on the height filled with the material of interest. What's more, the pressure in a sufficiently high container reaches an extremal value, regardless of the height.

The impossibility of using temperature as a rigorous thermodynamic parameter in an analysis of granular materials seriously hampers their investigation. Unlike gases and liquids in which temperature can be used to characterize the velocity of microcomponents of the medium, GM provides the only way to use the notion of granular temperature (GT) to evaluate the fluctuation velocity acquired by macroparticles in their relative motion under dynamic impact conditions. However, even in this special case, results of the implementation of this approach do not necessarily agree with the classical postulates of thermodynamics and hydrodynamics due to an inadequate reflection of the intricate dissipative nature of interparticle interaction effects [1].

Today, further progress in GM science is closely related to developments in various fields of soft condensed media physics. It accounts for much of the attention given to the analysis of GM properties in different states viewed as features of solid, liquid, and gaseous substances with unusual properties (see, for instance, [1]).

Fundamental differences between the properties of GM and those of conventional liquids and gases are attributed in Ref. [1] not only to the mesoscopic scale of GM particles but also to their inelastic interactions. The inelasticity of GM particles accounts for the loss of a certain amount of energy that is very difficult to identify with regard to the properties of the particles and conditions of their interaction. Determination of the dissipative constituent of particle-particle interaction is complicated by the necessity to take into account the influence of their relative velocities and collision angles on friction and restitution coefficients as well as the ratio of two constituent components (fluctuational and rotational) of the kinetic energy. Inelastic particle–particle interactions are associated with cluster formation characteristic of granular materials, i.e., the accumulation of particles showing a certain short-range order in their positioning. Cluster formation radically changes conditions for interparticle interaction characteristic of spatially homogeneous granular media.

The discussion of GM properties in Ref. [8] is confined to an analysis of a static problem related to the description of the three-dimensional state of an assembly of solid particles precipitating from the fluid flow. The authors emphasize that GM displays properties of both solids and liquids, but the interpretation of these states is open to question.

For many decades, the attention of researchers has been focused on separating and mixing nonuniform particles under conditions of vibrational impact on granular media and their shear flows [9–25]. Suffice it to mention the recurrent discussion of the mechanisms behind the floating up of a large particle regardless of its density in a layer of smaller particles under the action of vertical vibrational oscillations. Now, this phenomenon is commonly called the Brazil nut effect [14].

To verify and further develop theories providing a basis for GM research, investigators have recently shown special interest in the statistical properties of granular mixtures and their components in the framework of statistical physics. The growing number of experimental data made possible the theoretical interpretation of the Brazil nut effect using statistical analysis methods. The authors of Ref. [14] adapted the Monte Carlo method widely employed in statistical physics to show how the mechanism underlying the local formation of craze voids can lead to the separation of large particles with their concentration on the layer surface. The floating of a large particle is explained by the action of the void formation mechanism (arching, i.e., the formation of arches under large particles with a high probability of their filling with smaller ones). Because the probability of filling voids with small particles increases as their size decreases, the floating up speed of large particles increases too with their size.

However, most authors studying particle separation according to the size in vibrofluidized GM point out that it is strongly influenced by a variety of other factors: particle density, cohesive and dissipative properties, characteristics of the medium filling the free volume, etc. Specifically, there is a tendency toward a rise in the floating up speed of individual large particles with their density [15]. Such effect can be accounted for by the strengthening of particle inertial properties, which facilitates their percolation through the assembly of the surrounding particles. It can be concluded that the physical mechanism of the Brazil nut effect can not be explained only in terms of geometric analysis and models of spatial dislocation of nonuniform particles. At the same time, the Monte Carlo method finding wide application in statistical physics has been successfully used to explore granular media in which the particle separation dynamics are assumed to be independent of the dynamics of their collisions accompanied by energy dissipation [14, 16].

To generalize the results of a vast amount of research, a fundamental physical theory is needed that is still unavailable, even for the vibro-fluidization of homogeneous granular materials [26]. Luckily, we are witnessing certain progress in this field, as far as the use of a molecular kinetic theory is concerned, to describe pseudo-fluidized systems showing high-intensity relative particle motion (rapid shear flows). In this case, the particle-to-particle interaction occurs largely in a binary collision mode, and the use of a formal analogy between the granular medium and the gas — taking account of mechanical energy dissipation in particle collisions — permits predicting parameters of the pseudo-fluidized layer [27–29]. However, prognostication is based on the assumption that energy dissipation is relatively insignificant, particles are smooth, and their mutual positions are random both before and after collision [30]. The temperature of a granular medium is not infrequently determined with certain allowances, e.g., on the assumption of its homogeneity in the near-border layer. Serious problems are encountered in determining boundary conditions radically different from those for liquids [30, 31].

In this context (as correctly noted in Ref. [30]), further progress in research of particle separation by size depends, as before, on the use of an approach combining experimental methods, theoretical analysis, and mathematical simulation. Such an approach to the study of the behavior of GM particles under complex hydrodynamic conditions has been extensively used in a number of recent studies [22–24].

A variety of physical mechanisms have been proposed to explain separation and mixing in shear flows and vibrating layers of nonuniform particles [9–25]. They assume GM properties characterizing the material as an unusual liquid or a nonordinary gas. At any case, it was shown that, unlike conventional fluids, in which mixtures tend to be uniformly distributed after stirring or layer shifting, nonuniform GM exhibits a separating tendency contrary to the entropy principle. Certain separation (segregation) mechanisms will be considered and subjected to an analysis in what follows below.

## 2. Shear flows in granular materials

In general, the mesoscopic scale of GM elements does not allow their flows to be regarded as homogeneous or classical continual models to be used for describing their hydrodynamic properties. Stresses in shear flows of liquid media can be considered to be a result of averaging over a huge number of mutual interactions between molecules through the shear surface per unit time. In GM flows, such averaging is less justified due to the mesoscopic scale of interacting particles and interaction time [1].

For this reason, two idealized models are used to describe GM flows simulating slow and rapid shear flows. Idealization implies that a slow shear flow occurs as a result of irreversible quasi-static plastic deformations accompanied by shear rate-dependent dilatancy of the flow. In practice, the latter gives rise to various hydrodynamic effects, e.g., flow density waves [32], and seriously complicates the mathematical description of such flows.

GM particles in rapid shear flows move exchanging kicks. Rapid shear strains are accompanied by marked flow dilatancy, depending in a complex manner on particle properties and shear rate. Another factor markedly complicating the mathematical description of such flows is the tendency of the particles toward sporadic cluster formation, which radically changes the conditions of particle–particle interaction in the flow [1].

Rapid gravity-driven flows of granular media have been extensively investigated for many decades with special reference to their consistent patterns and interactions between nonuniform particles [33–41]. Particular attention to these aspects of GM behavior is due to the fact that many natural events and technological processes involving granular materials occur in the rapid shear flow regime and are accompanied by well apparent manifestations of particle interaction effects that exert a strong influence on the flow dynamics and kinetics of the related processes [42–45]. In rapid shear flows, GM acquires a chaotically distributed fluctuation component of the velocity, in addition to translational velocity in the direction of shear.

Under these conditions, shear stresses are largely generated as a result of impact momentum transfer through the shear surface during particle collisions [35]. Rather long-lasting contacts of particles interacting in the sliding and rolling friction regimes at a relatively low shear rate give way to short-term point contacts between colliding particles at a high shear rate. The specific character of particle-particle interaction dynamics in a rapid shear flow of granular media reveals its formal analogy with molecular gas dynamics. For this reason, a granular medium in the rapid shear flow regime is frequently referred to as gas of solid particles ('gas-solid suspension' or 'gas solid flow,' to borrow C E Brennen's terminology [38]), and the kinetic energy acquired by the particles in their chaotic vibrations is regarded as a parameter proportional to granular temperature.

Interactions of particles with each other and with the medium filling the interparticle space in a rapid shear flow of GM are accompanied by manifestations of nonuniform particle mixing and separation [19–25, 43–45]. An analogy between a rarefied granular medium and a dense gas under rapid shear conditions provides a basis for predicting a strong dependence of particle interaction kinetics not only on kinematic but also on microstructural characteristics of the medium. Indeed, the obvious relationship between the particle fluctuation velocity and the free path length, on the one hand, and the structural-kinematic characteristics of the rapid shear flow, on the other hand, suggests the necessity to take comprehensive account of these factors in an analysis of particle interaction dynamics under the relevant conditions.

However, a review of the overwhelming majority of publications reporting research on shear flow dynamics, e.g., [33–44], shows that an analysis of particle interaction effects disregards the structural nonuniformity of granular materials on the assumption of their incompressibility. It can be accounted for by the fact that granular media are usually analyzed under conditions that seemingly exclude an appreciable structural nonuniformity. Furthermore, possibilities of identifying parameters of GM structural nonuniformity are greatly limited in the majority of practically important cases [39, 44, 45].

Meanwhile, many authors (see, for instance, [44–46]) emphasize the necessity to use comprehensive and reliable data on GM flow dynamics in an analysis of particle interaction effects. In this respect, highly representative results have been obtained in a extensive study on separation and mixing effects [44]. The present article is designed to analyze separation mechanisms, allowing a consideration of the dependence of the process kinetics on a combination of dynamic, structural, and kinematic parameters of shear gravity-driven flows. In addition, methods for predicting these parameters are discussed.

## 3. Mechanisms and kinetics of interaction effects of nonuniform particles in shear gravity-driven flows

Effects of particle-to-particle interaction in granular materials exposed to rapid shear when particle nonuniformity can not be disregarded are conventionally divided into two groups: separation effects and mixing effects [18–25, 43–46]. In the general case, the physical nature of separation effects varies widely, depending on the dynamic conditions of particle–particle interaction. They are defined by the generic term 'separation'. This term is also used to describe technological processes designed to separate mixtures into constituent components. Another term widely used in the Russian and foreign literature is 'segregation' (from the Latin word segregatio); it largely serves to designate spontaneous separation effects arising from interactions of nonuniform particles in granular media.

One of the first mechanisms elucidated to explain separation in shear GM flows was reported in [9]. According to this mechanism, particles colliding in a gravity chute either slow down (e.g., small particles) or continue to move (big particles) depending on their inertial properties. Many researchers [12, 18, 21] agree with the author of [10] that the principal particle separation mechanism in a GM shear gravity-driven flow consists of interparticle percolation. According to percolation mechanism, the shear flow is considered a set of sieves characterized by different probability of penetration for particles with different properties.

The percolation mechanism agrees in many respects with the kinetic sieving mechanism proposed by later authors [13, 19, 20, 47]. Both mechanisms underlie the majority of the separation dynamics models [13, 18–21, 47]. However, the authors of [21] argue that the general model of separation dynamics with a high predictive potential remains to be developed. The absence of such a model appears to be due to the lack of models simulating mechanisms of the process suitable for predicting both the direction and the intensity of the movements of nonuniform particles under the variable hydrodynamic conditions of their contact.

Reference [44] reports on a study on the separation dynamics of particles of various sizes and densities in GM shear gravity-driven flows. The separation dynamics were analyzed with reference to their different mechanisms for particles differing in size and density. Separation in terms of density was simulated based on the buoyancy mechanism [42], which is formally analogous to the mechanism behind the floating up of bodies submerged in a fluid. The kinetics of particle size separation were simulated based on the mechanism of particle separation under the action of the gravity-induced lithostatic pressure gradient [43]. In accordance with this mechanism, stresses generated by gravity in a shear flow are distributed in some way between particles of different sizes in proportion to certain individual partial pressure coefficients, the values of which in general do not coincide with the relative concentration of properly-sized particles. It was conjectured in [47] that particles for which the proportion of partial stresses is larger than their local relative concentration in the mixture move into the flow region with lower lithostatic pressure. In contrast, particles in which the proportion of partial stresses is smaller than their local relative volume concentration eventually find themselves in the region with higher lithostatic pressure. In all the studies cited above, the effect of particle mixing was simulated based on a quasi-diffusion model.

In the absence of a solid theoretical foundation for the prediction of the dependence of separation and mixing kinetic characteristics on kinematic and structural parameters of the flow, kinetic coefficients were approximated by constants [42–44, 46, 47]. As a result, coefficients of the particle separation rate according to the size and density, as well as the quasi-diffusional mixing coefficient, were defined as fitting coefficients for the purpose of mathematical simulation. However, results of later research, e.g. [48–50], give evidence that the above coefficients strongly depend on the shear rate in the flow volume, which is highly variable in the general case. The same results suggest that taking account of the dependence of separation and quasi-diffusion coefficients on the shear rate improves the agreement between theory and experiment. However, the elucidation of the dependence of separation and diffusion coefficients on shear flow parameters remains a major challenge faced by researchers working in this field [44].

Moreover (as rightly noted in Ref. [44), in spite of the flexibility of the recently proposed mathematical models of separation [48–50], the reasoning behind them fails to take into account effects of the kinematic heterogeneity of shear flows. As a result, the models disregard particle separation effects produced by spatial nonuniformity of the shear rate and GT directly related to it. The data reported in Refs [44, 45, 51–54] also suggest the necessity to take into consideration effects of separation associated with spatial nonuniformity of the parameters of GM shear flows.

In Refs [44, 54], the magnitude of separation fluxes of differently-sized particles is considered to be proportional to the gradient of the kinetic stress component, with kinetic and contact stress constituents being distinguished among local stresses generated in a GM shear flow. The shear-related kinetic stress component is believed to appear as a result of pulse transfer in particle fluctuations. It is assumed that separation of nonuniform particles is a result of a different distribution of contact and kinetic stresses over GM components for particles of different sizes. The development of mathematical models of particle segregation according to size [43, 47] and density [42] enabled the authors of [44] to propose a mathematical description of separation under the conditions of spatial nonuniformity of the shear flow:

$$\rho^i(v^i - v) = \frac{(\psi^{\mathrm{c},i} - \psi^{\mathrm{k},i})}{c_{\mathrm{D}}} \frac{\partial \sigma_{yy}^{\mathrm{k}}}{\partial y} + \frac{(\phi_{\mathrm{m}}^i - \psi^{\mathrm{c},i})}{c_{\mathrm{D}}} \rho g \cos\zeta - D \frac{\partial \rho^i}{\partial y}, \qquad (1)$$

where $\rho$, $\rho^i$ are the local values of bulk density of the mixture and its component $i$, respectively, $g$ is gravitational acceleration, $v$, $v^i$ are the mean local velocities of the mixture and its component $i$ in separation direction $y$, respectively, $\phi_{\mathrm{m}}^i$ is local mass concentration of the $i$-th component of the mixture, $\psi^{\mathrm{c},i}$, $\psi^{\mathrm{k},i}$ are partial contact and kinetic stress coefficients for the $i$-th mixture component, respectively, $\sigma_{yy}^{\mathrm{k}}$ is the local value of the kinetic stress component, $\zeta$ is the angle of the shear flow inclination to the horizon, $D$ is the quasi-diffusional mixing coefficient, and $c_{\mathrm{D}}$ is the separation (segregation) resistance factor.

In accordance with this mathematical description, the component separation flux associated with the shearing flow of the bulk material is a result of the coupling of elementary separation fluxes and the flux of quasi-diffusional mixing of components. The elementary separation fluxes are regarded as resulting from the nonuniform distribution of the contact and kinetic stress constituents over mixture components nonproportional to component concentrations. In this case, gravity acts as the driving force of the separation fluxes due to the nonuniform distribution of contact stresses. The separation fluxes resulting from the nonuniform distribution of the kinetic stress constituent is defined as a parameter proportional to the gradient of the kinetic stress component. In other words, the former separation flux relates to the local inhomogeneity of the medium, while the latter one is a consequence of spatial nonuniformity of the shear rate in the flow of the material.

The development of the proposed description of separation process dynamics and the analysis of its prognostic potential in [44] were based on the assumption of high solid phase concentration and its uniform distribution in the flow. However, such an assumption is hardly justified bearing in mind the particular attention given to the shear rate nonuniformity effect in a flow, because any significant change of the shear rate in a granular medium is accompanied by an equally significant alteration of its microstructural characteristics. Such a correlation is confirmed by the results of an experimental study of structural and kinematic characteristics in shear flows of noncohesive inelastic particles at different strain rates. Relevant data were obtained in [55] by an X-ray method under the conditions of rapid shear flow in a rough chute. The strain rate dependence of the solid phase volume fraction (Fig. 1) was documented with the use of a conveyer shear cell [56]. Results of many studies (see, for instance, [34, 38, 45, 51, 55, 56]), confirm that solid phase

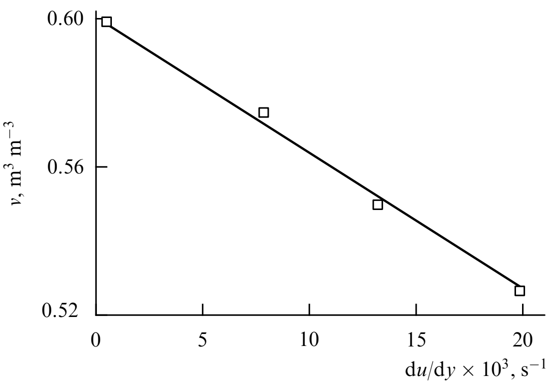


**Figure 1.** Solid phase volume fraction $v$ versus the quasi-plastic deformation rate in a layer of ceramic spheres $6.6 \times 10^{-3}$ m in diameter in a conveyer shear cell (from Ref. [56]).

concentration is an important factor determining the dynamics of granular medium flows and effects of particle-particle interaction. In this context, the structural homogeneity of shear flows assumed in a separation dynamics study [44] may be one of the main causes of the observed significant discrepancy between theoretical and model results.

One more factor supposedly responsible for the inadequacy of the description of the separation process [44] is the presumed equality of the kinetic coefficients for elementary separation fluxes in Eqn (1), the values of which are inversely proportional to the resistance coefficient $1/c_{\mathrm{D}}$. The presumed equality of the kinetic segregation coefficients appears highly questionable in light of the critical difference between the expressions for the driving force of separation fluxes resulting from spatial nonuniformity of the shear flow (the first two terms on the right-hand side of the equation).

Particle separation dynamics in GM shear gravity-driven flows is described in [51, 52] from the point of view formally analogous to that underlying the description of the process in [44]. The nonuniform particle distribution dynamics are represented in [51, 52] as a totality of integrated fluxes of convective transfer $\mathbf{j}_{\mathrm{c}}$, separation (segregation) $\mathbf{j}_{\mathrm{s}}$ due to local medium inhomogeneity, separation $\mathbf{j}_{\mathrm{m}}$ due to spatial nonuniformity of structural and kinematic parameters of the medium flow (migration), and quasi-diffusional mixing $\mathbf{j}_{\mathrm{d}}$:

$$\frac{\partial(c\rho_{\mathrm{b}})}{\partial t} + \mathrm{div}\,(\mathbf{j}_{\mathrm{c}}) = -\mathrm{div}\,(\mathbf{j}_{\mathrm{s}}) - \mathrm{div}\,(\mathbf{j}_{\mathrm{m}}) - \mathrm{div}\,(\mathbf{j}_{\mathrm{d}})\,. \qquad (2)$$

However, the description of particle separation fluxes arising from the local and spatial inhomogeneity of the medium and its flow is based, unlike that in [44], on the assumption of quite different mechanisms of the process. Specifically, the mechanism of separation in a shear flow is used to describe segregation due to local medium inhomogeneity. This mechanism suggests that any particle differing in properties from shear flow particles of a conventionally homogeneous medium acts, while in such flow, as a stress concentrator. Aggregates (clusters) of conventionally uniform particles form sporadically near the concentrator. When it interacts with such clusters under effect of the gravity-driven shear, it acquires a transversely directed momentum that displaces the concentrator in the direction in which stresses in the shear flow become relaxed, i.e., towards the open surface of the layer or towards its bottom. Also, the direction of particle movement is determined by the forces of gravity and friction, as well as by impact momenta. This allows us to express the dependence of separation kinetic characteristics on the combination of physicomechanical properties of medium components (size, density, roughness, elasticity) and structural-kinematic parameters of the flow (the solid phase volume fraction and the shear rate).

A separation flux resulting from spatial nonuniformity of the granular medium shear flow has been described based on the assumption of the particle quasi-diffusional separation mechanism called migration [51]. It is believed that migration occurs as a result of different velocities of chaotic displacements of nonuniform particles. By analogy with thermobaric diffusion in dense gases, rapidly moving particles migrate into the high-void region of the flow with a large void volume fraction, while slow particles migrate into the region with a high concentration of the solid phase. To recall, kinetic characteristics of migration are calculated with due regard for the differences among the physicomechanical properties of the particles. Mechanisms of separation in a shear flow and of migration will be considered at greater length in the concluding sections of this review.

## 4. Methods for predicting dynamic and structural-kinematic parameters of rapid gravity-driven flows of granular materials

The authors of [44] agree that the lack of reliable data on structural and kinematic parameters of the shear flow hampers the development of model representations of nonuniform particle interaction dynamics. The absence of a relevant theoretical framework hinders elucidation of separation and mixing characteristics, which necessitates the use of certain constants. Evidently, kinetic characteristics can not be identified based on the shear rate alone, especially when it varies along the flow and somehow correlates with the solid phase concentration.

At present, a variety of methods are available to study GMs under different conditions and clarify special aspects of their behavior. For example, magnetic resonance imaging, X-ray computed tomography, high-speed video filming, and ultrasonic scanning have proven highly efficient in research on structural characteristics of granular materials [5]. In many cases, however, detailed information on the structural and kinematic characteristics of the flow can not be obtained without virtual experiments [22–24, 44].

GM shear flows are described using either the discrete element method (DEM), a numerical technique to simulate the behavior of a population of independent particles, taking into account their interactions, or the finite element modeling technique for predicting the state of a particle assembly as a continuum [22–24, 41, 57–60]. These approaches being fundamentally different, each has a preferred area of application. The former is largely used to predict structural transformation dynamics in granular materials and simulate experimental studies under conditions allowing for idealization of internal and external impacts on individual particles [41, 44]. Continual approaches are employed to describe the dynamics of flows formed under complicated boundary conditions; as a rule, they are based on the assumption of the incompressibility and homogeneity of granular materials [58, 59]. However, some authors analyzing the causes of the inadequacy of continual models (see [44, 61]) came to the conclusion that it is necessary to abandon averaging parameters of a shear flow in its cross section to ensure an adequate physical description. According to [60], the solution of such a problem can be facilitated by using hydrodynamic models based on the phenomenological approach and taking account of microstructural properties of granular materials. However, such models have yet to be developed [41, 61].

The present review includes a discussion of the phenomenological description of the relationship between structural and kinematic characteristics of gravity-driven granular media flows in a rough chute [55]. The discussion is accompanied by an analysis of behavioral patterns of rapid granular flows in a vibrating rough chute [45], with the vibrational impact being regarded as a means to affect structural and kinematic characteristics of a gravity-driven flow and thereby to control nonuniform particle separation and mixing.

The proposed phenomenological description establishes the relationship among dynamic, structural, and kinematic characteristics of a rapid GM gravity-driven flow. It can be useful for predicting particle velocity and volume fraction profiles in rapid GM shear flows, either in combination with classical momentum, energy, and matter conservation equations [38, 57–61] or with the use of additional experimental data.

The description of the relationship between structural and kinematic properties of GM flows is based on the formal analogy between a granular medium under rapid shear conditions and a dense gas. In accordance with this analogy, a granular medium in the state of a solid-particle gas is characterized by the parameters of state formally analogous to those of a dense gas. To this effect, an analogy between dilatancy, the kinetic energy of all forms of particle relative movements, and the analog of hydrostatic pressure of the granular medium, on the one hand, and the molar volume, temperature, and pressure of the dense gas, on the other hand, is assumed. As a result, the following dependence formally analogous to the known law of the dense gas state is used to describe the relationship between structural and kinematic parameters of GM gravity flows in a chute [55]:

$$p\bar{\varepsilon} = \chi\theta \,, \tag{3}$$

where $p(y) = \int_{h-y}^{h} \rho(1-\varepsilon(y))g\cos\alpha\,\mathrm{d}y$ is the analog of hydrostatic pressure, $\bar{\varepsilon}(y) = (\varepsilon(y) - \varepsilon_0)/(1-\varepsilon(y))$ is GM dilatancy, $\varepsilon$ is the porosity (void volume fraction) of the medium, $\varepsilon_0$ is the medium porosity under conditions of free compact (disordered) packing of uniform spherical particles, $\varepsilon_0 = 0.4$, $\chi$ is the coefficient of the granular medium state equation, $y$ is the coordinate perpendicular to the chute surface, $g$ is the gravitational acceleration, $\alpha$ is the chute inclination angle to the horizon, and $\theta$ is the granular temperature.

GT is traditionally considered [36, 38] to be a parameter proportional to the mean squared particle fluctuation velocity. In the framework of the above analogy, GT is represented as the particles' kinetic energy characterizing the totality of various forms of their relative movement in a shear flow [45]. To recall, a similar representation of GT as particle kinetic energy can be found in other publications (e.g., [30, 62]). The expediency of taking into account various forms of relative motion of particles to determine GT ensues from the assumption that each form makes a certain contribution to flow dilatancy.

Thus, $\theta$ on the right-hand side of Eqn (3) is the kinetic energy the particles have due to their reciprocal displacements in the shear flow. The product on the left-hand side of Eqn (3) corresponds to the work performed by the particles upon a shift of the granular medium and leading to its dilatancy per unit solid phase volume. For this reason, coefficient $\chi$ can be defined as a characteristic reflecting the ratio of the work done for dilatancy in the case of a gravity-induced shift of the medium to the kinetic energy of particles acquired in their relative movements in the flow.

Determining temperature in a shear GM flow as the particles' kinetic energy in their reciprocal displacements implies consideration of various forms of particle motion in a relative system of coordinates moving in the direction of the shift with the mean particle speed in the same direction. In this way, it is possible to estimate the variation in medium temperature under the effect of different external impacts (physical fields) on the medium shear flow. Justification for such an approach to determining GT as an additive parameter is confirmed by a study (Ref. [30]) demonstrating the pronounced anisotropic character of GT.

An object of interest in the present paper is GT in a rapid shear flow in a rough chute experiencing the integral action of gravity and vibrational oscillations. GT is analyzed together with effects of flow structure transformation, separation, and mixing of nonuniform particles caused by the combined impact.

In this case, GT is determined in [45] as the overall effect of gravity and vibration on the particle kinetic energy supporting the particles' relative motion in the flow:

$$\theta(y) = \theta_{\rm v}(y) + \theta_{\rm g}(y)\,, \tag{4}$$

where $\theta_{\rm g}$ is the kinetic energy generated by gravity-driven shear and $\theta_{\rm v}$ is the vibration-induced kinetic energy.

The component of the kinetic energy acquired by the particles under the effect of the gravity-driven shear $\theta_{\rm g}$ is determined as a composite quantity, taking account of elementary forms of reciprocal particle displacements due to the difference among their mean velocities in the shear direction $\theta_{\rm sh}$, chaotic fluctuations of particles $\theta_{\rm fl}$, and transverse quasi-diffusional mass transfer $\theta_{\rm tr}$ [45]:

$$\theta_{\rm g} = \theta_{\rm sh} + \theta_{\rm fl} + \theta_{\rm tr}\,. \tag{5}$$

The GT component caused by the relative velocity of particles with mass $m$ in the shear direction is calculated taking into consideration the shear rate $\mathrm{d}u/\mathrm{d}y$ and the mean distance between the centers of mass of the particles $bd$ in the form

$$\theta_{\rm sh} = \frac{1}{2} m(bd)^2 \left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2, \tag{6}$$

where $b$ is the geometric parameter depending on porosity (void volume fraction) in the layer of the material: $b = [\pi/(6(1-\varepsilon))]^{0.33}$.

The temperature component due to chaotic movements of particles in space with a certain mean fluctuation velocity $V'$ is defined as

$$\theta_{\rm fl} = \frac{1}{2} m(V')^2\,. \tag{7}$$

The shear flow of particles is accompanied by their transverse mass transfer [35] underlain by a physical mechanism formally analogous to diffusional mass transfer [45]. With this in mind, the GT component due to the transverse mass transfer is determined for the case of a two-dimensional shear flow as the product of the transverse particle quasi-diffusion coefficient and the shear rate. Because the transverse quasi-diffusion coefficient is proportional to the particle fluctuation velocity and the mean distance between the particles $s$ [63], the GT component resulting from the transverse mass transfer can be represented in the form

$$\theta_{\rm tr} = \frac{1}{2} msV' \frac{\mathrm{d}u}{\mathrm{d}y}\,, \tag{8}$$

where $s = (b/b_0 - 1)d$ is the interparticle distance, and $b_0$ is the value of the geometric parameter $b = [\pi/(6(1-\varepsilon))]^{0.33}$ corresponding to the hexagonal (most compact possible) packing of the particles ($\varepsilon = 0.2595$ for uniform spherical particles).

The mean particle fluctuation velocity $V'$ in expressions (7) and (8) is calculated [35, 52] as a function of shear rate based on the energy balance under the stationary medium flow conditions:

$$V' = \varphi bd \frac{\mathrm{d}u}{\mathrm{d}y}\,, \tag{9}$$

where $\varphi$ is the factor reflecting the influence of dissipation effects on the particle fluctuation velocity, taking into account phase interaction conditions in the shear flow. The balance between the energy generated by a shift of the granular medium and energy dissipation during particle interaction with the medium filling the interparticle space in contact particle–particle interactions provides a basis for expressing the $\varphi$ factor as [35]

$$\varphi = \left[\frac{(1+k)(0.05+0.08\mu)(1+s/d)}{[3sC/2d]\,\rho_{\rm f}/\rho + k_{\rm E}}\right]^{1/2},$$

where $k_{\rm E}$ is the proportion of the kinetic energy of the particles dissipated in their interactions, $C$ is the coefficient of hydraulic resistance to particle displacements in the medium filling the interparticle space [35], $\rho_{\rm f}$ is the density of the medium filling the interparticle space, and $k$ is the restitution coefficient.

The energy balance equation is derived with reference to the combined particle collision hypothesis [52] permitting us to integrate the advantages of the classical impact hypotheses of Newton and Rouse. It made possible in [52] overcoming drawbacks inherent in Rouse's oblique impact hypothesis, used earlier in Ref. [35] to estimate kinetic energy losses in particle collisions in a rapid gravity flow. According to the hypothesis proposed in [45, 52], the fraction $k_{\rm E}$ of the particles' kinetic energy dissipated in their collisions is

$$k_{\rm E} = (1-k^2) + \frac{1}{2}\lambda - \frac{1}{8}\mu^2(1+k)^2 - \frac{1}{8}\lambda^2 + \frac{2}{\pi}\mu(1+k) - \frac{2}{3\pi}\mu\lambda(1+k)\,, \tag{10}$$

where $k$ is the restitution coefficient, $\lambda$ is the coefficient of the tangent velocity reduction in particle–particle collisions determined by the method proposed in [52], and $\mu$ is the friction coefficient.

An analysis demonstrates that all components of the relative particle velocity taken into consideration to formulate expressions (6)–(8) are (bearing in mind (9)) derivatives of the shear rate proportional to its first degree. Therefore, it is logically sound that the respective GT components are determined depending on the squared shear rate.

The vibrational GT component $\theta_{\rm v}$ in expression (4) (the kinetic energy of particle reciprocal displacements generated by the vibrating chute) is determined on the assumption that a granular medium composed of inelastic spherical particles prone to undergo cohesion exhibits the properties of an elastic compressible system under rapid shear flow conditions. This assumption follows from the mesoscopic properties of bulk materials, in which individual particles have characteristics of solid bodies, and the assemblage of the particles can display, under certain conditions, properties typical of solid particle gases [38]. Such a medium moving in a vibrating rough chute that undergoes harmonic oscillations experiences sequentially alternating periods of contraction and dilatancy with a

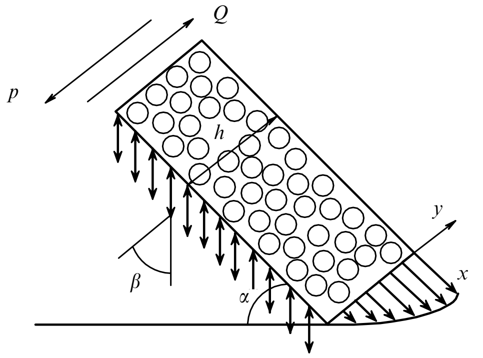


**Figure 2.** Schematic illustration of the calculation of a quasi-thermal flow and granular medium temperature in a gravity-driven flow in a vibrating chute.

frequency corresponding to that of chute oscillations. The quasi-thermal energy introduced into the medium in the period of compaction causes dilatancy of the solid particle gas at the following stage of oscillations.

The component of the kinetic energy of particles moving relatively to one another, $\theta_v$, is due to the action of the vibrating chute on the medium gravity-driven flow; it is calculated as illustrated in Fig. 2.

According to this scheme, the quasi-thermal flow $Q$ generated by the vibrating chute is directed oppositely to the vector of hydrostatic pressure $p$. Clearly, the intensity of the quasi-thermal flow directed from the bottom to the open surface of the layer is proportional to hydrostatic pressure and depends on parameters of the vibrational impact (direction, frequency, and vibration amplitude).

When vibrational acceleration exceeds the gravitational one, the action of the vibrating chute on the particle flow is periodically interrupted. It is assumed that the chute breaks away from the moving particle layer at instant $t_0$ when the layer is maximally contracted [45], corresponding to the point of time at which the chute surface coordinate $y$ is equal to the minimal layer thickness.

In accordance with the analogy between a granular medium undergoing rapid shear and the solid particle gas as an elastic compressible system discussed in a preceding section, the medium that lost contact with the chute passes to the state of dilatancy, in which it remains till contact is resumed. The influence of the chute on the particle flow becomes apparent again at the point of collision of the vibrating chute with the granular medium layer in the increasing dilatancy state [45]. Dilatancy occurs in accordance with the dynamics of gravitational action on the layer of particles and their quasi-diffusional interaction. The quasi-diffusional character of particle–particle interaction in a vibrating GM layer was documented in [64, 65]. Under such conditions, the coordinate of the resumed contact point and the respective resumption time $t_c$ are calculated by solving a set of two equations describing the motion of particles that lost contact with the chute and chute displacements, respectively.

The quasi-diffusional flow resulting from the action of the vibrating chute on the moving granular medium layer during their contact time $t_c - t_0$ is calculated in [45] as the sum of normal $Q_n(0)$ and tangential $Q_t(0)$ flow components:

$$Q(0) = Q_n(0) + Q_t(0)\,. \tag{11}$$

The magnitude of the quasi-thermal flow generated by the vibrating chute inside the granular medium layer in the solid particle gas state is determined based on the postulate proposed in [45], in accordance with which the magnitude of the quasi-thermal flow is proportional to the GT vibrational component $\theta_v$, the particle concentration per unit surface perpendicular to the quasi-thermal flow ($b^{-2}d^{-2}$), and frequency $\omega$ of the pulses imparted to the material flow by the vibrating base:

$$Q(y) = b^{-2}(y)d^{-2}(y)\theta_v(y)\omega\,, \tag{12}$$

where $b$ is the geometric parameter of the layer (a function of porosity or void volume fraction) fluxes (6), and $d$ is the mean particle diameter.

Dissipation of the mechanical energy during pulse transfer by the particles in the layer accounts for the reduction in the quasi-thermal flow from the bottom to the free surface of the layer as

$$Q(y) = Q(0) - \int_0^y \Delta Q(t)\,dt\,, \tag{13}$$

where $\Delta Q(y)$ is the specific dissipation energy per unit volume of the granular medium flow. The specific dissipation energy is calculated in [45] as the fraction of the kinetic energy of vibrating particles dissipated in their collisions due to friction and elasticity effects. The use of the oblique impact hypothesis (10) [52], permitting us to integrate the advantages of the classical impact hypotheses of Newton and Rouse with those of postulate [12], leads to the expression for the assessment of the specific volume dissipation energy [45]:

$$\Delta Q(y) = k_E\theta_v(y)\big(b(y)d(y)\big)^{-3}\omega\,.$$

Substituting this expression into (13), taking account of (12) and its rearranging by differentiation, separation of variables, and subsequent integration, yield [45] the dependence describing a quasi-thermal flow generated by the vibrating base in the GM gravity-driven flow:

$$Q(y) = Q(0)\exp\left(-\int_0^y k_E\big(b(t)d(t)\big)^{-1}\,dt\right). \tag{14}$$

Dependence (14) defines the attenuation dynamics of the quasi-thermal flow $Q(0)$ penetrating into the layer from the direction of the vibrating chute. A joint analysis of expressions (10) and (14) makes it possible to estimate the key role of particle physicomechanical properties in the propagation of the energy of chute vibrational oscillations in a rapid GM gravity flow. The intensity of absorption of the vibrational oscillation energy due to elasticity and friction effects of particle collisions is determined using coefficients of restitution $k$, interparticle friction $\mu$, and tangent velocity reduction in particle-to-particle collisions $\lambda$, taking into consideration the structural characteristics of the flow dependent on the layer height.

The known patterns of variation of quasi-thermal flow magnitude (14) generated by the vibrating chute makes it possible to express, using (12), the GT vibration component as a function of layer height:

$$\theta_v(y) = \frac{Q(y)\big(b(y)d(y)\big)^2}{\omega}\,. \tag{15}$$

To determine the profile of the temperature vibration component $\theta_v(y)$ in accordance with (14) and (15), the values of geometric parameter $b(y) = [\pi/(6(1-\varepsilon(y))]^{0.33}$ and mean volume size of a particle $d(y) = (c(y)/\rho_1 d_1 + (1-c(y))/\rho_2 d_2)(c(y)/\rho_1 + (1-c(y))/\rho_2)^{-1}$ are needed. These geometric characteristics are calculated using void volume fraction $\varepsilon(y)$ and component concentration distribution profiles in a granular medium flow. However, information about these parameters of gravity-driven flows of an inhomogeneous granular medium at the macrolevel remains difficult to access [44].

A brief analysis of the currently available methods for the predication of structural and kinematic characteristics of gravity-driven flows is presented at the beginning of the present section. In Section 5 we discuss an experimental analytical method [45, 52, 55] used to determine void volume fraction $\varepsilon(y)$, velocity $u(y)$, GT $\theta(y)$, and nonuniform particle distribution $c(y)$ profiles in rapid gravity-driven flows of a granular medium in a vibrating rough chute. The theoretical basis of the method is provided by the equation of state (3), taking into account dynamic effects of gravity-driven shear and vibrational oscillations.

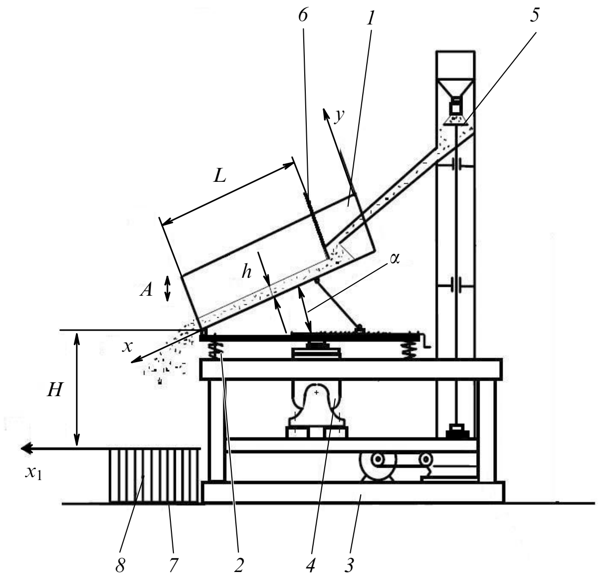


**Figure 3.** Layout of the experimental setup: *1* — open channel; *2* — springs; *3* — frame; *4* — vibratory drive; *5* — dosage device; *6* — slide-valve; *7* — cuvette (tube); *8* — cells; *A* — amplitude of vibrational oscillations.

## 5. Analysis of predictive properties of the experimental analytical method for the study of gravity-driven flow parameters

The method is based on the consideration of a granular medium undergoing a rapid gravity-driven shear as a gas of solid particles on the assumption of the relationship between dynamic and kinematic parameters of the shear flow described by the equation of a granular medium state (3). The equation is used in combination with the experimentally found function of horizontal distribution of material falling down the chute [45, 52, 55].

The method proves useful in research on structural and kinematic parameters and interaction effects of nonuniform particles in rapid GM gravity-driven flows in a chute, including flows undergoing the influence of vibration and hydrodynamic pressure of a transverse gas flow [45, 51, 52, 66]. In the present review, the potential of the method is analyzed based on the results of investigations of flows with different particle sizes and densities in a vibrating rough chute using the experimental setup shown schematically in Fig. 3 [45]. The test unit consists of an inclined open channel of a rectangular cross section attached to the frame by elastic supports at an angle that can be controlled with respect to the horizon. The channel has smooth side walls and a rough bottom, with the roughness equaling half of the largest GM particle diameter. There is a cuvette (tube) at distance $H$ right below the discharge threshold of the channel; it is divided by transverse partitions into cells of equal size. The channel is rigidly connected to the electric vibratory drive that imparts to the channel vertical oscillations with controllable amplitude and frequency.

The technique of the experiment is as follows. A dosage device is used to load GM particles directly into the inclined channel at the given parameters of its vibration and to fill cuvette cells with falling particles in the steady flow regime. Experimental data needed to determine velocity, the void volume fraction, and nonuniform particle distribution profiles in the flow contain information on the layer height $h$, particle fall height $H$, chute inclination angle to the horizon $\alpha$, cuvette filling time $t$, and function $G(x_1)$ of falling material distribution among the cuvette cells (along the horizontal coordinate $x_1$).

The equations allowing the determination of velocity $u(y)$, void volume fraction $\varepsilon(y)$, pressure $p(y)$, and GM $\theta(y)$ profiles in a rapid gravity-driven flow in a rough chute based on the granular medium state equation (3) were derived in [63]:

$$u = \frac{x_1 - y\sin\alpha}{\cos\alpha\sqrt{\big(H + y\cos\alpha - (x_1 - y\sin\alpha)\tan\alpha\big)(2/g)}}\,, \qquad (16)$$

$$u\big(y(x_1)\big)\rho\big(1-\varepsilon(y)\big) = G(x_1)\,. \qquad (17)$$

In these equations, $y(x_1)$ is the particle coordinate in the layer, depending on its coordinate on horizontal $x_1$ after falling down from the chute (see Fig. 3). Expressions (3)–(5), (15)–(17) form a closed system of equations for velocity $u(y)$, void fraction $\varepsilon(y)$, pressure $p(y)$, and interconnection function $y(x_1)$ profiles. It is solved with the boundary condition at the flow base corresponding to the condition of adhesion of its particles to the rough chute surface

$$u(0) = 0 \quad \text{at} \quad y = 0\,. \qquad (18)$$

This boundary condition suggests the absence of slip of the particles along the chute surface when the coordinate $y$ of their center of mass coincides with the coordinate ($y = 0$) of roughness peaks, the size of which corresponds to half of the largest GM particle diameter. The adequacy of the granular medium state equation (3) and the above method of its application to predict the set of parameters of rapid gravity flows is confirmed in [55] with the use of the X-ray technique shown in Fig. 4.

A remarkable feature of the experimental analytical method is the possibility of obtaining information not only on structural and kinematic characteristics of a flow but also about the profiles of nonuniform particle $c(y)$ distribution in the flow. In studies of nonuniform particle interaction effects,

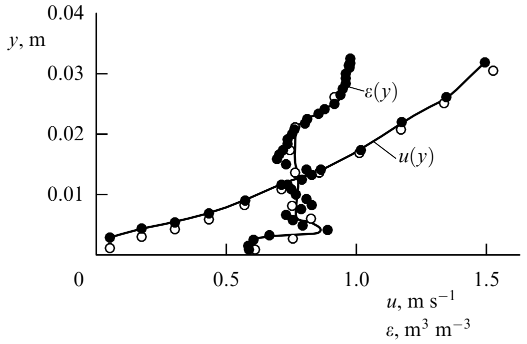


**Figure 4.** Velocity $u(y)$ and void fraction $\varepsilon(y)$ profiles in a rapid gravity-driven flow of ceramic spheres in a rough chute. Black dots — X-ray technique, circles — experimental analytical method.

**Table.** Characteristics of the mixtures of model granular materials

| Characteristics of mixtures | Mixture 1 | | Mixture 2 | |
|---|---|---|---|---|
| Particle material | Beads (glass) | | Beads (glass) | Silicagel |
| Particle size $d$, $10^{-3}$ m | $3<d<3.5$ | $5<d<5.5$ | $5<d<5.5$ | $5<d<5.5$ |
| Particle concentration, kg kg$^{-1}$ | 0.8 | 0.2 | 0.95 | 0.05 |
| Particle density, kg m$^{-3}$ | 2500 | 2500 | 2500 | 1015 |
| Natural angle of repose, degrees | 26 | 26 | 26 | 31 |

the potential influence of their initial random distribution in the flow is prevented by choosing a large enough length of the rough chute. Validity of the assumption of a negligibly small air drag for determining a falling particle trajectory (hence, the relationship between the coordinates $y(x_1)$) is confirmed experimentally by comparing $c(y)$, $u(y)$, $\varepsilon(y)$, and $p(y)$ profiles for different heights of the chute above the cuvette $H$. To implement the method, the system of equations (3)–(5), (15)–(18) is numerically solved using the method of successive approximations [67] and first approximation $\varepsilon(y) = \varepsilon = \text{const}$.

## 6. Analysis of separation mechanisms and kinetics based on results of mathematical and physical simulation

Mechanisms and kinetic patterns of the separation process due to local and spatial inhomogeneity of granular media were described in Section 3. The present section is focused on their comparative analysis based on the results of experimental and analytical modelling of velocity, solid phase volume fraction, and nonuniform particle concentration distribution profiles in a rapid GM gravity-driven flow in a rough chute [45]. The profiles were obtained with and without imposition of vibrations using the experimental analytical method described in Section 5. The model materials were mixtures of particles differing in size (mixture 1) and density (mixture 2), as shown in the table above.

Experimental analytical profiles of velocity, solid phase volume fraction ($v(y) = 1 - \varepsilon(y)$), and nonuniform particle concentration distributions in a rapid gravity-driven flow in a vibrating rough chute for mixtures 1 and 2 are presented in Fig. 5.

An analysis of the results reveals a very close analogy between the velocity profiles of the two mixtures having a parabolic shape (as in Fig. 4). This feature, especially apparent against the background of radical differences among the profiles of nonuniform particle distribution, appears to be of importance in the context of model representations of separation (segregation) set down in Ref. [44]. The authors of [44] argue that gradients of kinetic stress and lithostatic pressure constitute the driving force of the process. As the kinetic stress component is directly related to the shear rate, the direction of its gradient to the bottom of the layer (similar to that of the lithostatic pressure gradient) remains unaltered in the layer volume. Concentration distribution profiles of particles differing in size and density present at extremely high levels in the central part of the layer (Fig. 5) are in contradiction with model assumptions of segregation in Ref. [44].

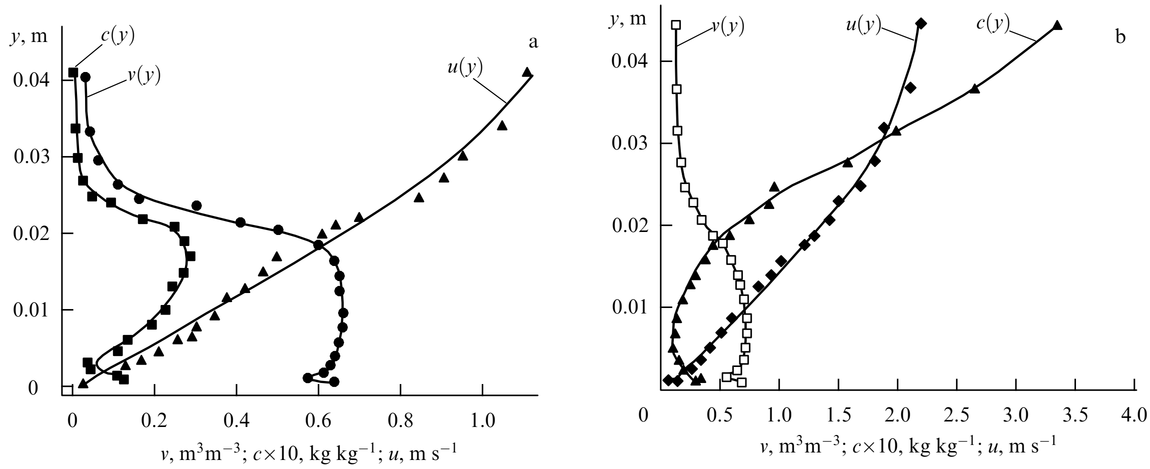


**Figure 5.** Velocity $u(y)$, solid phase volume fraction $v(y)$, large (a) and low-density (b) particle concentration $c(y)$ profiles determined by experimental analytical method in rapid gravity-driven flows of particles differing in size (mixture 1) (a) ($\alpha = 30°$) and density (mixture 2) (b) ($\alpha = 30°$) in a vertically vibrating rough chute: $\omega = 15$ Hz, $A = 0.002$ m.

In accordance with the classification of gravity-driven flows proposed in [34], the main features of the obtained velocity profiles are characteristic of 'sliding' (developed shear) flows. Such a flow pattern is observed at rough chute angles to the horizon close to and slightly larger than the natural angle of repose of a given material. The solid phase volume fraction vertical distribution function $(v(y) = 1 - \varepsilon(y))$ suggests an invariably maximum solid phase concentration in the central part of a flow.

As quasi-hydrostatic pressure decreases toward the flow free surface, the solid phase volume fraction gradually reduces to zero at the layer boundary. The solid phase concentration also decreases in the opposite direction from the central part of the flow. The decrease in the concentration toward the layer bottom is followed by its rise in the immediate proximity to the chute surface, which can be accounted for by the absence of slip in particles whose center of mass coordinate corresponds to the coordinate of roughness peaks on the chute surface. The observed particle concentration distribution patterns in a rapid gravity flow in a chute with the maximum level in the layer center were described earlier in Refs [31, 34, 51, 52, 68]; they were attributed to the formation of a particle cloud in the upper part of the layer and the existence of an intense shear region in the zone adjacent to the layer bottom.

Of special interest as regards both model materials is the striking manifestation of the analogy between solid phase volume fraction and nonuniform particle concentration distribution profiles. Evidently, the observed analogy is a consequence of nonuniform particle separation by mechanisms whose driving force shows a much stronger correlation with the solid phase volume fraction gradient than with the shear rate gradient. Indeed, while the shear rate in the flows of both mixtures undergoes largely monotonic variations, nonuniform particles distributed in the layer readily respond to the changes in solid phase concentration. Small low-density particles of mixtures 1 and 2 (see the Table) acquire a high fluctuation velocity in mutual collisions and become concentrated in the flow regions with a large void volume fraction. In contrast, big and dense particles having a relatively low fluctuation velocity after collisions are associated with the parts of the flow containing high solid phase concentrations.

Physical mechanisms underlying the formation of the observed nonuniform particle distributions were used in the development of separation kinetics models; they are represented by effects of shear flow segregation and migration. The first and the second of these effects are conditioned by local and spatial flow inhomogeneities, respectively [51, 52, 69]. Separation of nonuniform particles due to these effects are included in the general separation dynamics equation (2) (see above). The magnitude of separation fluxes was deduced [51, 52, 69] from the general kinetic patterns of chemical technological processes as the product of the driving force and the kinetic coefficient.

In a separation flux caused by local medium inhomogeneity (segregation flux), a particle with properties unlike those of the surrounding particles of a conventionally uniform shear flow functions as a stress concentrator. The degree of stress concentration depends not only on the relationship between properties of nonuniform particles but also on the particle-to-particle interaction conditions (shear rate and solid phase concentration) [63, 69]. It was proposed in [63] to describe the dependence of the magnitude of a segregation flux on the totality of particle physicomechanical properties and flow parameters in terms of the shear flow segregation mechanism. This mechanism operates at relatively high values of the solid phase volume fraction ($v \geqslant 0.25$) when particles are capable of retaining short-range order in their mutual arrangement. It promotes cluster formation around the concentrator in a shear flow. A particle interacting with the cluster receives a pulse for displacement, the direction of which points in the separation direction. In accordance with this mechanism, the driving force of separation is defined as an excess total moment of gravity, friction, and impact pulses $\Delta M$ acting on a control component particle in a shear flow of a conventionally homogeneous granular medium [52, 69]. Characteristics of the particles of such a medium are defined as mean volumetric ones. Such an expression of the driving force of the process makes possible the quantitative evaluation of the influence of particle nonuniformity on segregation intensity with regard to the totality of properties (size, density, roughness, and elasticity). A method for determining the segregation rate coefficient $K_s$ was proposed in [52, 69]. These papers also report the results of research giving evidence that coefficient $K_s$ can be regarded as a kinetic constant for a wide range of shear flow parameters and particle properties. This fact can be viewed as indirect confirmation of the adequacy of model representation of the process mechanism. The results of the study permit the magnitude of a segregation flux to be expressed as

$$\mathbf{j}_s = K_s c \rho_b \Delta M \,. \tag{19}$$

The described kinetic patterns of separation can be used to explain the shape of the nonuniform particle concentration profiles in the central part of the flow where the solid phase volume fraction $v > 0.25$. Due to relatively high and uniform values of the solid phase volume fraction, the central flow zone is dominated by the effect of shear flow separation (segregation). Because the excess moment of force $\Delta M$ has a positive value for large low-density particles under such conditions [69], they tend to move into the flow region with low hydrostatic pressure, i.e., the upper part of the central zone (Fig. 5). This leads to a rise in the concentration of large low-density particles in the direction from the bottom to the free surface of the layer.

The effect of particle separation produced by spatial nonuniformity of the granular medium flow referred to as migration has a quasi-diffusion nature and prevails in the flow regions with a small solid phase volume fraction $v < 0.25$ [51]. The physical mechanism of migration is formally analogous to the thermobaric diffusion effect in a gas mixture [70]. In accordance with this effect, molecules with a high fluctuation velocity migrate to the region providing conditions for their chaotic motion with a large free path length. The migration driving force and velocity coefficient were determined taking into account the physical nature of quasi-diffusional separation of particles with different chaotic motion velocity under conditions of spatial nonuniformity of the flow structure. The driving force of migration was determined in [51, 52, 69] to be the rate at which the mean interparticle distance $s$ changes in direction $y$ of the separation flux: $\partial(\ln s)/\partial y$.

The migration coefficient is expressed in [52] as the magnitude of the resultant quasi-diffusional flow of particles differing in size, density, elasticity, and roughness for the case in which the gradient of the transfer potential is unity. The rate of change of the mean interparticle distance $s$ is taken as the transfer potential in relation to the quasi-diffusional separation of particles. The expression for calculating the migration coefficient is derived on the assumption of equality of the free path lengths of nonuniform particles [51, 52]. The validity of this condition is confirmed in experiment and can be attributed to the chaotic motions of the particles being derivatives of the relative particle velocity in the shear direction. Because this velocity is higher than the mean fluctuation velocity and practically equal for all nonuniform particles, their mean path length can be assumed invariant and equal to the mean interparticle distance $s$. For particles differing in the aggregation of physicomechanical properties, the migration coefficient is calculated as follows [52]:

$$D_{\mathrm{m}} = \frac{\bar{m}(c)(\overline{V'})^2}{2Fk}\left(\frac{d_1^2 k_1}{m_1 \bar{d}^2} - \frac{d_2^2 k_2}{m_2 \bar{d}^2}\right), \qquad (20)$$

where $F = \overline{V'}/s$ is the particle collision frequency, $\bar{d} = cd_1 + (1-c)d_2$ is the mean particle diameter, $\bar{m}(c) = \pi/[(c/(d_1^3\rho_1) + (1-c)/(d_2^3\rho_2))6]$ is the mean particle mass, $d_i$ is the diameter of $i$-type particles, $k$ is the mean restitution coefficient for colliding nonuniform particles, $k_i$ is the restitution coefficient for particles of the $i$th type, and $m_i$ is the mean mass of an $i$-type particle.

For particles of different sizes, densities, elasticities, and roughnesses, the quasi-diffusional migration flow is expressed as

$$\mathbf{j}_{\mathrm{m}} = D_{\mathrm{m}} c\rho_{\mathrm{b}} \frac{1}{s}\,\mathrm{grad}\, s = c\rho_{\mathrm{b}} \frac{\bar{m}(c)(V')^2}{2Fk} \times \left(\frac{d_1^2 k_1}{m_1 \bar{d}^2} - \frac{d_2^2 k_2}{m_2 \bar{d}^2}\right)\frac{1}{s}\,\mathrm{grad}\, s\,. \qquad (21)$$

In accordance with kinetic equation (21), a migration flux of small, elastic, smooth, low-density particles acquiring a relatively high speed of chaotic motion after mutual collisions coincides with the direction of the mean interparticle gradient, i.e., it is directed to flow regions containing a large void fraction. Conversely, particles having a relatively low fluctuation velocity (large, dense, rough, inelastic ones) migrate against the mean interparticle distance gradient, i.e., they move to flow regions with a large solid phase volume fraction. Similar distribution dynamics of particles differing in size and density are observed in rapid GM gravity flows on a vibrating rough chute (Fig. 5). In both cases, chaotically moving particles with a high fluctuation velocity migrate to the most rarified peripheral regions of the flow. Minimal concentrations of such particles are registered in the central part of the flow with a high solid phase concentration.

The described separation mechanisms suggest a strong dependence of the process kinetics on the structural and kinematic parameters of the shear flow, which means that the separation kinetics can be controlled by varying flow parameters, e.g., by vibration impacts. To this end, a special study was carried out to assess the influence of vibration on structural and kinematic properties of a rapid GM gravity flow and effects of nonuniform particle separation. The study included experimental and analytical characteristics of the solid phase volume fraction and control particle concentration distribution profiles of model materials on a vibrating rough chute at different vibration parameters. Mixtures of particles of different sizes (mixture 1) and densities (mixture 2) were used as model materials (see the Table for their characteristics).

The profiles were determined for the cases of high (HF) and low (LF) frequency vertical vibrations of the rough chute, as well as in the absence of vibrational oscillations. The chute vibration frequencies were chosen based solely on the vibratory drive technical characteristics at the vibrational acceleration $a_{\mathrm{v}}$ equaling $2g$.

The results of the study presented in Figs 6 and 7 illustrate the fundamental difference between vibrorheological effects of HF and LF vibrations in rapid GM gravity flows on a rough chute. A comparison of the solid phase volume fraction profiles observed in the flows on the vibrating steady chute gives evidence that HF vibrations promote structural uniformity of the layer in both model materials owing to a relatively high particle concentration in the flow. In contrast, LF vibrations increase structural inhomogeneity of the flow at a rather high value of the void volume fraction in the layer.

The above kinetic patterns of segregation and migration effects (19)–(21) give reason to conclude that HF vibrations form flow conditions favoring the predominance of particle size separation [63, 69]. LF vibrations create conditions for the intense migration (quasi-diffusional separation) of parti-

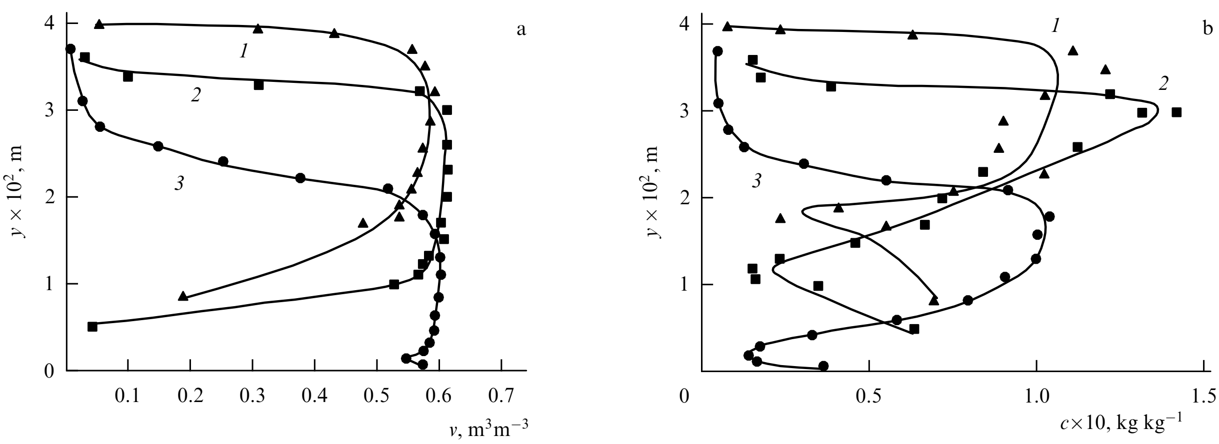


**Figure 6.** Experimental analytical profiles of solid phase volume fraction $v(y)$ and large particle concentration distribution $c(y)$ in a rapid gravity-driven flow of a binary mixture of particles differing in size (mixture 1, see the table) on a vibrating rough chute ($\alpha = 28.5°$, $a_{\mathrm{v}} = 2g$) at vibration frequencies of 15 Hz (*1*), 50 Hz (*2*), and 0 (*3*).

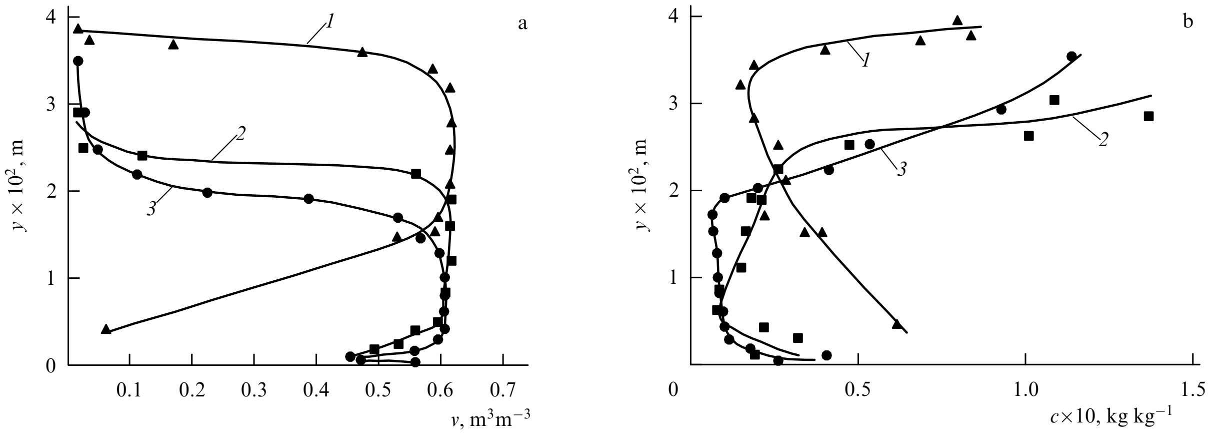


**Figure 7.** Experimental analytical profiles of solid phase volume fraction $v(y)$ and lower-density particle concentration distribution $c(y)$ in a rapid gravity-driven flow of a binary mixture of particles differing in density (mixture 2, see the table) on a vibrating rough chute ($\alpha = 27°$, $a_v = 2g$) at vibration frequencies of 15 Hz (*1*), 50 Hz (*2*), and 0 (*3*).

cles characterized by a different set of physicomechanical properties (size, density, roughness, elasticity) in the absence of the dominant influence of the size-based difference [51, 52].

These observations are confirmed by the shape of the control particle concentration distribution profiles $c(y)$ in flows of mixtures 1 and 2 (curves *2* and *3* in Figs 6 and 7). The prevalence of the segregation effect is most clearly manifested in particles of different sizes (see Fig. 6). In this case, HF vibrations give rise to a relatively wide plateau of high solid phase concentrations in the flow (curve *2*). The conditions for particle-particle interaction at the plateau facilitate intense shear flow separation (segregation) and lead to a monotonic rise in large-particle concentration in the direction of the layer free surface. A very similar increase in segregation intensity under the action of HF vibrations is observed in mixture 2 containing particles of various densities (curve *2* in Fig. 7). Here, as in mixture 1, the intense segregation effect takes place in the solid phase concentration region of the plateau formed by HF vibrations. However, due to less intense segregation of particles by density than by size, the concentration of light particles increases in the plateau region in the direction to the free surface with relatively low intensity. At the same time, the intensity of particle separation by density sharply increases in the flow region above the plateau directly adjacent to the layer free surface due to imposed effects of segregation and migration effects (quasi-diffusional separation). In this region, low-density particles migrate in the direction of the mean interparticle distance gradient that coincides with the direction of their segregation.

LF vibrations of the rough chute contribute to the formation of nonuniform particle concentration distribution profiles in the case of a predominance of the effect of their quasi-diffusional separation (migration). This inference is based on the fact that the driving force of migration is the mean interparticle distance gradient, directly dependent on the free volume fraction in the layer. For this reason, the direction of the solid phase volume fraction gradient is opposite to that of the interparticle distance gradient. In this case, the observed analogy between the shapes of the solid phase volume fraction and control particle concentration distribution profiles (the mirror analogy for less dense particles of mixture 2 in Fig. 7) serves to confirm the predominance of the migration effect under the influence of LF vibrations.

The observed analogy mentioned earlier in connection with an analysis of the results shown in Fig. 5 confirms the quasi-diffusion nature of the nonuniform particle separation effect at high solid phase volume fraction gradients. The effect consists of the counterflow motion of particles having different fluctuation velocities in flow regions with a void volume fraction (mean interparticle distance) gradient. Small, smooth, low-density particles acquiring a high fluctuation velocity in mutual collisions migrate toward the void volume fraction gradient present in the layer. Conversely, larger, dense, and rough particles having a relatively low fluctuation velocity travel in the direction of the solid phase volume fraction gradient. Importantly, some cases examined in the preceding paragraphs (see Figs 6 and 7) reveal alternative variants of nonuniform particle transverse motion along the layer height. This suggests the absence of direct correlation between the directions of separation fluxes and shear rate gradients. Particles of different sizes and densities in mixture 1 and 2 flows can travel both toward the shear rate gradient and in the opposite direction. The observed effect is in conflict with segregation model assumptions presented in [44].

To recall, all authors studying rapid gravity-driven flows report the observation of well apparent solid phase volume fraction gradients in both the top and bottom parts of the flows. Moreover, concentration gradients occur in the presence and absence of vibrational oscillations (Figs 5–7). They manifest themselves most strikingly in response to LF vibrations. The zone with a large void volume fraction in the lower part of the flow forms as the granular medium temperature grows exponentially in the direction of the layer bottom, in accordance with dependences (14) and (15). The reduction in hydrostatic pressure in the direction of the open surface of the flow in accordance with the equation of state (3) is accompanied by dilatancy, which leads to the formation of a rarefied zone near the layer surface. Because solid phase volume fraction gradients in these zones have opposite directions, the quasi-diffusional separation (migration) flows in the top and bottom parts of the layer are directed oppositely. This observation may be a hypothetical answer to the question of why not all but just a few large and low-density particles concentrate on the surface of the vibro-fluidized layer and provide one more explanation of the reverse Brazil nut problem (RBNP) [15, 71, 72].

An analysis of the observed phenomena leads to the conclusion that the migration of nonuniform particles due to the interaction and reciprocal quasi-diffusion is one of the main physical effects of particle separation under the conditions of irregular distribution of solid phase concentrations. The intensity of migration of particles differing in the totality of physicomechanical properties increases in a rapid thin-layer granular flow under the influence of LF vibrations of the rough chute.

The set of data on structural and kinematic flow characteristics that are possible to obtain by the described method can be used for mathematical simulation of nonuniform particle separation dynamics on a vibrating rough chute. Modeling separation dynamics of particles differing in terms of the totality of physicomechanical properties (size, density, roughness, elasticity) is possible [52] by integration of the differential equation that describes the relationship between elementary control component transfer flows, including those caused by convection, quasi-diffusional mixing, and separation (segregation (19) and migration (21)):

$$\frac{\partial(c\rho_{\rm b})}{\partial t} = -\frac{\partial(uc\rho_{\rm b})}{\partial x} + \frac{\partial}{\partial y}\left[\rho_{\rm b}\left(D_{\rm dif}\frac{\partial c}{\partial v} - cD_{\rm m}\frac{\partial \ln s}{\partial y} - K_{\rm s}c\Delta M\right)\right]. \quad (22)$$

The quasi-diffusional mixing coefficient $D_{\rm dif}$ in Eqn (22) is calculated in [52, 63, 69] as follows, taking account of the formal analogy to the molecular-kinetic theory:

$$D_{\rm dif} = \frac{1}{2}\, s\overline{V'}. \quad (23)$$

Coefficient of migration $D_{\rm m}$ is calculated as a function of particle properties and structural and kinematic characteristics of the flow using dependence (20). The segregation coefficient $K_{\rm s}$ in Eqn (22) is the sole kinetic characteristic determined experimentally as a constant by the method described in [52, 69]. The driving force of the segregation process $\Delta M$, a kinetic parameter in Eqn (19), is calculated as an excess moment of gravity, friction, and impact pulses acting on the control component particle in a shear flow of nonuniform particles [73].

The boundary conditions for Eqn (22) are formed on the assumption of the absence of transverse material fluxes at the upper ($y = h$) and lower ($y = 0$) boundaries of a moving particle bed:

$$D_{\rm dif}\frac{\partial c}{\partial y} = cD_{\rm m}\frac{\partial \ln s}{\partial y} = K_{\rm s}c\Delta M\big|_{y=0,\,h} = 0\,. \quad (24)$$

The initial condition in the general case can be given as

$$c(0, x, y) = f(x, y)\,. \quad (25)$$

Equation (22) with boundary (24) and initial (25) conditions is integrated numerically, making use of the Crank–Nicolson difference scheme. Figures 8, 9 present the results of modelling control component concentration profiles in rapid gravity flows of mixtures differing in particle size (mixture 1) and density (mixture 2, see the Table) on a vibrating rough chute. The modelling was performed on the initial condition corresponding to the uniform distribution of mixture components in the flow.

The results of modelling for the cases of HF and LF vibrations of the chute are presented in Figs 8 and 9 in comparison with the respective experimental results. The comparison shows the adequacy of all these data. They were assessed by calculating confidence intervals at a 95% level and comparing adequacy dispersions and reproducibility with the use of Fisher's F-test. The adequacy of the modelled concentration profiles confirms the validity of both the general separation dynamics equation (22) and the results of a determination of structural and kinematic parameters of the flow by the proposed experimental analytical method. Such a conclusion is justified, because the modelling of concentration profiles is based on the initial data, including particle velocity and volume fraction profiles in granular media flows on a vibrating chute.

## 7. Summary. Conclusion

Modern granular media physics is a central area of condensed state research focused on soft condensed matter [5]. It studies materials comprising macroscopic particles in which atoms are arranged in macroscale structures significantly larger than individual atoms but much smaller than the total volume of the bulk material. Characteristics of GM as large enough assemblages of macroscopic particles give reason to regard

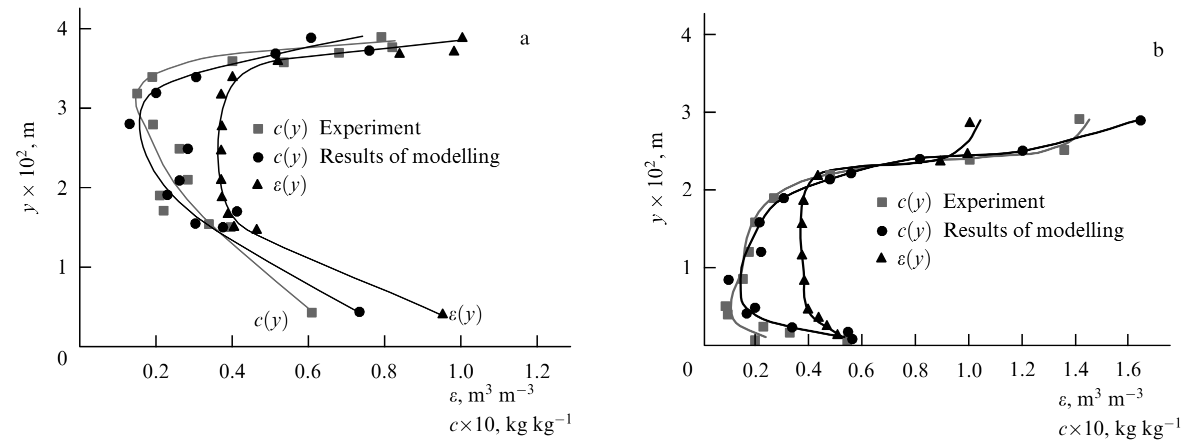


**Figure 8.** Void fraction $\varepsilon(y)$ and concentration distribution $c(y)$ profiles of low-density silicagel granules in the gravity-driven flow of a mixture of uniformly-sized bead and silicagel granules (a) at LF chute vibrations ($\omega = 15$ Hz, $a_{\rm v} = 2g$), (b) at HF chute vibrations ($\omega = 50$ Hz, $a_{\rm v} = 2g$).

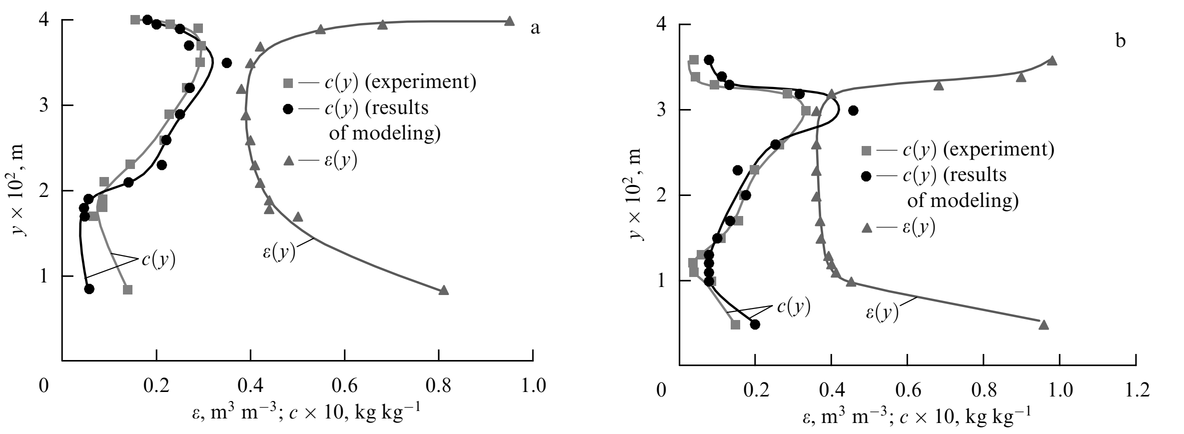


**Figure 9.** Void fraction $\varepsilon(y)$ and concentration distribution $c(y)$ profiles of large particles in the gravity-driven flow of a mixture of bead granules (a) at LF chute vibrations of ($\omega = 15$ Hz, $a_v = 2g$), (b) at HF chute vibrations ($\omega = 50$ Hz, $a_v = 2g$).

them as an additional state of matter. The unique mesoscopic properties of GM allow it to be identified as a solid, gas, or liquid, depending on the dynamics of the surrounding conditions [1].

Granular materials are characterized by the following typical properties [5]: disorder and heterogeneity combined with a tendency to structuring, marked potential for dissipation, manifestly nonequilibrium state, and well apparent nonlinear reaction to external force impacts. The diversity of properties accounts to a great extent for the lack of general consensus as regards what particle assemblage can be adequate for GM as a whole in terms of properties and for the absence of adequate statistical mechanics for the description of such materials [5].

Approaches to the study of crystalline materials, liquids, and the majority of other forms of soft condensed matter are inapplicable to GM research. The unusual behavior of granular matter not infrequently makes it difficult to associate it with a variety of other materials.

An analysis [1] shows that the unique properties of GM are due to two main features of particle interaction, viz. dissipative effects of static and dynamic friction in inelastic collisions and the absence of a pronounced temperature dependence. The latter feature hampers GM analysis from the point of view of classical thermodynamics.

One of the most striking manifestations of unusual GM properties is exemplified by the effects of particle size separation, e.g., in rotating and vibrating containers. Due to the absence of adhesive forces between the particles under such conditions, they must intermix in the container in accordance with the entropy principle. The observed separation suggests the predominance of dynamic effects in GM that cause its systems to deviate from thermodynamic equilibrium.

The phenomenon of nonuniform particle separation in granular media is a subject of undying interest as far as its kinetics and mechanisms are concerned. The effects of mixing and separation of nonuniform particles under the influence of vibrational impacts on granular media and their shear flows have for many years been in the focus of research activity [9–25]. However, the general theory of particle separation remains to be developed. Moreover, discussions regarding the physical nature of certain particle separation effects continue into the present period of time, as exemplified by debates about the mechanism behind the floating up of a large particle regardless of its density in a layer of smaller particles under the action of vertical vibrational oscillations. Today, this phenomenon is commonly called the Brazil nut effect [14]. Moreover, there is no proper understanding of the causes that prevent the emergence of some large particles on the surface of the vibrating layer in accordance with the reverse Brazil nut effect.

The mesoscopic scale of GM constituent elements does not permit regarding their flows as homogeneous or using classical continual models for their description. Shear deformations are accompanied by pronounced dilatancy of the flow, depending on particle properties and shear rate. This accounts for the use of two idealized models for the description of GM flows, viz. the slow shear flow model and the rapid shear flow model. Idealization implies that a slow shear flow originates from irreversible quasi-static plastic deformations. In rapid flows, granular medium particles move while exchanging impact pulses at relatively high dilatancy values. The specific dynamics of particle interaction in a granular medium under rapid shear suggest their formal analogy to molecular gas dynamics. For this reason, a rapid shearing flow of a granular medium is frequently referred to as 'solid particle gas' [38].

Many modern authors studying separation in granular medium flows emphasize the difficulty accessing comprehensive and reliable data on structural and kinematic flow parameters. Therefore, their conclusions are not infrequently based on information about flow dynamics obtained in virtual experiments with the use of the finite element method. An analysis of the known mechanisms of particle separation gives evidence that the overwhelming majority operate on the basis of an exclusively local inhomogeneity of granular media. However, the analogy between the granular medium and the dense gas makes it possible to predict separation effects arising from heterogeneity of structural and kinematic parameters of the medium. Indeed, a granular medium in the state of an inhomogeneous solid particle gas must produce separation effects formally analogous to molecular thermobaric diffusion. This hypothesis is confirmed by the results of analyses of separation kinetics and mechanisms rooted in the local and spatial inhomogeneity of the granular medium composition and structural and kinematic parameters of its shear flow.

An analysis was conducted using experimental and theoretical data on the profiles of velocity, solid phase volume fraction, and nonuniform particle concentration distribution in rapid GM gravity-driven flows on a rough chute. The profiles were determined for mixtures of particles of different sizes and densities with the use of the experimental analytical method based on the formal analogy between a granular medium under rapid shear and a dense gas. This physical analogy establishes the relationship among dilatancy, quasi-hydrostatic (lithostatic) pressure, and GT. Granular temperature is defined as a constituent component of the particle kinetic energy related to different forms of relative motion of particles. Four elementary forms of relative particle displacements resulting from gravitational and vibrational impacts have been considered: chaotic fluctuations, transverse mass transport, relative shear, and vibrational displacements.

To obtain information for an analysis of the prognostic properties of different segregation mechanisms, results of mathematical simulation of the process in a rapid GM gravity-driven flow on a vibrating chute were used. The results of modelling were obtained using the general segregation dynamics equation, taking into account separation fluxes arising from both medium inhomogeneity under the local conditions of particle-to-particle interaction and spatial nonuniformity of structural and kinematic parameters of the flow.

A separation flux caused by local heterogeneity of the granular medium (traditionally called segregation) is described in terms of the mechanism of separation in a shear flow. In accordance with this mechanism, each particle of an inhomogeneous medium becomes a stress concentrator in a shear flow. The system tends to relax the stress condition thus induced and makes the particle move to the flow region, where relaxation occurs. The degree of stress concentration determining the separation (segregation) rate is considered to be the driving force of the process. The degree of stress concentration is defined as an excess moment (either positive or negative) of forces of gravity, friction, and impact pulses acting on the control particle, as opposed to the moment of analogous forces acting on a particle of a conventionally homogeneous medium. Such an approach to the expression of the driving force of segregation (taking account of the modulus and sign of the excess moment of force) makes it possible to predict the process speed for particles differing in the totality of physicomechanical properties (size, density, roughness, and elasticity). Local structural and kinematic parameters of the flow (shear rate and solid phase volume fraction) are taken into consideration in driving force calculations. The physical properties of the particles of a conventionally homogeneous medium are determined as volume-averaged quantities, depending on their local concentration in the flow.

In accordance with the mechanism of separation due to the spatial nonuniformity of structural and kinematic parameters of a shear flow, separation of nonuniform particles occurs owing to different speeds of their quasi-diffusional displacements [51, 52]. Under the structural heterogeneity conditions of the flow, the differences among the velocities of chaotically moving particles give rise to a quasi-diffusional separation flux (migration). By this mechanism, formally analogous to molecular thermobaric diffusion of particles, those capable of high-velocity chaotic motion (small, smooth, and low-density) migrate into the flow region containing a large void volume fraction. Conversely, large, dense, and rough particles having a relatively low velocity of chaotic motion concentrate in the region with a large solid phase volume fraction.

Separation mechanisms underlain by local and spatial inhomogeneity of the granular flow provided a basis for the development of a general mathematical model of the process that includes not only separation fluxes but also convective and quasi-diffusional mixing fluxes. The segregation coefficient is the only experimentally determined kinetic constant of the model [52, 69].

Results of mathematical modelling and experimental studies of separation dynamics under the conditions of marked structural nonuniformity of GM flows on a vibrating rough chute confirm the proposed hypothesis of the process mechanism. Experimental and calculation data were integrated to analyze profiles of velocity, solid phase concentration, and distribution of concentrations of particles differing in size, density, along the layer height on the vibrating rough chute. The results of the analysis led to the following conclusions.

The marked heterogeneity of the granular flow microstructure caused by low-frequency vibrational oscillations is responsible for the quasi-diffusional separation (migration) of nonuniform particles according to size and density, without predominance of the former. Because solid phase concentration gradients in the top and bottom parts of a gravity flow are directed oppositely, nonuniform particle migration flows are directed either to the central region of the layer or to its peripheral part. Therefore, extreme concentrations of nonuniform particles are observed in the central region of the layer or simultaneously near its base and free surface. This mechanism of formation of the nonuniform particle concentration field may serve as a hypothetical explanation of such physical phenomena as the reverse Brazil nut effect.

The observed effects of nonuniform particle separation give evidence of the absence of a direct correlation between the directions of separation fluxes and the shear rate gradient. Particles of various sizes and densities can travel in a flow both toward the shear rate gradient and in the opposite direction, at variance with model assumptions of segregation, suggesting a proportionality between the driving force of the process and the shear rate gradient.

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