Interaction of a wave with an accelerating object and the equivalence principle

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Abstract. The paper presents a new view of the so-called accelerating matter effect. It was stated earlier that the effect is optical by its nature and is essentially a change in the frequency of the wave transmitted through a refractive sample that is moving with acceleration. But as follows from a simple consideration based on the equivalence principle, the opinion that the effect is related only to the phenomenon of refraction is unjustifiably narrow, and the change in frequency must appear during scattering by any object moving with acceleration. Such an object can be an elementary scatterer or any device that transmits a narrow band signal.

The first brief mention of the effect that later was dubbed the accelerating matter effect (AME) was by all probability made in the thesis by Mikerov [1]. Considering the possibility of filling a trap for ultracold neutrons (UCNs) without depressurization, he proposed using a membrane moving harmonically, co- and counterpropagating with the UCNs. He found that the energy of UCNs should change after they pass through the oscillating membrane. Being unpublished, this result remained unknown for a long time.

In 1982, Tanaka [2] proposed a solution of the problem of electromagnetic wave transmission through a linearly accelerating dielectric plate and predicted that the frequency of the wave transmitted through such a sample differs from the frequency of the incident wave. For a single wave transmission through the plate, the change in its frequency is expressed as

$$\Delta\omega \cong \frac{\omega a d}{c^2}\,(n-1)\,, \qquad \frac{ad}{c^2} \ll 1\,. \tag{1}$$

Here, ω is the incident wave frequency, n is the refractive index, d is the plate thickness, a is the acceleration, and c is the speed of light. The possibility of observing this optical effect was discussed in [3], but, to the best of our knowledge, the relevant experiment has not been conducted because of the smallness of the effect.

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In 1993, a paper by Koval'sky [4] appeared, aimed at proposing a test of the equivalence principle in a neutron experiment of a new type. His theoretical approach was based on considering the neutron phase and group velocities and on computing the time it takes a wave to propagate between the respective points in a laboratory reference frame and a reference frame moving with an acceleration. In both cases, the wave was transmitted trough a refracting sample. As an intermediate result, the author concluded that the energy of a neutron changes after it crosses the sample moving with a moderate acceleration. The change in energy was expressed as

$$\Delta E = mad\left(\frac{1-n}{n}\right)\,, \tag{2}$$

where m is the neutron mass.

Later, this same question was considered by Nosov and Frank [5]. Their analysis, dealing with rigorous computation of the neutron velocity as it enters the sample, moves inside it, and escapes through the other surface, in fact relied on the classical approach. For the magnitude of the effect, the authors obtained a formula that agreed with Koval'sky's result (2).

We note that formula (2) was obtained in Ref. [5] under two important assumptions. It was assumed, first, that quantum effects are small, and second, that the motion of matter in which the neutron wave propagates does not influence the result, and therefore all the change in the wave function is due to moving sample boundaries.

The first brief announcement on the experimental observation of the change in neutron energy on passing through an accelerating sample appeared in 2006 [6]. Very soon, the experiment was repeated under improved conditions [7]. In both cases, the change in the energy of UCNs was measured as they traversed an oscillating sample. Time-varying sample acceleration was 60–70 m s^{-2}. For the energy transfer, found to be $(2-6) \times 10^{-10}$ eV, good agreement was obtained with formula (2).

Later, the acceleration and deceleration of neutrons passing through an oscillating plate made of refracting material were observed in an experiment sensitive to neutron velocity [8]. The results were also in satisfactory agreement with the predictions.

The question of the interpretation of the physical effect had a certain development. An optical approach to the description of its nature, going back to Ref. [2], naturally invoked the Doppler frequency shift under wave refraction into a moving medium [9]. The frequency of a neutron wave in a moving medium, measured in the laboratory reference frame, is

$$\omega_i = \omega_0 + (n' - 1)k_0 V\,, \qquad V \ll v_0\,, \tag{3}$$

where ω_0 and ω_i are the frequencies of the incident and refracted waves, n' is the wave refractive index in the reference frame of the moving sample, v_0 and k_0 are the velocity and wave number of the incident wave, and V is the velocity of the sample and its boundary.

If a neutron wave passes through a uniformly moving layer of material, the Doppler frequency shifts accompanying the wave transmission through its two boundaries are equal in magnitude but opposite in sign. The net frequency change equals zero in this case. For accelerated motion, the frequency shifts on the entry and exit surfaces are different, because during the time the wave propagates through the sample the boundary velocity acquires an increment $\Delta V = a\tau$, where a is the acceleration and τ the propagation time through the sample. In our case of a refracting sample, the difference in frequencies between the incident and transmitted waves is

$$\Delta\omega = ad\,\frac{1-n}{n}\frac{k_0}{v_0} = \frac{mad}{\hbar}\frac{1-n}{n}\,, \qquad V, at \ll v_0\,, \qquad (4)$$

and the energy change $\Delta E = \hbar\Delta\omega$ is given by (2).

Relatively recently, a change in the energy of cold neutrons passing through an accelerating sample was observed under conditions close to those of Bragg reflection [10, 11]. Explaining the nature of the effect, the authors resort to arguments that are close to those above. It seems obvious that the effect of energy change observed in these studies has essentially the same nature as the AME under refraction, although the details differ in many respects.

The optical approach to describing the effect proved to be fruitful and was later generalized to doubly refracting materials. In the case of neutron optics, the phenomenon of double refraction, well known in traditional optics, comes from the spin dependence of the refractive index. It was shown in [9] that as a neutron passes through doubly refracting matter moving with an acceleration, a nonstationary state develops with a precessing spin. A similar situation can occur when a two-component neutrino propagates through a layer of accelerating matter. In this case, the wave function of the resulting state changes such that the character of neutrino state evolution in its subsequent propagation in free space is essentially modified.

A conclusion on the existence of the AME, however, can be made without resorting to concrete optical computations, relying only on the equivalence principle (EP). In order to show this, the authors of Ref. [7] turned to the thought experiment by Koval'sky [4], but, in contrast to that author, relied on the idea that EP is certainly valid. We repeat their arguments here, for this is important for what follows.

Figure 1 depicts the case where a neutron moves from its source to a detector, accelerating under the action of gravity. Apparently, the neutron energy at the detection point exceeds its initial energy by the quantity mgH, where H is the difference between the heights of the source and detector, and g the acceleration of gravity. The placement of a refracting sample in the neutron path does not change this conclusion. We next turn to the case shown in Fig. 1b. Here, the source, the refracting plate, and the detector move all together with the acceleration $a = g$, being in a noninertial coordinate system, while the neutron motion observed in the laboratory frame is uniform.

By virtue of the EP, the results of all measurements in the noninertial system should be the same as in the inertial one with gravity, which is to the full extent also related to the
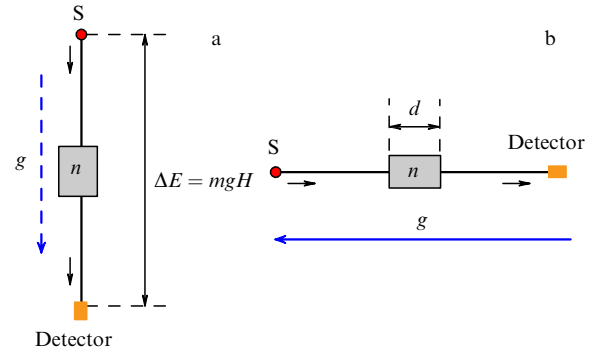
Figure 1. Neutron passage through a refracting sample (a) in an inertial frame in the presence of gravity and (b) in a noninertial frame.

energy measured by the detector, and hence the change in the neutron energy on the path from the source to the detector should equal mgH, as previously. In the trivial case where there is no sample, the statement above does not lead to any doubts. However, the appearance of a refracting plate moving with the same acceleration as the detector leads to the delay in the neutron passage time because the speed of a neutron in matter is smaller than in the vacuum (for definiteness, we assume $n < 1$). Disregarding a small difference in Δt for a resting and accelerating sample, we obtain

$$\Delta t = \frac{d}{v_0}\frac{n-1}{n}\,, \qquad (5)$$

where d is the sample thickness. During the delay time Δt, the detector continues to accelerate. The additional increment in its velocity compared to the case of a free neutron is $\Delta v = a\Delta t$.

Thus, if the role of the sample were reduced to only the time delay, then, at the moment the neutron reaches the detector, its velocity relative to the detector and hence the measured energy would be different from the values in the absence of the sample, which contradicts the EP. Thus, when the neutron passes through the accelerating sample, the time delay should be accompanied by a change in neutron energy so as to compensate the detector acceleration during the time Δt. Expressing this quantity as $\Delta E = mva\Delta t$, we arrive at formula (2).

Similar reasoning can also be used for photons. The time delay because of light refraction in a medium is

$$\Delta t = \frac{d\,(n-1)}{c}\,, \qquad (6)$$

where c is the speed of light. The Doppler frequency shift caused by an accelerating detector is, in the first order in v/c,

$$\Delta\omega \approx \frac{\omega\Delta v}{c} = \frac{\omega a\Delta t}{c}\,. \qquad (7)$$

Inserting (6) into (7), we obtain formula (1) by Tanaka.

Thus, relying solely on the EP and ignoring relativistic as well as nonstationary quantum effects, we can obtain an expression for the magnitude of AME that coincides with the results of more rigorous analysis for both light and a slow particle. Formula (7) written in the form

$$\Delta\omega = ka\Delta t\,, \qquad (8)$$

where k is the wave number, is valid in both cases.

We stress that in the consideration above we arrived at the conclusion on the change in the frequency of a wave

transmitted through an accelerating sample relying only on the assumption that the wave propagation times in the sample and in the vacuum are different, but in no way on the assumption that this difference occurs because of refraction in the sample matter. As a consequence, the conclusion stays valid if instead of a refracting sample we use an object moving with acceleration and transmitting a signal with delay.

The variety of such objects is immense. It is known, for example, that in quantum mechanics interaction is unavoidably connected to a time delay, which in the first approximation is described by a so-called group delay time (GDT)[1] [13–15]

$$\tau = h \, \frac{\mathrm{d}\varphi}{\mathrm{d}E} \,, \qquad (9)$$

where φ is the phase of the amplitude of interaction, e.g., scattering, and E is the energy. Thus, any elementary scatterer moving with acceleration should change its frequency; this change occurs in addition to the conventional Doppler effect and is proportional to the acceleration instead of velocity.

Any signal transmission involves a time delay; therefore, a device moving with acceleration and receiving a wireless signal would then transmit it at a different frequency. Such a device can be a receiver/transmitter of a radio or acoustic signal or a fiber optic line presenting an alternative to the refracting plate considered by Tanaka. It is only important that the element receiving the signal, i.e, the receiver, move with the same velocity as the transmitter. Their role can be played by a receiving/transmitting antenna or simply an end of a waveguide or fiber line.

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[1] It has long been referred to as the phase delay time.