

Evolution of close binary stars: theory and observations

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Abstract. We review modern concepts in the physics and evolution of close binary stars. The review is based, on the one hand, on numerical simulations of the evolution of their components and the processes that accompany that evolution and, on the other hand, on the entire set of observational information in all ranges of electromagnetic and gravitation-wave radiation. These concepts underlie modern astrophysics, the most extensive laboratory wherein the properties of matter in the Universe and the Universe itself are explored. We present the modern picture of the evolution of close binary stars, constructing which has been driving progress in the physics and evolution of astronomical objects for the last 50 years.

Keywords: binary stars, observational data, theoretical models, explosive processes, evolution, neutron stars, degenerate dwarfs, black holes, gravitational waves

1. Introduction

At the beginning of the previous century, in 1928, S I Vavilov began his preface to the papers by J Jeans and A Eddington published in Russian under the common title *Sovremennoye razvitiye kosmicheskoi fiziki* (The Modern Development of Space Physics) [1] with the following words: “The development of astrophysics in recent years should be of immense interest not only for astronomers but also for everyone involved in natural science, and primarily physics.” It should be acknowledged that these words remained extremely relevant and valid throughout the 20th century. The beginning of the new century, marked by the discovery that planets are a routine phenomenon in the stellar world, by understanding the role of gravitational waves in the evolution of close binary stars, and by the detection of events in which neutron stars and stellar black holes merge, confirmed that Vavilov’s words were not only as valid as before but also gained new importance.

The study of the physics and the evolution of binary stars plays a special role in modern astrophysics. Over the last 50 years, the analysis of the development of our concepts of astronomical objects and the phenomena accompanying their evolution shows that progress has been largely driven by designing a modern picture of the physics and the evolution of binary stars. The presence of a close satellite to a star does not complicate studying the evolution of close binaries; on the contrary, as the experience of astrophysicists of many countries collected over the past several decades shows, it enables transforming such stars from the object of study into an effective research tool. The high efficiency and performance of this tool is ensured by the interactions among close components, which enables them, in the course of evolution,

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to ‘exhibit’ their structure and the main forces that drive that evolution to the observer. The scenario-based approach developed in studying the evolution of close binary stars (CBSs) allows the observational data and theoretical model information on the physics and the evolution of these stars to be used in the most efficient way to construct the most comprehensive picture of their evolution, starting from the gravitational collapse of gas–dust clouds to the formation of final systems of compact objects: degenerate dwarfs, neutron stars, and black holes. The scenario-based approach is not only the most effective means to synthesize observational and theoretical information about the evolution of the objects under study but also a very productive technique to pose new conceptual questions, whose solution is necessary for progress in exploration of the physics of astronomical objects. Basically, this method can be traced back to the Darwinian method of studying the evolution of Earth’s biological tree of life based on the observed kaleidoscope of its manifestations.

The successes of modern astrophysics in the study of the physics and evolution of CBSs of various classes have been awarded with several Nobel Prizes in physics and presented in a number of comprehensive reviews and books. The Nobel Prizes were won for the discovery of neutron stars, whose rapid rotation is probably due to their predecessors’ belonging to CBSs. A secular decrease in the orbital period of tight systems containing radio pulsars as components was a signal that binary stars emit gravitational waves. The merger of stellar black holes in the astronomical area of experiments conducted by LIGO (Laser Interferometer Gravitational-Wave Observatory) [2] confirmed the validity of theoretical ideas regarding the evolution of the most massive CBSs. The assumption about the standard features of type-Ia supernovae, the probable products of mergers of degenerate carbon–oxygen components of CBSs, has enabled questioning the standard model of kinematic inertial expansion of the Universe [3]. We note, however, that, as is shown below, the ‘standard’ features of these supernovae remain ‘under suspicion’ because their nature and the properties of a particular model have not been finally clarified.

Owing to this impressive series of successes in the study of the physics and the evolution of CBSs, the work of several hundred astrophysicists from dozens of countries worldwide who study CBSs has become one of the most popular, productive, and promising areas of modern physics and astronomy. The results of this work, which are the subject of many books and extensive reviews, remain in the focus of attention of many symposia devoted to problems in astronomy. The goal of this review, which encompasses the physics and the evolution of binary stars of various masses, is to present the main results of this actively developing field of astrophysics to a broad community of physicists and astronomers. Special attention is paid to describing and analyzing the latest stages of the evolution of binary stars associated with the formation of compact remnants of their components: degenerate dwarfs, neutron stars, and stellar black holes. The merging of such components of binary stars under the effect of gravitational-wave radiation gives rise to a number of high-energy phenomena of stellar astrophysics: supernovae, gamma-ray bursters, etc. [4–7].

This review, perhaps, is not entirely complete, given that the problem of CBSs is very involved and multifaceted, while the scope of the review should have reasonable limits. In this regard, we apologize to the readers and refer them to the original articles presented in the list of references.

2. Main factors determining the structure and evolution of binary stars

The hydrostatic equilibrium of a star is maintained by the equality between gravitational forces and the total gas and radiation pressures. For degenerate dwarfs, the pressure of the degenerate electron gas plays the role of a force that counteracts gravity. The energy equilibrium in a nondegenerate star is maintained by the release of nuclear energy in its core and radiative energy losses. A large temperature gradient develops in cold stars with high absorption of matter, causing the formation of convective envelopes. Degenerate stars or their envelopes that can no longer maintain equilibrium between the generation of nuclear energy and its dissipation explode when the self-accelerating release of nuclear energy commences. This is the mechanism of explosions of novae, type-Ia supernovae, and gamma-ray bursters. A sequential change in the sources of nuclear energy in the massive-star core from hydrogen to silicon supports the star evolution until the final products are formed: a degenerate dwarf in the case of the initial mass of the star in the range $0.8M_{\odot}–8M_{\odot}$, a neutron star for initial masses $8M_{\odot}–30M_{\odot}$, and a black hole for large initial masses of main-sequence stars ($30M_{\odot}$ —about $150M_{\odot}$).

We now consider the main forces that control the distance between the components during the evolution of binary stars. Tidal friction forces ensure the exchange of angular momentum between the axial and orbital rotations of the components. The loss of mass by the components due to the stellar wind results in an increase in the semimajor axis a of the orbit of the systems in accordance with the relation $(M_1 + M_2)a = \text{const}$. The exchange of matter between components of conservative systems brings them closer to each other if the smaller-mass component accretes, and moves them further apart if the larger mass component accretes: $M_1 M_2 a^{1/2} = \text{const}$ if $M_1 + M_2 = \text{const}$. In addition to the external factors that change the angular momentum of a binary system (for example, collisions with field stars or interstellar-gas accretion), there are two more intrinsic efficient mechanisms of angular momentum loss: radiation of gravitational waves by the system and magnetic stellar wind of the components. Gravitational waves are of importance only for systems with small semiaxes of the orbits: $a/R_{\odot} \lesssim 3(M/M_{\odot})^{3/4}$, where M are the masses of comparable components. The magnetic stellar wind is essential only for systems that contain main-sequence stars with the mass $0.3M_{\odot}–1.5M_{\odot}$ for $a/R_{\odot} \lesssim 8(M/M_{\odot})$, where M is the mass of the components; notably, this probably fully determines the evolution of WUMa-type contact systems, cataclysmic binary stars, and X-ray binary stars with donors that are main-sequence stars with a small mass and relativistic accretors.

It is worth recalling the characteristic time scales of the driving forces listed above that control the evolution of binary stars. Nuclear time scales for the evolution of components are determined by the stock of nuclear energy and star luminosity. For $M_{\odot} \leq M \lesssim 100M_{\odot}$, the stars remain at the main-sequence stage for $\tau_N \approx 10^{10}(M_{\odot}/M)^{1.8}$ years, the duration limited by the energy of hydrogen conversion into helium, $\sim 6 \times 10^{18} \text{ erg g}^{-1}$, and star luminosity. Helium burning at the stage of giants or stars on a horizontal branch lasts for about one tenth of that time. The subsequent path to the iron cores of the most massive stars takes much less time due to the effective ‘discharge’ of the energy of the hot core of the star

due to neutrinos. The thermal time scale for star evolution is obtained by dividing its gravitational (thermal, due to virial equilibrium) energy by the luminosity L to yield $\tau_T \approx 3 \times 10^7 (M/M_\odot)^2 (R_\odot/R) L_\odot/L$ years, where R is the star radius. The time of the gravitational collapse of a supernova core, which begins when the gas cloud pressure weakens, is $\tau_G = (R/R_\odot)^{3/2} (M_\odot/M)^{1/2}$ hours. Gravitational collapse accompanies the formation of galaxies, stars, neutron stars, and stellar black holes.

In addition to the main time scales and processes noted above, which apply to almost all stars, several processes and scales can be indicated that only relate to certain groups of stars and systems, including a change in the star mass due to a loss of matter by the stellar wind or its accretion by a CBS component: $\tau_M \simeq M/\dot{M}$, where \dot{M} is the rate of accretion or gas loss. For the Sun, this time is $\sim 10^{14}$ years, a value that excludes the solar wind from among the factors of importance in the evolution of single stars. However, for massive stars with a solar abundance of metals, the time τ_M for $M \gtrsim 30M_\odot$ becomes comparable to the nuclear evolution time ($\sim 5 \times 10^6$ years) [5]. The evolution of red supergiants that precedes the formation of degenerate dwarfs and explosions of massive ($M \gtrsim 8M_\odot$) supernovae is also entirely determined by the stellar wind. Accretion is important at the stage of star formation and for the components of close binaries that actively exchange matter.

Three factors have been found for CBSs owing to which these systems efficiently lose orbital angular momentum: common envelopes, the magnetic stellar wind of components, and emission of gravitational waves by the tightest systems. These factors must be taken into account in analyzing the formation and describing the evolution of extremely close systems of compact final remnants that emerge from the evolution of components: degenerate dwarfs, neutron stars, and stellar black holes. The first of these factors is the common envelopes that emerge whenever the donor component loses matter at a rate greater than the accretor satellite is able to accept without filling up its Roche lobe. The physics of the common envelopes remains poorly studied. Today, the evolution of the system at this stage can be divided into two phases. At the first phase, the donor creates a common envelope of the system, filling it with the gas of its own envelope, usually according to its thermal time scale. As the density of the common envelope at the accretor orbit level increases to a value of the order of the average density of matter in this system ($\sim M/a^3$, where a is the semimajor axis of the orbit), the second phase begins, during which the orbital energy of the system ‘converts’ into acceleration and loss of matter in the common envelope, now on the dynamic time scale of the system (common envelope: $\sim a^{3/2} G^{-1/2} M^{-1/2}$). This time is $\sim (a/R_\odot)^{3/2} (M_\odot/M)^{1/2}$ hours. The rate of the common-envelope matter lost as a result turns out to be of the order of the orbital velocity of the system: $\sim 300(M/M_\odot)^{1/2} (R_\odot/a)^{1/2}$ km s $^{-1}$. The final semimajor axis of the system that now consists of a donor core and a satellite can be estimated using the condition of its orbital energy expenditure to the loss of the common envelope [8]:

$$a_f = \frac{M_2 M_n}{M^2} a_0, \quad (1)$$

where M_2 is the satellite mass, M_n is the donor-core mass, M is the initial donor mass, and a_0 is the initial semimajor axis of the system.

The very event of losing the common envelope in the tightest systems can manifest itself as a powerful explosion such as a ‘red nova’. Such systems lose their envelope with a mass of about M_\odot over a time of about a week to a month. The disintegration of the common envelopes of CBSs indicates the need for a careful search for the observable manifestations of burst phenomena in the world of stars with energies of $10^{44} - 10^{50}$ erg, i.e., energies that are intermediate between the burst energies of novae and supernovae [8]. Some of these stars were called ‘red novae’ [9, 10]. An impressive list of dozens of such events [11] shows that the characteristic time of their bright phase varies from several days to several years. We note that these values coincide with the characteristic dynamic times of the common envelopes indicated above. The expected frequency of disintegration of the common envelopes of binary stars in the Galaxy is about one event per year [4]. As a result, the emergence of a noticeable part (up to a half) of planetary nebulae may be due to the disintegration of the common envelope by a binary core. It has been established recently that, indeed, a noticeable fraction of planetary nebula cores are binary stars. We note that the expected expansion rate of the decay products of the common envelope is of the order of the orbital velocity of the binary that forms it ($v \approx 300(M/M_\odot)^{1/2} (R_\odot/a)^{1/2}$ km s $^{-1}$) and can significantly exceed the expansion rate of the envelopes of ordinary planetary nebulae. This mechanism ‘facilitates’ their identification, although it significantly reduces their lifetimes [9–11].

The η Car burst stands out among the bright transient phenomena known to astronomers. The bright phase lasted for about 20 years, after which, over the same period of time, the brightness decreased to a fraction of the pre-burst value [10]. In the last 100 years, the brightness has been returning to its original value. Interpreting this phenomenon as a dump of the common envelope of a massive close binary, we can estimate that the envelope mass is about $50M_\odot$ and its expansion rate is $\sim 10^8$ cm s $^{-1}$. The dimming that has followed the η Car outburst can be explained by the formation of dust in its ejected expanding envelope. Because the opacity of dust is 100 times greater than that of hot ionized gas, the duration of the dimming phase is about 10 times longer than the duration of the bright phase of the outburst, i.e., it lasts 200 years. The duration of the observed phase of brightness recovery is of the same order. Thus, it cannot be ruled out that the η Car activity over the last 200 years is a manifestation of the dumping of the common envelope of a very massive CBS. Simulation of the dumping of the common envelope and its observable manifestations remains a challenging and yet unresolved problem. This task is complicated by the need for three-dimensional modeling of gas dynamics and radiation transfer.

On several occasions in this review, we discuss situations where one of the components or the system as a whole dumps an envelope whose mass ranges from $10^{-6}M_\odot$ for nova envelopes to several tens of M_\odot for the common envelopes of massive binaries and supernova envelopes. To an external observer, such a dump looks like a radiation burst with a certain temperature T_e . The scale of the observable manifestation of such a burst can be estimated within an extremely simple single-zone homogeneous model. The expanding envelope luminosity can be represented as $L \simeq 4\pi\sigma T_e^4 R^2$, $R = vt$, where R is the envelope radius and t is its age. The luminosity increases until the envelope remains opaque, to reach a maximum at $\tau_{\text{opt}} = \kappa\rho R \approx 1$, where κ is the opacity,

$\rho = 3M/(4\pi R^3)$ is the average density of the envelope, and M is its mass. As a result, the envelope reaches its maximum brightness $L_m = 3\kappa\sigma T_e^4 M$ when its age is $\tau \simeq R/v$. Taking $\kappa \simeq 1 \text{ cm}^2 \text{ g}^{-1}$, we find $L_m/L_\odot \simeq 10^8 T_3^4 M/M_\odot$, where T_3 is the envelope radiation temperature in thousands of kelvins. To describe the observed optical outbursts in the Kashi list [11] with energies of $10^{44} - 10^{50} \text{ erg}$, which last from several days to several years, it is sufficient, for T_e of several thousand kelvins, to assume that the mass of the envelopes dumped in the evolution of binary and single stars ranges from $10^{-6} M_\odot$ to several M_\odot . Although the proposed model is extremely simple, it allows clarifying the nature of the light curve of any outburst associated with the dump of the envelope: novae, common envelopes, and supernovae.

Another time scale arises for systems in which one of the components is a main-sequence star with a mass $0.3M_\odot - 1.5M_\odot$ [4]. The boundary values for the mass are confirmed by observing the rotation speeds of FGKM dwarfs [12]. Stars with such masses consist of a convective envelope and a radiative core. Convection generates a stellar wind similar to the solar wind, and due to the magnetic field it becomes an efficient factor in the loss of the angular momentum by the star. In the tightest systems ($a \lesssim 10R_\odot$), as a result of spin-orbit interaction, this factor becomes the leading evolutionary mechanism with a characteristic time [4]

$$\tau_{\text{MSW}} \simeq 3 \times 10^6 \left(\frac{a}{R_\odot} \right)^5 \frac{M_1}{M_\odot} \left(\frac{R_\odot}{R_2} \right)^4 \left(\frac{M_\odot}{M_1 + M_2} \right)^2 [\text{year}], \quad (2)$$

where a is the semimajor axis of the system, M_2 and R_2 are the donor mass and radius, and M_1 is the accretor mass. Estimate (2) is based on Skumanicz's empirical relation [13] that describes the dependence of the rotation speed of solar-type main-sequence stars on their age. The physical 'content' of relation (2) is still unclear. The evolution of cataclysmic systems and low-mass X-ray binaries occurs according to this time scale.

And finally, the time scale that is the most popular today is the characteristic time for the components to merge under the effect of gravitational-wave radiation by their systems:

$$\tau_{\text{RGW}} = 10^8 \left(\frac{a}{R_\odot} \right)^4 \frac{M_\odot^3}{M_1 M_2 (M_1 + M_2)} [\text{year}]. \quad (3)$$

The merger of binary black hole components and neutron stars detected in the LIGO experiment boosted the popularity of this relation, discovered by Einstein [2, 13–16]. The mergers of stellar black holes and neutron stars [2, 15] observed in the LIGO experiment made it possible to visualize the final stages of the evolution of massive CBSs, which were previously predicted by a scenario-based analysis of their evolution [16]. It turned out later that the emission of gravitational waves is a decisive factor in the evolution of all sufficiently compact binary systems that consist of the final remnants of stellar evolution: degenerate dwarfs, neutron stars, and stellar black holes.

3. Formation of binary stars and their basic initial parameters

Modern astrophysics suggests two main mechanisms for the formation of binary stars: the fission of rotating molecular gas clouds during gravitational collapse and inelastic collisions of stars during the formation of young star clusters. We

now consider some details of these scenarios. A numerical simulation of the gravitational collapse of stellar-mass gas clouds showed that a dense gas-and-dust core emerges in the process of collapse. With the onset of core opacity, a hydrostatically equilibrium gas-and-dust star is formed with a mass $\sim 0.01M_\odot$ and a radius $\sim 10^3 R_\odot$. As a result of an increase in the mass of this 'star' to $\sim 0.1M_\odot$ and due to the ongoing gas accretion, its temperature increases and evaporation of dust begins. After becoming transparent, the 'star' continues its collapse, acquiring a stellar size $\sim 3R_\odot$. It is shown below that the two-stage collapse of protostellar gas clouds of a solar chemical composition is also reflected in the observed parameters of binary stars.

To determine the distribution of the emerging binary stars over the main parameters, the masses of the components M and the major semi-axes of the orbits a , it was necessary to perform significant work, accounting for the multitude of effects of observational selection and the evolution time of the components. This work has been done, and it is actively ongoing at present at many astronomical centers. The current result for the rate of the formation of binary stars in our Galaxy is [4]

$$d^3v = 0.2 \, d \log \frac{a}{R_\odot} \frac{dM_1}{M_1^{2.35}} f(q) dq [\text{year}^{-1}], \quad (4)$$

where M_1 is the mass of the primary component, $f(q) = 1$ for $6(M_1/M_\odot)^{1/3} \leq a/R_\odot \leq 3000$ and $f(q) \sim q^{-2}$ for $3000 \leq a/R_\odot \leq 10^6$, and q is the mass ratio of the components. The value of the numerical coefficient in this equation is confirmed by modern estimates [17]. This function, which is worth a detailed analysis, suggests that most stars are at least binary. It follows from this formula, in particular, that about a third of the stars can be single stars and have planets [4]. The last estimate coincides with the results of the Kepler spacecraft that searched for planets using the eclipse method. The flat distribution with respect to the logarithm of the semimajor axis means that there is no distinguished scale in the sizes of binary systems, the scale of their angular momentum. The smallest semimajor axis of young binary stars, $6(M_1/M_\odot)^{1/3} R_\odot$, is determined by the size of their young components, $\sim 3R_\odot$, formed in the accretion mode. The maximum initial semimajor axis, $\sim 10^6 R_\odot$, is limited by the tidal effect of the parent stellar cluster and the Galaxy. The formation of most stars in star clusters with masses $\sim 10^3 M_\odot$ also determines the characteristic mass of stars as $\sim M_\odot$, which corresponds to the initial densities of gas clouds, the precursors of clusters, $\sim 10^4 \text{ cm}^{-3}$.

Particularly noteworthy is the lower boundary of the initial separation of the components of young binary stars: $a_{\text{min}} \simeq 6(M_1/M_\odot)^{1/3} R_\odot$. This boundary is in essence an evolutionary track of the tightest emerging binary stars with accreting components [4]. However, if accretion by such systems continues, they become super-contact at $M \gtrsim 5M_\odot$, i.e., their components are immersed into a common compact envelope. The physics of such systems remains unclear, but a natural suggestion is to identify such systems with Be(Of) stars whose masses fall in the range $5M_\odot - 30M_\odot$ [18]. Should this be the case, short-period brightness variations [19] could be associated with the rotation of the binary core in a compact common envelope and the shock waves generated by this rotation in the decretion disk, rather than be attributed to the pulsations of such stars in a high mode [20]. Such an option to explain at least part of the Be stars warrants the development of appropriate models. The fast, almost critical, rotation of

Be stars [21] and the formation of a Keplerian decretion disk [22] could then be explained by the diffusion of the angular momentum of the binary into the envelope. Another option for maintaining the maximum rotation velocities of Be stars during their evolution in the main sequence is the diffusion of the angular momentum of the contracting core of the star into its envelope [23]. We recall that the observed rotation velocities of Be stars may be underestimated due to the equatorial gravitational darkening of rapidly rotating stars. Thus, the analysis performed shows that the phenomenon of Be stars can be due to the binary nature of their cores as a mechanism for achieving and maintaining maximum rotation [23, 24].

Also of interest is the problem of the initial distribution of star masses. The slope of this function for stars with a solar abundance of heavy elements at $M \gtrsim M_\odot$ is about -2.35 . But if low-metallicity stars are included, it increases to -2 for all stars with masses $M_\odot - 100M_\odot$ [17–19]. The decrease in the fraction of massive stars with a solar abundance of metals is probably caused by the stellar wind of OB stars. In other words, in the case of a solar abundance of metals, the matter of a gas protostar with a mass M_g transforms into a star with a mass M_* according to the law $M_* \sim M_g^{0.74}$. The basic characteristic of the $dN \sim M^{-2} dM$ dependence is emphasized by the observation that it is valid for astronomical objects with masses $M_\odot - 10^{15}M_\odot$ [25]. The physical content of this relation is simple: it signifies the absence of a distinguished mass scale of the initial elements of a turbulent gas medium, i.e., a flat distribution of matter in the cloud-mass logarithm. The characteristic star mass $\sim M_\odot$ is determined by the Jeans criterion at densities $\sim 10^{-20} \text{ g cm}^{-3}$ and temperatures $\sim 10 \text{ K}$ in giant molecular clouds, precursors of stellar clusters. The maximum mass of stars, $\sim 100M_\odot$, is limited by the stellar wind of OB stars and the pressure of their radiation on the dust component of the gas–dust medium. Observations have shown that the maximum mass of stars decreases in a noticeable way as the abundance of metals in the parent galaxies increases [26].

The maximum star mass and the slope of the initial mass function as a function of the abundance of metals is due to a strong relation between the intensity of the stellar wind of OB stars and their abundance of metals. This dependence can be expressed for OB stars by the relation [26] $\dot{M}_{\text{OB}} \approx 10^{-9} (M_{\text{OB}}/M_\odot)^2 (Z/Z_\odot)^{3/4} M_\odot \text{ year}^{-1}$ or $\dot{M}_{\text{OB}} \approx 10^{-11} \times (L/L_\odot)^2 (Z/Z_\odot)^{3/4} M_\odot \text{ year}^{-1}$. The empirical relation for the rate of mass loss can be expressed for Wolf–Rayet (WR) helium-rich stars using the relations found in [27, 28]: $\dot{M}_{\text{WR}} \approx 10^{-12} (L_{\text{WR}}/L_\odot)^{1.3} (Z/Z_\odot)^{1.3} \times M_\odot \text{ year}^{-1}$. Simple estimates show that if the lifetime of the most massive stars is $\sim 3 \times 10^6$ years, stars with masses greater than $\sim 100(Z_\odot/Z)^{3/4} M_\odot$ cannot become red supergiants, having lost the hydrogen-rich envelope in the main sequence due to the stellar wind. As a result, they become WR stars after the main sequence. It turns out at the same time that for the formation and merger of the close components of binary black holes with masses M_{BH} , the abundance of metals in their OB precursors must satisfy the condition $Z < (30M_\odot/M_{\text{BH}})^{4/3} Z_\odot$. This last requirement is necessary for the expansion of the OB star after the main sequence and the formation of a common envelope, the disintegration of which brings the components together [26–29]. Owing to the growth of the maximum masses of stars with a decrease in metallicity, an option for the formation of supermassive stars and black holes emerges.

The ‘traces’ of the early stages of the evolution of components are reflected in the observed distribution of the resulting binary stars as a function of the mass ratio of their components. CBSs with $a \lesssim 3000R_\odot$ are formed as a result of the collapse of a hydrostatically equilibrium protostars mentioned above, and hence the masses of their components arising as a result of the collapse are comparable [25]. Wider binary stars are the result of an ‘independent’ collapse of gravitationally bound objects or a product of the gravitational capture of nearby stars during the formation of scattered clusters. This explains the mass distribution of components of wide systems, which actually coincides with the reference mass distribution of astronomical objects: $dN \sim M^{-2} dM$. Such is the current explanation for the apparent bimodality of the initial distribution of young binary stars as a function of the initial mass ratio of their components.

The initial distributions of the binary stars under discussion reflect the distribution of gas–dust clouds by their masses and angular momenta, the origins of which, unfortunately, remain unknown so far. This function underlies the numerical scenario-based simulations of binary stars and estimates of their number and the frequencies of various phenomena occurring in their lifetime. A statistical approach to the distribution by the separation of components, in particular, allows estimating the fraction of multiple systems of any multiplicity, taking the stability conditions for the orbits of their components into account. The accumulation of empirical data on the observed properties of young binary stars with various chemical compositions and masses will clarify the parameters of this basic function and elucidate the conditions for its occurrence [30].

The foundations of a qualitative approach to studying the evolution of CBSs are laid down in the scenario-based approach. A scenario describes the evolution of an object from the moment of its emergence to the moment of the formation of a final remnant that exists for a time exceeding the Hubble time ($\sim 10^{10}$ years). The scenario that includes all the essential observational events in the evolution of a given object is based on modern physically substantiated mathematical modeling of the object and the phenomena accompanying its evolution [31]. A quantitative approach to the study of the CBS evolution involves scenario-based simulations of their evolution. Already the first numerical models of the evolution of close binaries [32, 33] have demonstrated their efficiency in estimating the number of such stars at various evolutionary stages and the frequency of events that accompany their evolution. The numerical scenario-based model uses the ‘birth function’ of these systems (4) and a qualitative description of their evolution [31–33].

The evolution of single stars and the components of wide binary stellar systems. About a half of binary stars are wide systems, and their components, while evolving as single stars, barely interact with each other. Therefore, the study of the evolution of single stars is necessary for studying the evolution of the components of wide binary stars. We consider the main landmarks in the evolution of single stars of various masses from the moment of their emergence to the time when final long-lived (eternal?) products are formed: degenerate carbon–oxygen dwarfs, neutron stars, and stellar-mass black holes.

The source material for the emergence of stars is cold interstellar dust and gas clouds, whose collapse leads to the formation of stars. Observations show that stars emerge in

clusters with masses of $10^3 M_\odot$ – $10^5 M_\odot$. The emergence of ionized-hydrogen zones along with the formation of massive stars that ionize the hydrogen in the clouds causes the rapid dissipation of the gas component of the young-star cluster and the decay of most of the clusters formed [34]. The expansion rate of the ionized-hydrogen zones with a temperature of $\sim 10^4$ K is about 10 km s^{-1} . The parabolic velocity of clusters of stars and galaxies at $M \simeq 0.2 R^2$ [30] is $\sim 300[M/10^{11} M_\odot]^{1/4} [\text{km s}^{-1}]$. We find, as a result, that star clusters with masses less than $\sim 10^5 M_\odot$ usually decay after their gas has been removed by ionized-hydrogen zones. Star clusters of a larger mass remain gravitationally bound and can maintain the accretion of the gas surrounding them and regenerate the gas of old stars, thus becoming dwarf galaxies. The mass limit of $\sim 10^5 M_\odot$ can be determined in this way, which separates star clusters and dwarf galaxies, allowing repeated formation of stars in the latter.

As noted in the Introduction, the collapse of gas–dust clouds into a star is a two-stage process that ends with the emergence in the cloud core of a young and completely convective star with the mass $\sim 0.1 M_\odot$ and radius $\sim R_\odot$. Ongoing accretion increases the mass and radius of such a star in accordance with the law $R/R_\odot \approx 3 \times (M/M_\odot)^{1/3}$ [4]. As the mass of the star increases to $\sim 3 M_\odot$, its core becomes radiative, and the star contracts to a radius that corresponds to that of a main-sequence star: $R/R_\odot \approx (M/M_\odot)^{2/3}$ [4, 30].

As the gas envelope remnants deplete, a young main-sequence star of age zero emerges (Fig. 1). Its further evolution is primarily determined by its mass. For the most massive stars, their chemical composition is an important factor. The abundance of metals determines the stellar wind speed. For example, for WR stars with $-4 \leq \lg(Z/Z_\odot) \leq 0$, $\dot{M}_w \approx 10^{-4}(Z/Z_\odot)^{3/4} M_\odot \text{ year}^{-1}$ [35], while for massive OB stars, $\dot{M}_w \approx 3 \times 10^{-6}(Z/Z_\odot)^{3/4} M_\odot \text{ year}^{-1}$ [26]. As a result, massive stars with a solar abundance of metals Z and masses greater than $\sim 30 M_\odot$, due to the loss of the hydrogen-rich envelope in the main sequence, cannot become red supergiants after the hydrogen has burned out in their cores. In the

case of a solar abundance of metals, this significantly limits the options for creating common CBS envelopes by the stars and reduces the rate with which the components of close binary black holes merge [29]. At the same time, as noted in the Introduction, the intense stellar wind of stars with a solar abundance of metals changes the slope of the initial function of the star masses from $dN/dM \sim M^{-2}$ for stars with $Z/Z_\odot \approx 0$ to $dN/dM \sim M^{-2.35}$ for stars with $Z \approx Z_\odot$ [25].

Numerical simulations of the stellar evolution showed that after a helium-rich core is formed, the star expands and, having left the main sequence, becomes a giant (supergiant) (see Fig. 1). The lifetime of stars on the main sequence is $\sim 10^{10}(M/M_\odot)^{-1.8}$ years. The lifetime at the supergiant stage, which in the most part is the time during which helium burns in the core, does not exceed 10% of the lifetime in the main-sequence stars [25, 26, 29, 35]. A carbon–oxygen degenerate core is formed as a result in stars with masses less than $\sim 8 M_\odot$ at the red-supergiant stage. The high luminosity of this core ($\lg(L_{\text{CO}}/L_\odot) \simeq 1.6 + 3.8$ at $0.6 \leq M_{\text{CO}}/M_\odot \leq 1.4$) [36] and large opacity of the cold-envelope matter give rise to an intense stellar wind and, in a comparatively short time ($\sim 10^4$ years), a loss of the extended ($\sim 500 R_\odot$) and primarily hydrogen-rich envelope of the red supergiant. The result of this loss is the emergence of a planetary nebula, the remnants of the hydrogen-rich envelope ionized by a hot bright degenerate carbon–oxygen dwarf (see Fig. 1), whose mass can be estimated from the formula $M_{\text{CO}}/M_\odot = 0.6(M_{\text{MS}}/M_\odot)^{1/3}$. The star is usually screened at the stage of rapid loss of the extended envelope by the dust of the outflowing matter to turn from a red supergiant into an infrared one. The frequency with which planetary nebulae emerge in the Galaxy is $\sim 1 \text{ year}^{-1}$.

The fate of stars with initial masses exceeding $\sim 8 M_\odot$ is different (see Fig. 1). The threshold mass was estimated in [37]. The sequential burning of hydrogen, helium, carbon, oxygen, and silicon in their cores results in the formation of iron cores with masses $\sim 0.05(M_{\text{MS}}/M_\odot)^{1.4}$ [36]. The main mechanism for cooling massive star cores beginning with the stage of carbon depletion in their cores is the neutrinos [4, 36, 37]. The rate of energy loss from the core due to the neutrinos is then so high that to compensate for it, the energy flux from the layered sources surrounding the red supergiant core is directed inward, into the core! The neutrinos reduce the burning time of carbon and oxygen in the cores of the most massive stars to several tens to hundreds of years. The iron cores of massive stars collapse to form neutron stars and black holes. The red supergiant explosion that accompanies this collapse exhibits all the features characteristic of the type-II supernova explosions, which play a decisive role in the dynamics of the gas components of disk galaxies by controlling the density of the gas disk of these galaxies and the rate of star formation in them.

The problem of the initial threshold mass of stars producing neutron stars and black holes is still unsolved because it is unclear whether theoretical and empirical approaches to its estimation are correct [7, 38, 39]. The masses of observed neutron stars in close binary systems are contained within narrow limits, $1.2 M_\odot$ to $2 M_\odot$, while the masses of black holes are in the range $4.5 M_\odot$ – $35 M_\odot$ [7, 40, 41]. This observation suggests that the minimum initial mass of stars producing stellar-mass black holes is of the order of $30 M_\odot$ [7, 39, 40]. The dependence of this quantity on the initial abundance of stellar-gas metals is not excluded. The remaining significant uncertainty ($\sim 10 M_\odot$) of this threshold mass is a consequence

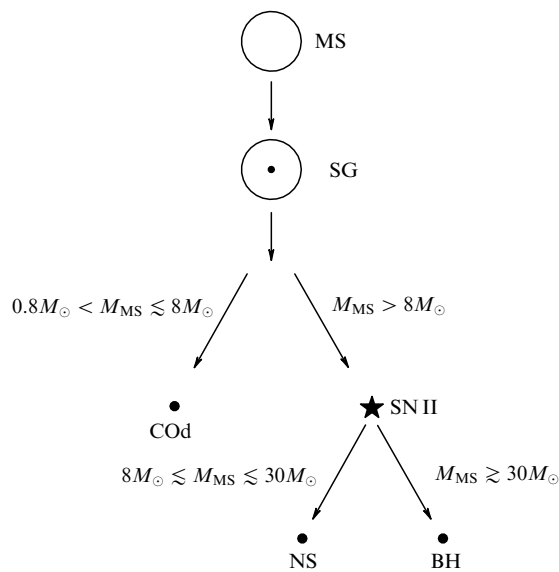


Figure 1. Evolution of single stars and components of wide stellar systems. The initial masses are $0.8 M_\odot$ – $150 M_\odot$. MS is the main sequence, SG is a supergiant, SN is a supernova, COd is a carbon–oxygen degenerate dwarf, NS is a neutron star, and BH is a black hole.

of the indeterminate physics of the current models of stellar evolution and insufficient observational base of relativistic objects. The relation between the black-hole mass and the initial mass of the star is determined by the formula $M_{\text{BH}}/M_{\odot} \simeq 0.05(M/M_{\odot})^{1.4}$ [4].

The rate of explosions of massive type-II supernovae in the Galaxy can be estimated for $M > 8M_{\odot}$ using Eqn (4) to be ~ 0.045 per year. The rate of black hole formation is ~ 0.008 per year. We assume that the rate of star formation in the Galaxy is of the order of one solar mass per year. About 1% of all the matter that is converted into stars then eventually becomes part of stellar-mass black holes. Equation (4) for the above relations between the masses of carbon–oxygen cores M_{CO} and black holes M_{BH} with the initial masses of stars enables determining their mass function based on the original mass function of the stars. As a result, we have $dN/dM_{\text{CO}} \sim M_{\text{CO}}^{-5}$ and $dN/dM_{\text{BH}} \sim M_{\text{BH}}^{-2}$. Estimates of the observed mass function of dwarfs confirm that most of them are in the small-mass region. The mass function of the observed black holes has not yet been reliably established due to the low statistics.

4. Evolution of close binary stars of small and moderate masses

4.1 Evolution of components of small-mass close binary stars that produce helium-rich degenerate dwarfs

Based on the general ideas about the evolution of single stars, we can proceed to analyzing the evolution of CBSs. Of importance for CBSs is R_{R} , the size of the Roche lobes of the components, which limits the size of their gravitational-effect zones: $R_{\text{R}} \approx 0.4(M_2/M_1)^{1/3}a$, where $M_1 \geq M_2$ are the masses of the components and a is the semimajor axis of the orbit. As the components fill their Roche lobes during evolution, the exchange of matter between them and the subsequent exposure of their dense cores begins. Owing to this, binary stars transform from an object of study into an efficient tool for exploring the structure and the evolution of stars. A study of the evolution of single stars showed that it results in the development of strong inhomogeneity and the exposure of dense cores. The density of helium-rich cores of low-mass stars, $0.8 \lesssim M_1/M_{\odot} \lesssim 2.8$, $a_0 \lesssim 500R_{\odot}$, increases in the course of evolution by almost four orders of magnitude, while the density of the cores of massive pre-supernova stars, by eight orders of magnitude. This allows considering the evolution of the cores and the envelopes of CBS components separately in a scenario-based analysis.

The evolution of stars with small initial masses ($0.8 \lesssim M_1/M_{\odot} \lesssim 2.8$, $a_0 \lesssim R_{\odot}$) after the main sequence reduces to an increase in the mass of the degenerate helium-rich core. It is accompanied by an increase in the size of the giant star itself, whose size R and luminosity L are determined by the mass of the degenerate helium-rich core: $R/R_{\odot} \approx 3000(M_{\text{He}}/M_{\odot})^4$ and $L/L_{\odot} \approx 2 \times 10^5 (M_{\text{He}}/M_{\odot})^{6.2}$, for $0.1 \leq M_{\text{He}}/M_{\odot} \leq 0.5$ [36, 42]. As the Roche lobe is filled by such a star due to the intense exchange of matter between the components, the system is immersed into a common envelope (Fig. 2). The dissipation of the common envelope gas due to the orbital energy of the binary core brings the components together in accordance with Eqn (1). For $a/R_{\odot} \lesssim 12(M/M_{\odot})^{0.8}$, the binding energy of the common envelopes of the tightest systems is so high that the components merge into a single giant with a helium-rich

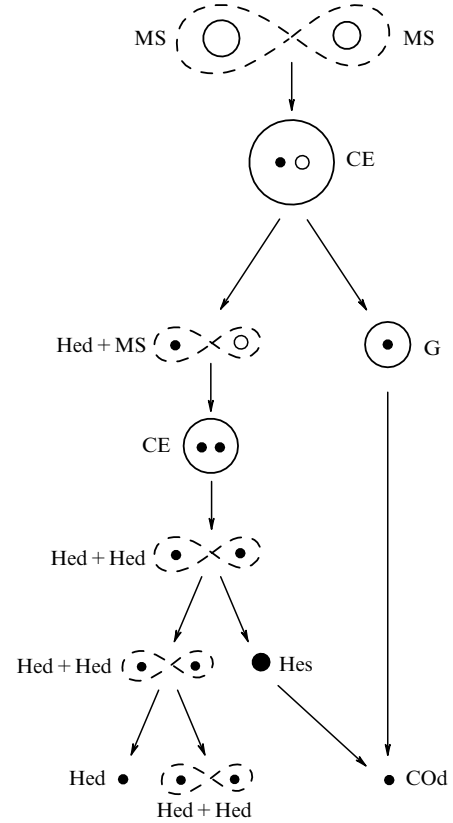


Figure 2. Evolution of low-mass CBSs: $0.8M_{\odot} \leq M \lesssim 2.8M_{\odot}$. The initial separation of components is $a \lesssim 500R_{\odot}$. MS is the main sequence, CE is the common envelope, G is a giant star, Hed is a helium-rich degenerate dwarf, Hes is a helium-rich star, and COd is a carbon–oxygen degenerate dwarf.

degenerate core. The further evolution of this giant is similar to that of an ordinary single star of the corresponding mass. After helium burns out, a carbon–oxygen core emerges, and the evolution ends after the remnants of the envelope are dumped as a planetary nebula, and a carbon–oxygen dwarf emerges.

Systems with $a/R_{\odot} \gtrsim 12(M/M_{\odot})^{0.8}$ remain gravitationally bound binaries after the dissipation of the common envelope (see Fig. 2). Tighter ‘final’ systems of degenerate helium-rich dwarfs that satisfy condition (3) can merge due to the emission of gravitational waves. Both classes of such systems have been well known for a long time [43]. The merger rate of helium-rich dwarfs in the Galaxy is $\sim 10^{-2} \text{ year}^{-1}$ [44]. The physics of the merger of degenerate helium-rich dwarfs is still insufficiently studied. Based on general considerations, we can only exclude the ignition of helium if the total mass of merging dwarfs is less than $\sim 0.5M_{\odot}$, because such is the mass of the helium-rich degenerate core of a giant at the beginning of the burning of helium in its core [36, 43, 44]. The merger of helium-rich dwarfs results in this case in the formation of a single rapidly rotating helium-rich dwarf (see Fig. 2). If the total mass of precursors is greater than $0.5M_{\odot}$, helium can ignite, and the most massive dwarfs can transform into a regular helium-rich star that ends its evolution after the stage of helium-rich RCrB-type supergiant as a carbon–oxygen dwarf (see Fig. 2). The conditions of explosive burning of helium in the massive product of the merger of helium dwarfs (which is not yet ruled out) are still unclear. Formally, the

nuclear energy released in the burning of $0.5M_{\odot}$ of helium is $\sim 10^{51}$ erg, a value sufficient for initiating the energy processes of a supernova and the complete destruction of the explosion products.

To summarize the discussion of the evolution of binary stars with the initial masses of the primary components in the range $0.8M_{\odot} - 2.8M_{\odot}$, we are facing the problem of the physics of the merger of helium-rich degenerate dwarfs of various masses. To resolve this problem, numerical simulation is needed that could elucidate both the course of the process and the nature of its final products. Relation (4) allows concluding that about 30% of all degenerate dwarfs formed annually in the Galaxy are helium rich. The observed distribution of degenerate dwarfs located close to the Sun confirms this estimate [45]. The total merger rate of degenerate dwarfs in the Galaxy is ~ 0.01 per year [45, 46].

It is worth saying a few words about the fate of wider low-mass systems of the specified mass range with $500 \lesssim a/R_{\odot} \lesssim 2000$, whose primary components fill their Roche lobe while already having a carbon–oxygen degenerate core with a mass of $\sim 0.6M_{\odot}$. The common envelope inevitable in this case brings the components closer to each other, as a result of which the second component cannot form a carbon–oxygen core. After the second stage with a common envelope, a wide long-lived system consisting of helium-rich and carbon–oxygen dwarfs arises that is barely able to ‘extend’ its life due to emission of gravitational waves in the Hubble time.

4.2 Evolution of close moderate-mass binary stars

The evolutionary scheme for binary stars with moderate masses, $2.8M_{\odot} \lesssim M \lesssim 8M_{\odot}$, is shown in Fig. 3. A characteristic feature of such stars is the formation in the main sequence of a helium-rich nondegenerate core of the mass $\sim 0.1(M_{\text{MS}}/M_{\odot})^{1.4}M_{\odot}$, and, after the helium has burned out in this core, the formation of a degenerate carbon–oxygen dwarf core with a mass $\sim 0.6(M_{\text{MS}}/M_{\odot})^{0.4}M_{\odot}$ [47]. Expansion of the primary component and filling up its Roche lobe initiates the exchange of matter between the components. As usual, two versions of this stage are possible (see Fig. 3). If the masses of the components are close to each other (the mass ratio $q_0 \gtrsim 0.5$), then an Algol-like semi-detached system is formed with a lifetime of the order of the donor’s thermal time scale. If, on the contrary, the initial masses of the components are very different ($q_0 \lesssim 0.5$), a common envelope emerges. The evolution of the system at the common-envelope phase is determined by the initial separation of the components. The fairly close components, having dumped part of the common envelope matter, merge, forming a single giant, whose evolution ends with the formation of a carbon–oxygen degenerate dwarf (see Fig. 3) [47].

After the exchange phase is completed, the evolution of the semi-detached system ends with the formation of a hot helium-rich nondegenerate star in a tight pair with a main-sequence star [47]. A helium-rich star with a mass $\sim M_{\odot}$ can once again fill its Roche lobe during its evolution, thus forming a helium-rich common envelope and passing the RCrB star stage, and transform into a carbon–oxygen or oxygen–neon degenerate dwarf [4]. The latter option is possible if the initial mass of the star is $\sim 6M_{\odot} - 8M_{\odot}$. The luminosity of the young degenerate dwarf is determined by its mass: $L/L_{\odot} \approx 6 \times 10^4 (M_{\text{CO}}/M_{\odot})^{0.5}$, and, as a rule, it is the core of the lost common envelope, a planetary nebula [48, 49].

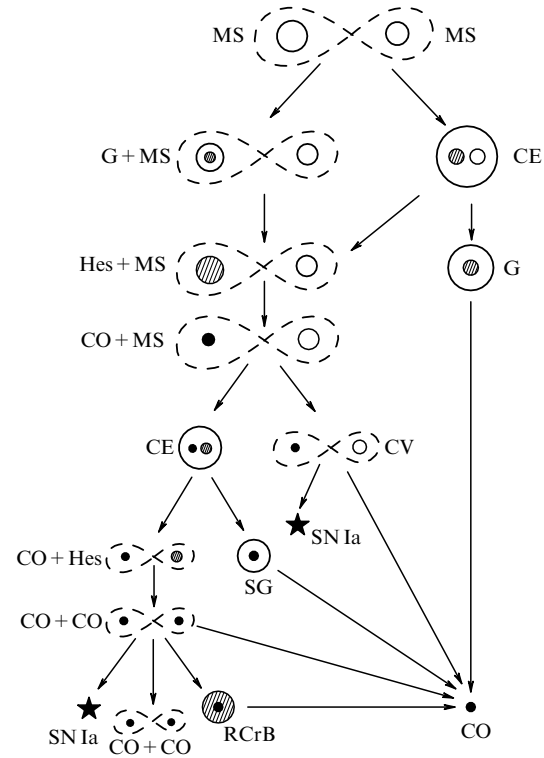


Figure 3. Evolution of moderate-mass CBSs: $2.8 < M/M_{\odot} \lesssim 8$, $a \lesssim 2000R_{\odot}$. MS is the main sequence stars, G is a giant with a helium-rich nondegenerate core, CE is the common envelope, Hes is a helium-rich nondegenerate star, CO is a carbon–oxygen (oxygen–neon) degenerate dwarf, CV is the cataclysmic system with the main-sequence donor star whose mass is less than the solar mass, SN Ia is a type-I supernova, SG is a supergiant with a carbon–oxygen degenerate core, and RCrB is a supergiant with an RCrB-type helium envelope.

The evolution of the second component in the selected range of initial masses results in the formation of a helium nondegenerate star in its core (see Fig. 3). As a semi-detached system is formed once again, a common envelope is usually formed again, ending in the emergence of either a single red supergiant when the components merge or a detached system of a helium-rich nondegenerate star and a carbon–oxygen dwarf. If the mass of the second component, a main-sequence star, is in the range $0.3M_{\odot} - 1.5M_{\odot}$ and $a \lesssim 10R_{\odot}$, then a cataclysmic semi-detached system emerges under the effect of the magnetic stellar wind of the main-sequence star and emission of gravitational waves by the system (see Fig. 3).

Cataclysmic systems, which deserve special attention as the first ‘tour guide’ to the world of interacting CBS components at the late stages of their evolution, exhibit the effect of a number of explosive mechanisms that accompany their evolution. The list of such processes includes cumulative instability of the accretion disk near the accreting dwarf and layered bursts of hydrogen burning in the envelope of a degenerate dwarf with the repetition period determined by its mass [50]. A characteristic feature of nuclear flashes is a high abundance of carbon, oxygen, and neon that exceeds that of the Sun by several dozen times. Owing to the high rate of nova outbursts in the Galaxy, up to 50 a year, attention is drawn to their evolution. At certain accretion rates ($\sim 10^{-7}M_{\odot} \text{ year}^{-1}$), hydrogen burns in the layer in a stationary mode with the buildup of a massive helium-rich envelope [50]. If the envelope mass is sufficiently large, a

helium burning flash can be so powerful that it causes detonation of the carbon–oxygen core and a type-I supernova explosion [50–52]. This is one of the (slightly modified) possible scenarios of the explosion of these supernovae. Initial hopes that a degenerate dwarf can attain the Chandrasekhar mass due to the stationary burning of hydrogen and helium in its envelope failed due to the mixing of hydrogen with carbon, as is demonstrated by the novae.

The exchange of matter in cataclysmic systems often occurs like an outburst [53, 54]. Moreover, two types of outbursts are known that accompany the accretion of matter from the disk: flashes and super-flashes. Flashes are probably a consequence of convective instability in the polar directions of the disk. By increasing the viscosity of the gas, convection causes a rapid dump of most of that gas onto the degenerate component of the system. The disk inevitably expands, accumulating the angular momentum of the gas accreted during the flash. The accumulation of matter at the edge of the accretion disk during ‘ordinary’ flashes eventually results in convective instability of the gas at the periphery of the disk. This process probably causes a super-flash, during which the bulk of the disk gas is accreted by a degenerate dwarf, while the angular momentum of this matter is partially converted into orbital momentum due to tidal effects and is partially lost by the system along with part of the disk gas. This is the possible nature of outbursts of dwarf novae that accompany the exchange of gas between the components of cataclysmic systems. Another reason for the onset of instability of the accretion-disk periphery may be the formation in this region of a tidal spiral wave that causes accretion instability of the disk [55]. The alternation of flashes and super-flashes of an accretionary nature is also observed in low-mass X-ray binaries, for example, in the 4U1705-44 system [53–57].

It has been found recently that some accreting degenerate dwarfs of cataclysmic systems have very large magnetic fields (up to 10^8 – 10^9 G) and fast rotation with a period of up to ~ 100 s. These are the parameters of the ARSco dwarf [58, 59]. The rapid rotation of the dwarf is likely a result of accretion. The presence of such a dwarf in a cataclysmic system can modify the nature of its evolution from the regular two-stage exchange in semi-detached systems to the ‘accretionless’ mechanism. The rotation energy of the degenerate ARSco dwarf is so large, $\sim 10^{48}$ erg, that it significantly exceeds the orbital energy of the donor: $\sim 2 \times 10^{47}$ erg for a mass $\sim 0.3M_\odot$. This, in principle, enables such a degenerate dwarf to transform in the presence of a strong magnetic field into a ‘fan’ that can eject all the gas supplied to it by the donor that fills its Roche lobe. It is likely in this case that all the phenomena that usually accompany gas exchange in semi-detached systems (disk formation, accretion instabilities of the disk, and accretion of donor material by a dwarf) are excluded. In the course of such evolution, the Galaxy becomes the ‘accretor’. It is still difficult to quantitatively estimate the formation rate of such systems due to the lack of observational data on the distribution of degenerate components of cataclysmic systems with respect to their rotation speed and magnetic field strength [58, 59].

The evolution of cataclysmic systems reduces to the depletion of donor matter. It occurs under the effect of the loss of angular momentum due to the stellar magnetic wind of the donor, whose orbital periods are longer than three hours, and under the effect of the radiation of gravitational waves by the system with orbital periods shorter than two hours [4]. As

the mass of a fully convective donor decreases to $\sim 0.1M_\odot$, its dynamic disintegration is possible. The reason for this can be the response of a fully convective star to the rapid loss of matter due to its expansion. As the donor mass decreases during the exchange, the time of the angular momentum transfer increases, and the donor becomes unstable to disintegration on the dynamic time scale and its transformation into a disk (envelope?) near a degenerate dwarf within a few minutes. This is probably the last phase of the evolution of cataclysmic systems and similar low-mass X-ray binary systems. The recent discovery of massive disk formation during the CKVul flash as a nova is probably the first example of the dynamic disintegration of a cataclysmic system donor [60].

However, it is now clear that a decrease in the donor mass to $\sim 0.1M_\odot$ is not a sufficient condition for its disintegration and the cessation of the evolution of a cataclysmic binary. The discovery of QZLib [52] with an orbital period of about 1.5 hours, the degenerate dwarf mass $\sim 0.8M_\odot$, and the donor mass $0.03M_\odot$ indicates that the donor may not collapse but ‘fall asleep’ for the Hubble time, hiding the system from the observer due to obvious selection effects. A similar phenomenon is apparently observed in the close system containing the neutron star PSRJ0636+5128 [61]. The orbital period of this system is also about 1.5 hours, the satellite mass is $\sim 0.017M_\odot$, and its radius is $0.076R_\odot$. This implies that the disintegration of the cataclysmic system donor is not mandatory, even on the Hubble time scale.

As a result of the second stage with a common envelope, either the components merge into a red supergiant with a carbon–oxygen core or a close binary system is formed from a helium-rich nondegenerate star and a carbon–oxygen dwarf. The tightest of these systems can form semi-detached systems as a result of radiating gravitational waves [62]. It seems possible in this case [51, 52, 61, 62] that the helium-rich layer accumulated by the dwarf explodes, the carbon of the dwarf itself detonates, and a type-Ia supernova subsequently explodes. As a result, the evolution of cataclysmic semi-detached binary systems of moderate masses with the depletion and probable disintegration of the donor when its mass is less than $\sim 0.1M_\odot$ usually ends with the formation of a single carbon–oxygen dwarf (see Fig. 3).

However, the evolution of the main part of a CBS belonging to the mass range under discussion results in the formation of close systems of carbon–oxygen dwarfs. In the case of large initial ratios of the component masses, the evolution of the less massive component results in the formation of a degenerate helium-rich dwarf. The tightest final systems with a major semiaxis less than $\sim 3R_\odot$ merge in the Hubble time under the effect of their gravitational radiation. The result of the merger of a helium-rich dwarf and a carbon–oxygen dwarf is probably the emergence of a helium-rich RCrB supergiant [8]. However, the process of the merger itself and its model are still obscure. It is only clear that the intense stellar wind inherent in stars of this type must lead to a relatively rapid loss of the helium-rich envelope and the emergence of a single carbon–oxygen dwarf.

The situation with the merger of carbon–oxygen degenerate dwarfs is different [4, 8]. It should be acknowledged that this problem, which enjoys the highest popularity, has attracted the full attention of astronomers over the past few decades. Hopes for an explanation of the ‘mysterious’ supernova explosions of this type (Ia) maintain the active interest of astrophysicists in this problem [8]. The problem of

type-Ia supernovae deserves a dedicated discussion and analysis. The explosions of these supernovae in elliptical galaxies, devoid of active stars with masses exceeding the solar mass, have long attracted attention to their physics. Their other important characteristic has for a long time been the ‘standard’ character of their light curves. An obvious solution to these puzzles is to invoke nuclear explosions of degenerate dwarfs whose mass is close to the Chandrasekhar mass. For the dwarf mass to increase to the maximum value, the presence of a satellite, a main-sequence star filling its Roche lobe [51] or a degenerate carbon–oxygen dwarf [4, 8], is assumed. Both scenarios are currently under intense study [7, 63]. Observable examples of close binary systems have been found where the main described scenarios are implemented. Numerical simulation of explosions using models of ever increasing complexity allows gradually expanding the range of the observable parameters of these supernovae [52, 64].

It is becoming increasingly clear that, within the limits of apparent ‘standardization’, there is a noticeable dispersion of the observed parameters of type-Ia supernovae that are being discovered today. Searches for possible causes of the observed incomplete uniformity and standardization of such supernovae suggest variations, in principle, in both the mass of exploding dwarfs and their chemical composition [47, 65]. In addition, incomplete burning of nuclear fuel during an explosion is also possible [7, 8, 51, 52, 63, 64]. It is now clear that the most massive dwarfs are oxygen–neon dwarfs with energies $\sim 6.4 \times 10^{17}$ erg g^{−1}, while the energy of carbon–oxygen dwarfs is 9.2×10^{17} erg g^{−1} [64]. This circumstance results in interesting implications for cosmology. The average mass of merging dwarfs decreases in scenario-based simulations with increasing the population age [44]. This implies that more massive oxygen–neon dwarfs begin to explode in young galaxies and, as indicated above, have a relatively low energy content. This can explain the observed relative ‘weakness’ of the first type-Ia supernovae, which is now attributed to the accelerated expansion of the Universe: $\Delta m_v = 0.5^m$ at the redshift $z = 1$ [66, 67]. This problem needs a thorough study. The scenario-based estimate of the current merger rates of galactic degenerate dwarfs with a total mass exceeding the Chandrasekhar mass is ~ 0.002 – 0.003 per year [44, 46], a value that is close to the estimates that follow from observations [46, 65–68].

More and more arguments are accumulating now in favor of the incomplete ‘standardization’ of the maximum luminosity of these supernovae. Type-Ia supernovae in S-galaxies with current star formation turned out to be $\sim 0.6^m$ brighter than the supernovae in E-galaxies [69]. Two well-studied Ia supernovae from NGC 3972, in addition to the difference in their brightness, exhibited an almost 40 percent difference $\sim 0.4^m$ in the mass of synthesized nickel [70]. The ‘hidden’ nonstandardization of SNIa draws attention to its possible reasons. The currently observed apparent bimodality of the mass distribution of degenerate dwarfs ($\sim 0.5M_\odot$ for helium-rich degenerate dwarfs and $\sim 0.7M_\odot$ for carbon–oxygen dwarfs [71]) has drawn the interest of researchers to the possible detonation of helium in the envelopes of massive degenerate accreting dwarfs [61, 72]. An obvious reason for the almost twofold difference in energy characteristics is still the corresponding difference in the masses of the dwarfs [61, 69, 71].

It was noted above that the difference in the energy characteristics of carbon and oxygen, in addition to being the cause of the nonstandardized nature of energy processes

in these supernovae, can also significantly ‘affect’ the parameters of the Universe. Large reserves of nonstandardization are ‘stored’ in the masses of exploding dwarfs. It was believed for a long time that a dwarf explodes when it reaches the maximum, standard Chandrasekhar mass. However, it is now clear that due to tidal preheating or explosion of the helium layer [65], it is possible that only a large-mass dwarf overheated by tides and an accretion jet explodes. Consequently, the observed difference in the brightness of ‘young’ and ‘old’ SNIa [69] may result from explosions of dwarfs of various masses and chemical compositions.

The nonstandardized nature of the observed manifestations of type-Ia supernovae has now justifiably become the subject of increased and comprehensive attention of observers. It was found, in particular, that the estimated mass of nickel produced in the explosion ranges within $0.1M_\odot$ – $0.8M_\odot$ [64]. The number of observed binary systems of degenerate dwarfs with short (< 0.4 days) orbital periods is growing, and one of them (V458 Vul) has the total mass of components that exceeds the Chandrasekhar limit [73]. The search for a numerical model of explosions of degenerate dwarfs [74] has significantly activated the study of the relations between the initial conditions of explosions, the observable parameters, and their manifestations [64, 73, 74].

A recent detailed analysis of the latest SNIa observations has shown that there are usually no signs of the second component, which is a good argument in favor of a model in which degenerate dwarfs merge [70]. On the other hand, a detailed study of the light curve of SN2018oh [72] showed possible signs of the presence of a component that fills its Roche lobe at a distance of $30R_\odot$ from the supernova. Thus, it may be concluded that modern observations successfully ‘supply’ content to the theoretical scenarios of this type of supernovae, which have been discussed over the past several decades.

To summarize the discussion of the evolution of moderate-mass CBSs, we can conclude that the application of modern ideas about their evolution to the interpretation of a wide range of classes of observed systems, stars, and phenomena occurring in the world of close binary stars enables understanding the causes of their emergence and the physics of a number of phenomena in the stellar world. It is especially worth noting that the discovery of close binary degenerate dwarfs is evidence of the decisive role that common envelopes play in the loss of orbital angular momentum by binary systems [65].

A simple analysis shows that SNIa not only mark the end of the evolution of some intermediate-mass CBSs but also probably determine the evolution of spheroidal stellar systems: E galaxies and bulges of S galaxies [30]. The rate of energy pumping into the interstellar gas of galaxies by these supernovae is $\sim 10^{41} M_{11}$ erg s^{−1}, where M_{11} is the mass of the galaxy in units of $10^{11} M_\odot$. The rate of cooling of the coronal gas of galaxies is $\sim 10^{40} M_{11}$ erg s^{−1}, while the rate of energy pumping into the gas component of the galaxy, which is necessary to maintain the galactic wind, is $\sim 10^{40} M_{11}^{3/2}$ erg s^{−1}. A comparison of these estimates shows that spheroidal galaxies with masses smaller than $\sim 10^{13} M_\odot$ can maintain the galactic wind, being ‘free’ from the gas of old stars while remaining E galaxies. And only massive ($M \gtrsim 10^{13} M_\odot$) CD galaxies can maintain star formation by preserving the gas lost by old stars.

5. Evolution of massive close binary stars

The evolution of binary stars with masses $M \gtrsim 8M_\odot$ has long been the focus of attention of astronomers because of the variety of high-energy phenomena in all spectral ranges that accompany their evolution. This attention has been substantially rewarded and ‘heated up’ in recent years by the discovery in the LIGO experiment of several events of mergers of stellar-mass black holes [2, 75, 76]. These discoveries have won the next in the series Nobel Prizes awarded for exploring binary stars.

We now consider the main stages of the evolution of a massive CBS with initial masses of its components above $\sim 8M_\odot$ (Fig. 4). It is of interest that the masses of components evolving in the framework of the scenario presented in Fig. 4 are also limited from above. The reason for this limitation is the stellar wind of OB stars, the strength of which is determined, as indicated in the Introduction, by the star mass M and the abundance of metals Z [26]. Therefore, stars with a mass greater than $\sim 100(Z_\odot/Z)^{3/4}M_\odot$, having lost their hydrogen-rich envelope, can no longer expand and follow the main scenario after a helium-rich core has formed in them (see Fig. 4). As a result, each of the components evolves independently according to the scheme OB-star–WR-star–SN Ib-supernova–relativistic remnant [2].

The expansion of the primary component after the formation of its helium-rich core when the initial masses of the components are close to each other ($M_2 \gtrsim 0.5M_\odot$) makes the system switch to the Algol stage with a lifetime of the order of the thermal time scale of the star, which corresponds to the Kelvin–Helmholtz scale $t_{KH} \simeq 3 \times 10^5 R_\odot / R$ years for a massive star of radius R . If $M_2 \lesssim 0.5M_\odot$, the expansion of the primary component immerses the system into a common envelope for probably nearly the same time (see Fig. 4). The merger of the closest components results in the formation of a single massive star. When its evolution ends after a type-II supernova explosion, a relativistic remnant (RR) emerges: a neutron star in the case $8M_\odot < M \lesssim 30M_\odot$ or a black hole. Preservation of wider systems after disintegration of the common envelope results in the emergence a helium-rich star with a luminosity $L_{He}/L_\odot \simeq 10^3 (M_{He}/M_\odot)^3$ and mass $\sim 0.1 (M_0/M_\odot)^{1.4}$ paired with a main-sequence star [4].

The most massive helium-rich stars in the phase of helium burning in their core become WR stars. The depletion of helium, carbon, oxygen, and silicon in the cores of such stars results in the collapse of the type-Ib supernova core, while the helium-rich envelope persists. If the helium-rich envelope is also lost due to the wind at the WR stage, such a supernova explosion is of type Ic. Binary stars of the WR + OB type have been well known to astronomers for a long time [5]. We now consider the condition for the system to survive if one of its components explodes as a supernova. The explosion is associated with the loss of a part of the pre-supernova envelope. The formal condition for the disintegration of binaries with circular orbits is $\Delta M > M_R$, where ΔM is the mass lost due to the supernova envelope and the mass defect during the formation of a relativistic remnant, and M_R is the total mass of the remaining components.

The first scenarios of massive CBS evolution encountered an obvious paradox related to the condition that the system persists when the second supernova explodes in it [16]. Radio pulsars (neutron stars) in binary systems had not yet been known at that time. The disintegration of the system during the second explosion naturally explained the high (several

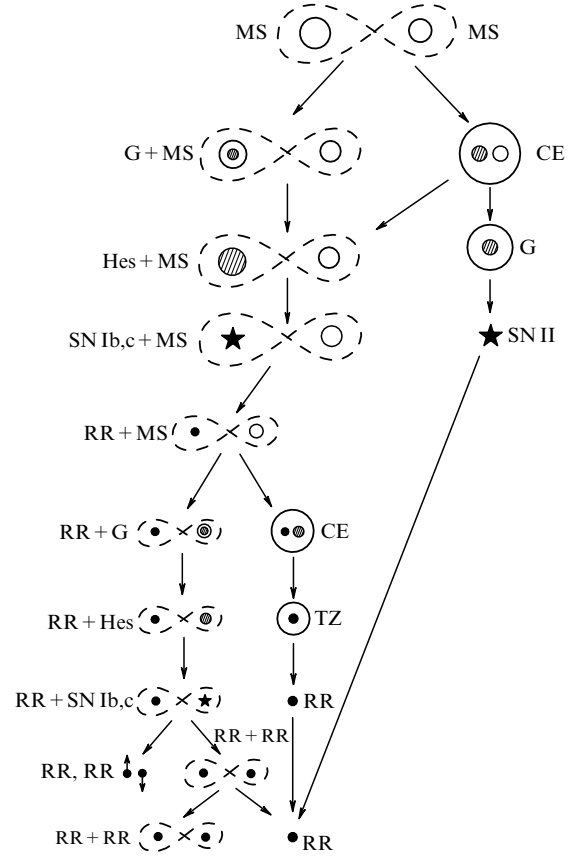


Figure 4. Evolution of massive close binary stars: $M \gtrsim 8M_\odot$. MS is a main sequence stars, G is a giant with a helium-rich nondegenerate core, CE is the common envelope, Hes is a helium-rich nondegenerate star, RR is a neutron star if $8M_\odot \lesssim M \lesssim 30M_\odot$ or a black hole with large initial masses of components, TZ is a Thorne–Zhytkow object, a supergiant with a relativistic core, SN Ib,c is a supernova with a helium envelope, and SN II is a supernova with a hydrogen-rich envelope.

hundred kilometers per second) spatial velocities of radio pulsars that had already been measured at that time. However, a simple analysis of the evolution of massive close binaries showed that for nearly a half of the systems with exploding components, the condition $\Delta M < M_R$ is satisfied and therefore they must persist! To resolve this paradox, an assumption was made that a supernova explosion is accompanied by giving the supernova product a ‘kick’ of several hundred kilometers per second. This kick, which destroys most massive binaries in both the first and second explosions, helps to reveal the reason for the high observed speeds of radio pulsars, which are almost 10 times higher than those of the parent OB stars, and the low degree of duplicity of radio pulsars [31].

The kick, whose physics remains unknown, may be due to incomplete symmetry of the supernova explosion, leading to one-sided ejection of part of its substance: $\Delta m v_{SN} \simeq M_{RR} v_k$, where Δm is the ejected mass, $v_{SN} \simeq 3000 \text{ km s}^{-1}$ is its speed, M_{RR} is the mass of the relativistic remnant of the supernova, and v_k is its final speed, the kick. The asymmetry may be caused by the supernova magnetic field or tidal forces. Obviously, there is room for the kick is an asymmetric ejection of a part of the mass defect with the neutrino. Assuming that the mass defect accompanying the formation of a relativistic remnant is $\sim 10\%$ of the remnant mass, we find that in the case of a unilateral neutrino ejection, the

remnant itself can gain a velocity up to $\sim 30,000 \text{ km s}^{-1}$. It is clear that such pulsar velocities are unknown. Turbulization of the star's core in the course of its collapse can possibly reduce the 'neutrino kick' to the observed value of several hundred kilometers per second, but it must be recognized that a definitive answer to this simple question is lacking.

The disintegration of CBSs in supernova explosions of their components produces stars with high spatial velocities. The magnitude of these velocities is determined by the orbital velocities of the components before the explosion. For systems consisting of a helium-rich star paired with a compact relativistic remnant (a neutron star or a black hole), this speed can be as high as $\sim 500 \text{ km s}^{-1}$ [5]. The decay products of close binaries can also attain even higher speeds, up to 10^5 km s^{-1} , if the binaries interact with a supermassive black hole in the galactic core [77].

The Roche lobe filling by a massive optical star results in an enhanced accretion of gas by the relativistic component and the emergence due to its high potential of a powerful X-ray source with the temperature

$$T \simeq 4 \times 10^5 \left(\frac{M}{M_\odot} \frac{\dot{M}}{M_\odot/\text{year}} \right)^{1/4} \left(\frac{R_\odot}{R} \right)^{3/4},$$

where M is the accretor mass and R is its radius [78, 79].

X-ray binaries have been explored in many studies performed in the 1970s–1980s (see [5] and the references therein). The expansion of the envelope of a donor that has a compact satellite usually results in the emergence of envelopes. This mechanism now seems indispensable for losing the 'excess' of the orbital angular momentum of binary stars and for obtaining extremely tight systems of relativistic remnants whose radiance was observed by LIGO. However, for common envelopes that contain relativistic remnants to be formed, it is necessary to prevent evaporation of the donor envelope due to the induced stellar wind. But this effect is not ruled out due to the immense energy emitted by the compact accretor. Formally, it follows from the energy conservation law that for the donor to 'evaporate' completely, the accretor only needs to acquire a fraction of the mass defined by $\Delta m/M \approx M/m(r/R) = 10^{-5}$, where Δm is the accreted mass of the donor with mass M and radius R , and m and r are the mass and radius of the compact accretor. This implies that in using the formal approach, we should be 'concerned' [78] that the stellar wind self-excited due to the accretion of only $\sim 10^{-5}$ of the donor matter can destroy it, thus skipping not only the common-envelope phase but also the semi-detached stage (see Fig. 4). Several factors can be listed that reduce the efficiency of such a donor evaporation by a close neutron star or black hole. First, the donor, even filling its Roche lobe, occupies only a few percent of the total solid angle, which, naturally, reduces the efficiency of evaporation. Second, the accretion of donor matter by the compact component of a binary star occurs in a disk, but the gravitational energy of the accreted gas is emitted mainly in the polar directions, thus bypassing the donor envelope and therefore significantly diminishing the strength of its induced stellar wind. And, third, the induced-wind matter has a large optical thickness, which also reduces the efficiency of donor evaporation by the component that accretes its matter.

Thus, to answer the question about the formation of a common envelope during the expansion of the donor of a relativistic accretor, it is necessary to solve a very involved problem that takes three of these factors into account at the

model level. With the results of such a study unavailable, we can only hope that only part of the hydrogen-rich envelopes of the donors in such systems are doomed to evaporation by their accretors (see Fig. 4). However, it is now clear that at least part of such systems are immersed into a common envelope, the deceleration of components in which results in the emergence of extremely tight final systems. Relativistic components of such systems merge under the effect of gravitational wave radiation, whose bursts were detected by LIGO [2]. The evolution of a massive CBS at the stage containing a common envelope results in the formation of either a supergiant with a relativistic core (a Thorne–Zhytkow object [80], TZ in Fig. 4), or a close binary consisting of a helium-rich nondegenerate component (a WR star) and a relativistic component. The evolution of a supergiant with a relativistic core (Thorne–Zhytkow object) after a relatively rapid loss of extended-envelope matter due to the intense stellar wind results in the emergence of a single neutron star or black hole. The study of the evolution of Thorne–Zhytkow-type objects is still hindered by the complexity of the physics of the accretion of envelope matter by the core and its evaporation. By analogy with ordinary red supergiants, we can only assume now that an intense stellar wind significantly reduces the lifetime of such a star, turning it into a bright infrared supergiant at $\dot{M} \gtrsim 10^{-6} [L/(10^5 L_\odot)]^{1/2} M_\odot \text{ year}^{-1}$. The evolution of neutron stars in close binaries results in the case of satellite matter accretion in the acceleration of their rotation, reducing the rotation period to a few milliseconds. Accretion and acceleration are especially efficient for long-lived satellites of small masses that produce helium-rich remnants [81].

We continue reviewing massive close binary systems (see Fig. 4). In sufficiently tight systems, the intense stellar wind of the OB component creates the conditions for the accretion of part of its matter by the relativistic satellite. As a result, a powerful X-ray source emerges whose luminosity is $L_X/L_\odot \approx 10^{12} \dot{M}/M_\odot \text{ year}^{-1}$. Its lifetime is determined by the time it takes to double the donor radius, and, for massive main-sequence stars, this can be as long as several million years. The removal of the hydrogen-rich donor envelope exposes the helium-rich nondegenerate core of the donor with a radius several times smaller than the initial radius of the donor, a main-sequence star [16]. If the mass is sufficiently large, the helium-rich star is a WR type. The depletion of nuclear fuel in the core of a helium-rich star results in an outburst of a type-Ib or Ic supernova if the donor has lost not only remnants of the hydrogen-rich envelope but also its helium-rich envelope with the wind, thus exposing the carbon-rich core. As a result of the evolution of a massive close binary, a tight system of two relativistic remnants (neutron stars or stellar-mass black holes) emerges if the loss of matter during the explosion of the second supernova in the system did not exceed half its mass and the kick was small. Otherwise, the second explosion destroys the system, and the relativistic remnants of the evolution of the components acquire spatial velocities of the order of the orbital velocities of the components before the explosion. This is the rationale that has been used for a long time to explain the origin of the observed high speeds of radio pulsars, rotating neutron stars.

The fate of the remaining components of binary systems of relativistic objects (black holes and neutron stars) is extremely interesting. It became clear long ago that the components of the tightest systems of this kind with major semiaxes of the orbits smaller than $\sim 3(M/M_\odot)^{3/4} R_\odot$

eventually merge due to the loss of orbital energy as a result of emission of gravitational waves [82–85]. The first estimates of the rates of such phenomena of the three types NS + NS, NS + BH, and BH + BH in our galaxy were obtained using a scenario-based program developed by Tutukov and Yungelson [84]: $\sim 3 \times 10^{-4}$ per year, $\sim 10^{-5}$ per year, and $\sim 10^{-6}$ per year, respectively. Such estimates, given the remaining uncertain parameters of the scenario-based model, have so far not yielded the values of these rates themselves but rather their orders of magnitude. The rate of black hole mergers in the Galaxy observed by LIGO, which is estimated based on their distances and their merger rates, is $\sim 3 \times 10^{-6}$ per year [23, 41, 46, 82, 84, 85]. The estimated distance to a pair of merged neutron stars is approximately 10% that for merged black holes in the LIGO experiment [41]. Given also that there are five black-hole merger events per neutron star merger event recorded by LIGO, it can be concluded that the rate of neutron star merger per unit volume is approximately 200 times that of the merger of stellar-mass black holes per unit volume. This implies that the value $\sim 6 \times 10^{-4}$ per year can be taken for the Galaxy as a preliminary observational estimate of the rate of neutron star merger. Thus, the very uncertain current estimates of the merger rates of the components of the systems under discussion agree fairly well with their theoretical counterparts. However, we must be aware that both theoretical and ‘observational’ estimates are still insecure due to the remaining uncertainty of theoretical estimates and the low statistics of LIGO events. Much work should be done to refine these estimates.

The black hole mergers result in the emergence of the brightest energy sources in the Universe after the Big Bang. Dividing the radiated energy $\sim 0.1Mc^2$ by the time of the black hole coalescence, we can estimate the intensity of the energy they emit in the form of gravitational waves:

$$L_{\text{GW}} \simeq 0.1 \frac{c^5}{G} \simeq 4 \times 10^{58} \text{ erg s}^{-1}. \quad (5)$$

It is noteworthy that the ‘brightness’ of the signal of a merger of black holes of comparable masses is independent of their mass M . The duration of the merger event is several orbital periods: $10^{-6}(M/M_\odot)$ s, owing to which such events can be identified and the masses of the merging black holes can be estimated. The observation of a stellar-mass black hole merger now paves the way to the observation of the merger of supermassive black holes with masses of $10^3 M_\odot$ – $10^{10} M_\odot$ in merging galaxies [30]. The black holes that are components of X-ray binaries have masses of $5M_\odot$ – $15M_\odot$, while the mass of black holes detected by LIGO [41] is $15M_\odot$ – $50M_\odot$. The reason for the apparent mismatch is the difference in the observational selection effects for these two families of black holes. While the best-represented black holes are detected in the X-ray range, the most ‘abundant’ merger events captured by LIGO are examples of the active merging of the most massive binary black holes that are formed at distances of several Gpc from very massive low-metallicity stars (less than 0.1 of the solar metallicity).

The evolution of the rotation of stars and their compact final remnants requires at least a brief discussion. The characteristic rotation periods of the main-sequence stars with radiative envelopes and masses greater than $\sim 1.5M_\odot$ are about 1 day: $P_{\text{MS}} = 10^5$ s [86]. If an assumption is made that solid-body rotation persists to the stage of supergiants, the rotation period of the latter would be $P_{\text{SG}} \simeq P_{\text{MS}}(R_{\text{SG}}/R_{\text{MS}})^2$. Assuming that the angular momentum of

the core is conserved during its collapse as a young neutron star is formed, we obtain the rotation period of a young neutron star, the product of the evolution of a single massive star, as $P_{\text{NS}} = P_{\text{MS}}(R_{\text{SG}}/R_{\text{MS}})^2 \times (R_{\text{NS}}/R_{\text{C}})^2$, where R_{SG} and R_{MS} are the radii of the giant (supergiant) and the main-sequence star and R_{NS} and R_{C} are the radii of the neutron star and the collapsing core of the giant (supergiant). Taking $R_{\text{SG}}/R_{\text{MS}} = 100$ and $R_{\text{NS}}/R_{\text{C}} = 10^{-3}$ as an estimate, we find $P_{\text{NS}} = 10^3$ s, a value that significantly exceeds the characteristic rotation periods ~ 1 s of the observed radio pulsars. It is apparent, however, that the components of close binary systems rotate prior to the supernova explosion with a period of the order of the orbital period P_{orb} ; therefore, in this simple model, the rotation periods of neutron stars, the products of the evolution of their components, are $P_{\text{NS}} = P_{\text{orb}}(R_{\text{NS}}/R_{\text{C}})^2$. If $P_{\text{orb}} \simeq 1$ –10 days, then P_{NS} is 0.1–1 s, a value that is close to the observed rotation periods of young radio pulsars. This observation arguably implies that most observed pulsars are products of the evolution of massive close binary systems [87].

Black holes can form during the collapse of the cores of the components of such systems, which occurs due to their rapid rotation in two stages. Supernovae with helium-rich envelopes are so compact and rotate so rapidly, and the collapse of the core is so deep, that the angular momentum of the core is too large for a single black hole. As a result of the collapse of the core of a component of such a system, a binary black hole may emerge with the major semiaxis of the orbit of the order of several black hole radii. The radiation of gravitational waves by such tight systems would lead to the merger of their components within a time of the order of several seconds. The assumption that the angular momentum of the core is conserved during its collapse helps in assessing the cause of and condition for the occurrence of two-stage collapse during the formation of black holes in close binary systems with helium pre-supernovae that are close to filling up their Roche lobes and with compact satellites. If the mass of a helium-rich star is $\sim 10M_\odot$, the orbital period is $\sim 10^3$ s. Due to the compression of the core of the star during its collapse by four orders of magnitude, its rotation is accelerated by a factor of $\sim 10^8$, as a result of which the rotation period of a young black hole is $\sim 10^{-5}$ s, a value that is comparable to the period of critical rotation of black holes of stellar masses. Consequently, it becomes possible to consider the conditions under which the presented two-stage scenario can materialize in the formation of black holes in the evolution of massive extremely close binary stars during a second supernova explosion in such systems.

It is now clear, however, that the model displayed is not a ‘mandatory’ one. When used to estimate the rotation of degenerate dwarfs produced by single stars, it yields rotation periods $\sim 10^5$ days, a value that apparently disagrees with the observational estimates of these periods. On the other hand, the assumption that the angular momentum of single-star cores is conserved during the formation of degenerate dwarfs yields ~ 100 s for their rotation periods, a value that rules out the validity of such an approximation. Exploring the evolution of the rotation of stellar cores remains a challenging and important problem, whose solution would yield new insights into the evolution of stars of various masses.

An analysis of the observed rotational velocities of degenerate dwarfs shows that the evolution of rotation of a dense star core cannot be described either under the assumption that the entire star rotates as a solid body or under the assumption that the angular momentum of its core

remains constant. If it is assumed that the star rotates as a solid body, the supergiant phase decelerates the core rotation by a factor of $\sim 10^5$ compared with the rotation of a main-sequence star. But the angular momentum of the degenerate carbon–oxygen core of the star is conserved, its rotation is accelerated by a factor of almost ~ 100 in comparison with the initial one. Observations of the rotation of degenerate dwarfs with masses $0.6M_{\odot}–0.9M_{\odot}$ showed that the periods of their axial rotation fall in the range 0.3–3 days [90], a value that is close to the initial periods of rotation of main-sequence stars with corresponding masses [87]. A comparison of the observed rotation speeds of degenerate dwarfs with the maximum ones within the two approaches described above shows that none of the two limit options is realized, and the problem of the evolution of rotation of stellar cores remains unresolved [88].

6. Multiple systems: a clue to understanding the mechanisms of the origin of close binary stars

We now turn to the description of the observational manifestations of close binary and multiple systems. As noted by Batten [89], because the number of parameters characterizing a multiple system is much larger than that for a binary system, the relations between the parameters of multiple systems (ratios of orbital periods, mass ratios, relative orbit orientation, etc.) contain important information on the mechanisms of formation of binary systems.

Modern observational data on ternary stellar systems and systems with greater multiplicity, as well as their evolutionary analysis, are presented in book [5]. Statistical studies of multiple systems are based on the analysis of data of the corresponding catalogs [90–98]. Catalog [98] offers data on 612 physical multiple systems whose multiplicity ranges from three to seven; those systems are, with some exceptions, hierarchically arranged. For each subsystem, the catalog contains orbital periods, angular separation of components, and mass ratios.

We first consider the characteristics of multiple systems that contain eclipsing binary systems [99]. Catalog [90] provides information on 80 multiple systems that contain eclipsing pairs. Most of them are ternary systems in which the third star is visually detached from the eclipsing binary system. The third component can only be distinguished in some cases using spectroscopic techniques or speckle interferometry. For example, the famous Algol eclipse system, the first eclipse binary, which was discovered by English amateur astronomers Edward Pigott and John Goodricke in 1782 and 1783, turned out to be a spectrally ternary system whose total spectrum exhibits pronounced lines related to the presence of a third star in the system.

The Chambliss catalog [90] also contains higher-order systems in addition to ternary systems. Quaternary systems can exhibit various hierarchies. In the case of hierarchy 2 (the system is organized according to the $2+2$ scheme), two close pairs are essentially detached (like a quaternary system containing two eclipsing binary systems: BV and BW Draconis). In the case of hierarchy 3 (the system is organized according to the scheme $(2+1)+1$), the hierarchical ternary system has a remote component (an example is η Ori).

Quinary and senary systems are much less common. An example of a quinary system is HR3337, which contains an eclipsing binary system with a period of 2.50 days. An

example of a senary system is the bright Castor star, which contains the eclipsing binary YYGem. Castor A and Castor B are spectroscopic binaries with respective periods 9.213 and 2.928 days. The period of their orbiting relative to each other is 450 years. The visual satellite, the eclipsing binary YYGem system, is located at an angular distance of 71 s, which corresponds to a minimum separation at a distance of about 1000 astronomical units (a.u.) and the orbital period of at least 15,000 years.

There are also systems of the Orion Trapezium type that are considered to be multiple systems containing many stars separated by comparable distances. It is believed that such systems are unstable and decay in relatively short times [34]. The Orion Trapezium itself is the central part of a rich star cluster in the Orion constellation that contains two young eclipsing systems: BMOri and V1016Ori. Another example of an eclipsing binary system in the NGC1502 star cluster is the SZCam system that has a visual satellite ADS2984A.

Statistics of multiple systems in [99] shows that about 20 to 30% of all binary stars are at least ternary and about 20 to 30% of ternary systems are quaternary, etc. The orbit of the third star in most ternary systems is noncoplanar to the tight-pair orbit. Although the orbit of the eclipsing pair is usually circular, the orbit of the third, distant component is almost always elliptical. The most characteristic case is the V772Her system, in which the eccentricity of the third-star orbit is 0.958. The ratio of the orbital periods P_2/P_1 and the ratio of the major semiaxes a_2/a_1 in ternary systems vary from system to system over a very wide range. For example, in three ternary systems that contain B stars (λ Tau, VV Ori, and IUAur), the ratio P_2/P_1 is relatively small, respectively equal to 8.35, 80.8, and 162. But in most cases, the value of P_2/P_1 is several thousand or more. The third component in the IUAur ternary system causes strong perturbations in the orbit of the eclipsing binary system, which results in significant changes in the inclination of the eclipsing binary orbit at characteristic times of several dozen years. We note in this regard that eclipses sometimes cease in some eclipsing systems (for example, in the SSLac system) [100], which can be due either to the ellipticity of the orbit and the rotation of its major semiaxis or to a change in the inclination of the orbit of the eclipsed binary system caused by the disturbing effect of the third star in the system. The eclipsing systems that contain early B stars belong most often to multiple systems. It turns out that short-period eclipsing WUMa-type systems are often also members of multiple systems. Examples of such systems are GZAnd, 44iBoo, VWCep, AACet, CCCom, BVDra, BWDra, AKHer, AMLeo, HTVir, AWUMa, and WUMa. Among them, BVDra, BWDra, AKHer, and AACet are quaternary systems, GZAnd is a quinary system, and the rest are ternary systems.

It has been found that close binary systems of the RSCVn type and cataclysmic binary systems rarely belong to ternary systems or systems with greater multiplicity. These data are not consistent with the conclusions of the theory, according to which cataclysmic binaries originate from WUMa-type systems.

We now proceed to a discussion of the Tokovinin catalog data [98]. Figure 5 shows the periods of the wide subsystems P_L (in a logarithmic scale) versus the periods of the corresponding close subsystems P_S . The straight line in the plot displays equal periods, $P_L = P_S$. The absence of systems near this straight line is a definite indication that the stability criterion for multiple systems [101] is satisfied: for most

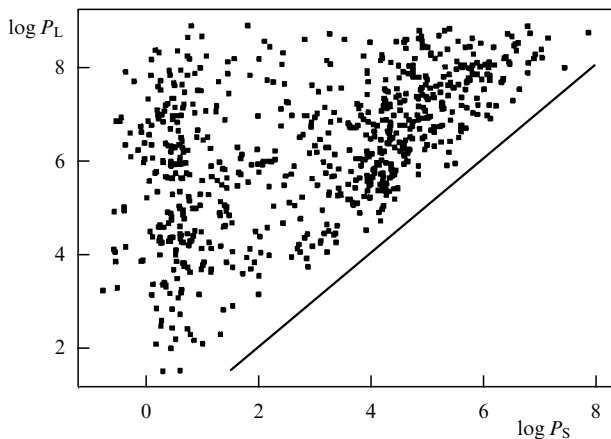


Figure 5. Relation between logarithms of orbital periods for short-period (period P_S) and long-period (period P_L) subsystems at appropriate hierarchical levels. The periods are measured in days. The systems whose multiplicity is greater than three are displayed on the plot with more than one dot. The solid line corresponds to equal periods. (Taken from [98].)

multiple systems, $P_L > 10P_S$. The P_L values are limited from the side of large values ($P_L > 10^6$ years) by the processes of disintegration of multiple systems. The disintegration is due to their interactions with the molecular clouds of the Galaxy and individual stars [102], as well as the Larson limit [103], which sets the conditions under which clusters of young stars are formed at the stage that precedes the main sequence.

The shortest periods, $P_S \simeq 0.3$ days, correspond to contact binary systems. As can be seen from Fig. 5, multiple systems occupy almost the entire triangular region in the upper left part of the $\log P_S$ – $\log P_L$ plot. No other correlation between the periods P_S and P_L apart from this feature is found. In particular, there is no distinguished value of the ratio of the periods P_L/P_S or the dependences of P_L or P_L/P_S on the mass of the primary component. The vectors of angular orbital momenta of wide and close subsystems were shown in [98] to exhibit a weak trend to coincide. All these results should be taken into account in theoretical simulations of the formation of binary stellar systems, as well as stellar systems with greater multiplicity.

A comparative statistical analysis of ternary and quaternary systems was performed in [104], and various scenarios of the formation of multiple stars, including binary stars, were analyzed. A typical quaternary ϵ Lyr system consists of two wide visually binary systems that move in an even wider relative orbit (a 2+2 hierarchy). Such a configuration of quaternary systems is shown to be typical in the Galaxy. In [104], the statistical characteristics of 81 quaternary systems with the 2+2 hierarchy and 724 ternary systems are studied in detail. The main conclusions are as follows.

In 42% of quaternary systems arranged according to the 2+2 hierarchy, the mass ratio of components in the outer orbit exceeds 0.5, and the periods of the inner orbits differ by a factor not greater than 10. The distributions of periods of inner systems in ternary and quaternary systems are similar to each other and, possibly, bimodal. The mass ratios of the components in the inner orbits do not correlate with the internal periods. The statistics on the outer orbit periods and the mass relations in the ternary and quaternary systems are different. The mass ratios of the components in the inner orbits and outer orbits in ternary and quaternary systems do not correlate with each other. The angular momenta for the

inner and outer orbits, as well as the corresponding periods in ternary and quadruple systems with periods of internal systems of more than 30 days, show some correlation. The directions of the angular momentum vectors of large and small orbits in systems with a small ratio of external and internal periods are correlated with each other, while for systems with a large ratio of periods, such a correlation is absent. The author of [104] concludes that the observed statistical properties of ternary and quaternary systems do not agree with the dynamic decay model of small star clusters, and the dominant process in the formation of multiple star systems is not the N -body dynamics. At the same time, the entire set of statistical data on ternary and quaternary systems is consistent with the model of cascaded fragmentation stimulated by rotation of the disk with the subsequent migration of protostars formed in this way.

Thus, the entire set of observational data on binary systems and systems with greater multiplicity indicates that the cascade fragmentation initiated by rotation with subsequent evolution caused by accretion and/or migration is the preferred and most typical mechanism of formation of multiple (and binary) stars. This qualitative scenario of the formation of such stars requires further quantitative study in the 3D gas dynamics of a multicomponent medium taking the magnetic field effect into account.

The decay of unstable ternary systems with a small ratio of major semiaxes results not only in the emergence of closer binaries but also in the guaranteed emergence of a single star. An essential feature of the decay of the closest of the ternary stars is the acceleration of an ejected single star to velocities of the order of parabolic velocities on the surfaces of the components that remain in the coupled binary system. If the remaining system consists of main-sequence stars, these speeds can be as high as several hundred kilometers per second. The decay of compact ternary systems consisting of degenerate dwarfs can result in the emergence of dwarfs at speeds of several thousand kilometers per second. The acceleration effect increases if binary stars interact with a supermassive black hole in the galactic core. The decay of such ternaries enables even main-sequence stars to be accelerated to relativistic speeds. Thus, the decay of tight ternary systems is an efficient generator of the rare high-speed stars yet to be discovered.

7. Variety of close binary systems and their observational manifestations

Observations show that the fraction of multiple (including binary) systems among the stars in the Galaxy exceeds 60–70% [4]. As Martynov has repeatedly emphasized [105], virtually any combination of components can be found in binary systems. Close binary systems can include main-sequence stars, subgiants, giants, supergiants, WR-type stars, Be stars, white dwarfs, neutron stars, black holes, brown dwarfs, planets, and pulsating stars, such as Cepheid variables. The variety of close binary systems is immense. Their periods range from very short — 18 minutes (AMCVn), 46 minutes (GPGCom), and even 321 s (RXJ0806.3+1527, a system of two degenerate dwarfs) — to very large (for example, 27.2 years for the ϵ Aur system). The sizes of orbits in close binary systems vary widely: from $0.05R_\odot$ to several dozen astronomical units (see, e.g., reviews [106, 107]). The orbital periods of merging close binary black holes in recently discovered gravitational-wave binary systems vary from 0.1 s

to a value of the order of several thousandths of a second at the moment that immediately precedes the merger and the formation of a new black hole of large mass [2, 75, 76]. The observed orbital period immediately before the merger in the gravitational-wave binary system of two merging neutron stars is about 0.001 s [15]. We now consider observational manifestations and the evolutionary status of close binary systems of various types.

7.1 Detached main-sequence systems

According to modern concepts [4, 108, 109], a binary star system is referred to as a close system if the exchange of matter between its components occurs at some stage of its evolution. In stars with masses greater than $0.7M_{\odot}$, the nuclear evolution time is less than the age of the Universe ($\sim 1.4 \times 10^{10}$ years). Because the star radius increases on average during its nuclear evolution, a more massive star in the system may be the first to fill its critical Roche lobe (referred to as the Roche lobe below), and mass transfer from the more massive star to the less massive star begins in the system. In that case, the relative distance between the stars decreases, which facilitates further mass exchange. In the case of stars with masses less than $0.7M_{\odot}$, whose nuclear evolution time is greater than the age of the Universe, the nuclear evolution of the star can be ignored. The main factor driving the evolution of low-mass close binary systems is the reduction in the distance between the components caused by the loss of energy and angular momentum by the system due to the emission of magnetic stellar wind and gravitational waves.

According to the modern classification [110], detached main-sequence systems are referred to as DM-systems (Detached Main-Sequence Systems). These systems contain main-sequence stars on the Hertzsprung–Russell spectrum–luminosity diagram, which are far from having filled Roche lobes. However, according to the definition given above, they can be referred to as close systems because mass exchange can occur in such systems in the future (if the initial distance between the stars in the physical binary system is not too large). In addition, detached systems include young stars at the stage prior to the main sequence that do not fill their Roche lobes (pre-MS systems, for example BMOri).

The study of detached main-sequence systems allows obtaining basic data on the masses, radii, and luminosities of stars and determining their age and chemical composition [5]. We note that these characteristics, information about which is of fundamental importance for studying the internal structure of stars and their evolution, can be extracted from analyzing light curves and radial velocity curves, regardless of the distance to the system. In addition, the exploration of detached eclipsing binary systems makes it possible to determine the limb darkening coefficients for stars of various spectral classes, which is of importance for testing theories of stellar atmospheres. The study of stars eclipsed by exoplanets is especially promising. Owing to space missions COROT (CONvection ROTation and planetary Transits) and Kepler, it is now possible to obtain eclipsed light curves, in this case with an accuracy of $10^{-4} - 10^{-5}$ stellar magnitudes (the accuracy of the eclipse curves obtained from the surface of Earth is $10^{-2} - 10^{-3}$ stellar magnitudes). The analysis of the curves of eclipse of stars by exoplanets (the depth of which is relatively small, an order of 10^{-2} stellar magnitudes) allows obtaining important information about the radii of exoplanets and the radii of eclipsed stars, and determining the limb darkening coefficients of stars belonging to various spectral classes (see

the review in book [5]). Spectral studies of eclipses of stars by exoplanets also enable the exploration of the chemical composition of atmospheres of exoplanets, which is of importance for astrobiology.

By studying the rotation of the semimajor axis of elliptical orbits in detached eclipsing systems, it is possible to obtain valuable information about the mass distribution in the body of the star and even estimate the distribution of the angular velocity of its rotation along the radius [111, 112].

Methods of multi-slit spectroscopy and the results of photometric observations of stellar fields, part of the program for searching for gravitational microlensing effects, have been used recently for a massive determination of the parameters of many eclipsing systems by means of the joint interpretation of light curves and radial velocity curves (see, e.g., [113]). It was shown for several dozen detached massive ($M_1 + M_2 > 12M_{\odot}$) systems that both components in these systems belong to the main-sequence strip, and neither the main (more massive) nor secondary components have moved far from the initial main sequence corresponding to zero age. The authors of [113] used evolutionary calculations [114] to compare the observed characteristics of massive stars in detached systems with theoretical ones for the stellar matter metallicity $Z = 0.004$, characteristic of the Small Magellanic Cloud. Positions of the primary and secondary components were compared in the ‘star mass logarithm versus the acceleration of gravity’ plot. Because stars in a close binary system have the same age, their characteristics must correspond to isochrones, lines of the same age. Moreover, because there has been no mass exchange in a detached close binary system, each component should ‘belong’ in this case to the isochrone that corresponds to its mass. Because the mass ratio averaged over several dozen investigated detached systems is close to unity (~ 1.2 , most likely the effect of observational selection), the difference among star evolution rates can be disregarded. Therefore, both stars in a detached close binary system should in most cases belong to the same isochrone, which is observed for several dozen detached close binary systems [113], thus confirming the conclusions of the evolutionary theory. Another confirmation of the theory is the good agreement between the evolutionary mass of the star M_e (obtained using evolutionary tracks) and the observed dynamic mass M_d determined from the radial velocity curves of close binary systems. These conclusions have been made both for massive close binaries ($M_1 + M_2 > 12M_{\odot}$) and for stars in moderate-mass close binaries ($M_1 + M_2 \lesssim 12M_{\odot}$).

Another rigorous test of the stellar evolution theory is to compare the positions of stars, detached binary components, with the corresponding isochrones on the mass–luminosity plot. In detached systems, the components must lie no lower than the zero-age sequence. As observations and their analysis show, this is true in the overwhelming majority of cases in detached close binary systems: within the error with which stellar parameters are determined, almost all stars in a detached close binary system under study lie either on the zero-age sequence or above it. In general, as the authors of [113] conclude, the agreement between the parameters obtained from observations and theoretical model parameters of stars is very good in the case of detached close binaries for 40 stars out of the 44 studied. The parameters of detached close binary systems are summarized in book [5], where they are also compared with model parameters. The positions of stars in detached close binary systems in the Hertzsprung–Russell diagram are shown in Figs 6 and 7.

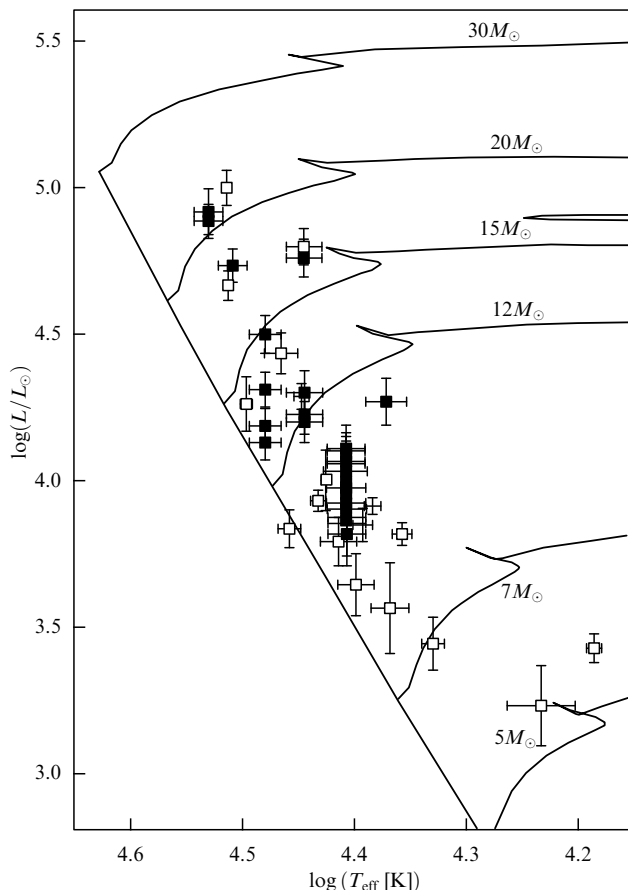


Figure 6. Location of primary and secondary components of detached massive ($M_1 + M_2 \geq 12M_\odot$) systems (21 systems in the Small Magellanic Cloud) on the Hertzsprung–Russell diagram. Displayed are the initial main sequence and evolutionary tracks of single stars with masses $5M_\odot$, $7M_\odot$, $12M_\odot$, $15M_\odot$, $20M_\odot$, and $30M_\odot$ and metallicity $Z = 0.004$. (Taken from [113].)

As regards O stars and early B stars belonging to the main sequence (see, e.g., [115]), there is the so-called mass problem: the masses of stars of this type estimated from evolutionary tracks on the Hertzsprung–Russell diagram turn out to be systematically larger than those found from spectroscopic analysis or from the stellar-wind theory [116]. The mass problem can be associated with the mixing induced by stellar rotation, which can be especially fast in relatively short-period massive close binary systems. If the effect of mixing and enrichment of the star with helium [115] is taken into account, the evolutionary tracks shift towards an increase in the effective temperature T_{eff} , and the farther the star is from the initial main sequence, the stronger the effect. In particular, the segment of the track that corresponds to the final stage of the main-sequence phase for a star with $M = 20M_\odot$ can be shifted in T_{eff} by 20% relative to the segment calculated without taking mixing into account.

7.2 Semi-detached systems

Semi-detached (SD) systems (S-systems), which are classical Algol-type systems, enable exploration of the characteristics of stars at various stages of evolution (the stages of the main sequence and subgiants). A paradox has long been noticed in systems such as Algol (the Algol paradox), which consists of the following. The less massive component fills its Roche lobe and departs from the main sequence with the characteristics

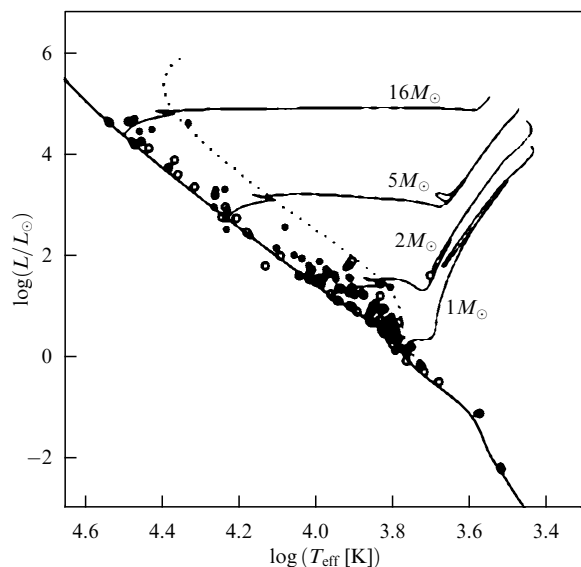


Figure 7. Location of primary and secondary components of detached moderate-mass ($M_1 + M_2 \gtrsim M_\odot$) Algol-type systems on the Hertzsprung–Russell diagram. Displayed are the initial main sequence and evolutionary tracks of single stars with masses $1M_\odot$, $2M_\odot$, $5M_\odot$, and $16M_\odot$. The dashed line shows the upper boundary for main-sequence stars. (Taken from [119].)

of a subgiant, while the more massive component does not fill its Roche lobe and is a main-sequence star [117, 118]. Because the rate of nuclear evolution of the star is approximately proportional to its mass cubed, it seemed paradoxical that a less massive star manages to overtake its more massive neighbor in evolutionary development (stars in a close binary system have the same age). This paradox is now easily resolved in the theory of close binary evolution with mass exchange: the currently less massive star of the pair (a subgiant) was previously more massive than its satellite, but it evolved faster than the satellite, filled its Roche lobe first, and lost a significant fraction of the mass due to mass exchange.

To date, a wealth of observational material has been accumulated on SD close binary systems, whose interpretation in modern close binary models based on the synthesis method (see, e.g., [5]) allowed detailed verifications of the theoretical conclusions about mass exchange in close binary systems. Secondary, less massive matter donor components in massive SD systems show clear signs that they are the stars that have evolved and lie to the right of the main sequence and have experienced a loss of mass due to the flow of matter in the close binary system [113] (Fig. 8). At the same time, the primary accretor components occupy a strip in the Hertzsprung–Russell diagram that is characteristic of stars close to the main sequence [113]. The primary components in SD systems increase their mass due to mass exchange.

A comparison of the characteristics of the primary components (T_{eff} , R , L , etc.) with those of single stars of the same masses allows verifying whether the primary components have reached thermal equilibrium after the stage of rapid mass exchange and an increase in their mass. It was shown in [113] that the luminosities and effective temperatures of the primary recipient components in massive SD systems are in good agreement with the luminosities and temperatures of equilibrium single stars of the same masses. Such good agreement indicates that the primary recipient components in massive SD systems managed to reach thermal

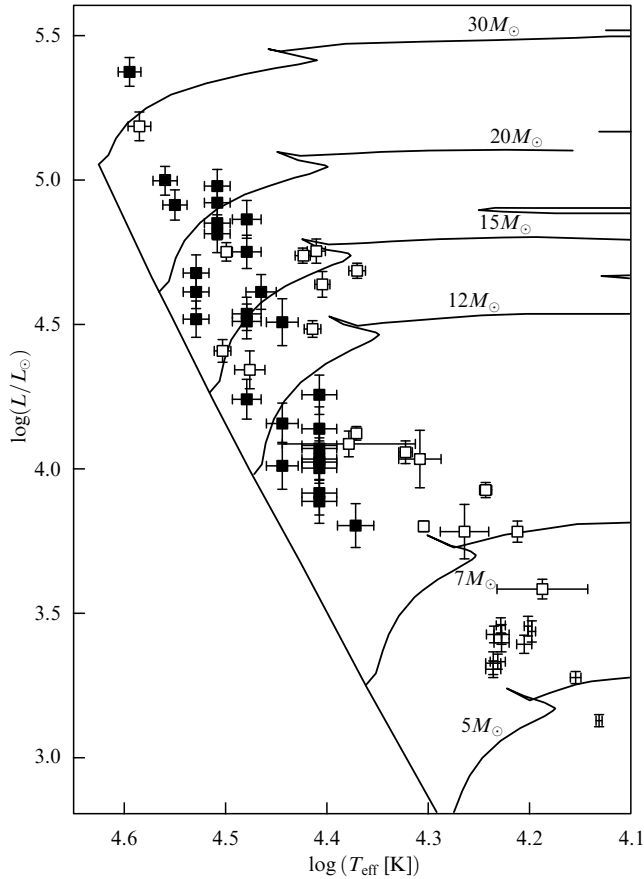


Figure 8. Location of primary and secondary components of semi-detached massive systems (28 semi-detached and one contact system in the Small Magellanic Cloud) on the Hertzsprung–Russell diagram. Displayed are the initial main sequence and evolutionary tracks of single stars with masses $5M_{\odot}$, $7M_{\odot}$, $12M_{\odot}$, $15M_{\odot}$, $20M_{\odot}$, and $30M_{\odot}$ and metallicity $Z = 0.004$. (Taken from study [113].)

equilibrium after mass exchange and a rapid increase in their mass.

The evolutionary status of moderate-mass stars in Algol-type close binary systems ($M_1 + M_2 \gtrsim M_{\odot}$) was verified using a large amount of statistical material (74 detached systems and 61 SD systems) in [119]. Similarly to massive SD systems, in moderate-mass SD systems, the primary components lie on the Hertzsprung–Russell diagram near the main sequence (Fig. 9). The secondary components, subgiants, lie much to the right of the main sequence in moderate-mass SD systems (unfilled circles in Fig. 9), exhibiting significant excesses of radii and luminosities compared to those that correspond to their masses. The average characteristics of the detached and semi-detached Algol-type moderate-mass CBSs are displayed in Table 1, as borrowed from [119].

The orbital angular momentum of the SD Algol-type systems is on average just over one third that of the detached systems. The secondary components of Algol-type SD systems feature significant excesses of radii and luminosities. It is of interest to determine the factor due to which the secondary components exhibit excessive luminosity, although the surface temperatures of these components are relatively low. The point is that their cores are enriched with helium, and, in the region of small masses of the secondary components of a close binary system (less than M_{\odot}), their temperatures decrease with the decrease in the star mass much

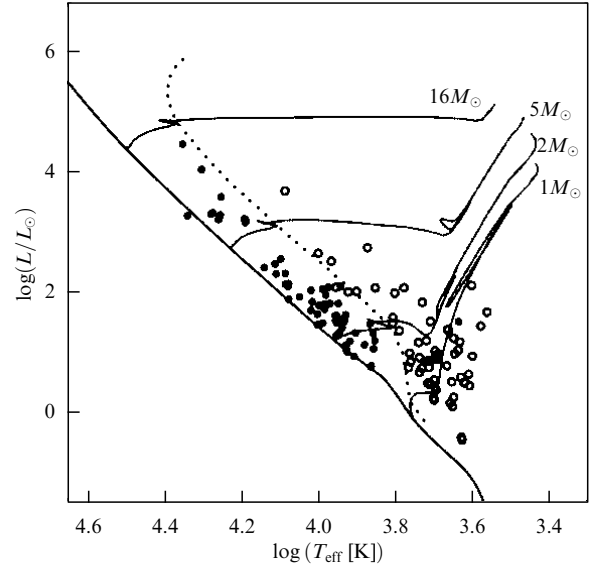


Figure 9. Location of primary and secondary components of semi-detached moderate-mass ($M_1 + M_2 \gtrsim M_{\odot}$) Algol-type systems on the Hertzsprung–Russell diagram. (Taken from [119].) Displayed are the initial main sequence and evolutionary tracks of single stars with masses $1M_{\odot}$, $2M_{\odot}$, $5M_{\odot}$, and $16M_{\odot}$. The dashed line shows the upper boundary for the main sequence.

more slowly than the temperatures of single stars of the same masses. Therefore, the observed excesses of luminosities are especially large for low-mass secondary components of close binary systems (with masses less than M_{\odot}).

The primary and secondary components of the detached Algol-type close binary systems ($M_1 + M_2 \gtrsim M_{\odot}$) are described by almost identical mass–luminosity laws:

$$L \sim M^{3.9}.$$

At the same time, the primary components of SD Algol-type systems obey the mass–luminosity dependence with a significantly lower exponent:

$$L \sim M_1^{3.2},$$

while the secondary components of SD Algol-type systems exhibit an even lower exponent in the mass–luminosity law due to the presence of a degenerate helium-rich core:

$$L \sim M_2^{1.5}.$$

These differences reflect differences in the evolutionary tracks of single stars and stellar components of close binary systems: single stars evolve with constant mass and a radius that can change with no restrictions whatsoever, while stars in a close binary system are stars with variable mass and a radius whose change is constrained due to tidal forces acting in the binary system.

The distributions of the orbital angular momentum in an SD Algol-type systems with periods $P_{\text{orb}} > 5$ days and $P_{\text{orb}} < 5$ days are different. This may be due to the difference between the mechanisms of matter transfer from the secondary donor star (in the nuclear scale of evolution time) to the primary more massive star: the formation of a rotating gas disk around it or the direct impact of a gas stream on the surface of the recipient star [119]. The numerical values of the

Table 1. Average characteristics of detached and semi-detached Algol-type systems * (taken from study [119]).

Type	Number of systems	Orbital period P_{orb} , days	$\langle q \rangle = \left\langle \frac{M_2}{M_1} \right\rangle$	$\langle M_1 \rangle, \langle M_2 \rangle$	$\langle M \rangle = \langle M_1 \rangle + \langle M_2 \rangle$	$\langle R_1 \rangle, \langle R_2 \rangle$	$\langle T_1 \rangle, \langle T_2 \rangle$	J'
Detached close binary systems	72	5.2	0.88	$3.6M_{\odot}$ $3.1M_{\odot}$	$6.7M_{\odot}$	$2.8R_{\odot}$ $2.4R_{\odot}$	11,242 K 10,400 K	15.0
Semi-detached close binary systems	61	6.1	0.27	$3.7M_{\odot}$ $1.0M_{\odot}$	$4.7M_{\odot}$	$3.2R_{\odot}$ $5.3R_{\odot}$	11,066 K 5441 K	4.6

* $J' = J/k$, where $k = 1.24 \times 10^{52}$, J is the orbital angular momentum of the system,
 $J = \left(\frac{G^2}{2\pi} \right)^{1/3} \frac{q}{(1+q)^2} M^{5/3} P^{1/3} = 1.24 \times 10^{52} q (1+q)^{-2} M^{5/3} P^{1/3}$,
 where P is the orbital period (in days), M_1 and M_2 are the mass of the primary and secondary component (in solar mass units), $q = M_2/M_1$, and $M = M_1 + M_2$.

parameters of detached and semi-detached Algol-type moderate-mass close binaries are given in book [5].

Subtle evolutionary effects related to the rotation of stars in massive close binary systems, which are driven by the interaction between components by means of tides, mass exchange, and merger, are described in [120], where the population synthesis method was used to calculate the distribution of the parameters of massive close binaries in the Galaxy, including the distribution of rotational speeds of the components. The secondary star that accretes the primary-star matter was shown in most cases to receive a significant angular momentum and quickly spin up to a critical (Kepler) equatorial rotation speed 500–600 km s⁻¹. Importantly, to increase the speed of star rotation to a critical value, only a few percent of its mass has to be accreted. Further, after the star is spun up to the critical speed, accretion continues, because the excessive angular momentum of the accretion disk is carried away by tidal interactions. As emphasized in [120], the rotation of close binary components is of importance for their evolution, the occurrence of star mergers, and the formation of single rapidly rotating massive stars (in particular, Be stars), in which anomalies in the surface chemical composition can be observed due to the meridional circulation of matter. Thus, the observed evolutionary statuses of stars in massive and moderate-mass close binary systems show good overall agreement between theory and observations.

7.3 WUMa-type contact close binary systems and their relation with chromosphere-active RSCVn-type close binary systems

Enhanced stellar activity is observed in systems such as RSCVn, most of which are detached. This activity is associated with manifestations of magnetic fields: spots and other active regions of the chromosphere, corona, and flares. Because the RSCVn-type close binary systems exhibit the highest surface and magnetic activity of the components due to the presence of magnetic fields, the effect of braking by a magnetic stellar wind is in this case significant [4, 121]. Therefore, it may be assumed that it is from RSCVn-type systems that the WUMa-type contact systems are formed. The parameters of contact and RSCVn-type systems are given in [122] and in [5]. Huang [123] was the first to suggest that the braking of a detached close binary system by the magnetic stellar wind can substantially bring components together and form a contact system. He also applied the magnetic braking mechanism proposed by Schatsman [124] and developed by Mestel [125] to binary systems.

Skumanicz [126], in studying the rotation of dwarf stars of late spectral classes with deep convective envelopes, which belong to open clusters of various ages, discovered an empirical relation between the equatorial rotation velocity of the star V_e and its age t :

$$V_e = \lambda \times 10^{14} t^{-1/2} [\text{cm s}^{-1}],$$

where t is the time in seconds and $\lambda \simeq 1$. According to this formula, the axial rotation speed of a star rapidly decreases with increasing age, a phenomenon associated with the ablation of the angular momentum of the star by the magnetic stellar wind.

If a close binary system contains low-mass stars with convective envelopes, the alignment of the axial rotations of components and orbital revolution caused by tidal dissipation of energy occurs in a time much shorter than that of the nuclear evolution of stars. Therefore, the deceleration of the axial rotation of stars due to the outflow of magnetic stellar wind from those stars is transferred into orbit and causes a loss of the orbital angular momentum by the close binary system. Moreover, because the rate of mass loss to the magnetic stellar wind is very low (for the Sun, it is $\dot{M} \simeq 10^{-14} M_{\odot} \text{ year}^{-1}$), it can be assumed that the component stars of a low-mass close binary system evolve with a constant mass. The evolution of the system is then conservative regarding mass, but not conservative with regard to angular momentum. It is of importance that the constituents of RSCVn-type stars themselves spin up; this is a mechanism that drives their activity.

Another mechanism of the formation of short-period close binary systems, including WUMa-type contact systems, proposed in [127, 128], is based on the model of a hierarchical ternary system and the so-called Kozai mechanism [129]. According to this mechanism, if a long-period orbit in a ternary system is tilted by an angle greater than 39° with respect to the plane of the short-period orbit, Kozai cycles arise in which the inner orbit becomes heavily eccentric, while the period and major semiaxis of this orbit remain almost constant. When the star passes the periastron of the generated elliptical orbit, a secular decrease in the size of the orbit of the internal binary system occurs due to the increase in tidal dissipation of the orbital motion energy, until the orbit becomes circular with an orbital period of several days. This mechanism is operative for stars of both late and early spectral classes in the case where braking by the magnetic stellar wind is not essential. If the components of the internal system are stars in the late spectral class, then braking by the

magnetic stellar wind can additionally bring them closer, leading to the formation of a WUMa-type contact system and next to the merging of the components and the formation of a single rapidly rotating star (a so-called blue straggler). An example of such a star is ABDra, a very rapidly rotating dwarf star of spectral class K whose age is about 5×10^7 years [128].

We emphasize that as was shown in [130], up to 50% of contact binary WUMa-type systems have a third satellite component. Given the effects of observational selection, it may be assumed that almost all WUMa-type systems are ternary (or feature a higher multiplicity). Thus, the formation of contact WUMa-type systems occurs both under the effect of their ternary nature and due to the loss of the orbital angular momentum in the outflow of magnetic stellar wind.

Statistical studies [131] have shown that the maximum in the distribution of contact systems by orbital periods is in the range of periods 0.31–0.40 days for W-type systems and 0.35–0.40 days for A-type systems. It is generally accepted that the A- and W-type contact systems have respective radiative and convective envelopes. The border-line temperature for energy transfer in systems of these two types in WUMa-type star envelopes is ~ 7200 K.

7.4 Special types of classical close binary systems

Massive SD systems, active Algols (systems similar to WSer and β Lyr), are known that can be used as laboratories for studying the rapid stage of primary mass exchange. These systems belong to the final stage of primary mass exchange, which immediately precedes the late evolution stage of close binary systems [132]. An example of such a system is RYSct, an OB+OB system, which, according to [133], directly precedes the formation of an OB+WR system. Due to the rapid nature of this stage, the number of known WSer- and β Lyr-type systems is small. The catalog of late close binary systems [134] contains about 10 such systems, while the total number of known eclipsing binary systems is several dozen thousand. The less massive BO star with a mass of $(8–10)M_{\odot}$ that shows an excess of helium in the spectrum is now completing the primary mass exchange stage in the RYSct system and is at the stage of helium core exposure and transformation into a WR type star. The mass of the OB satellite in the RYSct system is $\sim 35M_{\odot}$ (it increased at the fast stage of primary mass exchange in the thermal evolution time scale). Around the massive OB satellite, a geometrically thick disk is formed that consists of matter not yet deposited on the star, which shields the OB star photosphere. This phenomenon explains the absence of absorption lines in the spectrum of the more massive component of the system (Fig. 10). We note that an excess of helium in the spectrum of the massive β Lyr close binary system, which indicates that this system underwent a rapid stage of primary mass exchange, was first revealed in the pioneering work of Boyarchuk [135].

There are also systems containing two subgiants (DR systems). These are detached systems in which both components are subgiants that do not fill their Roche lobes. These systems are also referred to as the ARLac-type systems or the RSCVn-type systems discussed above. Information about systems of this type is collected in book [5] as well as in catalog [136]. The hotter component in such detached close binary systems usually belongs to the F–G spectral class. Enhanced emission lines are observed in the spectrum of such systems in phases outside the eclipses, a phenomenon that indicates the chromospheric activity of the components. It is

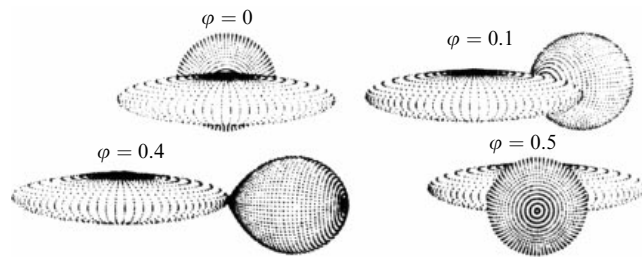


Figure 10. Eclipses in the RYSct system in a model with a geometrically thick disk that surrounds a more massive OB star formed at the concluding stage of the initial mass exchange. (Taken from study [133].)

generally assumed that the mass exchange in such systems consisting of two subgiants is currently negligible. Exploration of such detached systems containing subgiants is of particular interest to the theory of evolution close binary systems.

There are also systems that contain giants and supergiants (DG systems). These are DSs with large orbital periods in which at least one component has evolved away from the main sequence (for example, to the stage of a supergiant or a late-type giant, as is the case, in particular, in the VVCep system). The evolutionary stage of such systems is not completely clear [137]. If the secondary, less massive, component is on the main sequence, then such long-period systems can be assumed to be at the stage that immediately precedes the beginning of primary mass exchange.

In systems with a mass deficit of components (such as RCMa), both components exhibit an excess of luminosity, radii, and temperatures given their masses. None of the components of such systems can be attributed to the main sequence [138]. There are also special systems [139] in which both components fill their Roche lobes, but their surfaces do not touch each other due to asynchronous rotation of at least one component (due to which the size of the corresponding Roche lobe is smaller than that in the case of synchronous axial and orbital rotations).

We also note that a new subclass has been distinguished recently among SD systems: those whose components oscillate with characteristic periods $P_{\text{puls}} = 20–300$ min (see review [140]). This subclass is denoted as oEA (oscillating Algol-type eclipsing binaries): to date, several dozen oEA systems have been discovered. In these systems, pulsation variability is due to the main, more massive, star accreting matter from the donor component. The pulsation mode is affected by the nature of the accretion of matter.

Finally, the discovery of exoplanets that are components of close binary systems should also be mentioned. This discovery is a problem for the theory of evolution of close binary systems with mass exchange, requiring a dedicated theoretical study. New methods and results on the development of 3D gas dynamics of close binaries [141, 142] create hope for further progress in our understanding of the physics of mass transfer in them and the evolution of interacting binary systems.

7.5 Close binary systems at the late evolutionary stages

Late close binary systems are systems at the stage after the initial mass exchange. The catalog of such systems [134] contains about 650 systems of this type. To date, the number of known late close binary systems has increased to several thousand. After the initial rapid mass exchange is completed,

late close binary systems may contain a WR star, a white dwarf, a neutron star, or a black hole as peculiar components.

Studies of late close binary systems are of special interest to astrophysics and basic physics for the following reasons: the chance to rigorously test the theory of evolution of stars with variable mass; the opportunity to discover and explore basically new objects (neutron stars and black holes); the possibility of testing the general relativity (GR) theory (binary pulsars, merging black holes, and neutron stars in gravitational-wave binary systems, etc.); and the availability of vivid observational manifestations that are due to both the extreme properties of the peculiar component (WR star, white dwarf, neutron star, black hole) and the processes of mass exchange and gas accretion (accretion disks, X-ray radiation, gravitational wave radiation, jets, new phenomena, etc.). Three-dimensional gasdynamic models of matter flow in interacting close binaries [141, 142] enabled a better understanding of the physics of close binary systems.

7.5.1 Massive late close binary systems. Binary black holes and neutron stars can form in massive close binary systems ($M_1 + M_2 \gtrsim 20M_\odot$) at the final stages of evolution; their mergers result in powerful bursts of gravitational wave radiation [2, 15, 16, 75, 76].

It has long been known (see, e.g., [4]) that the cores of stars with masses greater than $60M_\odot$ enter the zone of pulsation instability stimulated by the production of electron-positron pairs. The amplitude of such pulsations and its implications are the subject of active numerical research. A recent study [143] has shown that large-amplitude pulsations can develop in pre-supernova stars with such masses, resulting in a loss of a significant part of their hydrogen-rich envelope before a supernova explosion. As a result, such a supernova is immersed into a massive dense gas-dust envelope. Unfortunately, the observable manifestations of such supernovae are not yet known, which complicates the adaptation of existing multi-parameter models. A thorough study of the mass spectrum of merging black holes of stellar masses that takes the effects of instrumental selection into account in this case will help clarify the role of pulsation instability in the evolution of the most massive (about $100M_\odot$) pre-supernovae.

Mass spectra of stellar black holes and neutron stars merging under the effect of gravitational wave radiation and the rates of such events have been explored in a large number of studies using scenario-based simulations. This technique enables tracing the evolution of a close binary star from its emergence to the formation of final objects based on the initial parameters of the systems (see Eqn (4)) and modern concepts of the evolution of close system components and the systems themselves. It is also necessary to understand how the star formation rate evolves in the Universe, because in observing the merger of relativistic components of close binaries, we ‘trace’ the evolution of the stellar composition of the Universe throughout the entire Hubble time.

The very first scenario-based models made it possible to estimate the merger rates of neutron stars and black holes, which coincided by an order of magnitude with the estimates obtained in the LIGO experiment. Detailed simulation of black hole mergers [144], in particular, enabled the discovery that the most massive black holes have an advantage among distant events of this kind because they have a large ‘reserve’ of time. Another circumstance that gives the ‘advantage’ to the most massive black holes must be noted: the most distant

events correspond to the early stages of the evolution of stellar composition, high rates of star formation, and low metallicity of the original stars. Such stars, as has been noted, produce a ‘weakened’ stellar wind, but are more numerous and have the largest masses due to the initial mass function. All these circumstances combined make it possible, given the instrumental selection, to understand the reasons why the most massive LIGO black holes are more abundant than the black holes of X-ray binary systems.

WR + OB systems. After completion of the primary mass exchange, the initially more massive star loses the bulk of the hydrogen-rich envelope due to mass exchange. A WR star is formed: an exposed nondegenerate helium-rich core with a small hydrogen-helium envelope and a powerful stellar wind ($\dot{M} \simeq 10^{-5}M_\odot \text{ year}^{-1}$) enriched in helium and CNO-cycle products. The secondary component of the system increases its mass and remains a main-sequence OB star with normal chemical composition (because its mass increased due to accretion of the hydrogen-rich envelope of the mass-losing satellite).

Mass exchange at the initial mass ratio close to unity can be regarded as being approximately conservative, although the mass loss in the form of stellar winds and the effects of wind collisions can result in deviations from the conservative regime. We emphasize that if the initial masses of the components differ strongly (the initial mass ratio $q = M_2/M_1 \leq 0.3$) and orbital periods are large, the evolution mode of a massive close binary with a common envelope can be realized, which ultimately results in a strong reduction in the orbit size and significant loss of mass by the system.

For short-period massive close binaries with a high speed of axial rotation of the components, M-evolution can be realized [145], when a meridional circulation of matter occurs in the body of the star due to its fast axial rotation. As a result, the envelope of the more massive star is enriched in helium, and the star starts exhibiting WR-star features even before it fills its Roche lobe. Such a star, which has an almost homogeneous chemical composition enriched in helium, unlike regular chemically inhomogeneous stars, does not increase its radius in the process of nuclear evolution, but rather decreases it. Therefore, it never fills its Roche lobe in the process of evolution. The tidal synchronization time is known to be very strongly dependent on the star radius R (as $(R/a)^{-6}$, where a is the radius of the relative orbit) [146]. Due to the relatively low rotation speed of the less massive star, the meridional circulation of matter in its body may be inefficient, and the star can develop significant inhomogeneity of its chemical composition as hydrogen burns out in its core. Because the enrichment of helium in the star’s envelope is in this case insignificant, the less massive star expands, and it may be the first to fill up its Roche lobe and start losing mass through the inner Lagrange point. Such a scenario for close binary evolution radically differs from the classical one, according to which the more massive system star is the first to fill up its Roche lobe. The M-scenario can be used to naturally explain the formation of massive ($M > 10M_\odot$) black holes in very tight X-ray binary systems, such as M33X-7 ($P_{\text{orb}} = 3.45$ days) and IC10X-1 ($P_{\text{orb}} = 1.43$ days). The case of M-evolution may also be of importance for classical close binaries, such as the WR20a system, which consists of two very massive WR stars ($M_1 \approx M_2 \approx 80M_\odot$) with a surface excess of helium and burning of hydrogen in the center, as well as for very close binary WR + O systems, such as CQ Cep, CX Cep, and the

very massive binary WR + O system HD311884 [147]. The M-evolution of close binary systems is of particular interest as an explanation of the existence of massive ($M > 10M_{\odot}$) black holes in X-ray binary systems. We note that the idea of matter mixing in the interiors of massive stars was used in [148–150] to explain massive black holes in X-ray binary systems.

Binary WR + OB systems are a well-studied class of objects (see catalog [147]). Parameters of the 41 WR + OB systems are presented in book [5]. The orbital periods of the known WR + OB systems are between ~ 1.6 days and ~ 4800 days. The eccentricity of the orbits is $e \approx 0$ for $P \leq 14$ days and $e = 0.3–0.8$ for $P > 70$ days. The mass ratio $q = M_{\text{WR}}/M_{\text{OB}} = 0.17–2.67$. The masses of WN stars (WR stars with enhanced nitrogen lines) are in the range $(4–60)M_{\odot}$, while the masses of WC stars (stars with enhanced carbon lines) are in the range $(5–30)M_{\odot}$. The average mass of WN stars is $21M_{\odot}$, while for WC stars it is $13M_{\odot}$, which emphasizes the important role of mass loss in the formation of the latter stars. We here exclude the masses of WN stars in the very massive binary system WR20a (WN6ha + WN6ha, $P = 3.675$ days, $e = 0$, $M_1 = 83M_{\odot}$, and $M_2 = 82M_{\odot}$) [151] and in other similar systems (WR21a, NGC3603-A1, R145) because both the characteristics and evolutionary status of these WR stars apparently differ from those of the classical WR stars that are helium-rich remnants of massive stars. An analysis of eclipses in the WR + OB systems shows that the radii of hydrostatic ‘cores’ of classical WR stars are relatively small: $\lesssim (3–4)R_{\odot}$ at masses $\sim (10–15)M_{\odot}$, and the temperatures of the ‘cores’ are large: $T \geq 45000$ K [5]. These parameters are in good agreement with the model of WR stars as helium-rich remnants of initially massive stars with small hydrogen-rich envelopes [16].

Observational arguments have been presented in [152, 153] in favor of the hypothesis that most WR stars in WR + OB systems were formed as a result of mass exchange, rather than due to a strong mass loss in the form of stellar wind by the originally more massive component. The distributions of orbit eccentricities for WR + OB systems and detached OB + OB systems (precursors of WR + OB systems) have been shown to be radically different. The transition period (from elliptical to circular orbits) P_{tr} for detached OB + OB systems is $(2–2.8)$ days, while P_{tr} for WR + OB systems it is close to 14 days, i.e., 5–7 times larger. Because WR + OB systems are often associated with young open clusters, groups of early stars, and also HII regions whose age is relatively young ($\sim 10^6–10^7$ years), a large transitional period of WR + OB systems indicates that they were subject to an additional mechanism of orbit circularization different from tidal dissipation of orbital motion energy in dynamic tides with radiation damping, which is characteristic of OB + OB systems [146]. This additional mechanism was apparently the exchange of matter in the initial OB + OB systems. As noted above, the massive interacting close binary system RYSct is an example of the formation of a WR + OB system from an OB + OB system under the effect of the mass exchange mechanism.

Quiet (‘sleeping’) X-ray binaries. After a WR star explodes as type-Ib/c supernova in a WR + OB system, a relativistic object (a neutron star or black hole) is formed together with a regular main-sequence OB star that does not fill its Roche lobe. The supernova explosion results in a high spatial velocity of the center of mass of the emerging system and a significant eccentricity of the orbit. The binary system remains in this case gravitationally bound if less than half of

its total mass is ejected during the supernova explosion, which is realized when the less massive component of the system (a WR star) explodes. If the secondary component fails to significantly spin up during the initial mass exchange, and it is not a Be star with a strong equatorial wind, the accretion of matter from the radially outgoing wind of the OB star onto the relativistic object occurs at a very low rate, and a bright X-ray source does not emerge in such a system. It is in this way that a ‘sleeping’ X-ray binary system (quiet X-ray binary) is formed.

The discovery of a close binary consisting of massive OB and Be stars containing radio pulses as components [154–156] proved that ‘sleeping’ X-ray binary systems do exist. The OB or Be star in these systems does not fill its Roche lobe, while the presence of a radio pulsar in the system is evidence that there is no accretion onto the relativistic object in those systems. For example, in a quiet X-ray binary system consisting of a radio pulsar PSR1259-63 and a massive Be-star SS2883, the orbital period is about 5.8 years, and the orbit eccentricity is very large, $e = 0.97$. This is evidence that a supernova explosion occurred in the system. In another quiet X-ray binary system that contains a B star and the PSRJ0045-7319 radio pulsar, the orbital period is ~ 51 days and the orbit eccentricity is $e = 0.80$. To date, about 10 such ‘sleeping’ X-ray binary systems have been discovered.

Massive transient X-ray binary systems with Be stars. These systems feature relatively large orbital periods and significant orbit eccentricities. The orbital periods of Be/X-ray binary systems are systematically large compared with the orbital periods of quasistationary massive X-ray binary systems (such as CygX-1 or CenX-3) because after the B-case mass transfer (according to [157]) the star with the smaller mass (a B star) leaves a much smaller relative portion of its mass in the form of a helium-rich star than a larger mass star does, and there is no common envelope. In the B-case, therefore, the increase in the orbital period in the case of conservative mass transfer in the mass transfer model is much larger for the close binaries containing donor stars with smaller masses than in the case of massive donors with the same initial mass ratio.

Catalog [134] contains data on 24 of the best-studied massive X-ray Be-transient systems. A more complete list of X-ray Be transients (about 100 objects) is presented in [158]. X-ray bursts of such systems occur due to an increase in the rate of accretion onto the relativistic object in passing near the orbit periastron, when it plunges into the strong equatorial stellar wind of a rapidly rotating Be star. The ‘discharge’ of accretion disks that can occur in such systems at the time when the periastron is passed by can also serve as a cause of activity.

The orbital periods of Be/X-ray transients are $P = 10^1–10^3$ days, and the orbit eccentricities are $e = 0.2–0.8$. Optical stars—rapidly rotating Be stars ($v \sin i = 70–450$ km s $^{-1}$, where i is the orbit inclination)—gained an additional angular momentum when matter flowed from the initially more massive component in the system. These stars feature a powerful equatorial stellar wind stimulated by the Be-star rotation. X-ray sources are the accreting neutron stars and black holes. To date, 184 X-ray binary systems are known that contain a rapidly rotating star of the spectral class Be as the optical donor star [159]. In the vast majority of cases, the accreting relativistic objects in such systems are neutron stars, many of which are X-ray pulsars with pulsation periods of 0.07–6000 s. Recently, black holes have been discovered

paired with Be stars. For example, a black hole paired with a Be star (X-ray binary MWC656 with an orbital period of about 60 days) was discovered in [160]. The black hole mass is $\sim 5M_\odot$, while the Be star mass is $\sim 13M_\odot$. The final stage in the evolution of such binary Be + BH systems is the merger of the black hole with the neutron star, which should result in a burst of gravitational radiation [160]. The estimated rate of detection of such gravitational-wave bursts using the state-of-the-art advanced laser gravitational-wave antennas LIGO and Virgo can be as high as one merger every several years.

During an X-ray burst, the luminosity of the Be/X-ray transients reaches $10^{38} - 10^{39} \text{ erg s}^{-1}$, and the burst duration is ~ 30 days. X-ray spectra are comparatively hard ($k_B T \simeq 15 \text{ keV}$). X-ray luminosity in the quiet state is $\sim 10^{33} - 10^{34} \text{ erg s}^{-1}$.

Quasistationary massive X-ray binary systems. When an optical star in the process of nuclear evolution approaches the boundaries of its Roche lobe, the stellar wind stimulated by tidal forces from the relativistic object results in the formation of an accretion disk around the relativistic object. The accretion of matter from this disk onto the relativistic object causes the emergence of a bright X-ray source with the quasistationary luminosity $\sim 10^{36} - 10^{38} \text{ erg s}^{-1}$. Several dozen such quasistationary massive X-ray binary systems are known [134] with relatively short orbital periods ($P = 1.4 - 9$ days) and orbit eccentricities close to zero ($e = 0 - 0.1$). Some of these systems exhibit a long-period, possibly precession-related, variability with a period of 30–300 days. The optical components in such systems are massive OB supergiants close to filling up their Roche lobes, while the X-ray sources are accreting neutron stars and black holes. The rotation periods of neutron stars (X-ray pulsars) are $\sim 0.7 - 600$ s, and the average luminosity is $10^{36} - 10^{39} \text{ erg s}^{-1}$. Examples of systems with black holes are CygX-1, LMCX-3, LMCX-1, and M33X-7. Naturally, none of these massive ($M > 3 M_\odot$) X-ray sources is an X-ray pulsar.

A new class of objects has been singled out recently among massive X-ray binary systems: fast X-ray transients with O and B supergiants (several dozen systems). They are characterized by the occurrence of short bursts of X-ray radiation that last several hours [161, 162]. X-ray luminosity in the outburst maximum is $10^{36} - 10^{37} \text{ erg s}^{-1}$, while in a quiet state it is $10^{32} \text{ erg s}^{-1}$. The spectral properties of X-ray radiation are similar to those of X-ray pulsars in massive X-ray binary systems. The nature of short X-ray bursts in these systems is not completely clear as yet. Perhaps they are a result of the accumulation instability of accretion disks near relativistic components (for more details, see [163]). It was shown in [164] that bright bursts ($10^{38} - 10^{40} \text{ erg s}^{-1}$) in fast X-ray transients with OB supergiants can be caused by sporadic transitions between various accretion regimes in a quasispherical envelope that surrounds a slowly rotating magnetized neutron star. It is assumed that the enhancement of the matter accretion rate in this case may be due to sporadic captures of the magnetized plasma of the stellar wind of the OB supergiant.

After the completion of the evolution of massive quasistationary X-ray binary system, binary black holes, binary neutron stars, and neutron-star–black-hole pairs can emerge. If these pairs of relativistic objects are sufficiently tight, then, because the time of their merger due to gravitational-wave radiation is shorter than the age of the Universe (1.4×10^{10} years), their merging can result in the observed powerful bursts of gravitational-wave radiation.

Binary $WR_2 + C$ systems. After a secondary mass exchange in a massive close binary system, a $WR_2 + C$ system is formed that consists of a ‘second-generation’ WR_2 star and a relativistic object accreting the stellar wind of the WR_2 star. An example of the precursor of such systems is the SS433 object, which is a massive X-ray binary system, a micro-quasar, at an advanced stage of evolution when a regular massive star has completely filled its Roche lobe and supplies the gas to the relativistic object on the thermal time scale of its evolution [165, 166]. Formed around the relativistic object in this system is a precessing, optically bright, supercritical accretion disk, from the central parts of which relativistic ($v = 0.26c$) jets burst. Thus, there is a fast phase in the SS433 system of secondary mass exchange with the rate $\dot{M} \simeq 10^{-4} M_\odot \text{ year}^{-1}$, owing to which the SS433 system can be considered the precursor of the $WR_2 + C$ system. A WR_2 star will emerge several dozen thousands years later in place of a regular massive star that outflows onto a relativistic object.

We emphasize that the SS433 object is an example of a massive binary system with intensive secondary mass exchange, when a common envelope is not formed and the loss of mass and angular momentum occurs via the formation of a supercritical accretion disk with the subsequent intense outflow of matter from it in the form of stellar wind and relativistic jets. Recent studies of the stability of mass transfer in massive close binary systems [167, 169] have shown that mass transfer through the inner Lagrange point L_1 in the nozzle model is limited by gasdynamic and thermodynamic effects. In the SS433 system, the optical star, which is the donor of matter, can be in the regime of stable overflow of its internal critical Roche lobe and even lose mass through the outer Lagrange point L_2 . Observational arguments are presented in [169] in favor of the conclusion that the relativistic object in the SS433 system is a black hole.

The discovery of a WR star in the peculiar short-period X-ray binary system CygX-3 [170] provided evidence of the actual existence of $WR_2 + C$ systems. Two more systems of this type have recently been discovered: IC10X-1 in the IC10 galaxy, which consists of a nitrogen-rich WR star belonging to an early spectral class (WNE) and a possible black hole with an orbital period of 1.46 days, and the NGC300X-1 system (WN5 + C, $P = 1.35$ days). The small orbital period in the CygX-3 system (~ 4.8 h) can be an indication that the system passed through the evolutionary stage with a common envelope. An observed characteristic of the CygX-3 system is the X-ray luminosity $\sim 10^{38} \text{ erg s}^{-1}$ (in the range 1–60 keV). This is a gamma-ray, X-ray, infrared, optical, and radio source. Radio flashes with a maximum flux of up to 20 Jy are observed, as are relativistic radio jets ($v \simeq 0.35c$) similar to SS433. The spectral class of the optical star is WN3-7, the apparent stellar magnitude of the CygX-3 system is $V \simeq 23^m$, the total interstellar absorption is $A_V = 15^m$, the distance is ~ 8 kpc, and the infrared stellar magnitude is $K \simeq 12^m$. (For more details on the CygX-3, IC10X-1, and NGC300X-1 systems, see book [5].) It was noted in [171] that because the CygX-3 system has passed the evolutionary stage with a common envelope and has a very short orbital period, Thorne–Zhytkow objects [80] must also exist in the Galaxy; they correspond to the case of a relativistic object falling along a spiral into the center of an optical star during evolution in a common envelope. Reports have been published recently that a reliable candidate to a Thorne–Zhytkow object [172] was discovered based on anomalies in

chemical composition: it is a fully convective red supergiant with a neutron star in the center (see also review [173]).

Gravitational-wave binary systems. The outstanding discovery by the LIGO observatory of gravitational waves generated by the merger of black holes in binary systems [2, 15, 75, 76] provided new opportunities for studying the origin of binary black holes in the process of evolution of massive close binary systems. By analogy with X-ray binary systems that have been discovered using the X-ray radiation formed during the accretion of matter, it is natural to refer to the close binaries detected using gravitational-wave radiation as gravitational-wave binary systems.

By September 2018, the LIGO observatory discovered five events of mergers of gravitational-wave binary systems of the ‘black hole + black hole’ type [2, 75, 76], while joint observations performed by the LIGO and Virgo gravitational-wave observatories led to the discovery of another pair of merging black holes, as well as a pair of merging neutron stars [15]. The discovery of a burst of electromagnetic radiation from merging neutron stars (in the gamma-ray, X-ray, and optical ranges) allowed measuring the velocity with which gravitational waves propagate in space; it was found to coincide with the speed of light to within $\sim 10^{-15}$. The gravitational-wave radiation emitted at the stage of the merger of two black holes and the ring-down stage provides information about the highly nonlinear large-scale dynamics of space–time curvature. Therefore, a detailed study of these gravitational-wave signals (with a sufficiently high signal-to-noise ratio) will allow verifying the nonlinear equations of GR in all their complexity. It is quite possible that on this path the final evidence will be obtained that the event horizon exists for black holes.

On November 30, 2018, the LIGO and Virgo team published the first Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs [174] (GWTC-1). Having thoroughly processed the results of observations made using LIGO and Virgo antennas, the authors of [174] determined and refined the parameters for a total of 10 gravitational-wave binary systems with black holes: GW150914, GW151012, GW151226, GW170104, GW170608, GW170729, GW170809, GW170814, GW170818, and GW170823, as well as one gravitational-wave system with neutron stars, GW170817 (Table 2). The authors of [174] note that they have so far not detected a gravitational wave signal from the merger of a black hole and a neutron star.

Table 2 shows that the masses of black holes prior to the merger fall in the range $(7.6–50.6)M_{\odot}$, while after the merger they vary from $17.8M_{\odot}$ to $80.3M_{\odot}$. The dimensionless angular momenta of the black holes after the merger are in the range 0.66–0.81. In two systems (GW151226 and GW170729), the angular momenta of black holes before merging are insignificantly different from zero (in other cases, they are close to zero within the error limits). The least massive pairs of merging black holes are GW151226 ($13.7M_{\odot} + 7.7M_{\odot}$) and GW170608 ($10.9M_{\odot} + 7.6M_{\odot}$). The most massive merging pairs of black holes are GW170729 ($50.6M_{\odot} + 34.3M_{\odot}$) and GW170823 ($39.6M_{\odot} + 29.4M_{\odot}$). The authors of [174] report estimated of merger rates for compact object pairs in the Universe: $110–3840 \text{ Gpc}^{-3} \text{ year}^{-1}$ for pairs of neutron stars, $9.7–101 \text{ Gpc}^{-3} \text{ year}^{-1}$ for pairs of black holes, and $< 610 \text{ Gpc}^{-3} \text{ year}^{-1}$ for pairs that consist of a black hole and a neutron star.

Table 3 contains the masses of some black holes in gravitational-wave binary systems with black holes. For comparison, it also displays the characteristics of the most massive X-ray binary systems with black holes, as well as the parameters of some of the most massive ‘classical’ CBSs in our Galaxy and the Large Magellanic Cloud (LMC), the galaxy that is closest to us. The large masses of black holes in the gravitational-wave binary system that was discovered first, GW150914 ($36M_{\odot} + 29M_{\odot}$), are beyond the black-hole mass range in well-known X-ray binary systems, $(4–16)M_{\odot}$ (see, e.g., [5]). This observation triggered a discussion of the mechanisms of formation of close binary black holes that are different from the classical scenarios (see [29, 175, 176] and the references therein).

However, the discovery by the LIGO observatory of a second gravitational-wave burst, GW151226, associated with the merger of black holes with masses of $14M_{\odot}$ and $7.5M_{\odot}$, which are consistent with the masses of black holes in X-ray binary systems, $(4–16)M_{\odot}$, showed that the classical scenario of the evolution of isolated massive close binary systems [4] is apparently able to explain the emergence of such close pairs of black holes. As noted in [177], because the black holes with masses of $36M_{\odot}$ and $29M_{\odot}$ in the GW150914 system must be formed from very massive stars with masses $\sim 100M_{\odot}$, whose number is very low in the Galaxy and other nearby galaxies, the formation rate of such massive black holes in the Galaxy and nearby galaxies is also very low. This is why the black holes with masses greater than $\sim 30M_{\odot}$ have not yet been discovered in X-ray binary systems.

Table 2. Parameters of 11 gravitational-wave binary systems from the GWTC-1 catalog [174].

Event	Types of components	m_1/M_{\odot}	m_2/M_{\odot}	m_{f}/M_{\odot}	a_{f}	$d_{\text{L}}/M_{\text{pc}}$
GW150914	BH + BH	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	$63.1^{+3.3}_{-3.0}$	$0.69^{+0.05}_{-0.04}$	430^{+150}_{-170}
GW151012	BH + BH	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$35.7^{+9.9}_{-3.8}$	$0.67^{+0.13}_{-0.11}$	1060^{+540}_{-480}
GW151226	BH + BH	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	$20.5^{+6.4}_{-1.5}$	$0.74^{+0.07}_{-0.05}$	440^{+180}_{-190}
GW170104	BH + BH	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	$49.1^{+5.2}_{-3.9}$	$0.66^{+0.08}_{-0.10}$	960^{+430}_{-410}
GW170608	BH + BH	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	$17.8^{+3.2}_{-0.7}$	$0.69^{+0.04}_{-0.04}$	320^{+120}_{-110}
GW170729	BH + BH	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	$80.3^{+14.6}_{-10.2}$	$0.81^{+0.07}_{-0.13}$	2750^{+1350}_{-1320}
GW170809	BH + BH	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	$56.4^{+5.2}_{-3.7}$	$0.70^{+0.08}_{-0.09}$	990^{+320}_{-380}
GW170814	BH + BH	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	$53.4^{+3.2}_{-2.4}$	$0.72^{+0.07}_{-0.05}$	580^{+160}_{-210}
GW170817	NS + NS	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	≤ 2.8	≤ 0.89	40^{+10}_{-10}
GW170818	BH + BH	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	$59.8^{+4.8}_{-3.8}$	$0.67^{+0.07}_{-0.08}$	1020^{+430}_{-360}
GW170823	BH + BH	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	$65.6^{+9.4}_{-6.6}$	$0.71^{+0.08}_{-0.10}$	1850^{+840}_{-840}

Table 3. Comparison of the parameters of gravitational-wave binary systems, massive X-ray binaries, and ‘classical’ massive close binaries.

Gravitational-wave binary systems with black holes (BH)			
System	Types of components	Masses of components/ M_{\odot}	Initial stellar masses/ M_{\odot}
GW150914	BH + BH	36 + 29	100 + 80
GW151226	BH + BH	14.2 + 7.5	50 + 30
GW151012	BH + BH	23 + 13	75 + 50
GW170729	BH + BH	51 + 34	150 + 80
Massive X-ray binary systems containing a black hole (OB + BH)			
System	Spectral class of components	Masses of components/ M_{\odot}	Orbital period/day
CygX-1	O9.7Iab + BH	19.16 + 14.81	5.60
LMCX-1	O(7–9)III + BH	30.6 + 10.3	3.91
M33X-7	O(7–8)III + BH	70.0 + 15.6	3.45
‘Classical’ binary systems			
System	Spectral class of components	Masses of components/ M_{\odot}	Orbital period/day
HD193793	WC7 + O4-5	27 + 60	2900
HDE311884	WN6 + O5V	51 + 60	6.24
WR20a	WN6h + WN6a	83 + 82	3.69
HD92740	WN7h + O9III-V	55 + 21	80.34
NGC3603-A1	WN6a + WN6	116 + 89	3.77
Melnick 34 (LMC)	WN5h + WN5h	139 + 127	155.1
R136-38 (LMC)	O3V + O6V	56 + 30	3.39
UWCMa	O(7–8)If + O8f	43.5 + 32.5	4.39

Because gravitational-wave telescopes can detect signals at distances of several hundred Mpc from an immense number of galaxies, the probability of detecting signals from the merger of the most massive black holes increases significantly. Moreover, because the sensitivity of gravitational-wave telescopes (LIGO, Virgo) is limited, the gravitational-wave signals from the merger of the most massive black holes should be the first to be detected, as is indeed the case (see Table 2). Moreover, because gravitational-wave bursts come from massive binary systems that merged in the early epochs of the Universe, when the metallicity of stellar matter was small (and hence the effect of stellar mass reduction due to stellar wind radiation was small), black holes originated from very massive stars. This circumstance additionally increases the probability of detecting gravitational-wave bursts from the merger of the most massive stellar black holes. For two massive black holes to merge under the effect of gravitational-wave radiation in a time shorter than the age of the Universe, the distance between them should not exceed $\sim 50R_{\odot}$ [29]. However, such a compact pair of black holes cannot be formed in the classical scenario of the evolution of very massive close binaries of a solar chemical composition, because the initial stars with masses $\sim 100M_{\odot}$ have very large radii ($R > 20R_{\odot}$) and produce very strong stellar winds, which increase the separation of components. The large initial radii of the stars that are precursors of massive black holes imply that the initial separation of the components in the corresponding ‘classical’ close binaries should be very large ($> 50R_{\odot}$). At the same time, due to the radial loss of matter by stars in the form of stellar wind (the Jeans mode of

the loss of matter), separation between the components increases in a secular fashion to significantly exceed the critical value $50R_{\odot}$ over time. As a result, a pair of black holes with a large separation ($> 50R_{\odot}$) is formed in the process of the evolution of such a massive close binary system, for which the black-hole merger time is longer than the Hubble time (1.4×10^{10} years).

Various possibilities for the formation of rather compact pairs of black holes were considered in [29]: the evolution of close binaries with a common envelope, the low metallicity of massive stars, such that the rate of mass loss in the form of a stellar wind is insignificant, the evolution of close binaries in a dense molecular cloud, etc. Specific scenarios for the evolution of isolated massive close binary systems with the formation of compact pairs of black holes and the emergence of gravitational-wave bursts were computed in [175, 176, 178, 179]. The stage of the gravitational-wave binary system (observable merging black holes and merging neutron stars) is a very late stage in the evolution of massive close binary systems.

Thus, the discovery of gravitational-wave binary systems made it possible, across a very wide time interval, to trace the evolution of massive close binary systems from two massive main-sequence stars to two relativistic objects that subsequently merge under the effect of gravitational-wave radiation. It is remarkable that virtually all stages of the evolution of massive close binaries over this very wide time interval can be satisfactorily described by theoretical models, and the results and predictions of their simulation can be confirmed by specific observational events.

7.5.2 Low-mass late close binary systems. The evolution of low-mass close binaries is determined by the mechanisms of angular momentum outflow from the system due to the outflow of the magnetic stellar wind and the emission of gravitational waves. Components can degenerate in the process of evolution of low-mass close binary systems.

X-ray novae. In X-ray novae, the accretion of matter from the accretion disk is directed toward the central relativistic object. The main reason for X-ray outbursts is instability in the accretion disk, which results in a sharp increase in the turbulent viscosity of its matter. The reason for this is that while the rate with which matter is supplied to the accretion disk from the low-mass optical star is relatively low, its temperature in a quiet state, due to gravitational-energy release in the disk, is lower than the hydrogen ionization temperature (~ 7000 K). Minor changes in the rate of matter supply to the disk associated, for example, with the activity of a star filling up its Roche lobe, result in a change in its temperature, the hydrogen ionization degree in the disk, and the opacity of matter in it. This causes the emergence of instability, an increase in turbulent viscosity in the disk and hence an increase in the rate of accretion onto the relativistic object, which results in a powerful burst of X-ray radiation (see, e.g., review [180] and the references therein). Instability can also arise in the disk when the density and optical thickness of matter in the low-viscosity accumulating disk reach some critical values. In this case, turbulence is also generated in the disk, which leads to an increase in the matter accretion rate and then to a burst.

The number of known X-ray novae is close to 100. The orbital periods of these systems lie in the range 0.2–33.5 days, and the eccentricity of the orbits is $e = 0$. Optical donor stars are MKA and late B dwarfs, subgiants, and giants filling up their Roche lobes. The relativistic objects are neutron stars with a weak magnetic field and black holes. Two-thirds of the known black holes with stellar masses in X-ray binary systems have been discovered in X-ray novae. The duration of X-ray bursts is of the order of several months. The X-ray luminosity increases during the burst by a factor of 10^2 – 10^6 to reach the Eddington value 10^{37} – 10^{38} erg s $^{-1}$. The X-ray luminosity in a quiet state is $\sim 10^{31}$ – 10^{33} erg s $^{-1}$. The quiet state can last several years. The X-ray spectrum during the burst is soft ($k_B T \approx 2$ keV) compared with that of Be/X transients ($k_B T \approx 15$ keV) (see above). Examples of X-ray novae with black holes are A0620-00, GS202+338, GS1124-68, and GRS1915+105. Observed in some X-ray novae (GROJ1655-40, GRS1915+105, etc.) are radio bursts and collimated relativistic jets (the v/c ratio can be as high as 0.98).

We note the recent discovery of an extremely fast decrease in the orbital periods of X-ray novae with black holes A0620-00, XTEJ1118+480, and Nova Mus 1991 [181, 182], the explanation of which requires the use of a model of an optical donor star with a strong magnetic field (of the order of several dozen kG) (see [181]) or a model of a binary system with a circumstellar envelope [185]. The presence of a variable linear polarization of infrared radiation from X-ray novae A0620-00 and Swif J1337.2-0933 in a quiet state was discovered in [184]. This is interpreted as a manifestation of synchrotron radiation from relativistic jets that exist in X-ray novae even if they are in a quiet state. It was found in [185, 186] that the X-ray nova A0620-00, even in the quiet state, undergoes transitions from the passive stage (low flickering) to the active one (high flickering, increase in average brightness by 0.2^m – 0.3^m). It was shown in [186] that in switching

from the passive stage to the active one, the A0620-00 system ‘traces’ the instability of the optical donor-star atmosphere.

The X-ray nova V404Cyg, which is a low-mass X-ray binary system with a black hole, flashed in 2015 after 25 years of ‘silence’ [187, 188]. This system previously flashed repeatedly; at least three outbursts were recorded: in 1938, in 1956 (according to archival photographic observations), and in 1989 (as follows from observations aboard X-ray space observatories [189], as well as from ground optical observations [190]). The V404Cyg flash in 2015 turned out to be a very unusual event: the light curves in the X-ray and optical ranges exhibit fast and irregular changes [188], owing to which this flash is very different from the V404Cyg flashes observed in 1938 and 1989. These results are of great interest to the theory of unsteady disk accretion of matter onto black holes.

Reviews [173, 191, 192] provide information on the masses of black holes in X-ray binary systems and their observational manifestations. An analysis of X-ray spectra of a number of X-ray binary systems containing neutron stars and black holes is presented in [193], and the criteria by which a neutron star can be distinguished from a black hole are specified (see also [194]).

X-ray novae are formed from massive close binaries with a very large initial mass ratio, where a massive star with $M_1 \approx (10-50)M_\odot$ (the precursor of a relativistic object) is close to a low-mass star of a late spectral class with the mass $M_2 = (0.5-1)M_\odot$. This is followed by mass exchange at the common envelope stage, the explosion of the formed WR star as a supernova, the formation of a relativistic object, a reduction in the system orbit size due to the common envelope, deceleration by the magnetic stellar wind, and the radiation of gravitational waves by the system [195]. Another mechanism is provided by a ternary-system model (see a review in [5]). As a massive close binary system evolves, a Thorne–Zhytkow object is formed, which has a large radius, of the order of the red-supergiant radius. If a third satellite star with a mass less than M_\odot is located near the initial massive close binary system (at a distance of several astronomical units), then it could be captured by the envelope of the Thorne–Zhytkow object and spiral to approach its center, the accreting relativistic object. The envelope dissipates as a result, and the third low-mass star decreases its orbital period to $\lesssim 1$ day. This can result in the emergence of a low-mass X-ray binary system consisting of an optical star of mass $\lesssim M_\odot$ and a relativistic object. Because the number of ternary systems among close binaries is quite large (see above), the probability of such a scenario is not small.

Bright X-ray binary systems of the galactic bulge. Several dozen such systems with relatively short orbital periods ($P \leq 1-10$ days) and the orbit eccentricity $e = 0$ are known [5, 134]. The X-ray luminosity is quasistationary on average ($L_X \approx 10^{36}$ – 10^{38} erg s $^{-1}$), but changes over time in an irregular manner; the amplitude of such variability can be as high as two orders of magnitude. Optical donor components are low-mass GM stars ($M \leq 1.5M_\odot$) that fill their Roche lobes. Relativistic objects are neutron stars with a weak magnetic field. Quasiperiodic oscillations (QPOs) of X-ray radiation have been discovered for many X-ray binaries of this type, for example, the QPO frequency is $\nu_c = 5.9$ – 6.4 Hz for the ScoX-1 system, $\nu_c = 5.2$ – 6 Hz for CygX-2, and $\nu_c = 20$ – 24 Hz for 4U1758-25. High-frequency QPOs with the oscillation frequency up to 1 kHz are also observed. (For more information on systems of this type, see [5].)

X-ray bursters. X-ray bursters are primarily observed in the galactic bulge and globular clusters. Catalog [134] contains data on several dozen such objects, low-mass X-ray binary systems. The duration of X-ray flashes is 1–40 s. The maximum X-ray luminosity is $\sim 10^{37}$ erg s $^{-1}$. Optical companions are low-mass stars of the late spectral classes that fill up their Roche lobes. Two types of X-ray bursters are distinguished. In type-1 X-ray bursters, the X-ray burst is caused by a thermonuclear explosion of matter accumulated during accretion on the surface of a weakly magnetized neutron star. In type-2 X-ray bursters, the X-ray burst originates from cumulative instability in the accretion disk (for more details, see [5]).

Cataclysmic binary systems. The phenomenon of a cataclysmic binary (the transition from a quiet state to a long-lasting burst of optical radiation) is associated with unstable accretion of matter from the disk onto the central white dwarf. This is a widespread and well-studied type of interacting close binary systems [134, 195, 196]. Cataclysmic binary systems consist of a low-mass GM star (luminosity classes III–V) that fills its Roche lobe and an accreting white dwarf. Several hundred such systems are known. The orbital periods are in most cases small, $P < 1$ day (depending on the donor-star luminosity class); the eccentricity of the orbits is $e = 0$. The cataclysmic binary systems can be divided into three large classes depending on the magnitude of the magnetic field H of the white dwarf.

1. If $H < 10^5$ G, an accretion disk is formed around the white dwarf that has a boundary layer on the equatorial surface of the white dwarf. The accretion of matter from the disk onto the white dwarf occurs in the boundary layer.

2. If $H \approx 10^5 - 10^6$ G, the accretion disk is destroyed in the inner parts by the magnetic field of the rotating white dwarf, its magnetosphere. Moreover, the outer parts of the accretion disk persist, and the accretion of matter from the disk occurs along the magnetic field lines to the magnetic poles of the white dwarf. An intermediate polar phenomenon is observed in this case: an example of it is the well-known DQHer system, in which regular pulsations of optical and X-ray radiation caused by the rotation of a white dwarf have been detected.

3. If $H = 10^7 - 10^8$ G, the radius of the white dwarf magnetosphere exceeds that of the binary-system orbit. An accretion disk is not formed around the white dwarf. Mass is transported from the ‘regular’ GM star and matter accretes directly along the field lines of the white dwarf magnetosphere to its magnetic poles. A polar phenomenon is observed. The optical and X-ray luminosities of the polar are usually modulated by the orbital period of the binary system; a strong variable circular and linear polarization of the radiation modulated by the orbital period is observed (the axial rotation of the white dwarf in such systems is synchronous with its orbital revolution). A survey of data on polars and intermediate polars is presented in [197].

Systems have been discovered recently wherein the dwarf’s magnetic field is $10^8 - 10^9$ G, and the white dwarf, upon mass exchange, has spun up to a very short rotation period, 30–118 s, as a result of which there is currently no accretion onto the white dwarf (the AEAquarii and ARSco systems (see [58, 59, 198–201])). The white dwarf is observed in these systems in the ejecting-pulsar regime.

Cataclysmic binaries are divided into three types depending on the amplitude of optical flashes: novae (orbital periods $P \approx 0.05 - 250$ days the flash amplitude $\Delta V > 11^m$), recur-

rent novae ($\Delta V \approx 7^m - 11^m$, the interval between flashes $\Delta t \approx 20 - 80$ years), and dwarf novae ($\Delta V \approx 2^m - 6^m$, $\Delta t \approx 10 - 100$ days). Outbursts of novae and, apparently, recurrent novae are associated with thermonuclear burning of matter accumulated during accretion on the white dwarf surface. Outbursts of dwarf novae are associated with the release of gravitational energy in the process of enhanced matter accretion from an unstable accretion disk onto a white dwarf.

Catalog [134] also summarizes information on pre-cataclysmic binary systems consisting of a white dwarf and a ‘regular’ red dwarf star that has not yet had enough time to fill its Roche lobe. This catalog also contains information on binary white dwarfs (orbital periods $P \approx 0.02 - 0.05$ days). A list of 59 detached close binaries with white dwarfs and known orbital periods ($P \approx 0.0272 - 30.09$ days), as well as known total masses of components, is presented in [202], where possible evolutionary scenarios for such systems are described.

A review of interacting binary white dwarfs, AMCVn-type stars, is given in [203]. Known currently are 25 binary systems that are classified as AMCVn-type systems or as candidates for this type of system. The orbital periods of these systems are very short; they can be as low as ~ 3 min. Therefore, such systems can be intense sources of gravitational-wave radiation. AMCVn-type stars can also be the precursors of type-Ia supernovae (for more details, see [204, 205]).

A catalog of 1602 pre-cataclysmic binary systems that are in the evolutionary stage after the common-envelope stage (according to the Sloan Digital Sky Survey (SDSS)), as well as the results of a statistical analysis of systems of this type, is presented in [206]. A summary of data on 118 hot sub-dwarfs paired with satellites, white dwarfs, or main-sequence stars is contained in [202]. The orbital periods of such systems fall in the range 0.073–127 days.

Luminous red novae. A new class of cataclysmic binary systems has been discovered recently: luminous red novae (LRN) associated with the formation of common envelopes in low-mass late close binary systems [11, 207–210]. The difference between red novae and ordinary ones is that their spectra do not contain lines of elements that accompany thermonuclear processes on the white dwarf surface; they also differ by the mass of dumped envelopes. It is assumed that the dump of the common envelope due to the transfer of the orbital angular momentum of stars to the envelope manifests itself as a flash of a peculiar nova, a luminous red nova.

The absolute stellar magnitude of a red nova at the maximum brightness is very high: $M \leq -10^m$. While traces of thermonuclear processes are not seen in the red-nova spectrum, a characteristic plateau on the light curve is observed that lasts 20–30 days, with a duration of the flash of about 2–3 months. The total explosion energy is very large, $E \simeq 10^{48}$ erg. Examples of red novae are V838Mon, V1309Sco, and M31LRN2015. It was shown in [211] that the precursor of the V1309Sco red nova was a contact binary system with an orbital period of ~ 1.7 days, which provides convincing evidence that the merging of the two components and the dumping of the common envelope cause the flash by a luminous red nova. This conclusion is also confirmed by the observation that the rate of red nova bursts is one per 20–40 years per galaxy like our Galaxy or the M31 galaxy, a value that is consistent with theoretical estimates of the rate with which common envelopes with a radius $(6 - 10)R_\odot$ are formed [4]. The MASTER robotic telescope system (Mobile Astronomical System of Telescopes-Robots) was used in

[210] to discover the LNR2015 red nova in the M31 galaxy, obtain detailed light curves, develop a hydrodynamic model of the flash, and study the evolution of this red nova. The multicolor light curves of this red nova are shown to agree with the initial common envelope radius $10R_{\odot}$, the mass of merging stars $3M_{\odot}$, and the explosion energy 3×10^{48} erg. A conclusion was made that the novae can be divided into two classes: classical novae with thermonuclear processes on the surface of an accreting white dwarf and rarer objects, red novae associated with mass loss in the form of common envelopes.

Symbiotic binary systems. Catalog [134] contains data on several dozen of the best-studied symbiotic binary systems with known orbital periods. These systems consist of a red giant (spectral class G–M) and a white dwarf or hot subdwarf (radius $(0.01–1)R_{\odot}$ and temperature $(150,000–30,000)$ K) that accretes a slow ($v \approx 10–30$ km s $^{-1}$) radial stellar wind of the red giant. The second star, the accretor, is in some cases a main-sequence star or even a neutron star. Orbital periods are large, ranging from 70 days to several dozen years. The eccentricities of the orbits are $e = 0–0.3$. The red giant is sometimes a variable Mira-type star. Symbiotic binaries can be divided into three groups, depending on the optical flash amplitude: 1) classical symbiotic binaries with the flash amplitude $\Delta V \approx 2^m–3^m$ (CICyg, ZAnd, etc.), 2) recurrent novae with the flash amplitude of $\Delta V \approx 5^m–7^m$ (TCrB, RSoph), and 3) symbiotic novae with the flash amplitude $\Delta V \approx 6^m–10^m$ (AGPeg, HMSge, RRTel). Symbiotic novae are sometimes referred to as slow novae; flashes from them have been observed only once. The flashes are caused in most cases by the thermonuclear burning of hydrogen plasma accumulated on the white dwarf surface as a result of accretion of matter from the slow wind of the red giant. Such flashes can last several dozen years. Shorter flashes ($\Delta t < 1–3$ years) are caused by the release of gravitational energy in the process of nonstationary accretion. Some symbiotic binary star systems exhibit collimated jets with velocities $\sim 10^2$ km s $^{-1}$ observed during optical flashes and radio flashes.

The class of late close binary systems referred to as barium systems and systems containing S stars are close to symbiotic stars in their evolutionary status. According to modern concepts, these are long-period binary systems in which the initially more massive component has strongly evolved, turned into a white dwarf, and transferred part of the envelope matter enriched in thermonuclear merger products to the satellite, which has now evolved to the stage of a red giant of spectral class KM [4]. This mechanism explains the anomalous chemical composition of the red giant, in particular, an excess of barium. These systems make it possible to study nucleosynthesis and mass loss in highly evolved stars in detail. In particular, S stars exhibit an excess of chemical elements formed in the S process of thermonuclear merger in the interior of stars.

“Ultra-soft” X-ray binary systems. About ten such systems are known, most of which have been discovered in the Large Magellanic Cloud. These systems consist of a low-mass star filling its Roche lobe and an accreting white dwarf. Orbital periods are ~ 1 day. The main feature of such systems is the high quasistationary X-ray luminosity, $\sim 10^{37}–10^{38}$ erg s $^{-1}$, combined with a very soft X-ray spectrum: $k_B T \approx 20–50$ eV. This luminosity is due to stationary thermonuclear burning of matter accreted from the disk onto the white dwarf surface. The X-ray luminosity of the

accreting white dwarf is close in this case to the Eddington limit. For the thermonuclear burning to be stable, the accretion rate must be bounded within a narrow range, $\dot{M} \approx (1–4) \times 10^{-7} M_{\odot}$ year $^{-1}$ [4]. Given that there are no flashes or dumps of matter, it is assumed that white dwarfs in such systems can increase their mass in the accretion process to the Chandrasekhar limit and collapse into a neutron star (ONe dwarfs), producing a type-Ia supernova. A typical example of such a system is CAL83/4U0543-682. (For more details, see [5, 134].)

7.5.3 Luminous blue variables. Some luminous blue variables (LBVs) are among the most powerful stellar objects in the Universe with regard to stationary optical radiation. LBVs are characterized by strong photometric variability (the amplitude up to several stellar magnitudes) at characteristic times ranging from several months to several hundred years. Some LBV objects are surrounded by small nebulae formed by the matter ejected from the star, and there are indications that the ejected material is enriched with products of thermonuclear nucleosynthesis. The corresponding mass loss rate can be as high as $10^{-2} M_{\odot}$ year $^{-1}$. Lists of known LBVs are presented in [134, 212–214].

Three suggestions have been made regarding the nature of LBV objects: a single, very massive star ($M > 100M_{\odot}$) with manifestations of nonstationarity; a massive close binary system at the common-envelope evolutionary stage after the stage of an X-ray binary system; a Thorne–Zhytkow object [80], which is a single massive star with a relativistic object in the center. The discovery by the Hubble Space Telescope of collimated jets from an LBV object, η Car, as well as the discovery of periodic X-ray eclipses in this object with a period of 5.54 years, is evidence in favor of LBV objects modeled as close binary systems. The discovery of a WR star in the CygX-3 X-ray binary system with a very short orbital period (~ 4.8) [170] suggests the existence of a significant number of Thorne–Zhytkow objects in the Galaxy [171]. A search for such objects has recently led to the discovery of a real candidate for Thorne–Zhytkow objects [172], a red supergiant featuring significant anomalies in chemical composition.

7.5.4 Radio pulsars in binary systems. To date, about 200 radio pulsars are known in close binary systems; the parameters of the most studied systems are given in [5, 134].

Orbital periods are in the range $P \approx 0.07–1300$ days; eccentricities of the orbits are $e = 0–0.97$; the companions are neutron stars, white dwarfs, B and Be stars, and planets. The periods of axial rotation of the pulsars are $0.0016–1$ s. Most short-period, millisecond pulsars are companions of binary systems representing recycled pulsars, which partially accumulate the orbital angular momentum of the close binary system in the process of secondary mass exchange [215]. There are also radio pulsars in binary systems (for example, PSR1057+20), wherein the satellite, a white dwarf, evaporates, heated by the relativistic corpuscular wind of the pulsar. The discovery of two pulsars in the J0737-3039AB binary system [216], which confirmed the prediction of recycled pulsars [215], enables various tests of general relativity.

According to the evolutionary theory (see, e.g., [16, 195]), pulsars in binary systems with elliptical orbits and relatively massive companions (neutron stars, massive white dwarfs, Be stars) were formed as a result of a supernova explosion. Pulsars with circular orbits and relatively low-mass satellites

(small-mass white dwarfs) probably emerged as a result of the collapse of a white dwarf that increased its mass to the Chandrasekhar limit in the process of mass flow from the satellite, a nondegenerate low-mass star, and subsequent accretion. The strong increase in the orbital period of such a system is due to the transfer of matter on the nuclear evolution time scale from the less massive regular star to the more massive white dwarf.

To date, the masses of more than 60 neutron stars in binary systems have been measured. The average mass of 20 neutron stars in 10 binary systems consisting of two neutron stars is $\bar{M}_{\text{NS}} = (1.32 \pm 0.1)M_{\odot}$, and the dispersion of masses, equal to $0.1M_{\odot}$, is not caused by errors in their determination (in binary radio pulsars where satellites are neutron stars, the neutron star masses are measured very accurately on the basis of relativistic effects), but rather reflects the actual physical dispersion of the neutron star masses.

The periods of axial rotation of pulsars in binary systems are known to be reduced due to the large angular momentum delivered by the accreted matter [217]. To ‘spin up’ a neutron star to the millisecond-pulsar stage, it is sufficient to accrete $\sim 0.2M_{\odot}$ of the matter in a binary system. Observations confirm this prediction: for 19 millisecond pulsars with axial rotation periods $P_{\text{spin}} < 20$ ms, the average mass is $1.548M_{\odot}$, while for 40 ordinary pulsars with $P_{\text{spin}} > 20$ ms, it is equal to $1.423M_{\odot}$. A difference in average masses is observed: the millisecond-pulsar mass is $0.13M_{\odot}$ on average, greater than that of an ordinary pulsar, which confirms conclusions of the theory [217]. Data on the masses of radio and X-ray pulsars are summarized in book [5].

To clarify the equation of state of neutron matter, it is of importance to have information about the maximum observed mass of a neutron star. A number of binary systems are known in which the mass of a neutron star exceeds $1.8M_{\odot}$; these are radio pulsars in binary systems with white dwarfs: J0348+0432 ($M_p = (2.01 \pm 0.04)M_{\odot}$, $P_{\text{spin}} = 39.123$ ms); J614-2230 ($M_p = (1.97 \pm 0.04)M_{\odot}$, $P_{\text{spin}} = 7.95$ ms); B1516+02B ($M_p = (2.08 \pm 0.19)M_{\odot}$, $P_{\text{spin}} = 3.15$ ms); J1748-2021J ($M_p = (1.88^{+0.02}_{-0.08})M_{\odot}$, $P_{\text{spin}} = 80.34$ ms); and J1748-24461 ($M_p = (1.91^{+0.02}_{-0.01})M_{\odot}$, $P_{\text{spin}} = 9.57$ ms). Thus, observational data on five binary systems in which accurately estimated masses of neutron stars are about $2M_{\odot}$ enable confidently rejecting the so-called soft equation of state of neutron matter, according to which the maximum mass of a neutron star is $\sim 1.5M_{\odot}$ (see details in book [5]).

8. Achievements of observations and theory. Tasks for further research

The analysis of the development of modern ideas about the physics and the evolution of binary stars that we presented above allows identifying the main stages and achievements on this path and outlining a number of problems whose solution will eventually contribute to gaining deeper insights into the physics of the processes that accompany the evolution of these systems. The results of the multilateral and fruitful activities of a large multinational team of astrophysicists create hope that the work will be successfully continued, and the current problems with the physics and the evolution of binary stars will be resolved. Successes in advancing our ideas about the physics and the evolution of stars were ensured by the progress in modern astrophysical tools: increased sensi-

tivity, extended spectral range of electromagnetic radiation, detection of gravitational-wave radiation, and enriched physical content of modern models that describe the structure and the evolution of stars and the main phenomena that accompany that evolution.

The following points can be considered as clear successes in understanding the physics and the evolution of binary stars.

I. Analyses of the observed properties of binary stars, which includes thorough consideration of the effects of observational selection. These analyses have allowed finding the distribution of young binary stars by masses of primary components, separation of the components, and the mass ratios of the components (see Eqn (4)). This function is now a basis for developing modern scenario-based models that describe the evolution of a family of binary stars and enable estimating the number of such stars of various types and rates of events that accompany the evolution of binary stars in the Galaxy and the Universe.

II. Development of modern scenarios and scenario-based programs (scenario machines) that represent models of binary star evolution. These scenarios are based on the entire set of observational data on the physics of stars at various stages of their evolution and on the results of extensive numerical simulation of the evolution of components from their formation in the process of accretion to explosion of massive stars as supernovae and to the formation of planetary nebulae of various types at the end of the evolution of moderate-mass stars. Modern astrophysics has shown that massive SNII, SNIb, and SNIc supernovae are the leading factor that governs the rate of star formation in disk S galaxies and their evolution.

III. Development of the modern physics of various supernova types. Type-II supernovae were identified with the collapse of the cores of massive red supergiants that causes the high-speed dump of the hydrogen-rich extended envelope of the supergiant. Formed as a result of such explosions are dense final remnants: neutron stars and stellar black holes. Massive supernovae themselves are a leading factor in the evolution of the gas disks of galaxies. Explosions of type-Ib,c supernovae herald the end of the evolution of massive stars with compact helium envelopes, the hydrogen-rich envelopes of which were lost either during the evolution of these stars as part of a close binary system or due to intense stellar wind. Explosions of type-Ia supernovae are caused, in a number of remaining possible scenarios, by thermonuclear explosions of degenerate dwarfs. These supernovae are now an important instrument of modern cosmology. It turns out that these supernovae maintain the galactic wind of E galaxies, thus suppressing star formation in those galaxies.

IV. The achievements of modern astrophysics, which should be especially noted, in the development of ideas about the role of gravitational-wave radiation in the evolution of a number of close binary systems with compact components. In particular, studies on the merger of degenerate dwarfs yielded an explanation of the physics of type-I supernovae. Explorations of the merger of neutron stars and the components of binary black holes in the LIGO experiment made it possible to create a solid foundation for the physics of gamma-ray bursters and gamma-ray astronomy.

V. Development of modern models of the evolution of galaxies of various classes on the basis of modern ideas about the physics and the evolution of stars. The first steps have been made along this path, creating hope that a unified model

of the evolution of stars and galaxies will be developed over time that will enable connecting the properties of stellar composition with those of galaxies of various types.

We also list the most urgent problems whose solution will contribute to progress in understanding the evolution of stars. It is worth noting that progress not only resolves current problems in any area of human activity but also poses new challenges that usually arise from the successes achieved. Awareness of the essence of new problems is a reliable guide and engine of progress in any activity.

1. The study of the conditions under which the function that describes the distribution of young binary stars by the mass and separation of components is formed (see Eqn (4)). Of particular note is the initial mass function $dN/dM \sim M^{-2}$, which, as has become clear, is the initial mass function of not only stars but also star clusters, galaxies, and their clusters.

2. The development of the modern theory of stellar wind for stars of various types. Causes for the strong dependence on the abundance of metals should be found for OB stars and WR-type stars. Regarding red and infrared supergiants, it is necessary to study the contribution of radiation pressure and shock waves generated in convective envelopes to the formation of their wind.

3. The fulfillment of the urgent task to develop evolution-based models of type-Ia supernovae. The degree of standardization of these supernovae and possible secular effects that affect their brightness should be studied. The role of the chemical composition, rotation, and mass of the exploding dwarf in the observed manifestations of the explosion should be explored to apply them in modern cosmology.

4. Analyzing the scenarios illustrated in Figs 1–4. This analysis results in a conclusion that the evolution of the components of CBSs in the Hubble time ($\sim 10^{10}$ years) ends with the emergence of binary systems with the following components: main-sequence stars of small mass ($< M_{\odot}$), helium-rich nondegenerate stars with masses $0.4M_{\odot}–100M_{\odot}$, helium-rich degenerate dwarfs with masses $0.1M_{\odot}–0.5M_{\odot}$, carbon–oxygen degenerate dwarfs with masses $0.6M_{\odot}–1.1M_{\odot}$, oxygen–neon dwarfs with masses $1.1M_{\odot}–1.4M_{\odot}$, neutron stars with masses $\sim 1.4M_{\odot}$, and stellar black holes with masses $3M_{\odot}–50M_{\odot}$. The gravitational-wave radiation and the magnetic stellar wind for main-sequence stars with masses $0.3M_{\odot}–M_{\odot}$ reduce the distance between the closest of these components, thus converting the corresponding detached systems into semi-detached. As a result, 21 classes of semi-detached systems can be formed. While some classes of systems of this kind are widely discussed today, namely, binary degenerate carbon–oxygen dwarfs and SNIa, cataclysmic binary systems, X-ray binaries, NS + NS, and BH + BH, the remaining classes of possible close systems remain insufficiently popular and poorly studied, although some of them might be quite interesting. For example, the merger of a helium-rich dwarf with a carbon–oxygen star can lead to the formation of helium-rich giants of the RCrB type. A comprehensive study of 21 close binary types seems to be a task needed to expand our understanding of the latest stages of the evolution of these stars.

5. Exploring the evolution of CBSs at the common-envelope stage, which remains an important task. Despite the ‘efficiency’ of a simple approach based on energy characteristics (see Eqn (1)), the picture of the loss of the common envelope is still unclear, and the detailed under-

standing of the transformation of the binary core energy into the energy of an expanding envelope is unavailable. A most thorough study of the observed properties of the expanding envelopes of planetary nebulae and related objects is also necessary to determine involvement and role of binary cores in their formation.

6. A study of the physics of kick, an increment in the spatial velocity obtained by a young neutron star or black hole when they are formed in a supernova explosion. It seems that the clarification of the role that the neutrino pulse plays in increasing the spatial velocity of supernova explosion products and the study of the asymmetry introduced by the magnetic field into the initial potential of the pre-supernova may be possible directions for the initial search. It is not ruled out that incomplete burning of nuclear fuel in the process of explosion, turbulization of products of nuclear burning, and the asymmetry of envelope ejection caused by large-scale turbulence can contribute to the acceleration of the remnant.

7. A study of the interaction between supermassive black holes and CBSs, which is a special kind of the numerous mechanisms for the acceleration of stars of various types. Numerical simulations show that as a result of the decay of a close binary in the course of such an interaction, main-sequence stars, neutron stars, and black holes of stellar masses can emerge that move at the speed $v = v_0(M_{\text{BH}}/m)^{1/3}$, where m is the mass of the accelerated star, M_{BH} is the mass of the supermassive black hole, and v_0 is the orbital velocity of the star. It is clear that if $M_{\text{BH}} \sim 10^9 M_{\odot}$, even main-sequence stars can acquire a relativistic speed. It is worth studying both the process of such acceleration itself and possible observable manifestations for stars with such high spatial velocities. An interesting problem, for example, is the interaction between the envelope of a planetary nebula accelerated to a relativistic velocity and the galactic and intergalactic gas.

We also presented the main observational characteristics of various types of close binary systems in connection with modern ideas about their evolution, including the most recent data related to the discovery of gravitational waves from the merger of black holes and neutron stars in close pairs of relativistic objects. An important conclusion can be drawn from the above that the observed characteristics of close binaries are mostly in good agreement with the predictions of the theory of evolution involving mass exchange. This agreement becomes especially impressive if we take into account that the discovery of X-ray binary systems and, in particular, gravitational-wave binary systems made it possible to trace the evolution of close binaries in the widest range of evolutionary stages of stars and relativistic objects beginning with young stars that have not yet reached the main sequence to the close binary components at the latest stages of evolution, when the final products of stellar evolution (white dwarfs, neutron stars, black holes) are formed and the merging of relativistic objects occurs in tight pairs of neutron stars and black holes. These processes are accompanied by powerful bursts of radiation from gravitational waves, which are successfully detected by the state-of-the-art laser gravitational-wave observatories LIGO and Virgo. Amazingly, the theory provides a good explanation of the basic observational characteristics of close binary systems across this entire vast evolutionary range, and its predictions are in good agreement with the observations.

There are a number of new observational facts that stimulate the further development of the theory of evolution

of close binary systems. For example, stellar winds in massive close binaries can cause significant nonconservative mass exchange. As a result of fast axial rotation of stars in very close massive binary systems, the evolution of stars can occur that involves the mixing of matter in the star through meridional circulation, eventually leading to a scenario like the M-evolution of close binary systems (see Section 7.5.1).

An unusual, almost flat, mass distribution of stellar black holes in X-ray binary systems should also be noted: the number of known black holes in binary systems does not increase with a decrease in their masses (currently, the number of stellar black holes with known masses in X-ray binaries is approaching 30, making the statistics quite representative). This distribution should be confronted with the observation that the number of emerging massive stars in the Galaxy very rapidly (like M^{-2}) increases with decreasing the star mass. Moreover, a dip in the mass distribution of relativistic objects in the range $(2-4)M_{\odot}$ is apparently looming. Neither neutron stars nor black holes have been discovered so far in this mass range of binary systems. It can be shown [218, 219] that this fact is not related to the effects of observational selection. Most likely, the relativistic-object mass is determined not only by the parent-star mass but also by other factors including loss of mass by the star in the form of wind, rotation of the star's core, its magnetic field, and various instabilities that occur during collapse. All these factors and observational data should be taken into account by the theory (see the review in [5]).

To date, information has been collected on the masses of about 100 relativistic objects in X-ray and radio pulsar binary systems (about 30 black holes and more than 60 neutron stars). It became clear that neutron stars and black holes differ not only in mass but also in observational manifestations, in full agreement with the GR prediction about the absence of observable surfaces in black holes. Namely, in all cases where a relativistic object exhibits apparent signs of an observable surface (X-ray, radio pulsar, or type-I X-ray burster phenomena), its mass does not exceed $3M_{\odot}$, i.e., the absolute upper limit of the neutron star mass predicted by GR.

At the same time, none of the 30 massive ($M > 3M_{\odot}$) compact objects (black hole candidates) shows signs of an observable surface: it is neither an X-ray pulsar, nor a radio pulsar, nor a type-I X-ray burster, also in full agreement with the prediction of GR about the presence of an event horizon in black holes. These reliable results are an indirect indication of the absence of observable surfaces in known black holes and the presence of event horizons. Unfortunately, these results cannot be considered evidence of the existence of event horizons for the 30 black-hole candidates in X-ray binary systems because some neutron stars can also show no signs of observable surfaces. For example, if the rotation axis of a neutron star coincides with the magnetic-dipole axis, the X-ray pulsar phenomenon would not be observed in the process of accretion, and the massive neutron star may then be mistaken for a black hole. Moreover, a definitive judgement regarding the nature of an object apparently cannot be based on the absence of certain signs in it. For the existence of the event horizon of black holes to be finally confirmed, it is necessary to observe effects specific to black holes. As noted in [177], the remarkable discovery of gravitational-wave bursts from the merging of black holes in binary systems provides a conceptual option to finally prove the existence of an event horizon of black holes by observing

quasinormal oscillation modes of the newly formed more massive black hole at the stage of ringing down of the gravitational wave signal. Prospects for further gravitational-wave explorations in this area are quite exciting.

The nature of ultra-luminous X-ray (ULX) sources in other galaxies has become clearer in recent years. At least some of these sources proved to be X-ray pulsars or not very massive black holes ($M < 15M_{\odot}$), which are in the stage of highly supercritical accretion. The study of ULX sources poses new challenges to the theory of close binary evolution with mass exchange. New results obtained in 3D gasdynamic and magnetogasdynamic studies of mass exchange in close binary systems should be noted [141, 142]. These studies are very promising. For example, the evolution of close binaries on the dynamic time scale was studied in [220]. The distance between the components is shown to change at the beginning of the dynamic mass exchange not monotonically but in an oscillatory manner, although on average it decreases with time. A significant portion of the orbital angular momentum of the system (up to 20%) is accumulated in this case in the axial rotation of the components, while the oscillatory regime of changing the distance between them is associated with the angular momentum of component rotation being transferred into orbit and back due to tidal interactions. If mass is transferred in a close binary system not on the dynamic but on the thermal and nuclear time scales of evolution, the gas flow density is much lower, but the characteristic times of mass exchange are much longer. Therefore, a 3D gasdynamic analysis of the evolutionary change of the distance between the close binary components appears to be very relevant, even in the case of mass exchange on the thermal and nuclear time scales of close binary evolution.

Distinguished among the many stages of close binary evolution is the common-envelope stage, which is of particular importance for the formation of compact pairs of black holes and neutron stars that merge due to the emission of gravitational waves in a time less than the age of the Universe [29]. A detailed study of the evolutionary stage of close binary systems with a common envelope has not yet been completed [101, 221].

Reports have recently appeared (see, e.g., [222]) that discuss the validity of applying the Skumanicz law [126], which describes the rotation of single stars in clusters of various ages, to close binary evolution models. It was argued in particular that actual braking by a magnetic stellar wind in compact binary systems can be much weaker than what follows from the Skumanicz law. This issue requires a dedicated theoretical study.

Owing to the observations made by the Kepler spacecraft, several thousand exoplanets have been discovered around many stars in the Galaxy, including exoplanets in a number of eclipsing binary systems. The problem of the origin and stability of the orbits of such exoplanets requires a very detailed study. Moreover, because the orbital inclination in eclipsing binary systems is close to 90° , it is very likely that eclipses of the components of those systems by exoplanets can be observed, which makes it possible to accurately determine the masses and radii of such exoplanets, as well as to study the chemical composition of their atmospheres.

To conclude, we list the scientific results obtained in the study of late close binary systems that are of particular importance for both astrophysics and fundamental physics.

1. Radio pulsars have been discovered paired with massive B and Be stars, which proved the actual existence of quiet

(‘sleeping’) massive X-ray binary systems predicted by the evolution theory. In addition, observational evidence of the existence of massive satellite stars of radio pulsars in binary systems yields an empirical basis to assume the existence of radio pulsars paired with black holes. The discovery of a radio pulsar in a close binary system containing a black hole would enable experimental exploration of the space–time metric near the black hole based on the motion of the pulsar (which is a very accurate clock operating in the orbit).

2. Two radio pulsars have been discovered in the J0737-3039AB system, which confirmed the concept of recycled pulsars [215] and made it possible to comprehensively verify GR in the limit of a strong gravitational field (see review [223]).

3. Tight binary white dwarfs have been discovered. This discovery is of importance for the evolutionary theory that predicts the merging of white dwarfs under the effect of gravitational-wave radiation and the occurrence of type-Ia supernova explosions. A strongly magnetized ($H \sim 10^8$ G) white dwarf, which is a pulsar [201], has been discovered recently in the ARSco binary system. AEAquarii is apparently also a system that contains a pulsar, a white dwarf [198].

4. A WR star has been discovered in a very short-period X-ray binary system, CygX-3 ($P \simeq 4.8$ h). This discovery has proved that the massive close binary systems $WR_2 + C$ predicted by the evolutionary theory and containing a ‘second generation’ WR star paired with a relativistic object, actually exist. Two more systems of this type have been discovered recently: IC10X-1 and NGC300X-1 (for details, see [5]).

5. The SS433 object has been discovered, providing a new example of massive close binary evolution at the stage of secondary mass exchange. As follows from the observations, a common envelope is not formed in this case, and the mass and the angular momentum are taken away from the system by forming a supercritical accretion disk around the relativistic object and its subsequent ‘evaporation’ under the effect of a powerful radial stellar wind and collimated relativistic jets. The SS433 object proved to be the first example of a new class of microquasar objects (for more details, see [5]).

6. New results have been obtained in the interpretation of the eclipsed light curves of binary WR + OB systems V444Cyg and BAT99-129 [5]. They show that the ‘classical’ Population I WR stars have radii $R_{\text{core}} < 4R_{\odot}$, which are small compared with their masses ($\sim (10-15)M_{\odot}$) and high temperatures of hydrostatic WR ‘cores’ ($T_{\text{core}} > 45,000$ K). The small radius and high temperature of the WR star ‘core’ observed in the very short-period X-ray binary system CygX-3 [171] confirm these results. The data are convincing evidence that ‘classical’ WR stars are the exposed helium-rich cores of initially massive stars that have lost the bulk of the hydrogen-rich envelope. It then follows that WR stars should explode after several hundred thousand years as type-Ib/c supernovae and form neutron stars or black holes. WR stars were formed in the V444Cyg and BAT99-129 systems as a result of the loss by massive stars of their hydrogen-rich envelopes due to mass exchange between the components.

Helium enrichment of the outer layers of stars and the emergence of the WR phenomenon in massive short-period systems like the WR20a system (WN6ha + WN6ha, $P = 3.675$ days, $e = 0$, $M_1 = 83M_{\odot}$, $M_2 = 82M_{\odot}$), in which the components rotate rapidly, are apparently due to the meridional circulation of matter in the body of the star and the outflow of merger products from the central parts of the star to the surface.

Parameters of the Melnick34 system located in the Large Magellanic Cloud [224], which is a long-period close binary ($P \simeq 155$ days) consisting of two very massive WR stars (WN5h + WN5h, $M_1 = 139_{-18}^{+21}M_{\odot}$, and $M_2 = 127_{-17}^{+17}M_{\odot}$) that move in highly elliptical orbits ($e = 0.68 \pm 0.02$), have been published recently. There is no mass exchange in this system due to the large semimajor axis of the relative orbit (about 3 a.u.), and WR stars were formed as a result of radial mass loss in the form of stellar wind. Thus, the accumulation of observational data on massive close binary systems displays various ways of WR star formation.

7. Observational data have been obtained indicating that at least some compact ULX sources are either X-ray pulsars or black holes with relatively small masses ($\lesssim 15M_{\odot}$), which are in a highly supercritical accretion mode in massive X-ray binary systems at special evolution stages. It was shown in [225] that most ULXs in other galaxies, whose luminosity in the X-ray range is up to $\sim 10^{42}$ erg s $^{-1}$, are supercritical accretion disks in binary systems with black holes, similar to supercritical microquasar accretor SS433 in our Galaxy [165].

To date, three X-ray pulsars in binary systems that exhibit the ULX phenomenon have been discovered by the NuSTAR (Nuclear Spectroscopic Telescope ARray) orbital observatory (see review [226], which contains all necessary references). For example, M82X-2, the ULX source in the M82 galaxy, whose maximum X-ray luminosity exceeds 10^{40} erg s $^{-1}$, turned out to be an X-ray pulsar with a period of 1.37 s orbiting a donor star with an orbital period of ~ 2.5 days. The period of axial rotation of the X-ray pulsar decreases at the rate $\dot{P}_{\text{puls}} = -2 \times 10^{-10}$ s s $^{-1}$ [227]. The occurrence of coherent X-ray pulsations definitely indicates the presence of a neutron star whose observed luminosity greatly exceeds the Eddington limit. It is possible that an accretion to the magnetic poles of a neutron star with a strong magnetic field, which significantly differs from the spherical one, is observed in this case [228]. It was shown in [229] that ULX P13 in the NGC7793 galaxy is a close binary with an orbital period of about 64 days. The donor satellite in this system is a supergiant star belonging to spectral class B9Ia heated by powerful X-ray radiation from an accreting black hole, whose mass does not exceed $15M_{\odot}$. An intermediate-mass black hole ($\sim 10^3M_{\odot}$) has not been detected in this system. A red supergiant orbiting a black hole whose estimated mass is several tens of M_{\odot} was discovered [230] as an optical donor satellite in ULX J004722-252051 in the NGC253 galaxy. Study [231] detected quasiperiodic X-ray radiation oscillations of the ULXX-1 object in the M82 galaxy with the ratio of quasiperiods of 3:2, and the mass of the accreting black hole was indirectly estimated to be $400M_{\odot}$. Baryon jets were observed from the tentative black hole in the 4U1630-70 X-ray binary system, which may be an analog of the SS433 object [232]. Studies [233, 234] discovered relativistic baryon collimated outflows (jets) from ULX sources M81ULS-1 and NGC300ULX-1 with outflow velocities exceeding $(0.17-0.24)c$. (For more information on ULX sources, see review [226].)

8. Significant successes have been achieved in the optical identification of cosmic gamma-ray bursts, including linear polarization of intrinsic optical radiation that accompanies the gamma-ray burst [235]. A wealth of data has been accumulating in recent years that favors the model of long gamma-ray bursts as a phenomenon accompanying the collapse of the rapidly rotating core of a very massive star

and the formation of an extremely fast-rotating Kerr black hole with relativistic jets. The fast rotation of the massive-star core may be maintained in this model by the orbital motion of a nearby satellite in a short-period late close binary system [236, 237].

The short gamma-ray burst accompanying the gravitational-wave burst GW170817, which is due to the merger of two neutron stars, made it possible to study the physics of nucleosynthesis in the merger of two neutron stars in detail and to test the model of the formation of gamma-ray bursts and related relativistic jets. The arrival times of electromagnetic and gravitational-wave bursts was used to show that the propagation velocity of gravitational waves in space coincides with high accuracy with the speed of light.

9. The masses of more than 60 pulsars — neutron stars in binary systems — have been determined (see the review in [5] and the references therein). The masses of X-ray pulsars and radio pulsars, as well as the masses of type-I X-ray bursters and neutron stars in the gravitational-wave binary GW170817 system, do not exceed $3M_{\odot}$, the upper limit of the neutron star mass predicted by GR. The masses of millisecond pulsars are on average $\sim 0.13M_{\odot}$ greater than those of ordinary pulsars, whose periods are of the order of seconds. This observation is consistent with the conclusions of the theory of recycled pulsars regarding mass exchange in close binary systems. The number of discovered massive, $M_p \simeq 2M_{\odot}$, neutron stars is now five, owing to which the soft equation of state of neutron matter can be rejected.

10. An analysis of the secular reduction in the orbital periods of radio pulsars in the PSR1913 + 16, J0737-3039AB, PSR1534 + 12, and PSRJ0348 + 0432 binary systems provided indirect evidence of the existence of gravitational waves, encouraging researchers to create laser gravitational wave antennas. In 2015, the LIGO Observatory was the first to detect gravitational-wave signals from merging black holes in a binary system, which confirmed the validity of GR and brought the study of black holes to a qualitatively new level. This discovery was awarded with the Nobel Prize in 2017.

11. The masses of about 30 black holes in massive and low-mass X-ray binary systems have been determined. The angular momenta have been estimated for approximately 10 stellar black holes. It is remarkable that none of these massive ($M > 3M_{\odot}$) compact X-ray sources shows signs of an observable surface (it is neither an X-ray pulsar nor a type-I X-ray burster), in full agreement with the predictions of GR.

12. By December 2018, the LIGO and Virgo observatories had discovered gravitational-wave bursts from the merger of black holes in ten systems and measured the masses of black holes prior to (M_1, M_2) and after (M_{fin}) the merger (see Table 2).

The dimensionless spin (angular momentum) of newly formed more massive black holes, which falls in a range 0.66–0.81, is in all cases significantly less than unity (the case of extremely rapidly rotating, Kerr-type, black holes). This suggests that the spins of black holes prior to the merger were also significantly less than unity, and therefore the rotation of these black holes is far from critical. This problem requires a dedicated theoretical study.

Further observations of gravitational-wave radiation bursts at the advanced LIGO and Virgo observatories will dramatically increase the number of determinations of black-hole masses in gravitational-wave binary systems and clarify the evolutionary status and mechanisms of black hole formation in gravitational-wave binary systems. The study

of the final ringdown stage in gravitational-wave signals from merging black holes in binary systems with a larger signal-to-noise ratio and for a larger number of systems will hopefully enable exploring the quasinormal oscillation modes of newly formed more massive black holes and obtaining final evidence that the event horizons for stellar black holes actually exist [177].

9. Conclusion

We described the current state of the problem of close binary systems that has moved over the past half a century of research from a classical field of astronomy to the front line of modern natural science.

The problem of close binary systems has many facets in its current form. It encompasses such areas of science as astrophysics, celestial mechanics, the physics and evolution of stars, relativistic astrophysics, and cosmology. Studies of close binary systems are also of importance for fundamental physics and even for astrobiology (given the eclipsing studies of exoplanets and their atmospheres).

Three ‘golden ages’ can be distinguished in the research on close binary systems, in each of which this problem was boosted to a qualitatively new level.

The first relates to the 1950s–1970s, when models of the internal structure of stars began to emerge, and application of high-precision photoelectric techniques for observing close binaries and new methods for their interpretation commenced. The accumulation of reliable data on the basic parameters of stars from research on close binaries (masses, radii, and temperatures), which took place in those days, made it possible to perform rigorous observational tests of the theory of the inner structure of stars, thus helping to solve many problems with the physics and the evolution of stars.

The second age dates back to 1970–2014, when the X-ray astronomy era began, the theory of accretion was created, and the theory of close binary evolution with mass exchange was developed. A new field of science, relativistic astrophysics, was born; neutron stars and black holes were discovered as components of X-ray binary systems; massive studies of radio pulsars in binary systems commenced; exoplanets were discovered and searches for manifestations of life on those planets began; the origin of type-Ia supernovae, the standard ‘candles’ that were used to discover the accelerated expansion of the Universe, was explained within in the model of close binary systems; and the important role of close binaries as the main sources of gravitational waves was recognized.

The third ‘golden’ age began quite recently, in 2015, when the LIGO and Virgo observatories discovered gravitational waves. It is remarkable that the gravitational-wave sources turned out to be close binaries consisting of black holes, as had been predicted theoretically [238]. The discovery of electromagnetic radiation bursts associated with gravitational wave signals from the merger of binary neutron stars heralded the birth of gravitational-wave astronomy as part of multi-messenger astronomy, which led to a breakthrough in the study of relativistic close binary systems. It would be sufficient to note that in just two sessions of gravitational-wave observation, the masses of 30 black holes have been measured (20 before the merger and 10 after). This is as many black holes as researchers have been able to discover in X-ray binary systems over a long half-century of studies. It is gravitational wave astronomy that in the years to come will become the main source of information about the masses of

stellar black holes, and, in the not very distant future, about the masses of supermassive black holes based on the results of exploration of gravitational waves from binary galactic cores. By studying gravitational-wave signals from close binary systems, researchers will also be able to verify the nonlinear equations of general relativity in all their complexity [239] and, possibly, use direct observations of quasinormal oscillation modes to prove the existence of the event horizon of black holes [177].

Broad prospects for studies of close binary systems also open up in connection with the development of electromagnetic-wave astronomy: the upcoming launch of the James Webb Space Telescope and Spektr-RG space observatories and also owing to the commissioning of giant ground-based optical telescopes 20–39 meters in diameter (ELT (Extremely Large Telescope), TMT (Thirty Meter Telescope), and Magellan) and large survey telescopes (like LSST (Large Synoptic Survey Telescope)). Combined with the enhancement of the next-generation of gravitational-wave antennas, this advancement will provide unique opportunities for studying close binaries in multi-messenger astronomy [240–245], owing to which new outstanding discoveries are expected in this very promising field of research.

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References

1. Jeans J H *Nature* **118** 29 (1926); Eddington A S *The Internal Constitution of the Stars* (Cambridge: Cambridge Univ. Press, 1920); Translated into Russian: Jeans J H, Eddington A S *Sovremennoe Razvitiye Kosmicheskoi Fiziki* (Modern Development of Space Physics) (Moscow–Leningrad: Gos. Izd., 1928)
2. Abbott B P et al. (LIGO Scientific Collab., Virgo Collab.) *Phys. Rev. Lett.* **116** 061102 (2016)
3. Perlmutter S et al. *Astrophys. J.* **517** 565 (1999)
4. Masevich A G, Tutukov A V *Evolutsiya Zvezd: Teoriya i Nablyudeniya* (Evolution of Stars: Theory and Observations) (Moscow: Nauka, 1988)
5. Cherepashchuk A M *Tesnye Dvoynye Zvezdy* (Close Binary Stars) Pt. I, II (Moscow: Fizmatlit, 2013)
6. Iben I (Jr.) *Stellar Evolution Physics* (Cambridge: Cambridge Univ. Press, 2012)
7. Postnov K A, Yungelson L R *Living Rev. Relativ.* **17** 3 (2014)
8. Tutukov A V, Yungelson L R *Acta Astron.* **29** 665 (1979)
9. Soker N *Galaxies* **6** (2) 58 (2018)
10. Damineli A et al. *Mon. Not. R. Astron. Soc.* **484** 1325 (2019)
11. Kashi A *Galaxies* **6** (3) 82 (2018)
12. Rizzuto A C et al. *Astron. J.* **156** 195 (2018)
13. Skumanich A *Astrophys. J.* **171** 565 (1972)
14. Einstein A *Sitzungsber. Königl. Preuss. Acad. Wiss. Berlin* **1** 154 (1918)
15. Abbott B P et al. (LIGO Scientific Collab., Virgo Collab.) *Phys. Rev. Lett.* **119** 161101 (2017)
16. Tutukov A V, Yungelson L R *Nauch. Inform. Astrosoveta Akad. Nauk SSSR* (27) 58 (1973)
17. Duchêne G et al. *Mon. Not. R. Astron. Soc.* **478** 1825 (2018)
18. Zorec J, Levenhagen R, Chanville J *IAU Symp.* **215** 83 (2004)
19. Baade D et al. *Astron. Astrophys.* **620** A145 (2018)
20. Savonije G J *Astron. Astrophys.* **469** 1057 (2007)
21. Bastian N et al. *Mon. Not. R. Astron. Soc.* **465** 4795 (2017)
22. Meilland A et al. *Astron. Astrophys.* **464** 59 (2007)
23. Tutukov A V *Nauch. Inform. Astrosoveta Akad. Nauk SSSR* (11) 62 (1969)
24. Tutukov A V, Fedorova A V *Astron. Rep.* **51** 847 (2007); *Astron. Zh.* **84** 937 (2007)
25. Shustov B M, Tutukov A V *Astron. Rep.* **62** 724 (2018); *Astron. Zh.* **95** 765 (2018)
26. Vink J S *Astron. Astrophys.* **615** A119 (2018)
27. Crowther P A, in *Stellar Evolution at Low Metallicity: Mass Loss, Explosions, Cosmology. Proc. of the Conf., 15–19 August, 2005, Tartu, Estonia* (ASP Conf. Ser., Vol. 353, Eds H Lamers et al.) (San Francisco, CA: Astron. Soc. of the Pacific, 2006)
28. Hainich R et al. *IAU Symp.* **329** 171 (2017)
29. Tutukov A V, Cherepashchuk A M *Astron. Rep.* **61** 833 (2017); *Astron. Zh.* **94** 821 (2017)
30. Tutukov A V *Astron. Rep.* **63** 79 (2019); *Astron. Zh.* **96** 91 (2019)
31. Tutukov A V, Yungelson L R *Nauch. Inform. Astrosoveta Akad. Nauk SSSR* (27) 10 (1973)
32. Tutukov A V, Yungelson L R *Pis'ma Astron. Zh.* **14** 623 (1988)
33. Lipunov V M, Postnov K A *Sov. Astron.* **31** 228 (1987); *Astron. Zh.* **64** 438 (1987)
34. Tutukov A V *Astron. Astrophys.* **70** 57 (1978)
35. Onifer A, Heger A, Abdallah J, in *Hydrogen-Deficient Stars. Proc. of the Conf., 17–21 September, 2007, Tübingen, Germany* (ASP Conf. Ser., Vol. 391, Eds K Werner, T Rauch) (San Francisco, CA: Astronomical Society of the Pacific, 2008) p. 305
36. Iben I (Jr.), Tutukov A V *Astrophys. J.* **418** 343 (1993)
37. Williams B F et al. *Astrophys. J.* **860** 39 (2018)
38. Tutukov A V *Astron. Rep.* **49** 993 (2005); *Astron. Zh.* **82** 1109 (2005)
39. Lipunov V M *Phys. Usp.* **59** 918 (2016); *Usp. Fiz. Nauk* **186** 1011 (2016)
40. Kovetz E D et al. *Phys. Rev. D* **95** 103010 (2017)
41. Barack L et al. *Class. Quantum Grav.* **36** 143001 (2019)
42. Iben I (Jr.), Tutukov A V, Fedorova A V *Astrophys. J.* **503** 344 (1998)
43. Preece H P, Tout C A, Jeffery C S *Mon. Not. R. Astron. Soc.* **481** 715 (2018)
44. Bogomazov A I, Tutukov A V *Astron. Rep.* **53** 214 (2009); *Astron. Zh.* **86** 240 (2009)
45. Kilic M et al. *Mon. Not. R. Astron. Soc.* **479** 1267 (2018)
46. Tutukov A V, Yungelson L R *Mon. Not. R. Astron. Soc.* **268** 871 (1994)
47. Iben I (Jr.), Tutukov A V *Astrophys. J. Suppl. Ser.* **58** 661 (1985)
48. Paczyński B *Acta Astron.* **20** 47 (1970)
49. Paczyński B *Annu. Rev. Astron. Astrophys.* **9** 183 (1971)
50. Shara M M et al. *Astrophys. J.* **860** 110 (2018)
51. Whelan J, Iben I (Jr.) *Astrophys. J.* **186** 1007 (1973)
52. Pala A F et al. *Mon. Not. R. Astron. Soc.* **481** 2523 (2018)
53. Osaki Y *Publ. Astron. Soc. Jpn.* **41** 1005 (1989)
54. Osaki Y, Kato T *Publ. Astron. Soc. Jpn.* **65** (3) 50 (2013)
55. Bisikalo D V et al. *Astron. Rep.* **48** 588 (2004); *Astron. Zh.* **81** 648 (2004)
56. Phillipson R A, Boyd P T, Smale A P *Mon. Not. R. Astron. Soc.* **477** 5220 (2018)
57. Linford J D, Chomiuk L, Rupen M P, in *Science with a Next-Generation Very Large Array* (Astronomical Society of the Pacific Conference Series. Monograph 7, Vol. 517, Ed. E J Murphy) (San Francisco, CA: Astronomical Society of the Pacific, 2018) p. 271
58. Beskrovnaya N G, Ikhsanov N R *Adv. Space Res.* **55** 787 (2015)
59. Potter S B, Buckley D A H *Mon. Not. R. Astron. Soc.* **481** 2384 (2018)
60. Eyres S P S et al. *Mon. Not. R. Astron. Soc.* **481** 4931 (2018)
61. Kaplan D L et al. *Astrophys. J.* **864** 15 (2018)
62. Iben I (Jr.) et al. *Astrophys. J.* **317** 717 (1987)
63. Nomoto K, Kamiya Y, Nakasato N *IAU Symp.* **281** 253 (2013)
64. Dhawan S et al. *Mon. Not. R. Astron. Soc.* **480** 1445 (2018)
65. Shen K J et al. *Astrophys. J.* **854** 52 (2018)
66. Perlmutter S, Turner M S, White M *Phys. Rev. Lett.* **83** 670 (1999)
67. Hamuy M, arXiv:1311.5099
68. Woosley S E *Astrophys. J.* **878** 49 (2019)
69. Ashall C et al. *Mon. Not. R. Astron. Soc.* **460** 3529 (2016)
70. Tucker M A, Shappee B J, Wisniewski J P *Astrophys. J. Lett.* **872** L22 (2019)

71. Kilic M et al. *Mon. Not. R. Astron. Soc.* **479** 1267 (2018)
72. Dimitriadis G et al. *Astrophys. J. Lett.* **870** L1 (2019)
73. Postnov K A, Yungelson L R *Living Rev. Relat.* **17** 3 (2014)
74. Fink M et al. *Astron. Astrophys.* **618** A124 (2018)
75. Abbott B P et al. (LIGO Scientific Collab., Virgo Collab.) *Phys. Rev. Lett.* **116** 241103 (2016)
76. Abbott B P et al. (LIGO Scientific, Virgo Collab.) *Phys. Rev. Lett.* **118** 221101 (2017)
77. Dremova G N, Dremov V V, Tutukov A V *Astron. Rep.* **61** 573 (2017); *Astron. Zh.* **94** 580 (2017)
78. Van den Heuvel E P J, Heise J *Nature Phys. Sci.* **239** 67 (1972)
79. Tutukov A V, Yungelson L R *Nauch. Inform. Astrosoveta Akad. Nauk SSSR* (27) 86 (1973)
80. Thorne K S, Zytkov A N *Astrophys. J.* **212** 832 (1977)
81. Hui C Y et al. *Astrophys. J.* **864** 30 (2018)
82. Zel'dovich Ya B *Sov. Phys. Dokl.* **9** 195 (1964); *Dokl. Akad. Nauk SSSR* **155** 67 (1964)
83. Tutukov A, Yungelson L *IAU Symp.* **99** 377 (1982)
84. Tutukov A V, Yungelson L R *Mon. Not. R. Astron. Soc.* **260** 675 (1993)
85. Blinnikov S I et al. *Sov. Astron. Lett.* **10** 177 (1984); *Pis'ma Astron. Zh.* **10** 422 (1984)
86. Foucart F, Hinderer T, Nisanke S *Phys. Rev. D* **98** 081501(R) (2018)
87. Rizzuto A C et al. *Astron. J.* **156** 195 (2018)
88. Hermes J J et al. *Astrophys. J. Suppl.* **232** 23 (2017)
89. Batten A H *Binary and Multiple Systems of Stars* (Oxford: Pergamon Press, 1973); Translated into Russian: *Dvoynye i Kratnye Zvezdy* (Moscow: Mir, 1976)
90. Chambliss C R *Publ. Astron. Soc. Pacific* **104** 663 (1992)
91. Pribulla T, Rucinski S M *Astron. J.* **131** 2986 (2006)
92. Zakirov M M *Kinematika Fiz. Nebesnykh Tel* **25** 163 (2009)
93. Zakirov M M *Kinematika Fiz. Nebesnykh Tel* **26** 3 (2010)
94. Fekel F C (Jr.) *Astrophys. J.* **246** 879 (1981)
95. Anosova J P *Bull. Astron. Belgrade* (138) 13 (1988)
96. Popović G M *Bull. Astron. Belgrade* (144) 13 (1991)
97. Poveda A et al. *Rev. Mexicana Astron. Astrofis.* **28** 43 (1994)
98. Tokovinin A A *Astron. Astrophys. Suppl.* **124** 75 (1997)
99. Chambliss C R, in *Evolutionary Processes in Interacting Binary Stars. Proc. of the 151st Symp. of the International Astronomical Union, Cordoba, Argentina, August 5–9, 1991* (Eds Y Kondo, R F Sistero, R S Polidan) (Dordrecht: Kluwer Acad. Publ., 1992) p. 315
100. Milone E F, Stagg C R, Schiller S J, in *Evolutionary Processes in Interacting Binary Stars. Proc. of the 151st Symp. of the International Astronomical Union, Cordoba, Argentina, August 5–9, 1991* (Eds Y Kondo, R F Sistero, R S Polidan) (Dordrecht: Kluwer Acad. Publ., 1992) p. 479
101. Eggleton P *Evolutionary Processes in Binary and Multiple Stars* (Cambridge: Cambridge Univ. Press, 2006)
102. Weinberg M D, Wasserman I *Astrophys. J.* **329** 253 (1988)
103. Larson R B *Mon. Not. R. Astron. Soc.* **272** 213 (1995)
104. Tokovinin A *Mon. Not. R. Astron. Soc.* **389** 925 (2008)
105. Martynov D Ya *Sov. Phys. Usp.* **15** 786 (1973); *Usp. Fiz. Nauk* **108** 701 (1972)
106. Abt H A *Annu. Rev. Astron. Astrophys.* **21** 343 (1983)
107. Guinan E F, Engle S G *Astrophys. Space Sci.* **304** 5 (2006)
108. Paczynski B *Bull. Am. Astron. Soc.* **8** 442 (1976)
109. Paczynski B, in *Wolf-Rayet and High-Temperature Stars. Proc. from IAU Symp. No. 49, Buenos Aires, Argentina, August 9–14, 1971* (Eds M K V Bappy, J Sahade) (Dordrecht: D. Reidel Publ. Co., 1973) p. 143
110. Malkov O Yu et al. *Astron. Astrophys.* **446** 785 (2006)
111. Khaliullin Kh F, Khaliullina A I *Mon. Not. R. Astron. Soc.* **401** 257 (2010)
112. Khaliullin Kh F, Khaliullina A I *Mon. Not. R. Astron. Soc.* **411** 2804 (2011)
113. Hilditch R W, Howarth I D, Harries T J *Mon. Not. R. Astron. Soc.* **357** 304 (2005)
114. Girardi L et al. *Astron. Astrophys.* **141** 371 (2000)
115. Langer N *Astrophys. J.* **256** L17 (1992)
116. Herrero A et al. *Astrophys. J.* **261** 209 (1992)
117. Struve O *Stellar Evolution* (Princeton, NJ: Princeton Univ. Press, 1950)
118. Parenago P P, Masevich A G *Trudy Gos. Astron. Inst. im. P.K. Shternberga* **20** 81 (1950)
119. Ibanoglu C et al. *Mon. Not. R. Astron. Soc.* **373** 435 (2006)
120. de Mink S E et al. *Astrophys. J.* **764** 166 (2013)
121. Rodono M, in *Evolutionary Processes in Interacting Binary Stars. Proc. of the 151st Symp. of the International Astronomical Union, Cordoba, Argentina, August 5–9, 1991* (Eds Y Kondo, R F Sistero, R S Polidan) (Dordrecht: Kluwer Acad. Publ., 1992) p. 71
122. Eker Z et al. *Mon. Not. R. Astron. Soc.* **373** 1483 (2006)
123. Huang S-S *Annu. Rev. Astron. Astrophys.* **4** 35 (1966)
124. Schatsman E *Annu. Astrophys.* **25** 18 (1962)
125. Mestel L *Mon. Not. R. Astron. Soc.* **138** 359 (1968)
126. Skumanicz L A *Astrophys. J.* **171** 565 (1972)
127. Eggleton P P, Kisseleva L G *Astrophys. J.* **562** 1012 (2001)
128. Eggleton P P, Kisseleva L G *Astrophys. Space Sci.* **304** 73 (2006)
129. Kozai Y *Astron. J.* **67** 591 (1962)
130. Pribulla T, Rucinski S M *Astron. J.* **131** 2986 (2006)
131. Vanko M et al. *Astrophys. Space Sci.* **304** 133 (2006)
132. Plavec M J *IAU Symp.* **88** 251 (1980)
133. Antokhina E A, Cherepashchuk A M *Sov. Astron. Lett.* **14** 105 (1988); *Pis'ma Astron. Zh.* **14** 252 (1988)
134. Cherepashchuk A M et al. *Highly Evolved Close Binary Stars* Vol. 1 (Amsterdam: Gordon and Breach, 1996) p. 41
135. Boyarchuk A A *Sov. Astron.* **3** 748 (1959); *Astron. Zh.* **36** 766 (1959)
136. Svechnikov M A *Katalog Orbital'nykh Elementov, Mass i Svetimosti Tesnykh Dvoynykh Zvezd* (Catalog of Orbital Elements, Masses, and Luminosities of Close Binary Stars) (Irkutsk: Irkutsk Univ. Publ., 1986)
137. Guiricín G, Mardirossian F, Mezzetti M *Astron. Astrophys.* **54** 211 (1983)
138. Sarma M B K, Vivekananda Rao P, Abhyankar K D *Astrophys. J.* **458** 371 (1996)
139. Wilson R E *Astrophys. J.* **234** 1054 (1979)
140. Mkrtichian D E et al., in *Solar and Stellar Physics Through Eclipses. Proc. of the Conf., 27–29 March, 2006, Antalya, Turkey* (ASP Conf. Ser., Vol. 370, Eds O Demircan, S O Selam, B Albayrak) (San Francisco, CA: Astronomical Society of the Pacific, 2007) p. 194
141. Bisikalo D V, Zhilkin A G, Boyarchuk A A *Gazodinamika Tesnykh Dvoynykh Zvezd* (Gas Dynamics of Close Binary Stars) (Moscow: Fizmatlit, 2013)
142. Lukin V V et al. *Mon. Not. R. Astron. Soc.* **467** 2934 (2017)
143. Marchant P et al. *Astrophys. J.* **882** 36 (2019)
144. Mapelli M et al. *Mon. Not. R. Astron. Soc.* **487** 2 (2019)
145. de Mink S E et al., in *Binaries—Key to Comprehension of the Universe. Proc. of a Conf., June 8–12, 2009, Brno, Czech Republic* (Astron. Soc. of the Pacific, Vol. 435, Eds A Prsa, M Zejda) (San Francisco, CA: Astronomical Society of the Pacific, 2010) p. 179
146. Zahn J-P *Astron. Astrophys.* **57** 383 (1977)
147. van der Hucht K A *New Astron. Rev.* **45** 135 (2001)
148. Tutukov A V, Cherepashchuk A M *Sov. Astron.* **29** 654 (1985); *Astron. Zh.* **62** 1124 (1985)
149. Tutukov A V, Cherepashchuk A M *Astron. Rep.* **37** 159 (1993); *Astron. Zh.* **70** 307 (1993)
150. Tutukov A V, Cherepashchuk A M *Astron. Rep.* **41** 355 (1997); *Astron. Zh.* **74** 407 (1997)
151. Bonanos A Z et al. *Astrophys. J.* **611** L33 (2004)
152. Cherepashchuk A M, Karetnikov V G *Astron. Rep.* **47** 38 (2003); *Astron. Zh.* **80** 42 (2003)
153. Cherepashchuk A M *Astron. Rep.* **62** 567 (2018); *Astron. Zh.* **95** 602 (2018)
154. Kaspi V M et al. *Astrophys. J.* **423** L43 (1994)
155. Johnston S et al. *Astrophys. J.* **387** L37 (1992)
156. Johnston S et al. *Mon. Not. R. Astron. Soc.* **268** 430 (1994)
157. Kippenhahn R, Weigert A Z. *Astrophys. J.* **65** 251 (1967)
158. Raguzova N V, Popov S B *Astron. Astrophys. Trans.* **24** 151 (2005)
159. Ziolkowski J *Acta Polytech. CTU Proc.* **1** (1) 175 (2014); in *Proc. of the 10th Intern. Workshop on Multifrequency Behaviour of High Energy Cosmic Sources, Palermo, Italy, 2013*
160. Casares J et al. *Nature* **505** 378 (2014)
161. Sunyaev R A et al. *Astronomer's Telegram* **190** 1 (2003)

162. Grebenev S A, Lutovinov A A, Sunyaev R A *Astronomer's Telegram* **192** 1 (2003)
163. Sidoli L et al. *Mon. Not. R. Astron. Soc.* **400** 258 (2009)
164. Shakura N et al. *Mon. Not. R. Astron. Soc.* **442** 2325 (2014)
165. Cherepashchuk A M *Mon. Not. R. Astron. Soc.* **194** 761 (1981)
166. Margon B *Annu. Rev. Astron. Astrophys.* **22** 507 (1984)
167. Pavlovskii K, Ivanova N *Mon. Not. R. Astron. Soc.* **449** 4415 (2015)
168. Pavlovskii K et al. *Mon. Not. R. Astron. Soc.* **465** 2092 (2017)
169. Cherepashchuk A M, Postnov K A, Belinsky A A *Mon. Not. R. Astron. Soc.* **479** 4844 (2018)
170. van Kerkwijk M H et al. *Nature* **355** 703 (1992)
171. Cherepashchuk A M, Moffat A F J *Astrophys. J.* **424** L53 (1994)
172. Levesque E M et al. *Mon. Not. R. Astron. Soc.* **443** L94 (2014)
173. Cherepashchuk A M *Phys. Usp.* **59** 702 (2016); *Usp. Fiz. Nauk* **186** 778 (2016)
174. Abbott B P et al. (LIGO Scientific Collab., Virgo Collab.) *Phys. Rev. X* **9** 031040 (2019)
175. Bogomazov A I, Cherepashchuk A M, Lipunov V M, Tutukov A V *New Astron.* **58** 33 (2018)
176. Belczynski K et al. *Nature* **534** 513 (2016)
177. Cherepashchuk A M *Phys. Usp.* **59** 910 (2016); *Usp. Fiz. Nauk* **186** 1001 (2016)
178. Bogomazov A I *Astron. Rep.* **58** 126 (2014); *Astron. Zh.* **91** 180 (2014)
179. Abubekkerov M K, Antokhina E A, Bogomazov A I, Cherepashchuk A M *Astron. Rep.* **53** 232 (2009); *Astron. Zh.* **86** 260 (2009)
180. Cherepashchuk A M *Space Sci. Rev.* **93** 473 (2000)
181. Gonzalez-Hernandez J I, Rebolo R, Casares J *Mon. Not. R. Astron. Soc.* **438** L21 (2014)
182. Gonzalez-Hernandez J I et al. *Mon. Not. R. Astron. Soc.* **465** L15 (2017)
183. Xu X-T, Li X-D *Astrophys. J.* **859** 46 (2018)
184. Russell D V et al. *Mon. Not. R. Astron. Soc.* **463** 2680 (2016)
185. Cantrell A G et al. *Astrophys. J.* **673** L159 (2008)
186. Cherepashchuk A M et al. *Mon. Not. R. Astron. Soc.* **483** 1067 (2019)
187. Kuulkers E et al. *Astronomer's Telegram* (7647) (2015)
188. Bernardini F et al. *Astrophys. J. Lett.* **818** 5 (2016)
189. Syunyaev R A et al. *Astron. Lett.* **17** 123 (1991); *Pis'ma Astron. Zh.* **17** 291 (1991)
190. Goranskii V P et al. *Astron. Lett.* **22** 371 (1996); *Pis'ma Astron. Zh.* **22** 413 (1996)
191. Cherepashchuk A M *Phys. Usp.* **57** 359 (2014); *Usp. Fiz. Nauk* **184** 387 (2014)
192. Casares J, Jonker P G *Space Sci. Rev.* **183** 223 (2014)
193. Burke M J, Gilfanov M, Sunyaev R *Mon. Not. R. Astron. Soc.* **466** 194 (2017)
194. Titarchuk L, Shaposhnikov N *Astrophys. J.* **626** 298 (2005)
195. Shore S N, Livio M, van den Heuvel E P J *Interacting Binaries* (Berlin: Springer-Verlag, 1994)
196. Warner B *Cataclysmic Variable Stars* (Cambridge: Cambridge Univ. Press, 1995)
197. Andronov I L *J. Phys. Studies* **12** 2902 (2008)
198. Ikhsanov N R *Astron. Astrophys.* **338** 521 (1998)
199. Ikhsanov N R, Beskrovnaya N G *Astron. Rep.* **56** 595 (2012); *Astron. Zh.* **89** 659 (2012)
200. Bezkravnaya N G, Ikhsanov N R, in *Stars: From Collapse to Collapse, Proc. of a Conf., Nizhny Arkhyz, Russia 3–7 October 2016* (ASP Conf. Ser., Vol. 510, Eds Yu Yu Balega) (San Francisco, CA: Astronomical Society of the Pacific, 2017) p. 439
201. Marsh T R et al. *Nature* **537** 374 (2016)
202. Yungelson L R, Thesis for the Degree of Dr. Sci. (Phys.-Math.) (Moscow: Institute of Astronomy, Russian Academy of Sciences, 2011)
203. Solheim J-E *Publ. Astron. Soc. Pacific* **122** 1133 (2010)
204. Iben I, Tutukov A V, Yungelson L P *Astrophys. J. Suppl.* **100** 217 (1995)
205. Iben I, Tutukov A V, Yungelson L P *Astrophys. J. Suppl.* **100** 233 (1995)
206. Rebassa-Mansegras A et al. *Mon. Not. R. Astron. Soc.* **402** 620 (2010)
207. Kurtenkov A et al. *Astronomer's Telegram* (6941) (2015)
208. Kurtenkov A et al. *Astronomer's Telegram* (7150) (2015)
209. Williams S C et al. *Astrophys. J.* **805** L18 (2015)
210. Lipunov V M et al. *Mon. Not. R. Astron. Soc.* **470** 2339 (2017)
211. Tylenda R et al. *Astron. Astrophys.* **528** A114 (2011)
212. Conti P S, in *Observational Tests of Stellar Evolution Theory* (IAU Symp., No. 105, Eds A Maeder, A Rensini) (Dordrecht: D. Reidel, 1984) p. 233
213. Lamers H J G L M, de Loore C W H (Eds) *Instabilities in Luminous Early Type Stars: Proc. of a Workshop in Honour of Professor Cees de Jager on the Occasion of his 65th Birthday, Lunteren, the Netherlands, 21–24 April 1986* (Dordrecht: D. Reidel, 1987)
214. Conti P S, Crowther P A, Leitherer C *From Luminous Hot Stars to Starburst Galaxies* (Cambridge: Cambridge Univ. Press, 2008)
215. Bisnovatyi-Kogan G S, Komberg B V *Sov. Astron.* **18** 217 (1974); *Astron. Zh.* **51** 373 (1974)
216. Burgay M et al. *Nature* **426** 531 (2003)
217. Alpar M A et al. *Nature* **300** 728 (1982)
218. Cherepashchuk A M *Phys. Usp.* **46** 335 (2003); *Usp. Fiz. Nauk* **173** 345 (2003)
219. Cherepashchuk A M *Astron. Rep.* **45** 120 (2001); *Astron. Zh.* **78** 145 (2001)
220. D'Souza M C R et al. *Astrophys. J.* **643** 381 (2006)
221. Milone E F, Leahy D A, Hobill D W (Eds) *Short-Period Binary Stars: Observations, Analyses, and Results* (Berlin: Springer, 2008)
222. Politano M, Weiler K P *Astrophys. J. Lett.* **641** L137 (2006)
223. Bisnovatyi-Kogan G S *Phys. Usp.* **49** 53 (2006); *Usp. Fiz. Nauk* **176** 59 (2006)
224. Tehrani K A et al. *Mon. Not. R. Astron. Soc.* **484** 2692 (2019)
225. Fabrika S et al. *Nature Phys.* **11** 551 (2015)
226. Kaaret P, Feng H, Roberts T P *Annu. Rev. Astron. Astrophys.* **55** 303 (2017)
227. Bachetti M et al. *Nature* **514** 202 (2014)
228. Basko M M, Sunyaev R A *Astron. Astrophys.* **42** 311 (1975)
229. Motch C et al. *Nature* **514** 198 (2014)
230. Heida M et al. *Mon. Not. R. Astron. Soc.* **453** 3510 (2015)
231. Pasham D R, Strohmayer T E, Mushotzky R F *Nature* **513** 74 (2014)
232. Diaz Trigo M et al. *Nature* **504** 260 (2013)
233. Lui J-F et al. *Nature* **528** 108 (2015)
234. Kosec P et al. *Mon. Not. R. Astron. Soc.* **479** 3978 (2018)
235. Troja E et al. *Nature* **547** 425 (2017)
236. Tutukov A V, Cherepashchuk A M *Astron. Rep.* **47** 386 (2003); *Astron. Zh.* **80** 419 (2003)
237. Tutukov A V, Cherepashchuk A M *Astron. Rep.* **48** 39 (2004); *Astron. Zh.* **81** 43 (2004)
238. Lipunov V M, Postnov K A, Prokhorov M E *Mon. Not. R. Astron. Soc.* **288** 245 (1997)
239. Novikov I D, Frolov V P *Phys. Usp.* **44** 291 (2001); *Usp. Fiz. Nauk* **171** 307 (2001)
240. Cherepashchuk A M (Ed.) *Mnogokanal'naya Astronomiya* (Multimessenger Astronomy) (Fryazino: Vek-2, 2019)
241. "Extending the event horizon: multimessenger astronomy (Scientific session of the Physical Sciences Division of the Russian Academy of Sciences, 3 April 2019)" *Phys. Usp.* **62** 1125 (2019); "Razdvigaya gorizont sobytii: mnogokanal'naya astronomiya (Nauchnaya sessiya Otdeleniya fizicheskikh nauk RAN, 3 aprelya 2019 goda)" *Usp. Fiz. Nauk* **189** 1201 (2019)
242. Shakura N I et al. *Phys. Usp.* **62** 1126 (2019); *Usp. Fiz. Nauk* **189** 1202 (2019)
243. Bisikalo D V, Zhilkin A G, Kurbatov E P *Phys. Usp.* **62** 1136 (2019); *Usp. Fiz. Nauk* **189** 1213 (2019)
244. Postnov K A, Kuranov A G, Mitichkin N A *Phys. Usp.* **62** 1153 (2019); *Usp. Fiz. Nauk* **189** 1230 (2019)
245. Fabrika S N, Atapin K E, Vinokurov A S *Phys. Usp.* **62** 1162 (2019); *Usp. Fiz. Nauk* **189** 1240 (2019)