# Wigner's friend paradox: does objective reality not exist? 

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#### Abstract

It is shown that the lack of objective existence of the results of quantum measurements of the state of collapse of the state vector of a remote localized system cannot be proved by an experiment using the reality of violation of Bell's inequality in the Clauser-Horne-Shimony-Holt form. Arguments of a general nature and a specific calculation example confirming this conclusion are also given.


Keywords: Wigner's friend paradox, quantum nondemolition measurements, Bell's inequality, Clauser-Horne-Shimony-Holt inequality, quantum state vector, von Neumann projection postulate, no-communication theorem

## 1. Introduction

Recent years have seen a growth of interest in the elucidation of the ontological status of the wave function and quantum state vector, which are among the main notions of quantum mechanics. Manifestations of so-called quantum nonlocality and a variety of quantum paradoxes have not been given an indisputable and generally accepted consistent interpretation. In this regard, the so-called information interpretation, which was outlined by Niels Bohr [1], discussed in Physics-Uspekhi by V A Fock [2-5] and further developed, for instance, in Refs [6-23], has an increasing number of proponents.

The information interpretation denies the objective existence of the wave function and the state vector and assigns them the status of mathematical abstractions, whose sole role reduces to a calculation tool. This immediately removes a variety of questions about the majority of quantum paradoxes: for instance, there are no problems with the instantaneous collapse of the wave function, since it

[^0]Received 1 March 2020, revised 28 March 2020
Uspekhi Fizicheskikh Nauk 190 (12) 1335-1342 (2020)
Translated by E N Ragozin
does not exist in nature, nor with nonlocality, since all its manifestations yet again are associated with the unusual behavior of the state vector, etc.

The information interpretation is underlain by the hypothesis that the result of quantum system measurement depends on the information which the observer received in the course of experiment, or is potentially possible to obtain. For instance, when it is possible to learn which slit a quantum particle has passed through in the course of a two-slit experiment, the behavior of the quantum system changes radically: interference vanishes. This is a highly useful observation, since it permits predicting the result at a qualitative level without resorting to specific measurements. In comparison with the 'zero' 'obscurantism' of David Mermin with his well-known aphorism "Shut up and calculate," here clearly positive aspects of comprehending what is occurring emerge. However, this algorithm of information interpretation is to be used with great caution, for it may fail and cloud unfulfilled expectations, as shown in the second part of our paper.

So, it is assumed that information, on the one hand, is perceived as a result of observations and, on the other, changes the measured quantum system itself, since the system loses information. This is an idealistic theory, because the reality, according to this theory, is underlain by information rather than matter. In this respect, it is close to the positivism of the Copenhagen interpretation but goes much farther and actually questions the basics of the scientific method of cognition. Why do we cognize something if there is no objective reality?

The Copenhagen interpretation leaves freedom in choosing alternatives: on the one hand, the wave function may be assumed to be a real physical object, which experiences a collapse in the course of measurement, and on the other hand, it may be assumed not to be a real entity but merely an auxiliary mathematical tool, whose sole destiny is to furnish the possibility of calculating probabilities. The information interpretation adopts the latter possibility. In a sense it is consistent with Niels Bohr's viewpoint, which emphasized that the only thing that can be predicted is the results of


Figure 1. Experiment setup [25]. A pair of entangled photons from source $\mathrm{S}_{0}$ are sent to friends of Alice and Bob's (modes $a$ and $b$, respectively). They measure the photon polarizations in the basis of horizontal and vertical polarizations to obtain values $\mathrm{A}_{0}$ and $\mathrm{B}_{0}$. In doing this, they use their own sources of entangled photon pairs $\mathrm{S}_{\mathrm{A}}$ and $\mathrm{S}_{\mathrm{B}}$ (modes $\alpha, \alpha^{\prime}$ and $\beta, \beta^{\prime}$, respectively). Mixing and measuring are performed using polarization beam splitters (PBSs) as well as half-wave (HWPs) and quarter-wave (QWPs) plates. The photons of modes $\alpha^{\prime}$ and $\beta^{\prime}$ are detected by superconducting nanowire detectors (SCNWDs) in the photon count mode, while the photons of modes $\alpha$ and $\beta$ 'remember' the results of the friends' measurements, since the polarization of the pairs $\alpha, \alpha^{\prime}$ and $\beta, \beta^{\prime}$ is strictly correlated. Alice and Bob either repeat the friends' measurement $\mathrm{A}_{0}$ and $\mathrm{B}_{0}$ on removing the $50 \%$ beam splitters (BSs) or, on putting them in place, measure new quantities $\mathrm{A}_{1}$ and $\mathrm{B}_{1}$. Only the realization in which all six photons are recorded simultaneously is treated as informative: $a, b, \alpha, \alpha^{\prime}$, and $\beta, \beta^{\prime}$.
physical experiments, and additional questions therefore pertain to philosophy and not to science. He shared the philosophical conception requiring that science deal only with really measurable things.

As for the collapse, it is indeed the cardinal problem with the quantum theory; it is as if 'imposed' from the outside by von Neumann's projection postulate. Hugh Everett's manyworld interpretation [24] solves this problem radically: the collapse is utterly nonexistent, and all possible alternatives are distributed among parallel universes. Despite its fantastic character, Everett's hypothesis finds many adherents and is second only to the Copenhagen interpretation. True, its obviously positive result is the decoherence theory, which makes it possible to describe the interaction of a quantum system with a macroscopic measuring facility without going beyond the Schrödinger equation. However, this advantage can be taken advantage of even without resorting to the wasteful multiplication of the universes in the geometrical progression. This fact will be noted in the Conclusions.

But let us revert to the information interpretation. One of the arguments advanced in its favor is the so-called Wigner's friend paradox, which was experimentally borne out, according to Proietti et al. [25].

In brief, the heart of the paradox is as follows. Wigner's friend measures a quantum system, which is in the state of quantum superposition. As a result of this measurement, the state vector collapses and a certain result turns out. But Wigner himself does not know about it. Therefore, for him the system is in the superposition state as before. What really happened? Was there a collapse or not? If the friends do not communicate with each other, each believes in his own the 'objective' reality. Of course, the situation is highly simplified here, but this introduction serves the purpose of preparing the reader for an analysis of a rather intricate experiment [25].

## 2. Experiment which reproduces Wigner's friend paradox

A pair of polarization-entangled photons (Fig. 1) arrive at Alice and Bob's friends' places, each at one of the friends' separate laboratories. Each of the friends also has a source of polarization-correlated photons. The friends measure the
polarizations of their incoming photons and send to Alice and Bob each a photon from their generated pairs. Since the polarizations of the photons of each pair are strictly correlated, the photons sent to Alice and Bob carry information about the polarization measured by the friends, the result being encoded by values $\mathrm{A}_{0}$ and $\mathrm{B}_{0}$ of a dichotomic variable equal to +1 or -1 . Therefore, Alice and Bob themselves, who are also at different locations, may either obtain the same result - the $A_{0}$ and $B_{0}$ values of the dichotomic variables equal to +1 or -1 , depending on the polarization state of the recorded photons - or perform, according to the authors, measurements of whether the collapse of the superposition state of entangled photons occurred. To this end, with a small upgrade of the experimental facility - the introduction of additional beam splitters (BSs) into their configurationsAlice and Bob also obtain dichotomic values $A_{1}$ and $B_{1}$ equal to +1 or -1 .

So, in every measurement act there are quite definite values $\mathrm{A}_{0}$ and $\mathrm{B}_{0}$, i.e., collapse has objectively occurred. But Alice and Bob, when recording $A_{1}$ and $B_{1}$, observe quantum interference in this case, which ostensibly testifies to the opposite. How do the authors of Ref. [25] suggest making sure of this? They make up a Clauser-Horne-Shimony-Holt (CHSH) type of Bell's inequality of quantities $\mathrm{A}_{i}$ and $\mathrm{B}_{i}[26]$,

$$
\begin{equation*}
\mathbf{S}=\left|\left\langle\mathrm{A}_{1} \mathrm{~B}_{1}\right\rangle+\left\langle\mathrm{A}_{1} \mathrm{~B}_{0}\right\rangle+\left\langle\mathrm{A}_{0} \mathrm{~B}_{1}\right\rangle-\left\langle\mathrm{A}_{0} \mathrm{~B}_{0}\right\rangle\right| \leqslant 2, \tag{1}
\end{equation*}
$$

and it is violated in the experiment, which testifies to the absence of definite values of quantities $\mathrm{A}_{0}, \mathrm{~A}_{1}, \mathrm{~B}_{0}, \mathrm{~B}_{1}$ simultaneously, although they all are measured and known, including $\mathrm{A}_{0}$ and $\mathrm{B}_{0}$.

So, in the course of one and the same experiment, there are certain measured values $A_{0}$ and $B_{0}$, but the statistical observations of the averages appearing in inequality (1) testify that the measured values $\mathrm{A}_{0}$ and $\mathrm{B}_{0}$ cannot exist simultaneously with $A_{1}$ and $B_{1}$. But they were measured and they exist! Proceeding from this obvious contradiction, the authors of Ref. [25] arrive at the conclusion that objective reality does not exist, for one and the same experiment may not yield mutually exclusive results! Is everything correct here? The fundamental significance of this problem hardly needs emphasizing.

## 3. Features of the Clauser-Horne-Shimony-Holt inequality

To elucidate the implication of CHSH inequality violation, we turn to its simplest derivation [27, 28]. Let all four quantities $\mathrm{A}_{i}, \mathrm{~B}_{i}$ simultaneously have definite values $a_{0}, a_{1}$, $b_{0}, b_{1}$ equal to +1 or -1 . Then, they may make up the following expressions:

$$
\begin{align*}
a_{1} b_{1} & +a_{1} b_{0}+a_{0} b_{1}-a_{0} b_{0}=a_{1}\left(b_{1}+b_{0}\right)+a_{0}\left(b_{1}-b_{0}\right) \\
& =b_{1}\left(a_{1}+a_{0}\right)+b_{0}\left(a_{1}-a_{0}\right)= \pm 2 \tag{2}
\end{align*}
$$

whence there follows inequality (1). In this case, it is significant that the notion of definite values $a_{0}, a_{1}, b_{0}, b_{1}$ in the derivation of Bell's inequalities, including CHSH types, consists not of their determinancy - for they are random but of the simultaneousness of their existence in every act of measurement. Bell's inequalities are violated when only two quantities are measured out of four, or three out of six, or four out of eight, as in the Greenberger-Horne-Zeilinger (GHZ) paradox [29, 30]. This is related to the reason underlying the violation of classical Bell's inequalities: the observables that enter it are described by noncommuting operators in the quantum-mechanical approach [27]. That is why their simultaneous direct measurements are not performed, but the inequalities are made up of the pairs (CHSH), triples, or quadruples (GHZ) of the quantities that enter into them.

Does this mean that inequality (1) may be violated only when all definite values $\mathrm{A}_{i}, \mathrm{~B}_{i}$ are simultaneously absent? Far from it. Two would be sufficient, for instance $A_{1}$ and $B_{1}$, while $A_{0}$ and $B_{0}$ may be completely determined. Inequality (1) may also be violated in this case, as follows from expression (2), since both brackets may be nonzero, more precisely, indefinite.

Generally speaking, relation (2) does not imply any advantage of pair $\mathrm{A}_{0}$ and $\mathrm{B}_{0}$ in relation to pair $\mathrm{A}_{1}$ and $\mathrm{B}_{1}$, for they enter symmetrically. It is significant that the violation of CHSH-type Bell's inequality is not a sufficient condition for the indefiniteness of $\mathrm{A}_{0}$ and $\mathrm{B}_{0}$, which the authors of Ref. [25] selected proceeding from their role in the recorded and measured collapse, since it is precisely $\mathrm{A}_{0}$ and $\mathrm{B}_{0}$ which are that result of Alice and Bob's friends whose objectivity is questioned.

At the first stage of the experiment in Ref. [25], all observers (Alice, Bob, and their friends) measure the same quantities $A_{0}$ and $B_{0}$ to obtain, naturally, the same results. The average $\left\langle\mathrm{A}_{0} \mathrm{~B}_{0}\right\rangle$ is made up of them. Next, Alice and Bob insert additional beam splitters in their meters and move to the measurements of $A_{1}$ and $B_{1}$. In this case, all four quantities $A_{0}, B_{0}, A_{1}$, and $B_{1}$ are measured simultaneously ( $A_{1}$ and $B_{1}$ by Alice and Bob, $A_{0}$ and $B_{0}$ by their friends) and simultaneously acquire definite values $a_{0}, a_{1}, b_{0}$, and $b_{1}$. If the averages that appear in inequality (1) are made up of them, it certainly will not be violated on the strength of relation (2), since the simultaneous existence of definite values $a_{0}, a_{1}, b_{0}$, and $b_{1}$ is the sufficient condition for the fulfillment of inequality (1). Even if the beam splitter is installed only at Alice's or only at Bob's place, three quantities out of the four $\mathrm{A}_{0}, \mathrm{~B}_{0}, \mathrm{~A}_{1}$, and $\mathrm{B}_{1}$ will be simultaneously measured, and, again, on the strength of relation (2), inequality (1) cannot be violated, because one of the brackets in relation (2) will be zero. Why was it violated in Ref. [25]?

If we discard the possibility of some experimental error in Ref. [25], the only explanation of the emerging discordance
may be as follows: the average $\left\langle\mathrm{A}_{0} \mathrm{~B}_{0}\right\rangle$ at the first stage of the experiment is not identical to the average $\left\langle\mathrm{A}_{0} \mathrm{~B}_{0}\right\rangle$ at its subsequent stages. Why can this take place? The point is that only the detection of all six photons is considered to be informative in Ref. [25]. The rest of the realizations are simply discarded. Therefore, on changing the measuring conditions at Alice's and/or Bob's (insertion of BSs), a selection of measurement readings at their friends's occurs, and the average $\left\langle\mathrm{A}_{0} \mathrm{~B}_{0}\right\rangle$ may change.

Does this mean that there is no objectivity of measurements? By no means. Changing the measurer may naturally entail a variation of the measurement results. Objectivity might suffer only in the case of a truly nondemolition measurement, when Alice's and/or Bob's result would have no effect on the results their friends get. However, as shown in the next section, this is hardly possible, either. But first, we adduce additional arguments in favor of the above considerations.

There is another proof of inequality (1) reliant on a softer assumption of the simultaneous existence of not all four values $a_{0}, a_{1}, b_{0}$, and $b_{1}$, but of the existence of all elementary four-dimensional probabilities $P\left(\mathrm{~A}_{0}, \mathrm{~A}_{1}, \mathrm{~B}_{0}, \mathrm{~B}_{1}\right)$ [31]. Specifically, assuming that all probabilities are non-negative and add up to one, proceeding from the normalizing condition for all possible experiment outcome probabilities and writing out the averages appearing in inequality (1), for instance,

$$
\operatorname{Pab}(++)=(++++)+(+++-)+(+-++)+(+-+-),
$$

we ascertain that the sum of all averages entering into (1) is precisely equal to the doubled expansion of unity, i.e., to the doubled sum of all possible $P\left(\mathrm{~A}_{0}, \mathrm{~A}_{1}, \mathrm{~B}_{0}, \mathrm{~B}_{1}\right)$, whence there necessarily follows inequality (1) [31]. But for inequality (1) to be violated, the absence of the existence of not all, but only some, of $P\left(\mathrm{~A}_{0}, \mathrm{~A}_{1}, \mathrm{~B}_{0}, \mathrm{~B}_{1}\right)$ is sufficient.

In fact, if the quantum averages of these elementary probabilities are calculated as applied to the case of measuring the polarization state of an entangled pair of photons, as in the experiment in Ref. [25], only some of them will turn out to be negative [31], much like what takes place in the Wigner distribution.

What do these joint negative probability distributions signify? They link observables, some of which are described by noncommuting operators, for instance $\mathrm{A}_{0}$ and $\mathrm{A}_{1}$ when inequality (1) is violated [31]. That is why their direct measurements, like those of their probability distributions, are impossible. In this sense, similar elementary probabilities are void of operational meaning, as are negative probabilities in general.

How did the authors of Ref. [25] manage to obtain mutually exclusive results? This obviously happened, because there were different averages $\left\langle\mathrm{A}_{0} \mathrm{~B}_{0}\right\rangle$ at different stages of the experiment. Indeed, when additional BSs are inserted and all four observables are simultaneously measured, it is clear that they are all described by commuting operators, and inequality (1) cannot be violated. The violation may occur only when the operator of the observable $\mathrm{A}_{0}$ at the first stage of the experiment does not commute with $A_{1}$ at the next stages. The same with $B_{0}$ and $B_{1}$. If the observables are described by different operators at different experiment stages, it is clear that the observables themselves differ from each other.

From these simple considerations it obviously follows that the violation of inequality (1) does not testify to the
absence of objectively existing $\mathrm{A}_{0}$ and $\mathrm{B}_{0}$ and of the absence of the collapse of the initial quantum state vector. Proving this proposition would call for firmer grounds.

## 4. Some general considerations

Even the very possibility of a nondemolition measurement of the presence or absence of the collapse of the state vector in a remote localized system brings up several hardly resolvable questions. If the collapse occurs instantaneously (this is experimentally borne out: at least the collapse rate in Refs [32, 33] exceeded $c$ by several orders of magnitude), then, it being possible to implement this measurement, I can instantaneously transmit information with a supraluminal (FTL) telegraph, since the presence and absence of the collapse is encoded with dichotomic values corresponding to 1 bit. But this is prevented by the so-called 'no-communication theorem' [34], which is highly general in nature, so that this is hardly possible to overcome, as we see it.

Indeed, suppose that Alice and Bob perform the interference experiment of Ref. [25] before their friends detect the entangled pair of photons, i.e., prior to the collapse. Naturally, they obtain interference, which confirms the absence of collapse. But what if the collapse occurs prior to the measurement by Alice and Bob? In perfect accord with the 'no-communication theorem', nothing should change, otherwise an instantaneous FTL communication channel would be established between them and their friends.

So, even without delving into the intricacies of the experiment or the features of CHSH-type Bell's inequality, one can conclude that denial of the existence of objective reality cannot be proved proceeding from Wigner's friend paradox.

In fairness, it should be noted that treated as informative in the experiment of Ref. [25] is only the simultaneous detection of all six photons, i.e., neither Alice, Bob, nor and their friends, formally speaking, can remotely monitor the localized quantum system of the friends. But Wigner's friend paradox implies precisely this nondemolition measurement, which should be necessarily included when planning such experiments. At the same time, this naturally explains the possibility of obtaining nonmatching $\left\langle\mathrm{A}_{0} \mathrm{~B}_{0}\right\rangle$ at different stages of the experiment in Ref. [25].

These simple considerations may be confirmed by a specific example.

## 5. Unsuccessful attempt to implement FTL communication

Attempts have been repeatedly undertaken to implement FTL communication based on a remote nondemolition measurement of the instantaneous collapse of a state vector. The authors of $[35,36]$ came up with the configuration depicted in Fig. 2, which seemingly permits realizing this possibility. However, more careful calculations suggest the opposite, as shown below (see also Refs [37, 38]). We adduce them here, because they have a direct bearing on the problem of Wigner's nondemolition observation of his friend.

Consider the operating principle of the configuration. In a known time interval, a pair of entangled photons from a biphoton parametric scattering source irradiated by a laser pump are directed to observers A and B, i.e., the laser pump passes through a piezocrystal to give birth to a pair of entangled photons in it. One of them is sent to observer A and the other to observer B. The photons are polarizationentangled.

The observers have Wollaston prisms, which the photons are directed to - its own photon to each prism. It is basically possible to measure the polarization state of these photons using detectors $\mathrm{X}_{b}$ and $\mathrm{Y}_{b}$. However, observer B decides whether to carry out this measurement or not. If they make the measurement, this event is assigned the value 1 , and if not, 0 . The Wollaston prism rotation angles $\alpha$ and $\beta$ are taken to be equal, i.e., they are equally oriented in space relative to each other.

Next, at observer A the photon, which is divided into two channels, arrives at cubic nonlinear media. Sent in the opposite direction is a probe photon P , which is also divided into two channels, these channels being the arms of a MachZehnder interferometer for it. The probe photon $P$ exits the interferometer and is recorded by one of detectors $D_{1}$ or $D_{2}$. The difference scheme $\ominus$ permits measuring the cosine of the phase difference in the interferometer arms with the inclusion of the nonlinear interaction of the entangled and probe photons. After this measurement, observer A records the entangled photon with one of detectors $\mathrm{X}_{a}$ or $\mathrm{Y}_{a}$.


Figure 2. Configuration intended for measuring by observer $A$ the instant of state-vector reduction resulting from the collapsing measurement by observer B. Using the far left detectors $X_{a}$ and $Y_{a}$, observer A can determine which of the detectors $\left(X_{b}\right.$ or $Y_{b}$ ) of observer $B$ actuated if he performed the collapsing measurement. In this case, it is significant that observer $A$ should first perform the measurement by detectors $D_{1}$ or $D_{2}$ and only then by $X_{a}$ or $Y_{a}$.

The physical operating principle of the scheme relies on the fact that the measurement of one of the photons of the pair (by observer B) results in the collapse of the quantum state vector of the entire system of two entangled photons. The collapse occurs instantaneously, and therefore observer A equipped with the corresponding measuring system capable of recording this collapse (or its absence) would learn about the actions of observer B practically instantaneously, no matter how far apart they are.

How does the measuring system of observer A work? First of all, his actions should not entail the collapse of the polarization state superposition of the entangled photon coming to him; otherwise, the information about the actions of observer B would be lost forever on the strength of the 'nocommunication' theorem [34]. His measurement must therefore be a nondemolition one. On the other hand, he should somehow 'probe' the photon. In Ref. [39], it was rigorously proven (albeit as in several other studies, e.g., in Ref. [40]) that prior to the collapse the photon is present simultaneously in two spatially separated channels corresponding to orthogonal polarizations. If these channel are made the arms of a Mach-Zehnder interferometer, the photon is simultaneously present in both arms; otherwise, there would be no experimentally observed interference of single photons. After the collapse due to the fact that the measurement was made by observer B, the photon will be present in only one channel on the strength of the collapse of the state vector of the whole system of two entangled photons.

Next, let us place nonlinear transparent media with cubic Kerr nonlinearity in the interferometer arms. Assume that observer A wants to determine whether the entangled photon coming to him is in two or one interferometer arm without determining, in doing so, which specific arm it is in (otherwise, observer A will make a collapsing measurement earlier than observer B, if B has not done this measurement). So, this task of a nondemolition measurement may seemingly be accomplished with an additional illumination of the interferometer by probing radiation, which experiences a nonlinear cross interaction with the entangled photon in a Kerr medium.

So, what is the result? By performing a nondemolition measurement of the entangled photon, observer A would learn whether observer $B$ has made a collapsing measurement or not, which is equivalent to the transmission of one bit of information from B to A.

## 6. Basic relations

We now consider the formal procedure for describing the system.

Consider a pair of entangled photons correlated in polarization. Their state vector is

$$
\begin{equation*}
|\psi\rangle=\frac{1}{\sqrt{2}}\left(|1\rangle_{x}^{a}|1\rangle_{x}^{b}|0\rangle_{y}^{a}|0\rangle_{y}^{b}+|0\rangle_{x}^{a}|0\rangle_{x}^{b}|1\rangle_{y}^{a}|1\rangle_{y}^{b}\right) . \tag{3}
\end{equation*}
$$

Here, $|1\rangle$ are single-photon Fock states, $|0\rangle$ is a vacuum, superscripts ' $a$ ' and ' $b$ ' pertain to the first and second photons of the entangled pair, and the mutually orthogonal transverse directions $x$ and $y$ define the orthogonal polarization directions. The structure of this state vector is as follows: although the polarization directions $x$ and $y$ of each of the photons, ' $a$ ' or ' $b$ ', of the pair are equally probable, they are strictly correlated with each other, because their polarization planes always coincide in detection. Such states are usually
prepared using parametric light scattering (see Ref. [41] and references therein).

We direct each of the photons of the pair to the Wollaston prism, which divides mutually orthogonal polarizations into two separate channels. It operates, in fact, as a BS, and as a $50 \%$ BS for photons with absolutely random polarization.

Les us now turn to the nondemolition measurement of the first photon. We place cubic nonlinear media, in which phase self-modulation (PSM) occurs, in both output channels after the Wollaston prism. Since the $\hat{n}(t)$ operator in the PSM is time-invariant, the photon number in the PSM is a nondemolition observable and may be measured in a nondemolition way [42]. To the inputs of the cubic nonlinear media (for example, quartz fibers) we also apply, apart from the signals under measurement, weak probe modes $p_{1}, p_{2}$ of equal average intensity. Using their phase difference, we will endeavor to determine whether the first photon $(a)$ is in the superposition state prior to the 'strong' collapsing measurement of the second photon $(b)$ or in one of the channels after the reduction caused by this strong measurement.

For a probing mode, we take the single-photon Fock state $|1\rangle^{p}$. After a $50 \% \mathrm{BS}$, the superposition

$$
\left|\psi_{p}\right\rangle=\frac{1}{\sqrt{2}}\left(|1\rangle_{1}^{p}|0\rangle_{2}^{p}+|0\rangle_{1}^{p}|1\rangle_{2}^{p}\right)
$$

forms. Here, subscripts 1 and 2 relate to the interferometer arms.

After the production of a pair of entangled photons and their separation by polarization prisms at observers A and B, the quantum state of the system as a whole is described by the pure state with the vector

$$
\begin{align*}
& \left|\psi_{a b p}\right\rangle=\frac{1}{2}\left(\left(|1\rangle_{1}^{a}|1\rangle_{1}^{p}|0\rangle_{2}^{a}|0\rangle_{2}^{p}+|1\rangle_{1}^{a}|0\rangle_{1}^{p}|0\rangle_{2}^{a}|1\rangle_{2}^{p}\right)|1\rangle_{x}^{b}|0\rangle_{y}^{b}\right. \\
& \left.\quad+\left(|0\rangle_{1}^{a}|1\rangle_{1}^{p}|1\rangle_{2}^{a}|0\rangle_{2}^{p}+|0\rangle_{1}^{a}|0\rangle_{1}^{p}|1\rangle_{2}^{a}|1\rangle_{2}^{p}\right)|0\rangle_{x}^{b}|1\rangle_{y}^{b}\right) . \tag{4}
\end{align*}
$$

The effect of nonlinearity $\chi$, which is defined by operator $\hat{U}=\exp \left(-i \bar{\chi}_{a p} \hat{n}_{a} \hat{n}_{p} / 2\right)$, in the case of cross interaction (see, e.g., Ref. [40] and references therein) gives

$$
\begin{align*}
& \left|\psi_{a b p}^{\prime}\right\rangle=\frac{1}{2}\left\{\left[|1\rangle_{1}^{a}|1\rangle_{1}^{p}|0\rangle_{2}^{a}|0\rangle_{2}^{p} \exp \left(-\frac{\mathrm{i} \bar{\chi}_{a p 1}}{2}\right)\right.\right. \\
& \left.\quad+|1\rangle_{1}^{a}|0\rangle_{1}^{p}|0\rangle_{2}^{a}|1\rangle_{2}^{p}\right]|1\rangle_{x}^{b}|0\rangle_{y}^{b}+\left[|0\rangle_{1}^{a}|1\rangle_{1}^{p}|1\rangle_{2}^{a}|0\rangle_{2}^{p}\right. \\
& \left.\left.\quad+|0\rangle_{1}^{a}|0\rangle_{1}^{p}|1\rangle_{2}^{a}|1\rangle_{2}^{p} \exp \left(-\frac{\mathrm{i} \bar{\chi}_{a p 2}}{2}\right)\right]|0\rangle_{x}^{b}|1\rangle_{y}^{b}\right\} \tag{5}
\end{align*}
$$

In the Heisenberg representation, the action of the beam splitter, located in front of the detectors of the difference scheme, is described as $\hat{a}_{p}^{\prime}=\left(\hat{a}_{1}^{p} \pm \hat{a}_{2}^{p}\right) / \sqrt{2}$. Here, the plus sign corresponds to the first detector $\mathrm{D}_{1}$ and the minus sign to the second one $\mathrm{D}_{2}$. We then obtain the average number of photocounts of detector $D_{1}$ :

$$
\frac{1}{4}\left(2+\cos \frac{\bar{\chi}_{a p 1}}{2}+\cos \frac{\bar{\chi}_{a p 2}}{2}\right)
$$

and of $\mathrm{D}_{2}$ :

$$
\frac{1}{4}\left(2-\cos \frac{\bar{\chi}_{a p 1}}{2}-\cos \frac{\bar{\chi}_{a p 2}}{2}\right) .
$$

In the Schrödinger approximation, the quantum state of the field at the output of the Mach-Zehnder interferometer after the output beam splitter is given by the vector

$$
\begin{align*}
& \left|\psi_{a b p}^{\prime \prime}\right\rangle=\frac{1}{2 \sqrt{2}}\left\{\left[|1\rangle_{1}^{a}|0\rangle_{2}^{a}|1\rangle_{x}^{b}|0\rangle_{y}^{b}\left(\exp \left(-\frac{\mathrm{i} \bar{\chi}_{a p 1}}{2}\right)+1\right)\right.\right. \\
& \left.\quad+|0\rangle_{1}^{a}|1\rangle_{2}^{a}|0\rangle_{x}^{b}|1\rangle_{y}^{b}\left(1+\exp \left(-\frac{\mathrm{i} \bar{\chi}_{a p 2}}{2}\right)\right)\right]|1\rangle_{1}^{d}|0\rangle_{2}^{d} \\
& \quad+\left[|1\rangle_{1}^{a}|0\rangle_{2}^{a}|1\rangle_{x}^{b}|0\rangle_{y}^{b}\left(\exp \left(-\frac{\mathrm{i} \bar{\chi}_{a p 1}}{2}\right)-1\right)\right. \\
& \left.\left.\quad+|0\rangle_{1}^{a}|1\rangle_{2}^{a}|0\rangle_{x}^{b}|1\rangle_{y}^{b}\left(1-\exp \left(-\frac{\mathrm{i} \bar{\chi}_{a p 2}}{2}\right)\right)\right]|0\rangle_{1}^{d}|1\rangle_{2}^{d}\right\} . \tag{6}
\end{align*}
$$

Here, $|1\rangle_{1}^{d}|0\rangle_{2}^{d}$ and $|0\rangle_{1}^{d}|1\rangle_{2}^{d}$ are the states at the inputs of the detectors located in front of the difference scheme in Fig. 2, $|1\rangle_{1}^{d}|0\rangle_{2}^{d}$ corresponding to the actuation of the first detector $\mathrm{D}_{1}$, and $|0\rangle_{1}^{d}|1\rangle_{2}^{d}$ to the actuation of the second one $\mathrm{D}_{2}$. In the actuation of one of them, i.e., in the reduction of expression (6) to its two upper or two lower lines, one can see that the superposition $\left|\psi_{b}\right\rangle=(1 / \sqrt{2})\left(|1\rangle_{x}^{b}|0\rangle_{y}^{b}+|0\rangle_{x}^{b}|1\rangle_{y}^{b}\right)$ does not reduce to one of the components of this state, with the result that, in the general case, both $|1\rangle_{x}^{b}|0\rangle_{y}^{b}$, and $|0\rangle_{x}^{b}|1\rangle_{y}^{b}$ are present in each line of expression (6). So, this measurement is truly a nondemolition one. In this case, it is important that the numerical coefficients in expression (6) not turn out to be zero. Best of all, they should be equal in modulus. Then, the measurement performed by detectors $\mathrm{D}_{1}$ or $\mathrm{D}_{2}$ will be completely free from the information about which of the channels the photon of the entangled pair is present in.

The previous results with cosines also follow readily from expression (6).

What will happen when observer B makes the collapsing measurement of the polarization state? State $\left|\psi_{a b p}^{\prime \prime}\right\rangle$ reduces to either the first and third term of expression (6) or to the second and fourth. The detector actuation probabilities will turn out to be equal to either $(1 / 2)\left[1 \pm \cos \left(\bar{\chi}_{a p 1} / 2\right)\right]$ or $(1 / 2)\left[1 \pm \cos \left(\bar{\chi}_{a p 2} / 2\right)\right]$, where, as above, $\pm$ corresponds to either the first or the second detector, i.e., the upper sign corresponds to the first $\mathrm{D}_{1}$, and the lower one to the second $\mathrm{D}_{2}$. So, the pure state transforms into a mixed one with equal probabilities $(1 / 2)$ of both outcomes. This signifies that, proceeding from the measurement data, there is no way to distinguish the pure state $\left|\psi_{a b p}^{\prime \prime}\right\rangle$ from the mixed one with the probability $(1 / 2)\left[1 \pm \cos \left(\bar{\chi}_{a p 1} / 2\right)\right]$ or $(1 / 2)\left[1 \pm \cos \left(\bar{\chi}_{a p 2} / 2\right)\right]$ after observer B makes a 'strong' collapsing measurement, since averaging these probabilities, i.e., their summation with a weight $1 / 2$, yields the same probability as in the absence of the collapsing measurement by observer B.

We consider the last possibility that might lead to the desired goal. We make one more subsequent measurement by observer A using additionally introduced detectors $X_{a}$ and $\mathrm{Y}_{a}$, which are located in the leftmost part of Fig. 2; he will then be able to determine which of the detectors of observer B ( $\mathrm{X}_{b}$ or $\mathrm{Y}_{b}$ ) actuates when he performs a collapsing measurement. Preliminarily, it is required to set up nonlinear phase delays such that the cosines $\cos \left(\bar{\chi}_{a p 1} / 2\right)$ and $\cos \left(\bar{\chi}_{a p 2} / 2\right)$ differ from each other, but the numerical coefficients in all four terms in expression (6) are equal in modulus. This is achieved for $\cos \left(\bar{\chi}_{a p 1} / 2\right)=+\sqrt{2} / 2, \cos \left(\bar{\chi}_{a p 2} / 2\right)=-\sqrt{2} / 2$ (or vice versa). In this case, the actuations of detectors $D_{1}$ and $D_{2}$, which are located in front of the difference scheme in the lower part of Fig. 2, are probabilistically related to the actuations of detectors $\mathrm{X}_{a}$ and $\mathrm{Y}_{a}$, provided, of course,
observer B has preliminarily made the collapsing measurement. And if not, these actuations will be random. So, if detector actuations do not correspond to the probability law $(1 / 2)\left[1 \pm \cos \left(\bar{\chi}_{a p 1} / 2\right)\right]$ when an additional detector $\left(\mathrm{X}_{a}\right)$ of observer A actuates or $(1 / 2)\left[1 \pm \cos \left(\bar{\chi}_{a p 2} / 2\right)\right]$ when the other one $\left(\mathrm{Y}_{a}\right)$ does, observer A may conclude that observer B has not made the collapsing measurement. But, according to the above calculations, the probability laws are the same in both cases, although the presence and absence of collapse call for different calculation algorithms.

This example demonstrates how the seemingly well grounded scheme of the nondemolition measurement of the state vector collapse of a localized system fails on the strength of the 'no-communication' theorem. If so, the nondemolition measurement by Alice and Bob of the quantum state of their friends - whether or not they had a collapse - is hardly possible.

Also of interest is the fact that the erroneous conclusion about the feasibility of FTL communication resulted from considerations based on precisely the informational interpretation. In fact, when observer A does not find out which channel the photon is in after the polarization prism, the absence of this information, it would seem, gives him the capability to nondemolitionally measure whether observer B had a collapse or not, just as the absence of information of which slit a quantum particle went through does not ruin the interference in a double-slit experiment. But in this, more complicated, case, this simple qualitative model yields an erroneous prediction. On the one hand, this is a warning about using a certain amount of caution, and, on the other hand, this somewhat discredits the interpretation itself or at least limits its applicability.

## 7. Conclusions

What conclusion can be drawn from the above reasoning? Does it prove the inconsistency of the informational interpretation of quantum mechanics? Not at all. Should attempts to prove the absence of objective reality as applied to the wave function and state vector meet with success, all the remaining interpretations should be sent to the archive. However, as follows from the foregoing, that would be premature. The informational interpretation is just one hypothesis along with other consistent conceptions [43-49].

But is Wigner's friend paradox so insoluble in the framework of the traditional quantum-mechanical description? I believe that it does not invite some radically new approach or a cardinal change in the notions of the objectivity of quantum processes and measurement results. In fact, Wigner's friend has made a collapsing measurement and quite justly describes it using von Neumann's projection postulate. Wigner himself considers the friend's entire experimental facility, including his measurer, a single quantum system. In this case, there is no need to subject the measurement procedure to the action of projection postulate, the procedure being simply considered in the framework of decoherence effect. As pointed out in Refs [48, 49], there are two approaches to the description of decoherence.

The first is that off-diagonal terms vanish from the density matrix in a quantum measurement. This is mathematically described by the operator of projection on one of the basis states (collapse of the wave function). This situation is not described by the Schrödinger equation; the process is nonlinear.

In the second approach, introduced into consideration prior to the measurement are two independent superposition states - of the system and the environment, which plays the role of the instrument. Let the system be in the superposition of states $\left|a^{\prime}\right\rangle$ and $\left|a^{\prime \prime}\right\rangle$, and the environment (the instrument) of states $\left|b^{\prime}\right\rangle$ and $\left|b^{\prime \prime}\right\rangle$. The result of measurement - the state of the compound system, which comprises the initial system and its environment - is now described by the product of these vectors, i.e., by the linear operation corresponding to the evolution in accordance with the Schrödinger equation. Quantum correlation emerges between the previously independent subsystems, because the state of the initial system can be judged by the instrument state. By constructing the density matrix of the compound system after the measurement and taking the trace over all degrees of freedom of the instrument, we obtain the density matrix of the system being measured. In this matrix, the off-diagonal matrix elements are nonzero. However, the following reasoning shows that they are actually negligible.

A necessary property of a measuring instrument is a large (macroscopic) number of degrees of freedom, as well as the 'macroscopic distinguishability' of the states corresponding to different measurement results. Consequently, the corresponding wave functions depend on very many variables and differ by their functional dependences on a large number of these variables, the scalar product of these wave functions being hardly different from zero (to be more precise, it is exponentially small, with the exponent of the order of minus $10^{23}$ ), since the scalar product is the integral over a huge
(macroscopic) number of variables. In this case, even if the integral over each variable yields a factor only slightly smaller than unity, the full multiple integral will be close to zero. Therefore, we have the equality $\left\langle b^{\prime} \mid b^{\prime \prime}\right\rangle=\left\langle b^{\prime \prime} \mid b^{\prime}\right\rangle=0$ with a high degree of precision, with the result that the off-diagonal terms of the density matrix practically vanish. This is how decoherence, or superselection, occurs during measurement. The essence of decoherence was understood quite a long time ago. Wojciech Zurek deepened the analysis of the phenomenon and gave it a good name: environment-induced superselection [49] (see also Refs [8, 9, 15]).

Considering the corresponding models suggests that decoherence occurs (i.e., the off-diagonal terms vanish) exponentially fast. This proceeds as more and more degrees of freedom of the environment become entangled with the system under measurement. Therefore, one and the same measurement result is simply described in different ways by Wigner and his friend, which exhausts the paradoxicality of the situation.

We also note that FTL communication is impossible in the framework of special relativity; otherwise, I might write myself a letter to the past so as to avoid fatal errors [50]. I express my appreciation to A V Kaminsky, who drew my attention to this fact.

I am also grateful to M B Menskii, who actively favored publications on this subject in Physics-Uspekhi and showed interest in and benevolent attention to my papers and presentations. This paper is devoted to the blessed memory of Mikhail Borisovich Menskii.


Sir Roger Penrose (2020 Nobel Laureate in Physics) and Mikhail Borisovich Menskii, member of the Editorial Board of and author in the journal Physics-Uspekhi, after the seminar "Do we need new physics to explain the brain and consciousness?"
(1 April 2013, Institute of Philosophy, Russian Academy of Sciences, Moscow.)

This work was supported by the Russian Foundation for Basic Research (grant no. 18-01-00598A).

## References

1. Bohr N Phys. Rev. 48696 (1935); Usp. Fiz. Nauk 16446 (1936)
2. Fock V A, Einstein A, Podolsky B, Rosen N, Bohr N Usp. Fiz. Nauk 16436 (1936)
3. Fock V A Usp. Fiz. Nauk 453 (1951)
4. Fock V A Usp. Fiz. Nauk 62461 (1957)
5. Fock V A Sov. Phys. Usp. 8628 (1966); Usp. Fiz. Nauk 86363 (1965)
6. Klyshko D N Sov. Phys. Usp. 3174 (1988); Usp. Fiz. Nauk 154133 (1988)
7. Klyshko D N Sov. Phys. Usp. 32555 (1989); Usp. Fiz. Nauk 158327 (1989)
8. Kadomtsev B B Phys. Usp. 37425 (1994); Usp. Fiz. Nauk 164449 (1994)
9. Kadomtsev B B Dinamika i Informatsiya (Dynamics and Information) 2nd ed. (Moscow: Redaktsiya Zhurnala "Uspekhi Fizicheskikh Nauk", 1999)
10. Kadomtsev B B, Kadomtsev M B Phys. Scripta 50243 (1994)
11. Kadomtsev B B, Kadomtsev M B Phys. Usp. 39609 (1996); Usp. Fiz. Nauk 166651 (1996)
12. Kadomtsev B B Phys. Lett. A 210371 (1996)
13. Sokolov Yu L Phys. Usp. 42481 (1999); Usp. Fiz. Nauk 169559 (1999)
14. Menskii M B Phys. Usp. 43585 (2000); Usp. Fiz. Nauk 170631 (2000)
15. Kadomtsev B B Phys. Usp. 461183 (2003); Usp. Fiz. Nauk 1731221 (2003)
16. Vyatchanin S P, Khalili F Ya Phys. Usp. 47705 (2004); Usp. Fiz. Nauk 174765 (2004)
17. Menskii M B Phys. Usp. 50397 (2007); Usp. Fiz. Nauk 177415 (2007)
18. Khalili F Ya Phys. Usp. 59968 (2016); Usp. Fiz. Nauk 1861059 (2016)
19. Zheltikov A M Phys. Usp. 611016 (2018); Usp. Fiz. Nauk 1881119 (2018)
20. Zheltikov A M, Scully M O Phys. Usp. 63698 (2020); Usp. Fiz. Nauk 190749 (2020)
21. Pronskikh V S Phys. Usp. 63192 (2020); Usp. Fiz. Nauk 190211 (2020)
22. Brukner Č, Zeilinger A Acta Phys. Slovaca 49647 (1999)
23. Brukner C , Zeilinger A Phys. Rev. Lett. 833354 (1999)
24. Everett H (III) Rev. Mod. Phys. 29454 (1957)
25. Proietti M et al. Sci. Adv. 5 eaaw9832 (2019)
26. Clauser J F et al Phys. Rev. Lett. 23880 (1969)
27. Belinskii A V, Klyshko D N Phys. Usp. 36653 (1993); Usp. Fiz. Nauk 163 (8) 1 (1993)
28. Belinsky A V, Klyshko D N Phys. Lett. A 176415 (1993)
29. Greenberger D M, Horne M A, Zeilinger A "Going beyond Bell's theorem", in Bell's Theorem, Quantum Theory and Conceptions of the Universe (Fundamental Theories of Physics, Vol. 37, Ed. M Kafatos) (Dordrecht: Springer, 1989) p. 69
30. Greenberger D M et al Am. J. Phys. 581131 (1990)
31. Belinskii A V Phys. Usp. 37413 (1994); Usp. Fiz. Nauk 164435 (1994)
32. Gisin N Quantum Chance. Nonlocality, Teleportation and other Quantum Marvels (Cham: Springer Intern. Publ., 2014); Translated into Russian: Kvantovaya Sluchainost'. Nelokal'nost', Teleportatsiya i Drugie Kvantovye Chudesa (Moscow: ANF, 2016)
33. Salart D et al. Nature 454861 (2008)
34. Peres A, Terno D R Rev. Mod. Phys. 7693 (2004)
35. Belinsky A V, Zhukovskiy A K Moscow Univ. Phys. Bull. 71482 (2016); Vestn. Mosk. Univ. Ser. 3 Fiz. Astron. (5) 21 (2016)
36. Belinsky A V Elektron. Tekh. Ser. 3 Mikroelektron. (3) 94 (2018)
37. Belinsky A V Moscow Univ. Phys. Bull. 72638 (2017); Vestn. Mosk. Univ. Ser. 3 Fiz. Astron. (6) 127 (2017)
38. Belinsky A V Quantum Electron. 50469 (2020); Kvantovaya Elektron. 50469 (2020)
39. Belinsky A V Moscow Univ. Phys. Bull. 73351 (2018); Vestn. Mosk. Univ. Ser. 3 Fiz. Astron. (4) 12 (2018)
40. Belinsky A V Phys. Usp. 621268 (2019); Usp. Fiz. Nauk 1891352 (2019)
41. Klyshko D N Photons and Nonlinear Optics (New York: Gordon and Breach, 1988); Translated from Russian: Fotony i Nelineinaya Optika (Moscow: Nauka, 1980)
42. Belinsky A V Kvantovye Izmereniya (Quantum Measurements) (Moscow: BINOM. Lab. Znanii, 2015)
43. Frauchiger D, Renner R Nat. Commun. 93711 (2018)
44. Lazarovici D, Hubert M Sci. Rep. 9470 (2019)
45. Sudbery A Found. Phys. 47658 (2017)
46. Pusey M F Nat. Phys. 14977 (2018)
47. Khrennikov A Yu Theor. Math. Phys. 1571448 (2008); Teor. Matem. Fiz. 157 (1) 99 (2008)
48. Menskii M B Phys. Usp. 41923 (1998); Usp. Fiz. Nauk 1681017 (1998)
49. Zurek W H Los Alamos Science (27) 1 (2002)
50. Penrose R The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics (Oxford: Oxford Univ. Press, 1989); Translated into Russian: Novyi Um Korolya. O Komp'yuterakh, Myshlenii i Zakonakh Fiziki 4th ed. (Moscow: URSS, 2011); WikiReading: "Novyi Um Korolya. O Komp’yuterakh, Myshlenii i Zakonakh Fiziki", https://fil.wikireading.ru/86092

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