 

*METHODOLOGICAL NOTES*

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# On David Bohm's 'pilot-wave' concept

A V Belinsky

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## Contents



**Abstract. We consider the interpretation of quantum mechanics on the basis of the so-called 'pilot-wave' concept from the point of view of its adequacy in the light of both already-realized and possible and gedanken experiments, including those that involve photons. It is shown that this concept, despite undoubtedly being useful, can hardly ensure compliance of quantum-theory predictions with the postulate that particle coordinates and velocities objectively exist, while splitting the wave function into empty and nonempty wave packets seems to contradict to the results of feasible experiments and their interpretations.**

**Keywords:** orthodox interpretation, hidden variables, determinism, quantum superposition, quantum statistics

## 1. Introduction

D Bohm's 'pilot-wave' concept does not have many followers. Among specialists, however, it is traditionally believed that it is fundamentally irrefutable, since it relies on the Schrödinger equation, and its inferences are therefore in perfect agreement with experimental data. Furthermore, this interpretation of quantum mechanics was enthusiastically adopted by such a giant of 'quantum thought' as John Bell [1]:

"But why then had Born not told me of this 'pilot wave'? If only to point out what was wrong with it? Why did von Neumann not consider it? More extraordinarily, why did people go on producing 'impossibility' proofs, after 1952, and as recently as 1978? When even Pauli, Rosenfeld, and Heisenberg could produce no more devastating criticism of Bohm's version than to brand it as 'metaphysical' and 'ideological'? Why is the pilot wave picture ignored in text books? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism are not forced on us by experimental facts, but by deliberate theoretical choice?"

So, the interpretation clearly deserves attention, although certain doubt in its irrefutability still persists. To start with, we consider the main premises and propositions of this theory.

**A V Belinsky** Lomonosov Moscow State University, Faculty of Physics, Leninskie gory 1, str. 2, 119991 Moscow, Russian Federation
E-mail: belinsky@inbox.ru

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## 2. Schrödinger equation and Born's probabilistic interpretation

In 1926, Erwin Schrödinger, while developing Louis de Broglie's idea that quantum particles possessed wave properties [2], formulated the wave equation [3] for describing a quantum-mechanical system:

$$i\hbar \frac{\partial \psi}{\partial t} = \left( -\frac{\hbar^2}{2m} \Delta + V \right) \psi \,, \tag{1}$$

where $\psi$ is the wave function, $\hbar$ is the Planck constant, and $V(\mathbf{x}, t)$ is the potential field acting on the particle of mass $m$.

"Recognizing that [this] equation has the structure of a diffusion equation with an imaginary diffusion coefficient, Schrödinger relaxed his original requirement concerning the reality of $\psi$ and admitted complex-valued functions for what he called the mechanical field scalar $\psi$. ...Schrödinger concluded his paper with a discussion on the physical significance of $\psi$. He interpreted $\psi\psi^*$ as a weight function in configuration space that accounts for the electrodynamical fluctuations of the space density of the electric charges. He declared: The $\psi$ function has to do no more and no less than to offer us a survey and mastery over these fluctuations by a single differential equation. It has repeatedly been pointed out that the $\psi$ function itself cannot and may not in general be interpreted directly in terms of three-dimensional space..." [4].

The summer of 1926 saw the advent of the famous probabilistic interpretation by Max Born, which brought him the Nobel Prize:

"For Born probability, as far as it was related to the wave function, was not merely a mathematical fiction but something endowed with physical reality, for it evolved in time and propagated in space in accordance with Schrödinger's equation. It differed, however, from ordinary physical agents in one fundamental aspect: it did not transmit energy or momentum. Since in classical physics, whether Newtonian mechanics or Maxwellian electrodynamics, only an entity that transfers energy or momentum (or both) is regarded as physically 'real,' the ontological status of $\psi$ had to be considered as something intermediate... . Laws of nature, as Born and Heisenberg contended from now on, determined not the occurrence of an event, but the probability of the occurrence... .

Having interpreted $\psi$ as a probability wave in the sense just explained but realizing that $\psi$ can be expanded in terms of a complete orthonormal set of eigenfunctions... , Born had to ask himself what meaning to ascribe to the $c_n$ [coefficients of this expansion]? ... [It] suggested to Born that the integral $\int |\psi(q)|^2 \, dq$ has to be regarded as the number of particles and $|c_n|^2$ as the statistical frequency of the occurrence of the state characterized by the index $n$. To justify this assumption Born calculated 'the expectation value of the energy and obtained [for it] the [correct] energy eigenvalue'" [4].

## 3. Bohm's model

In 1952, David Bohm published two papers [5, 6], in which he proposed a nontrivial approach to further development of quantum mechanics. From a purely formal point of view, his proposition involved the passage from a single equation for the complex wave function to two equations for two real quantities: the amplitude $R(\mathbf{x}, t)$ and phase $S(\mathbf{x}, t)$ of the wave function. We denote

$$\psi = R \exp\left(\frac{\mathrm{i}S}{\hbar}\right) . \tag{2}$$

In the case of one quantum particle, we denote $R^2 = \rho$, and use expression (2) and the Schrödinger equation to obtain

$$\frac{\partial \rho}{\partial t} + \nabla\left(\rho \frac{\nabla S}{m}\right) = 0 , \tag{3}$$

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0 , \tag{4}$$

where $Q = \hbar^2 \Delta R/(2mR)$ is the so-called quantum potential.

In classical mechanics, function $S$ is interpreted as action, its time derivative $\partial S/\partial t$ as energy, and $\nabla S/m$ as velocity. Relations (3) and (4) may be treated as the continuity and energy balance equations, but in Eqn (4), a radically new term appears — *quantum potential Q*.

In the case of $N$ particles, it is possible to introduce the wave function

$$\psi = R(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N, t) \exp\left[\frac{\mathrm{i}S(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N, t)}{\hbar}\right] \tag{5}$$

and define the $3N$-dimensional trajectory in the configuration space, which describes the behavior of each particle in the system. The $i$th particle velocity is

$$\mathbf{v}_i = \frac{\nabla_i S(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N, t)}{m} . \tag{6}$$

As in the case of one particle, the quantum potential is defined with the use of quantity $R$:

$$U(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N) = -\frac{\hbar^2}{2mR} \sum_{k=1}^{N} \Delta_k R(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N) , \tag{7}$$

$\rho(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N, t) = R^2$ being equal to the density of representing points $(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N)$ in our $3N$-dimensional ensemble.

We emphasize again that the presence of the quantum potential distinguishes the quantum description from the classical one, in which there is no analogue to this term. In the general case, the quantum potential underlies the so-called entanglement between particles, i.e., the fact that individual trajectories, which have physical meaning in Bohm's interpretation, are not independent of each other and are not described by separate independent wave functions. It is highly significant that the quantum potential in the configuration space varies, as is commonly assumed, instantaneously as the wave function changes, and this mechanism is responsible for nonlocal correlations, which are highly characteristic of quantum mechanics. A human is prone to perceive such instantaneous changes rather as supraluminal information exchange [7].

The quantum potential nonlocality thesis is usually not emphasized anywhere. Expressed are only indirect considerations, which rely on the fact that, owing to the quantum potential, the coordinates of one particle of a quantum system turn out to be dependent on the coordinates of all other particles of the system. In particular, therefore, not only does the wave function control the particle motion, but also the particle exerts back action on the wave function of the system [8]. However, from this it does not logically follow that the nonlocal correlations are transmitted instantaneously and not, say, at the speed of light in a vacuum. It seems, however, that the violation of Bell's inequalities recorded between very distant observers [9] proves both.

On the other hand, the nonlocality of quantum correlations (irrespective of Bohm's model) is presently a commonly accepted fact (see, for instance, Refs [10–13]), which nevertheless seems to be paradoxical. In 2017, in Refs [14, 15] an attempt was made to explain this paradox by an effect of the relativity theory that is very close to the well-known twin paradox.

## 4. Formulation of dualism in Bohm's model

We dwell on two possible notions of the wave–particle dualism. Two approaches are possible [16]:

• "*A wave OR a particle*": Heisenberg, Pauli, Dirac, and many others believed that, depending on the experimental situation, one or the other approach should be chosen to describe the behavior of a quantum system. Electrons are associated with probability amplitudes. The corpuscular nature of the electron manifests itself when we measure its coordinate. According to Bohr, an object cannot be simultaneously both a wave and a particle (this is referred to as the

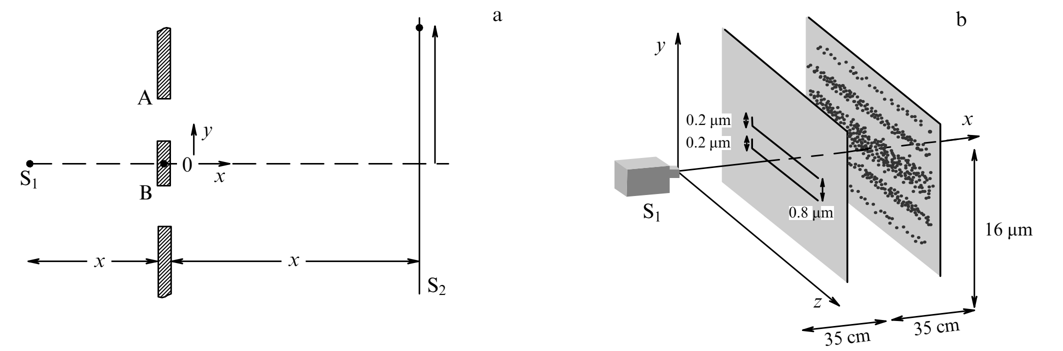


**Figure 1.** Configuration (a) and results (b) of the double-slit interference experiment [18, 19].

orthodox interpretation of quantum mechanics and the principle of complementarity).[1]

• "*A wave AND a particle*": de Broglie, and Bohm after him, believed that the notions of wave and particle merge at atomic scale lengths, where the 'pilot wave' guides the electron trajectory. Two objects, not one of them, simultaneously exist.

The difference between these two approaches may be easily seen by the example of the interpretation of the double-slit experiment, in which a low-intensity electron beam (so that the electrons are injected one at a time) is directed to an opaque surface with two slits. The discrete traces of electron hits are recorded on a detecting screen located an the other side of the surface. Even if it is assumed that the traces on the screen correspond to particles, they group into interference fringes characteristic of waves. Therefore, both wave (interference) and corpuscular (the points on the screen) behaviors are demonstrated.

According to the 'wave and particle' de Broglier–Bohm approach, the wave function (whose modulus gives the probability density that an electron is at some coordinate irrespective of the measurement process) passes through both slits. At the same time, a well-defined trajectory is associated with the electron. But this trajectory passes through only one of the slits. The final particle position on the detecting screen and the slit that the particle passes through are defined by the initial state of the particle. The initial state is not controlled by the experimenter, which gives rise to the effect of randomness of the detected image. The wave function controls the particle in such a way that it rarefies the particle traces in the destructive interference domain and concentrates them in the constructive interference domain, giving rise to interference fringes on the detecting screen. In this connection, Bell wrote [1]: "This idea seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored."

The Young double-slit experiment has long been a crucial experiment for interpreting the wave–particle dualism. This simple effect exhibits the two properties of a quantum phenomenon: the wave nature at the microscopic level related to the phenomenon of wave function interference and the corpuscular nature at the microscopic level related to the traces of collisions on the screen. Double-slit interference experiments have been carried out with massive objects like electrons, neutrons, cold neutrons, atoms, and recently also with coherent ensembles of ultracold atoms and with mesoscopic single quantum objects.

Correct numerical simulations of the double-slit experiment based on Bohm's interpretation were first carried out successfully in Ref. [18] and more recently in Ref. [19]. The quantum potential was calculated for the ordinary double-slit configuration, which comprised the electron source $S_1$, two slits, A and B, and the screen $S_2$. In the coordinate system with the origin at point 0, which is shown in Fig. 1a, the slit centers have coordinates $(0, y)$ and $(0, -y)$.

The slits are sufficiently long (infinite) along the $z$-axis (normal to the plane of the drawing), so that there is no diffraction along the $z$-axis. That is why only the wave function along the $y$-axis was considered in the simulation; the $x$ variable was treated classically as $x = vt$. The electrons emitted by the electron gun were represented by the same initial wave function

$$\psi^{0}(y) = (2\pi\sigma_0^2)^{-1/4} \exp\left(-\frac{y^2}{4\sigma_0^2}\right) \tag{8}$$

with a standard deviation $\sigma = 3$ μm. The method of continual integrals along Feynman trajectories permitted calculating the time-dependent wave function. The wave function in front of the slits is expressed as

$$\psi(y, t) = \left(2\pi s_0^{\,2}(t)\right)^{-1/4} \exp\left[-\frac{(y - vt)^2}{4\sigma_0 s_0(t)}\right], \tag{9}$$

where

$$s_0(t) = \sigma_0\left(1 + \frac{\mathrm{i}\hbar t}{2m\sigma_0^{\,2}}\right). \tag{10}$$

Behind the slits, the wave function is the sum of the wave functions of slits A and B:

$$\psi(y, t) = \psi_{\mathrm{A}}(y, t) + \psi_{\mathrm{B}}(y, t), \tag{11}$$

where

$$\psi_{\mathrm{A}}(y, t) = \int_{\mathrm{A}} \left(\frac{m}{2\mathrm{i}\hbar t_1}\right)^2 \exp\left[\mathrm{i}m\,\frac{(y - y_{\mathrm{a}})^2}{2\hbar\,(t - t_1)}\right] \psi(y_{\mathrm{a}}, t_1)\,\mathrm{d}y_{\mathrm{a}}, \tag{12}$$

$$\psi_{\mathrm{B}}(y, t) = \int_{\mathrm{B}} \left[\frac{m}{2\mathrm{i}\hbar t_1}\right]^2 \exp\left[\mathrm{i}m\,\frac{(y - y_{\mathrm{b}})^2}{2\hbar\,(t - t_1)}\right] \psi(y_{\mathrm{b}}, t_1)\,\mathrm{d}y_{\mathrm{b}}. \tag{13}$$

[1] An in-depth theoretical analysis of the complementarity relationship leads to the inequality $V^2 + D^2 \leqslant 1$, which limits from above the maximal values of the simultaneously determined parameters: the interference fringe visibility $V$ and the path distinguishability $D$ [17]. Evidently, the cases $V = 1$, $D = 0$ and $V = 0$, $D = 1$ are the limiting ones.

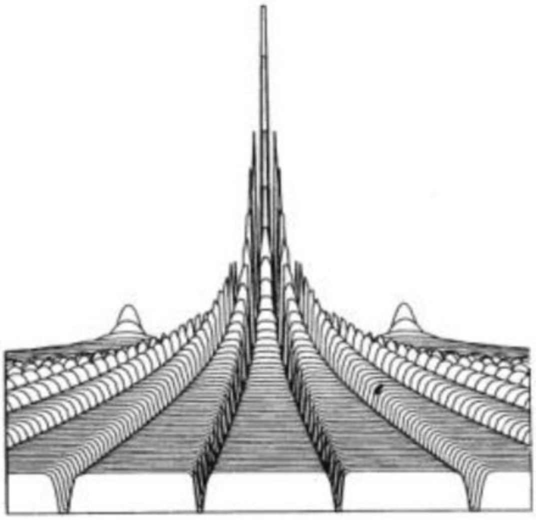

**Figure 2.** Quantum potential for two Gaussian slits relative to $S_2$ [18].

The probability amplitudes $\psi_A(y, t)$ and $\psi_B(y, t)$ were found by integration over all coordinates of the slits. The wave function was calculated using the method of continual integrals over trajectories, which served to obtain the quantum potential from expression (4) (Figs 2 and 3).

The trajectories were calculated by integrating in time the equation $\nabla S = mv$, which relates the $S$-function to the particle velocities in the usual way. Initially, the trajectories emanate from each slit in such a way that they are compatible with a single Gaussian slit. The sequential knees of the trajectories coincide with troughs in the quantum potential. The troughs appear because, when a particle falls into a trough domain, it experiences a significant action in the $y$ direction, which rapidly accelerates the particle from trough to trough in the domain where the 'force' becomes weak again. As a consequence, the majority of trajectories are located along the plateau to produce a bright interference pattern, while the troughs coincide with dark fringes.

Interestingly, according to Fig. 3, the Bohm trajectories cannot intersect, since the velocity field is single-valued with respect to the $x$-axis; otherwise, what determinacy can be spoken of at all (for more details, see Section 2.2 of Ref. [45] entitled exactly so: "The noncrossing rule")? It appears then, however, that the particles transmitted through the upper slit would never find themselves below the $y = 0$ plane, and vice versa. However, it is perfectly clear that these trajectories are quite possible due to diffraction by the slit. This is a serious problem, since it casts doubt on D Bohm's entire logic. This problem is discussed in Refs [8, 20, 45].

## 5. 'Surrealistic' Bohm trajectories

An interesting situation arises when a binary detector is placed near each slit to indicate whether a particle has passed through a given slit or not [20]. These detectors impart information of the path taken ('which way') and permit distinguishing trajectories of two types: those passing through one slit and those passing through the other. In this case, naturally, the interference pattern on the screen is lost (Fig. 4).

In the presence of these binary detectors, the upper-slit contribution now correlates with the 'which-way' information recorded by the upper detector, and the same applies to the lower slit and the lower detector.

But what slit did a particle pass through? Let us assume that the upper detector said 'yes' and the lower one said 'no'. Then, the probability of finding the particle trace on the lower half of the screen does not vanish completely, though none of the possible trajectories may intersect due to the single-valuedness of the velocity field [45]. Consequently, this case is possible when the particle passes through the upper detector and, hence, through the upper slit and then finds itself on the lower part of the screen, so that the corresponding Bohm trajectory passes through the lower slit. To state it in different terms, in the presence of binary detectors the Bohm particle trajectories may be characterized by contradictory behavior: they may originate in one slit, while the detector readings suggest that the particle passed through the other one. In brief, the Bohm trajectories are now 'surrealistic', not realistic.

In accordance with this theoretical model, an experimental study was performed by Mahler et al. [8], who also performed a theoretical analysis of the situation. The experiment was performed with two path-entangled photons using the 'weak' measurement technique (see, for instance, Ref. [21]). They showed that the trajectory of the first particle (its position and velocity) are indeed related to the behavior of the second, distant, particle, i.e. nonlocally. This, of course, is not new, but it gives confidence in the nonlocality of quantum correlations and the Bohm potential $Q$ in expression (4).

But can Bohm's interpretation be tested by experiments on photons? The point is that Bohm's concept was initially formulated for massive particles. However, if it remained

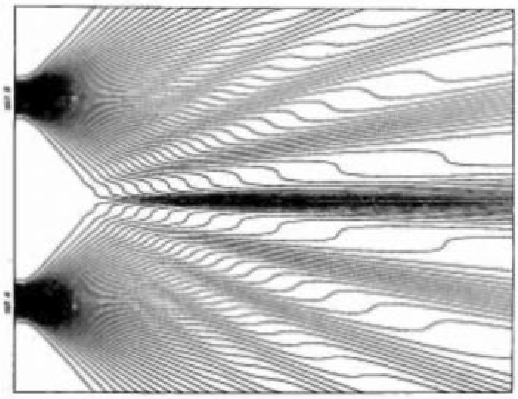

**Figure 3.** Ensemble of trajectories passing through two Gaussian slits [18].

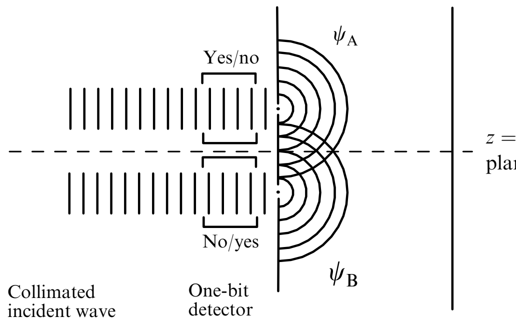


**Figure 4.** Double-slit interferometer with binary ('yes/no') detectors: a collimated plane wave is incident on the detecting screen through the slits. The interference on the screen vanishes due to information about particle paths. But can the trajectories intersect?

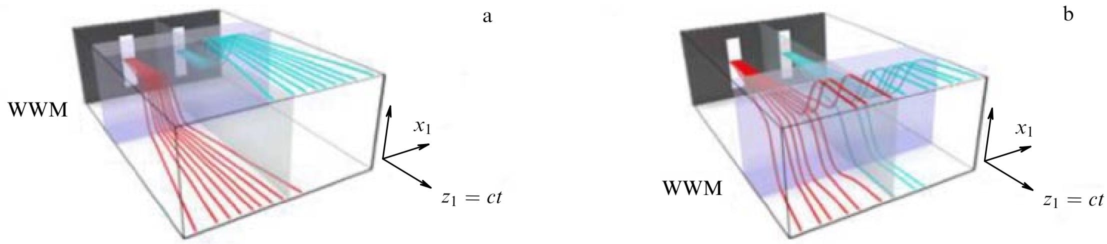


**Figure 5.** (Color online.) Bohmian trajectories in a double-slit experiment. The separation of the trajectories in the vertical direction shows which slit a particle comes from. In the histogram of Fig. 5b, ambiguity of trajectories appears, which is an indication that either the trajectories nevertheless intersect or the correspondence of the $|\mathrm{H}\rangle$ and $|\mathrm{V}\rangle$ states to their slits is not strict. (Borrowed from Ref. [8].)

such, it could not be regarded as the universal interpretation of quantum mechanics. And so its generalization for photons was a natural development of this concept. Section 3 of Ref. [45] reads:

"Bohmian mechanics was formerly formulated to describe massive particles, resulting very appealing to explain event-to-event experiments like those carried by Merli et al. [52], Tonomura et al. [53], or Shimizu et al. [54]. Now, there are also event-to-event experiments with light, such as those performed by Dimitrova and Weis [55], which claim for a treatment on equal footing. As shown by Prosser [56], this is actually possible by directly considering Maxwell's equations (which put electromagnetism at the same level of Schrödinger's wave mechanics [57])."

Therefore, experiments with photons are also representative relative to the Bohmian concept.

It is very simple to determine which slit the particle passed through: in the case of photons, mutually orthogonal polarizers can be placed in front of the slits, so that a passage through the upper slit will correspond, for instance, to state $|\mathrm{H}\rangle$ and through the lower slit to state $|\mathrm{V}\rangle$. If observations are made of electrons, these may be the two opposite spins. In any case, the states are mutually orthogonal, so that $\langle \mathrm{H}|\mathrm{V}\rangle = 0$.

The trajectories of a single photon (particle 1) are measured and post-selected by detecting the second photon entangled with the first one (particle 2). To this end, one of the entangled photons is directed to the trajectory-parameters measuring setup and the second photon is used as the source of control action, permitting or blocking the arrival of the first photon at the measuring setup.

The reading of a which-way information detector (Which-Way Measurement, WWM) can be taken, at the discretion of the experimentalist, at the moment when particle 1 is in the near region (Fig. 5a) or middle region (Fig. 5b). In this case, the reading for particle 2 is not taken until particle 1 arrives at the requisite region. The corresponding slits for the initial Bohmian trajectories are indicated in Fig. 5, and the height shows the WWM result (position $x_2$ of the second, 'control', particle, i.e., which slit it passed through). If a WWM reading is taken when particle 1 is in the near region, the Bohmian trajectories perfectly correlate with the value of this result. However if a WWM reading is taken when particle 1 is in the middle region, the Bohm trajectories correlate with the WWM result only at the edges of the diagram. Near the symmetry axis of the instrument, both WWM results are equally probable, irrespective of the slit in which the Bohmian trajectory originates. The authors of Ref. [8] attribute this to the fact that the state of particles transforms into the superposition of $|\mathrm{H}\rangle$ and $|\mathrm{V}\rangle$ as they recede from the screen with the slits, which is certainly true, and this results in ambiguity in determining which slit a particle went through. However, if the polarization state changed in the propagation of photons, their interference would be observed due to the gradual 'erasure' of the 'which-way' information, which is equivalent to the 'quantum eraser' effect (see, for example, Ref. [22]). This, of course, may not occur, since there are no physical reasons for this change. The superposition emerges only due to the overlap of scattering indicatrices of the slits, and the result of polarization measurements corresponds unambiguously to one slit or the other.

So, attempts to circumvent in this way the difficulty of interpreting 'surrealistic trajectories' do not meet with success.

## 6. Bohm's theory and the measurement problem

As early as Ref. [23], von Neumann drew special attention to the fundamental difference between the 'intrinsic' evolution of a quantum system (described by the time-reversible Schrödinger equation) and the 'reductional' evolution. The latter is usually irreversible and occurs in the measurement of the particle state. The measurement procedure results in the collapse of the quantum state, when the superposition of possible states is instantaneously, as is commonly believed, replaced, in the case of orthogonal measurements, with one and only one of the states of the superposition, in which the system was prior to the measurement. Von Neumann termed this measurement 'projective,' because the initial vector of state in the Hilbertian space instantaneously transforms ('reduces', or 'collapses') into one of its basis components in this space. This signifies that the initial (prior-to-measurement) wave function is replaced with one of the eigenstates of a specific projection operator $\hat{P}$. Unlike the deterministic wave-function evolution law defined by the Schrödinger equation, the collapse is not deterministic, since the final wave function is randomly selected among the eigenstates of operator $\hat{P}$. This description is reminiscent of some 'trick' [24], but most important, it arbitrarily 'isolates' the measured quantum system from the rest of the world.

Bohm came up with an entirely different approach to the measurement problem. In his theory, the measurement procedure is interpreted like any other interaction of particles, so that the above difficulties of orthodox interpretation simply vanish. In particular, the need to introduce projection operators vanishes. Here, the entire quantum system is described by the trajectory plus the wave function, and not only by the wave function. The wave function and the trajectory are both associated with the system as a whole, i.e., with the measured quantum system plus the measuring device. For this compound system, in the course of measure-

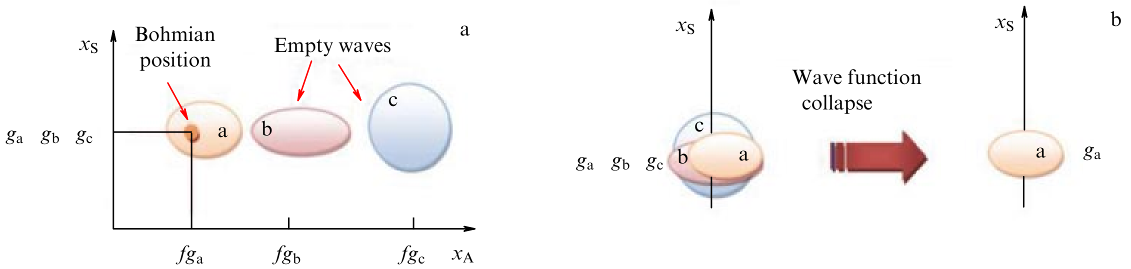


**Figure 6.** (Color online.) Diagrams that serve to explain the difference between the Bohmian interpretation and the orthodox one. (a) Bohmian explanation of measurement in space $[x_S, x_A]$ (system–device): of the nonoverlapping wave function, only the $g_a$ part, where the trajectory is present, is required for computing the evolution of the Bohmian system. (b) Orthodox explanation of measurement in space $[x_S]$ (system): the (system's) wave function collapses into the $g_a$ part in the measurement. (Borrowed from Ref. [16].)

ment in the common Schrödinger equation, a term appears in the Hamiltonian, which describes the interaction, so that the Schrödinger equation remains valid during the measurement as well as after interaction termination. It is this term that introduces coupling between the states of the system and the measuring device, thereby making it possible to effect the measurement. At the same time, in the measurement, changes occur in the states of the system being measured and the measuring device: their degrees of freedom, which were mutually independent prior to the measurement, turn out to be correlated (entangled) after the measurement. As pointed out by Bohm, in the measurement it is important that the system–device coupling be sufficiently strong and the interaction itself continue for a certain minimal time interval, but not so long as to give rise to distortions (in his book [25], Bohm provides, as a comparative analogue, too short or too long an exposure in photography).

A good example of measurement is provided by the double-slit experiment with a continuous spectrum of possible trajectories. According to Bohm's notions, a particle, unlike one in the orthodox interpretation of the quantum theory, owing to the effect of a nonlocal hidden parameter (the phase of the wave function), selects a specific trajectory — one of the possible trajectories in accord with the Schrödinger equation. In the course of experiment, the total wave function, which corresponds to the superposition of possible states, splits into wave packets equal to such trajectories in number, the difference between the packet centers increasing, so that they cease to overlap in space. As a result, the Bohmian trajectory corresponds to one of the packets, while all remaining packets turn out to be 'empty' (Fig. 6) (see also Ref. [6]).

The wave function splitting very much resembles Everett's many-worlds interpretation of quantum mechanics [26], with the difference being that, according to Everett, a particle belongs to all wave packets, but they are in different 'worlds' (see, for instance, Refs [27–29]), while according to Bohm, all packets except one are empty.

From the modern viewpoint, the measurement procedure and the evolution of state vectors are termed decoherence. Both the system M under measurement and the measuring device (or the environment) ε participate in this [30].

Before the measurement, the system M is characterized by the density matrix that corresponds to this state. The elements of the main diagonal of the matrix give the probabilities of obtaining each of the possible (basis) measurement results, while the elements that are outside of the main diagonal correspond to correlations (phase relationships) between the basis states. The environment ε is described in a similar way.

In the interaction of system M with the environment ε (i.e., in the measurement), entanglement of the degrees of freedom of M and ε occurs. Their overall density matrix, which could be resolved (factorized) into two factors corresponding separately to M and ε prior to the measurement, loses this property: a correlation arises between the degrees of freedom of M and ε which did not exist prior to the measurement. In particular, if M and ε were initially in pure states, at this stage each part of the compound system (M and ε) may only be described by the density matrix rather than by a separate state vector (Fig. 6).

The completion phase reduces to the decoherence process, which is characterized by the breaking of phase relations between separate states: the elements of the density matrix that are outside the main diagonal (correlation coefficients) damp down, while the elements of the main diagonal (the probabilities of basis states) do not change significantly. As a result, the system under measurement transforms into a statistical mixture of possible measurement results, which may formally correspond to the von Neumann projection operator if the density matrix is a projector.

## 7. Criticism of hypothetical wave-function splitting into empty and nonempty wave packets

If we follow Bohm's hypothesis of wave-function splitting into empty and nonempty wave packets, in the interference schemes with particle separation into two channels the particle itself must be present only in one channel; otherwise, it will have no trajectory at all. Notice that the Bohmian interpretation differs from the Feynman path formalism in quantum mechanics [31], according to which the transition probability between two points in phase space is calculated with the use of all possible paths connecting these two points. On the other hand, if we regard R Feynman's method as a purely mathematical way of treatment and do not endow it with interpretational meaning, this difference does not seem to be highly significant.

However, Bohmian mechanics asserts that each quantum particle follows its trajectory in a deterministic way. In this case, the notions of classical mechanics largely persist. Here, apart from verification of 'surrealistic' trajectories, we have a lucky chance to experimentally verify Bohm's hypothesis for photons, at least in the form of a gedanken experiment, by selecting an appropriate experimental configuration and using a usual quantum-mechanical calculation. Indeed, it is

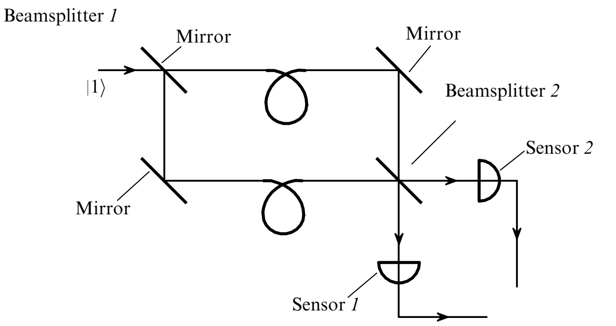


**Figure 7.** Mach–Zender interferometer with identical nonlinear fibers in its arms.

possible to advance a rigorous proof that a photon simultaneously is in both arms of a Mach–Zender interferometer [32, 33], i.e., to prove that all wave packets are nonempty and there is no predetermination that the particle is in one of them.

Let us place in the interferometer arms two identical cubic nonlinear media as phase delays, in which phase self-modulation (PSM) occurs, i.e., a variation in refractive index under the action of light. The role of such media may be played, for instance, by quartz fibers (Fig. 7). In the passage through this fiber, a photon must acquire an additional phase shift, which will inevitably show up in the interference result. However, for the nonlinear phase shift to occur, the photon itself and not simply its empty wave packet must be in the fiber, since the latter is void of energy needed to initiate the nonlinear PSM effect. According to Bohm, the entire particle energy is confined in precisely the particle, and the quantum state vector merely directs it to one side or the other.

Let us assume that the phase shifts in the arms are equal in the absence of radiation. Then, on delivering a single photon to the interferometer, we face the following alternative: either the photon passes through only one arm and the phase difference changes due to the nonlinear phase shift in this arm, or the photon passes through both arms and the phase shifts will be equal in both arms, so that the phase difference will remain invariable. In the latter case, the photon will appear at only one interferometer output.

The input monochromatic mode in the Fock state $|1\rangle$ will be described by the photon annihilation operator $\hat{a}_1$ and the vacuum mode $|0\rangle$ at the second input by operator $\hat{a}_0$. After the first 50% beamsplitter, we consider two modes described by operators $\hat{a}_2$ and $\hat{a}_3$ in the Heisenberg representation:

$$\hat{a}_2 = \frac{\hat{a}_1 + \hat{a}_0}{\sqrt{2}}\,, \qquad \hat{a}_3 = \frac{\hat{a}_1 - \hat{a}_0}{\sqrt{2}}\,. \tag{14}$$

Next, we take into account the action of Kerr nonlinearity. The stable transverse intensity distribution in the quartz fibers may be treated as the radiation mode and the four-photon process itself may be described by the single-mode Hamiltonian (see, for instance, Ref. [34] and references therein):

$$\hat{H} = \frac{\hbar}{2}\chi^{(3)}\hat{a}^+\hat{a}^+\hat{a}\hat{a}\,, \tag{15}$$

where $\chi^{(3)}$ is the cubic nonlinearity coefficient normalized to the number of photons. The nonlinear response is assumed to be instantaneous.

The corresponding operator of the quantum state evolution in the Schrödinger representation is expressed as

$$\hat{U} = \hat{I}\exp\left(-\,\mathrm{i}\,\frac{\bar{\chi}}{2}\,\hat{a}^+\hat{a}^+\hat{a}\hat{a}\right) = \hat{I}\exp\left[-\,\mathrm{i}\,\frac{\bar{\chi}}{2}\,\hat{n}(\hat{n}-1)\right], \tag{16}$$

where $\bar{\chi} = \chi^{(3)}t$, the evolution time $t$ is related to the fiber length $l = vt$, $v$ is the velocity of mode propagation in the fiber, and $\hat{n}(t)$ is the photon number operator.

In the Heisenberg representation, the mode field photon annihilation operator obeys the equation $\mathrm{i}\hbar\,\mathrm{d}\hat{a}/\mathrm{d}t = [\hat{a}, \hat{H}]$; hence, $\hat{a}(t) = \exp\left(-\,\mathrm{i}\bar{\chi}\hat{a}^+(0)\hat{a}(0)\right)\hat{a}(0)$ and, in our case,

$$\hat{a}_2' = \exp\left(-\mathrm{i}\bar{\chi}\hat{a}_2^+\hat{a}_2\right)\hat{a}_2\,, \quad \hat{a}_3' = \exp\left(-\mathrm{i}\bar{\chi}\hat{a}_3^+\hat{a}_3\right)\hat{a}_3\,. \tag{17}$$

Accordingly, we obtain two output interferometer modes:

$$\hat{a}_0' = \frac{\hat{a}_2' - \hat{a}_3'}{\sqrt{2}}\,, \qquad \hat{a}_1' = \frac{\hat{a}_2' + \hat{a}_3'}{\sqrt{2}}\,. \tag{18}$$

We find the average photon numbers at the interferometer outputs:

$$\langle\hat{n}_0\rangle \equiv \langle\hat{a}_0'^+\hat{a}_0'\rangle = 0\,, \quad \langle\hat{n}_1\rangle \equiv \langle\hat{a}_1'^+\hat{a}_1'\rangle = 1\,. \tag{19}$$

So, we observe interference with the zero phase difference, and a photon therefore is in both interferometer arms simultaneously.

Everything would be nice were it not for one regrettable circumstance. According to expression (16), a single photon in the Fock state $|1\rangle$ does not acquire a nonlinear phase shift, since for $n = 1$, as with any averaging over $|1\rangle$, $\hat{U} = \hat{I}$. This comes as no surprise, because PSM is a kind of frequency- and direction-degenerate nonlinear four-photon process, when two pump photons transform into precisely the same two photons, though with a phase shift. That is why a single photon may not carry out the PSM process.

What can be done? Let us deliver the superposition $|\psi\rangle = (1/\sqrt{2})(|12\rangle + |21\rangle)$ rather than $|\psi\rangle = (1/\sqrt{2})(|01\rangle + |10\rangle)$ to the two interferometer arms. These three-photon fields may be prepared as follows. First, by using a three-stage atomic transition to the ground state, whereby two of three emitted photons are degenerate, i.e., belong to one mode. Second, by using the nonlinear frequency down-conversion in a medium with a cubic nonlinearity $\chi^{(3)}$ [35] or due to a cascade process similar to that described in Refs [36, 37]. In this version, at the first stage, two photons are produced in the course of nondegenerate parametric scattering in a piezocrystal, for instance, $3\omega \to 2\omega + \omega$, and at the second stage one of the photons splits in a degenerate parametric process, i.e., with the production of a subharmonic: $2\omega \to \omega + \omega$. The thus-formed modes $a$ and $b$ are next delivered to the Mach–Zender interferometer arms with nonlinear fibers. The state vector at their input has the form

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}\left(|1\rangle_a|2\rangle_b + |2\rangle_a|1\rangle_b\right). \tag{20}$$

Upon the action of evolution operator $\hat{U}$ (16), we have

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|1\rangle_a|2\rangle_b\exp\left(-\mathrm{i}\bar{\chi}_b\right) + |2\rangle_a|1\rangle_b\exp\left(-\mathrm{i}\bar{\chi}_a\right)\right]. \tag{21}$$

At the interferometer output, according to expressions (18), the detection probability is proportional to the average number of photons:

$$\langle \hat{n}_0 \rangle \equiv \langle \hat{a}_0'^{+} \hat{a}_0' \rangle = \frac{1}{2} \left[ 1 + \cos\left( \bar{\chi}_a - \bar{\chi}_b \right) \right] ,$$
$$\langle \hat{n}_1 \rangle \equiv \langle \hat{a}_1'^{+} \hat{a}_1' \rangle = \frac{1}{2} \left[ 1 - \cos\left( \bar{\chi}_a - \bar{\chi}_b \right) \right] . \qquad (22)$$

When solving this problem in the Heisenberg representation, we obtain the same result.

So, for the same fibers, $\bar{\chi}_a = \bar{\chi}_b$, we have photocounts of only one detector, which signifies that interference does take place and both superposition states are simultaneously present in both interferometer arms. Consequently, there is neither state vector splitting nor empty wave packets, as in the case of single-photon states.

When we are dealing with a Mach–Zender interferometer, we can feed it with the Fock state $|3\rangle$, and the field in its arms will be described by the state vector

$$|\psi\rangle_0 = \frac{1}{2\sqrt{2}} \left( |0\rangle_a |3\rangle_b + \sqrt{3} |1\rangle_a |2\rangle_b + \sqrt{3} |2\rangle_a |1\rangle_b + |3\rangle_a |0\rangle_b \right) , \qquad (23)$$

and the detector operation probabilities will be unequal: 1/4 and 3/4. This also proves the presence of interference and the simultaneous presence of all components of quantum superposition in the interferometer arms.

There is one more, albeit indirect, proof of this fact, which relies on the incontestable presence of quantum superposition rather than on specific values of measured quantum observables. At the same time, the existence of quantum superposition is precisely in contradiction with the existence of one definite trajectory of a quantum particle or particles. The proof mentioned involves an experiment, so far a gedanken experiment, in which there is no specific phase difference between two entangled photons, but there is a complete superposition of all possible values of this phase difference [38].

In this connection, it is pertinent to note that the interference effect considered above, broadly speaking, may be attributed to the hypothetical "nonlocal effect of the empty arm." However, as shown in Ref. [38], nonlocal classical realism (in the sense of the existence of definite pre-measurement values of physical quantities) can be refuted. As applied to the interference effect considered above, this signifies that a particle may not be in one of the interferometer arms prior to its recording, since the particle residence in it would be its a priori determined spatial location. Therefore, the "nonlocal effect of the empty arm" in the context of nonlocal classical realism, which is inherent in the Bohmian concept, is unacceptable.

## 8. Von Neumann's hidden variables, Bohm's and Bell's objections

Bohm interpreted his resultant relations in the sense that they basically implied a denial of quantum indeterminism. Were the value of phase $S$ known, we would be able, as Bohm believed, to determine the 'individual' particle velocities $\mathbf{v}_i$ and find the 'individual' particle trajectories in the subsequent integration over time for given initial values. This brings up at least two fundamental questions.

• What is to be done with von Neumann's well-known theorem about hidden variables, which limits the determinism of the behavior of individual quantum particles?

• Is it generally correct in this case to speak about 'individual' particle characteristics?

However, is it possible to reconcile the principles of quantum mechanics with the deterministic description of quantum objects rather than a probabilistic one? Cited in the famous work of John von Neumann is the result obtained earlier by Robertson [39], which later received the name von Neumann's theorem on "no hidden variables." According to this theorem, the quantum theory, as follows from its basic principles, can give only a probabilistic description of quantum objects and not a deterministic one. Because more recently this theorem was repeatedly subjected to criticism and 're-interpretations' (see, for instance, Ref. [40] and references therein), we set forth its exact formulation.

Let us assume that two noncommuting self-adjoint Hermitian operators, $\hat{A}$ and $\hat{B}$, with $\hat{A}\hat{B} - \hat{B}\hat{A} = \mathrm{i}\hat{C}$, correspond to two quantum observables $A$ and $B$. Then, it is possible to rigorously prove the Heisenberg uncertainty relation:

$$\sqrt{(\Delta A)^2 (\Delta B)^2} \geqslant \frac{1}{2} |\bar{C}| , \qquad (24)$$

where $\Delta A$ and $\Delta B$ are the root-mean-square, fundamentally unremovable deviations of the individual results of measurements of $A$ and $B$ from their average values. In other words, since describing quantum phenomena calls for noncommuting operators, the theory that uses them may not be deterministic.

Bohm himself would not call into question the mathematical correctness of the formal apparatus of the quantum theory. However, he insisted that this description corresponded to an interim level of representing reality and not to the final one. Bohm's logic involved the following statement: yes, the statistical interpretation of quantum mechanics, which allows the use of noncommuting operators, is inherently consistent and is borne out by experiments. 'Hidden' (deterministic) variables are indeed not necessary for such a theory. But does this mean that a deeper level of description, also inherently consistent and allowing experimental confirmation, is impossible? By way of possible illustration, Bohm provided a comparison of phenomenological thermodynamics (in which figure macroscopic parameters — pressures, temperatures, etc.) with statistical physics, in which the *macroscopic* parameters are not postulated but appear as the result of action of an ensemble of *microscopic* degrees of freedom of individual atoms and molecules. It is common knowledge that the proponents of these conceptions waged a stubborn ideological struggle, and it was precisely the microscopic approach which was victorious.

It is also possible to make another analogy, which is closer, it seems, to Bohm's point. Consider a linear electric circuit with stationary harmonic currents and voltages. Here, we can distinguish two levels of analysis. At the 'fine' level one can operate on instantaneous values of currents and voltages (and measure them), and this is precisely what Bohm means. At the 'coarse' level, we have to speak only about the 'effective' root-mean-square (rms) values of the currents and voltages, which are quadratically averaged over the harmonic current cycle in every branch, as well as about phase angles between them. Then, we have a pragmatic, absolutely noncontradictory and complete theory of stationary harmonic processes, which, however, may in principle be deepened to the 'fine' level. Furthermore, this theory also makes use of complex quantities, which leads to several amazing analogies, including commutation relations, to quantum mechanics [41].

John Bell also raised objections to the universal validity of von Neumann's theorem. In the chapter "On the impossible pilot wave" of his book [1], Bell outlined constructive criticism of the premises of this theorem and referred to the model proposed by Bohm, which went beyond the framework of these premises and, in particular, introduced the phase parameter, which clearly exemplified von Neumann's 'hidden parameter.' "But in 1952 I saw the impossible done," is how J Bell expressed his ineffable surprise concerning the publication of D Bohm's paper. In his book [1], Bell expresses his astonishment at the silence of his teachers regarding the 'pilot wave' theory of de Broglie–Bohm. That is why Bell addressed the analysis of the situation from another side. Considering the thought experiment related to the well-known Einstein–Podolsky–Rosen (EPR) paradox, which deals with the separation of entangled quantum particles, Bell tried to ascertain whether the results of quantum theory and the corresponding experiments may be described with the aid of *deterministic hidden variables* [42]. It turned out that this is impossible in principle, unless it is assumed that *nonlocal* (supraluminal) correlations may exist between receding EPR particles (and/or between the detectors of these particles). As formally proved by Bell in the 1960s [42] and more recently verified experimentally by Aspect et al. in the 1980s, quantum mechanics (and Bohm's theory explicitly via quantum potential) is characterized by precisely *nonlocal* correlations in the entanglement [43, 44].

## 9. Are the velocities and trajectories in Bohm's model the characteristics of individual particles?

Are the velocities and trajectory points found by Bohm the instantaneous characteristics of precisely the *individual* particles? Bohm himself was convinced of this and did not even endeavor to pose the problem in a different way, because herein lay the sense of his interpretation. However, there are arguments in favor of a different viewpoint.

First, the Schrödinger equation itself, which was the starting point for Bohm, operates on a *statistic* ensemble, which has one distribution or another. Hence, it seems, it follows that any description obtained from the Schrödinger equation will also be statistical in nature.

Second, proceeding from the analogy with a 'quantum liquid' obeying the same Bohm equations, it was shown in [45] that "quantum fluxes cannot cross in configuration space... . Therefore, two or more Bohmian trajectories cannot cross through such a point at the same time... ." At the same time, it was theoretically and experimentally shown, as noted above, that individual quantum particles can 'jump' from one Bohmian trajectory to another with a nonzero probability, thus generating so-called 'surrealistic' trajectories [20]. Therefore, the 'group' Bohmian trajectories and 'individual' tracks of particle motion are not the same thing, which was demonstrated by the example of 'surrealistic' trajectories.

Indeed, as recognized by Madelung in 1926, from the time-dependent Schrödinger equation there follows an equation of the form of the hydrodynamic continuity equation, in which the density and velocity potential of a moving fluid appear. Elaborating on these ideas, Madelung showed that each eigen function (the solution of the wave equation), although depending on time, could be interpreted as a certain type of stationary flow. Since the hydrodynamic model also described other important features of the Schrödinger theory, Madelung assumed that it was possible to consider the quantum theory of atoms from this viewpoint. If the Bohm equations are interpreted as hydrodynamic equations, the trajectories obtained from these equations should not necessarily be regarded as the trajectories of real particles but rather as streamlines associated with a quantum liquid (we note that the Schrödinger equation typically describes in fact a degree of freedom rather than a 'true' particle).

This immediately brings up the question: what is a Bohmian trajectory? Is it the real path followed by a quantum particle or does it merely represent a quantum degree of freedom?

Consider a classical continuous medium consisting of many different particles (atoms, ions, molecules, etc.). All degrees of freedom are described by a system of differential equations, the number of equations coinciding with the number of degrees of freedom. Let our concern be not a microscopic but merely a macroscopic description of a medium by Euler or Navier–Stokes type equations, which phenomenologically describe the continuum evolution but pay no attention to the microscopic dynamics of its constituents. This underlies hydrodynamics. In this case, in the experimental study of medium behavior, it is common practice to follow the motion of certain particles, namely marker particles. These permit visualizing the flow dynamics in the motion along the streamlines, which coincide with energy transfer lines. For instance, when we want to observe the evolution of airflow, we may follow the propagation of cigarette smoke. To trace the flow of water, advantage can be taken of another liquid like ink or small floating particles like pollen or charcoal grit. On the cosmological scale, this hydrodynamic approach may also be employed with the use of stars and galaxies or their clusters as marker particles. Real separate quantum particles behave like corpuscles, although their distribution displays wave properties according to the Schrödinger equation or its Bohmian equivalents. It is therefore evident that the ensemble properties should be described collectively, i.e., with a distribution density function, whose role is played by the probability density in quantum mechanics or, at a more elementary level, by the wave function. This corresponds to the statistical Born interpretation for quantum mechanics [46].

## 10. Grössing's model

Recall the quotation from Ref. [4] given in Section 2: "Recognizing that this equation has the structure of a diffusion equation with an imaginary diffusion coefficient, Schrödinger relaxed his original requirement concerning the reality of $\psi$ and admitted complex-valued functions for what he called the mechanical field scalar $\psi$... ." Encountered here are two key words: 'diffusion' and 'complex-valuedness.' These two notions were addressed by the Austrian theorist Gerhard Grössing in his several papers an the construction of so-called *emergent quantum mechanics*. Grössing uses this term in reference to the quantum mechanics *emerging at the subquantum level* from inherently classical notions.

In Ref. [47] Grössing proposes the following model to describe the propagation of a quantum particle in the medium of 'zero-point vacuum oscillations.' The energy of this quantum particle is assumed to comprise two constituents. This first is the ordinary (constant) energy of a quantum oscillator (proportional to the oscillation frequency), and the second one is an *additional kinetic constituent caused by the fluctuating* (*harmonically varied*) *momentum of the particle*, which is continuously acquired and lost due to energy exchange with the vacuum. As a result, the particle motion acquires a Brownian character. Putting the kinetic energy of the 'vacuum thermostat' per degree of freedom equal to $k_B T/2$ and the average kinetic energy of the particle-oscillator to $\hbar\omega/2$ and equating them, then introducing the probability for the particle and expressing it in the ordinary way in terms of the real amplitude and phase of the wave function, Grössing obtains the macroscopic *diffusion* equation exactly coincident with the Bohm equations, where the said specific quantum potential arises automatically. This potential does not have a significant effect for a single free particle; however, in the case of several particles for diffusion fields, there immediately follows a new understanding of the basic properties of the quantum potential.

In this case, an important role is played by the particle's eigenfrequency, which (one should think) is at resonance with the vacuum oscillation frequency throughout the volume.[2] This is precisely the reason why *diffusion* waves emerge, whose properties are radically different from those of ordinary ('traveling') waves. In particular, attention should be paid to the *nonlocal properties of diffusion waves*. Since their 'propagation velocity' is unlimited, the original equation results in neither traveling waves and wavefronts nor the phase velocity. It is as if the entire domain were breathing in phase with the oscillating source. In the world of diffusion waves, there are only spatially correlated phase lags defined by the diffusion length.

Therefore, instead of the conventional analysis of the behavior of a single quantum particle (a photon or an electron) itself, we face the necessity of considering its 'diffusive' propagation in a medium made up of zero-point vacuum oscillations. As a matter of fact, this thought is not as unexpected as it might seem. The decoherence theory makes active use of the notion of the interaction of a quantum particle with a medium, when this medium 'measures' the particle and entangles with it. D I Blokhintsev, for instance, believed that the properties of quantum mechanics stem from the fact that there is no way of isolating a particle from surrounding world. Bodies radiate and absorb electromagnetic waves at any temperature above absolute zero. From the standpoint of quantum mechanics, this signifies that their position is continuously 'measured,' which entails the collapse of wave functions. "From this viewpoint, there are no isolated 'free' particles in itself," wrote Blokhintsev. "Possibly the nature of this impossibility of isolating a particle, which manifests itself in the apparatus of quantum mechanics, underlies the relationship between particles and the medium" [49].

Modern physics has focused closely on theoretical decoherence models and their experimental verification at the quantitative level. In particular, "A tractable model of the environment is afforded by a collection of harmonic oscillators ... or, equivalently, by a quantum field.... If a particle is present, excitations of the field will scatter off the particle. The resulting 'ripples' will constitute a record of its position, shape, orientation, and so on, and most important, its instantaneous location and hence its trajectory. A boat traveling on a quiet lake or a stone that fell into water will leave such an imprint on the water surface. Our eyesight relies on the perturbation left by the objects on the preexisting state of the electromagnetic field" [50].

In this case, it is significant that a purely classical description of quantum objects in the presence of the vacuum mode continuum leads to incorrect results. For instance, endeavors to numerically simulate atomic hydrogen in the framework of the Rutherford planetary model in the field of white vacuum noise do not yield stationary Bohr orbits [51].

## 11. Conclusions

To summarize, first of all, we note that Bohm did not formally enhance the quantum mechanical apparatus itself. He 'merely' came up with the idea to replace the Schrödinger equation for one complex-valued wave function with the equivalent system of two equations for two real functions — the amplitude and phase of the wave function.

On the other hand, this representation of the mathematical apparatus made it possible to view quantum mechanics from a different aspect.

- Prior to Bohm, as far as we can judge, the 'absolute' phase of the wave function was treated as a formal parameter void of physical significance. In the new representation, the Schrödinger equation was replaced by the continuity equation (for the probability density) and the Hamilton–Jacobi equation known from classical physics, which permits fixing the phase of the wave function for an individual trajectory (or group of trajectories). In the quantum analogue of the Hamilton–Jacobi equation, a new term appeared that is nonexistent in classical physics — quantum potential — which explicitly gave rise to the nonlocal correlations of quantum particles.
- Bohm interpreted separate trajectories fixed by specific values of phase of the wave function as the tracks of motion of individual quantum particles. However, theoretical arguments and experimental facts suggest that more likely we are dealing with classes of trajectories averaged over a given phase value, while the notion of an individual particle trajectory is highly controversial, the more so as 'surrealism' is also invoked here. Bohm's theory supposedly deals not with individual particles and their trajectories but with mass/energy transfer lines.
- Bohm adduced several arguments refuting the universal validity of von Neumann's theorem on hidden variables. His introduced 'absolute' phase of the wave function is a good example of a hidden (nonlocal) parameter.
- Bohm expressed his firm belief that, while quantum mechanics is a complete and noncontradictory theory, other physical theories are also possible, also complete and noncontradictory, which operate at a more subtle level of notions about space, time, and physical interactions.

So, Bohm's hypothesis undoubtedly possesses a number of attractive and useful components. However, it may hardly serve to substantiate the determinism of quantum processes and reconcile the predictions of quantum theory with the postulate of objectively existing particle coordinates and velocities. Furthermore, a wave function splitting into empty and nonempty packets is believed to be at variance with the results of feasible experiments, at least photon experiments, and their interpretations.

[2] Compare this with the constancy of the oscillation frequency (i.e., energy feeding) for the entire liquid volume in the experimental tray in the macroscopic experiments of Couder et al. [48] with 'walking droplets,' which have become widely known due to a close analogy between them and basic quantum motion effects. In particular, the waves occurring there are also diffusive in character.

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