

METHODOLOGICAL NOTES PACS numbers: **01.65.+g**, **04.20.–q**

Visible shape of moving bodies

B M Bolotovskii, G B Malykin

DOI: https://doi.org/10.3367/UFNe.2018.08.038407

Contents

Abstract. **We show that if an extended object moves with not only a relativistic but even a nonrelativistic speed, an observer at rest sees the shape of this object distorted, and the distortion depends on the way the object is observed. This phenomenon is due to different retardation times of light emitted by various parts of the object. Moreover, the observer at rest sees the spatial position and speed of objects in an incorrect way. If an extended object moves with a relativistic speed, the relativistic aberration phenomenon occurs, which was analyzed by Einstein. The essence of the effect is that the observer at rest sees the image of a moving small body rotated by some angle. The analysis of these phenomena reported in well-known papers by Terrell and Penrose fails to correctly address the effects related to different retardation times of light emitted by various parts of the extended object but coming to the observer at rest at the same time. In particular, it follows from their studies that the observer at rest sees the image of a moving extended object, for example, a cube or a sphere, not flattened in the direction of motion (as follows from the Lorentz transformation) but only 'rotated' by the relativistic aberration angle. We report correct expressions for the images of rods parallel and perpendicular to the velocity of motion as seen by an observer at rest. In particular, if a cube moving sufficiently fast passes by a remote observer at rest, the image of the cube face turned to the observer is contracted in the direction of motion in accordance with the Lorentz transformations, but is not 'rotated', while the image of its rear face (with respect to direction of motion) 'rotates' by some angle. The image of the cube is therefore distorted. A history of theoretical predictions and experimental observations of this phenomenon is presented. We discuss Gamow's relativistic street car paradox, which shows that Terrell's and Penrose's results are incorrect in the general case of motion of objects. Results of our study explain the Gamow street car paradox in an easily comprehensible way. Physical problems are presented that can be solved significantly more easily if the formulas for the relativistic aberration and light retardation effects are used. We show that assertions made by some astronomers regarding the observation of superluminal motion of some galaxies and supernova jets are incorrect because the effects discussed here were ignored in their calculations.**

Keywords: light delay, relativistic aberration, Gamow paradox, velocity of galaxies

B M Bolotovskii [(1)], **G B Malykin** [(2)]
[(1)] Lebedev Physical Institute, Russian Academy of Sciences, Leninskii prosp. 53, 119991 Moscow, Russian Federation
[(2)] Federal Research Center, Institute of Applied Physics, Russian Academy of Sciences, ul. Ul'yanova 46, 603950 Nizhny Novgorod, Russian Federation
E-mail: [(1)] bolot@td.lpi.ru, [(2)] malykin@ufp.appl.sci-nnov.ru

Received 17 May 2018, revised 5 August 2018
Uspekhi Fizicheskikh Nauk **189** (10) 1084 – 1103 (2019)
DOI: https://doi.org/10.3367/UFNr.2018.08.038407
Translated by M Zh Shmatikov; edited by A M Semikhatov

1. Introduction

A moving body can be observed in different ways. Sensors can be located in space that would respond when contacting a body passing by and thus provide information about the dimensions of the body and details of its motion. Another approach to observation is that an observer makes a judgement about the motion of the body by receiving acoustic, light, or radio signals that the body emits. To be more specific, we only consider light signals in what follows. Sometimes, this second method is the only feasible one, as is the case with astronomical observations. We must take into account, however, that this observation method can provide incorrect information about the location of the moving body and its shape and velocity. Even if an extended object moves with a nonrelativistic speed, an observer at rest sees the shape of the object distorted, the distortion being dependent on the observation method and the distance to the body. The same phenomenon occurs if a remote observer at rest measures the speed of a moving body and the rate of a clock on that body: the observer determines them incorrectly. If the speed of the body exceeds the light propagation speed (as may be the case in an optical medium with a certain refractive index), a number of new effects occur; for example, Vavilov–Cherenkov radiation is observed [1–9].

We note that the possibility of superluminal motion has been analyzed in detail in well-known studies by Ginzburg (1916–2009) [10] and Bolotovskii and Ginzburg [11] (see also [12–14]). Some specific features of superluminal motion have been explored in [15–26].

If a luminous small body moves with a relativistic speed, the phenomenon of relativistic aberration occurs, as a result of which an observer at rest sees the image rotated by some angle that depends on the relative speed of motion of the small body. The concept of relativistic aberration was introduced in 1905 by Einstein (1879–1955) in his seminal work [27] that laid the foundations of the special relativity theory (SRT). The development of SRT was completed in the studies by Minkowski (1864–1909), where the our-dimensional space–time was discovered. We note that Einstein himself did not use the term 'relativistic aberration'. However, Einstein closely related this phenomenon to the so-called Doppler effect, which is due, in turn, to relativistic time dilation on the moving object. The point is that the source of the Doppler effect and relativistic aberration is the same: the wave 4-vector (frequency and wave vector) experiences changes as a result of motion. Relativistic aberration in the sense of [27] implies rotation of the light beam emitted by the moving point that is detected by the observer at rest. The results in [27] are discussed in detail in Section 4.2.

Because a similar phenomenon, referred to as stellar aberration, is also observed in classical physics [28, 29], and the formulas for the stellar and relativistic aberration angle were different, fierce debates occurred between SRT supporters and critics [30–34].

If an extended luminous body moves with a relativistic (and sometime also nonrelativistic) speed, in addition to relativistic aberration, the effect of different retardations occurs for light signals that arrive to the observer simultaneously but were emitted by different points on the object at different times. As a result of this effect, the observer at rest sees the image of the moving body distorted, the length of the body side facing the observer being contracted in the direction of motion as prescribed by the Lorentz transformation. At the same time, the rear side, with respect to the body motion direction, 'rotates' by some angle, owing to which it becomes visible to the observer.

We show below that the effects related to different retardation times of light signals emitted by different parts of an extended and rapidly moving body that arrive simultaneously to an observer at rest affect the image viewed by the observer no less than the relativistic contraction of its longitudinal dimensions. We note that some SRT critics (see, e.g., [35]) erroneously believe that in considering the problem of the image of a rapidly moving body, the Lorentz transformations are not needed at all, and light retardation alone should be taken in account.

More than fifty years after [27] was published, Terrell (1923–2009) [36] and Penrose (b. 1931) [37] came to the conclusion that due to the relativistic aberration effect and light-signal retardation, the observer at rest sees the moving body not flattened, as follows from SRT, but only 'rotated' by a relativistic-aberration angle: the light quanta that arrive simultaneously to the observer were emitted by different points on the body at different times, namely, the points located farther from the observer emitted quanta earlier than the points located closer. The authors of [36, 37] asserted that the effect of complete compensation of the Lorentz contraction also occurs when the dimensions of the body are much smaller than the distance to it: the image of the body is not distorted but only rotated by some angle. This phenomenon was named the Terrell effect or the Terrell–Penrose effect.

The goal of this study is to analyze in detail the issues described above and some physical problems whose solution can be significantly simplified by using formulas for relativistic aberration and retardation of light signals. We show that in exploring this phenomenon, Terrell [36] and Penrose [37] did not quite correctly take the above effects related to the differences in light retardation into account. We present correct formulas for the angle of 'rotation' of the image of a rapidly moving extended object.

Another goal of this paper is to show that the assertions made by some astronomers that some distant galaxies move faster than light or that the jets ejected by exploding supernovae and rapidly expanding radio sources have a superluminal speed are incorrect. This apparent optical phenomenon can be explained by accounting for the relativistic aberration effect and different retardation times of light emitted by different points of the extended light source and simultaneously arriving at the observer at rest.

Prior to proceeding to the main topic of the paper — observation of the apparent shape and speed of moving bodies — we review the problem of superluminal motion.

2. Superluminal motion

Because we here consider the cases where a material body or an image move with a speed v (and both cases $v < c$ and $v > c$ are possible, where c is the speed of light in the vacuum), we briefly review the history of the problem. We note that some important physical problems involve the speed of light in a medium with a refractive index n, i.e., c/n (chromatic dispersion of n is disregarded here). The speed of material objects can be $v > c/n$, but not $v > c$. At the same time, the speed of images, for example, a running spot produced by a light beam or an electron beam, can be not only $v > c$ but even $v \gg c$.

2.1 Vavilov–Cherenkov radiation

Historical research by Kaiser [38], Tyapkin (1926–2003) [39], and Bolotovskii [40] into studies by O Heaviside (1850–1925) has shown that as early as 1888–1889, Heaviside analyzed the case of an electric charge moving with a velocity v in a medium whose refractive index is n and $v > c/n$ [41]. Heaviside showed in [41] that a charge moving faster than the speed of light c/n in an optical medium emits electromagnetic radiation. Initially, paper [41] was rather actively referred to (see, e.g., [42]); however, later it fell into oblivion for a long time. By the time Vavilov–Cherenkov radiation was discovered in 1934 [1, 2], Tamm (1895–1971) and Frank (1908–1990) proposed its theoretical description within a classical approach in 1937 [3], and Ginzburg developed the quantum theory of this effect in 1940 [43], paper [41] had been fully forgotten.

2.2 Superluminal electromagnetic 'light spots'

In 1900, Heaviside analyzed the problem of an electromagnetic pulse incident on a mirror at some angle. The planar front of the pulse runs in this setup along the mirror with a superluminal speed [44]. Actually explored in [44] was the motion of a superluminal 'light spot'. This study [44], similarly to [41], fell into oblivion for a long time. Frank actually re-discovered this effect in 1942 [45].

'Light spots' from rotating light sources were explored in 1972 in the well-known studies by Ginzburg [10] and Bolotovskii and Ginzburg [11] (see also [12 – 14]). It was shown in [10, 11] that in some cases the speed of 'light spots' can exceed the speed of light by many orders of magnitude. It is of interest that superluminal 'light spots' can be used to experimentally check SRT [20–22]. Superluminal 'light spots' result in the emergence of a virtual charge that runs along the conducting surface (for example, a metal mirror). This superluminal virtual charge induces Vavilov–Cherenkov radiation [10, 11]. This phenomenon has been recently applied in practice [46, 47]; corresponding references can also be found in reviews [23, 24].

2.3 Superluminal electron 'spots'. Gyrocons, bermutrons, lasertrons

The speed of a spot from a rotating electron beam or from an electromagnetic beam at a sufficiently long distance from the source can be arbitrarily large, also including the case $v > c$. This phenomenon has been applied in practice for more than 60 years.

As early as 1940, Neiman proposed a generator in which an electron beam performs a conical motion and hits the slit of a ring-shaped super-high-frequency (SHF) waveguide [48]. If the electron beam speed coincides with the speed of electromagnetic waves in the waveguide, generation can occur. In 1956, L A Rivlin (1922–2013) (see a publication about him in [49]) performed the generation of low-power SHF radiation using a ring-shaped waveguide with a slit into which the superluminal electron 'spot' was directed along a conical trajectory [50]. Rivlin was employed at that time at the 'classified' research institute Post Office Box No. 17, currently the Istok research and production company [49], and, unfortunately, his study [50] published as an internal report remained virtually unknown.

In the late 1970s, Budker (1918–1977) and coauthors used the same mechanism to develop a gyrocon, a superpower SHF generator with a superluminal 'spot' from a beam of relativistic electrons [51, 52]. Gyrocons can operate not only in the generation mode (continuous or pulse) but also in the modes of signal amplification and signal frequency multiplication. Gyrocons were improved at the Budker Institute for Nuclear Physics of the Siberian Branch of the Russian Academy of Sciences by applying a constant magnetic field that follows the rotating electron beam [53]. Also developed in the Budker Institute for Nuclear Physics were so-called lasertrons. In the device, a ring-shaped photo cathode illuminated by a light beam circularly running with the signal frequency was used to generate an electron beam circularly running with the signal frequency and directed into a wave resonant ring after being accelerated to relativistic energies [54].

Superluminal 'spots' have also been studied in the USA. In 1946, six years after the submission of application [48], the so-called bermutron was patented [55], which insignificantly differs from the gyrocon. The bermutron was produced as late as 1965 [56], nine years later than Rivlin's experiments [50]. As Tallerico notes in his report [57], there is some confusion in using the terms 'gyrocon' and 'bermutron'. To avoid misunderstanding, the term 'gyrocon' is now applied to amplifiers or SHF radiation generators, while the term 'bermutron' refers to SHF-radiation frequency multipliers [57]. We note that Tallerico and Wilson designed the lasertron in 1982 [58].

It is of importance for the problem that we discuss here that superluminal 'spots' can be used to create the image of an object (dot, line, or a two-dimensional or three-dimensional body) that moves with a subluminal or superluminal speed, or even with a speed that exceeds c/n in an optical medium. This observation conceptually enables performing a number of experiments where the shape and 'rotation' angles of rapidly moving images can be observed.

3. Apparent shape and speed of moving bodies

When an observer watches a moving body (with the naked eye or using a video camera), the body shape and dimensions are displayed to that observer in a distorted way. The reason is that the image is formed on the retina of the eye or the light-sensitive photo camera plane by the light signals that arrive at the same moment of time, but the signals actually come from different points of an extended body with retardation. The farther the part of a moving body is located from the observer, the longer the light signal travels. Due to the retardation effect, signals that come simultaneously to the observer were emitted by different points of the body at different moments of time. This does not cause any distortions in the apparent shape of the body if the body is at rest with respect to the observer. But if the body is in motion, the observer sees a distorted image, whose dimensions and shape can have little in common with the real ones. The apparent image of the moving body is determined by the difference in retardation times of the signals that come to the observer from different points of the body. The Lorentz transformation is also involved in forming the apparent shape of the body, but Lorentz contraction is masked by the retardation effect and can be observed in a pure form in only a few special cases.

3.1 The case where the speed of signal propagation exceeds the speed of motion of a body

A moving body can be observed in different ways. For example, sensors can be placed along the path of the moving body to respond when the body passes by their positions. If can the sensors are placed sufficiently densely, the observer can have detailed information about the motion of the body and, in particular, its position and speed at any moment of time.

It is of interest that Mandelstam (1879–1944), in discussing measurement of length in his remarkable lectures on the theory of relativity [59], proposed a method that differs from that considered by Einstein. The method is as follows. A certain length scale, say a rod one-meter long, is sequentially placed on a body. An observer moves along the object to be measured and sequentially places the 1-meter ruler on it. If the body is at rest, this method yields the same results as the method for measuring length introduced by Einstein. But if the body is in motion, the results yielded by the method of measurement proposed by Mandelstam differ from those obtained by Einstein's method. The second method for measuring length proposed by Mandelstam is essentially the one for measuring the length of a moving body by the remote observer that was considered above. Measurements are performed using a light pulse that propagates along the body. The length of the body is in this case given by the speed of light times the duration that the light pulse propagates along the body [59].

The method where length is measured by 'applying a standard ruler' and the measurement is performed by a remote observer, and the method described above become even more similar in view of a comment Mandelstam made in passing. He noted that the wavelength of a well-explored spectral line can be used as such a standard, mentioning the red line of cadmium as an example (later, after Mandelstam's death, an international metrology congress adopted the orange line of krypton as the length standard). [1]

If some processes occur on a body in motion, for example, a mechanical pendulum or electric circuit oscillations, and the sensors located along the path of the body can determine the phase and amplitude of those oscillations at each point of the path, the observer can use sensor readings to determine the oscillatory process amplitude and phase at any point on the path and the oscillation frequency. An analysis of sensor readings enables a sufficiently detailed description of the motion of the body and the phenomena that occur on that body.

An alternative method of observation is that the observer is located at some point and watches the body in motion and the events occurring on that body from that selected point. Whenever this observation method is chosen, the observer can trust neither his/her own eyes nor even the readings of devices located at the observation point. In other words, to understand what is going on, adjustment should be made to the apparent picture of the event. This becomes necessary due to the finite speed of the signal: the signals emitted by the body under observation propagate with a finite speed, and hence all information arrives to the observation point retarded, and the longer the distance between the body and the observation point, the larger is this retardation. While the signal travels to the observer, the body keeps moving; when the signal arrives at the observer, the body is already located at a quite different point. However, the observer sees the body in the position from which the signal arrived. This implies that the observer does not see the body at the point where it was located at the moment of observation, and the time shift depends on the distance to the body, i.e., the time shift also varies with time. Therefore, if this observation method is selected, the apparent location of the body, its apparent velocity, and the time rate of the processes occurring on the body differ from those values that follow from observations made using sensors located along the path of the body. If the body in motion cannot regarded as a point-like object, i.e., is an extended object, then its shape perceived by a remote observer ('apparent shape') also differs from the real shape [60–66]. The real shape here means the shape that is determined by a set of sensors that respond when coming into contact with the body surface.

To clarify these conclusions, we consider the following example. We assume that an observer located on the x axis at the point $x = 0$ watches some body moving uniformly. We also assume that the dimensions of the body are small and we can therefore regard it as a luminous and uniformly moving point. Let the equation of motion of the body be $x = vt$. We assume that the speed of the body v is positive. Then, if $t < 0$, the body approaches the observer from the $x < 0$ side, while if $t > 0$, the body moves away from the observer along the positive-x semi-axis. At $t = 0$, the body passes the point of reference, i.e., the point $x = 0$, where the observer is located. We can assume that the observer is located not at the point $x = 0$ but at some rather small distance from it, away from the x axis. The observation of the moving body then does not perturb its motion.

We first consider the result of observations as the point approaches the observer ($t < 0$). Let at a moment t the observer receive a light signal from the body in motion. This signal was apparently emitted earlier, at some moment of time $t' < t$. The body was located at the moment of emission at the point $x' = vt'$, at a distance $|vt'|$ from the observer. If the moment t when the signal was received is a negative quantity, the moment of emission t' is even more negative and therefore $|vt'| = -vt'$. The emitted signal passes the distance $|vt'|$ during the time $|vt'|/c$. Hence, the relation between the moment of emission t' and the moment of arrival t can be written as

$$t = t' - \frac{vt'}{c}\,, \tag{1}$$

whence

$$t' = \frac{t}{1 - v/c}\,. \tag{2}$$

The body is located at the moment t at the point $x = vt$, while the observer sees it at the point where it was located at t', i.e., at the point

$$x'(t) = vt'(t) = \frac{vt}{1 - v/c}\,. \tag{3}$$

While the signal emitted at the point $x' = vt'$ was traveling to the observer, the body moved to the point $x = vt$. The apparent position of the body is displaced with respect to the position at which the body was located at the moment of observation. At t, the body was located at the point $x = vt$, and the apparent position lags behind that point. The displacement is

$$x'(t) - x(t) = \frac{v}{c}\,\frac{vt}{1 - v/c}\,. \tag{4}$$

The apparent position of the body lags behind its real position if the real position is that determined using the sensors located along the path of the body.

[1] When this article had been completed, the authors became aware that as early as 1871, Maxwell (1830–1879) proposed to use spectral lines for metrological purposes.

We now consider how the remote observer determines the speed of the body. The apparent position of the body at a moment t is given by Eqn (3). At t_1, the observer sees the body at the point

$$x'(t_1) = vt'(t_1) = \frac{vt_1}{1 - v/c}\,. \tag{5}$$

The difference between Eqns (5) and (3) yields the distance between apparent positions of the body at moments t and t_1. We thus obtain the apparent speed of the body:

$$v' = \frac{v}{1 - v/c}\,. \tag{6}$$

Here, v is the speed of the body determined using a clock and a ruler placed directly on the trajectory, and c is the speed of the signal that is used to perform observations (in this case, the speed of light).

We recall that Eqns (3) and (6) were derived in the case where the body approaches the observer. Then, as follows from Eqn (6), the apparent speed of the body can significantly exceed not only the real speed of the body v but also the speed of light. For example, let the speed of the body be half the speed of light, $v = c/2$. It then follows from Eqn (6) that $v' = c$, i.e., the apparent speed is equal to the speed of light. If the speed of the body exceeds half the speed of light, the apparent speed becomes larger than the speed of light. As a result, the observer sees the body approaching with a superluminal speed. However, the observer should not trust his/her eyes in this case. If a body approaches the observer with the speed of light or a higher speed, it only becomes visible after it has passed by the observer. The body overtakes all the signals it emits.

The arguments presented above show that if a remote observer watches a moving body, he/she does not see the body at the point where it is located at the moment of observation, and the speed determined by the observer differs from the speed of the body at the moment of observation. These differences originate from the retardation effect.

There is another specific feature in observing a moving body from a remote point: the passage of time on the moving body, as perceived by the observer, differs from that on the observer's clock. This follows from Eqn (2). We assume that a body is approaching the observer and some event whose duration is Δt occurs on that body. We let $\Delta t'$ denote the duration of the same event measured by the observer. Equation (2) then yields

$$\Delta t' = \Delta t\left(1 - \frac{v}{c}\right). \tag{7}$$

This change in duration is in no way related to the Lorentz transformation, because both time intervals, $\Delta t'$ and Δt, are measured in the same reference frame. The reason for this change is purely kinematical.

Equation (7) has an important implication. We assume that an oscillatory process with a period T_0 occurs on the moving body; for example, the brightness of the moving body varies according to a harmonic law with the period T_0. It then follows from Eqn (7) that the period T_0' for the observer has the form

$$T_0' = T_0\left(1 - \frac{v}{c}\right). \tag{8}$$

Therefore, the process frequency for the observer is ω_0' and

$$\omega_0' = \frac{\omega_0}{1 - v/c}\,, \tag{9}$$

where ω_0 is the frequency of the periodic process that occurs on the moving body.

Equation (9) describes the Doppler change in the frequency of radiation emitted forward with respect to the direction of motion of the body. The problem under consideration does not enable deriving the Doppler formula in the case where the direction of radiation makes some non-zero angle with the velocity of the body, because the observer is located on the straight line along which the body moves. [2] But if the observation point is located away from the trajectory, it is easy to show that the frequency of the signal emitted by the moving body and propagating at an angle ϑ with respect to the velocity is

$$\omega_0(\vartheta) = \frac{\omega_0'}{1 - (v/c)\cos\vartheta}\,. \tag{10}$$

Thus, if a remote observer watches a moving body, he/she sees the body not in the place where it was located at the moment of observation and, moreover, the speed of the body determined by the observer differs from the real speed (real speed means the distance the body passes during a unit of time). And, finally, the duration of the events occurring on the moving body differs from the real one for the observer. The real duration of events here has the following meaning. We assume that synchronized clocks are placed along the trajectory of the body. The start and end of an event is then determined using the clocks that the body passes by at the corresponding moments of time.

Observation of the moving body from a remote point yields the apparent position of the body (we do not see the body in the place where it was at the moment of observation), the apparent speed, and the apparent time rate of the events that occur on the body.

In the foregoing, we considered the case where the body is approaching the observer. If the body is moving away from the observation point, the relation between the moment t' when radiation from the signal started and the moment t when it was received and the formulas for the apparent location of the body $x'(t)$, the apparent velocity v', and the apparent duration of events $\Delta t'$ are

$$t' = \frac{t}{1 + v/c}\,, \tag{11}$$

$$x'(t) = vt'(t) = \frac{vt}{1 + v/c}\,, \tag{12}$$

$$v' = \frac{v}{1 + v/c}\,, \tag{13}$$

$$\Delta t = \Delta t'\left(1 + \frac{v}{c}\right). \tag{14}$$

[2] There is a simple physical explanation of why Eqn (6) for the apparent speed v' of the moving body and Eqn (9) for the frequency of radiation from the moving body ω' altered due to the Doppler effect depend on the speed of the body v in the same way. We assume that the body moves along the x axis with the speed v, and, after some spatial interval is passed, a bulb flashes for a short time and the observer measures the frequency of those flashes. The frequency of the flashes then increases by the factor v'/v.

Consequently, the frequency ω_0' of the oscillatory process that occurs on a body moving away is related to the apparent frequency ω_0 as

$$\omega_0 = \frac{\omega_0'}{1 + v/c}\,, \tag{15}$$

in accordance with the formula for the Doppler effect when the source is moving away from the observer.

The analysis presented above disregards the dimensions of the moving object. In essence, it was assumed by default that the body under observation is a luminous point. If the moving object is an extended body that has a specific shape, the remote observer sees a body whose shape is distorted. The reason for the distortion is the same as in the observation of a luminous point: retardation of light signals. We assume that the extended body in motion emits light, i.e., that its surface consists of many luminous points. Signals arrive from different points of the body with different retardation times, as a result of which the apparent shape of the body is distorted with respect to its real shape.

We begin with a simple case that explains how apparent orientation and dimensions of a moving body change as the body moves past the observer. We consider a rectangular coordinate system (x, y) and a thin rod ab (of length d) parallel to the y axis (Fig. 1). The rod moves along the x axis with a velocity v. End a of the rod moves along the x axis, and hence the equation of motion of point a can be represented as $x_{\text{a}} = vt$, $y_{\text{a}} = 0$. The equation of motion of the other end of the rod, point b, has the form $x_{\text{b}} = vt$, $y_{\text{b}} = d$. This end of the rod moves along a straight line parallel to the x axis at a distance d from the rod. Thus, the rod moves along the x axis with the velocity v and is oriented perpendicular to the velocity (i.e., parallel to the y axis). However, as shown below, the observer sees the rod not parallel to the y axis but tilted with respect to it at some angle α. The observer makes a conclusion about the orientation of the rod by viewing positions of its ends a and b.

The observer is located at point P at a distance L from the origin (point O). We let θ denote the angle between the x axis and line OP that connects the point of observation with the origin (see Fig. 1). We assume that the moving rod is watched using parallel beams that make an angle θ with the x axis (and hence with the velocity v of the rod).

At a moment t, the observer receives a signal emitted by a point of the moving body at an earlier moment t'. The relation between t and t' has the form

$$t = t' + \frac{R(t')}{c}\,, \tag{16}$$

where $R(t')$ is the distance between this point of the body and the observation point at the moment t'. We refer to Eqn (16) as the retardation equation. In receiving the signal from the moving body at the moment t, the observer sees this body at the position where it was located at an earlier moment t'. The retardation equation relates these two moments of time.

We can now derive the retardation equations for points a and b, i.e., for the ends of the moving rod:

$$t = t_{\text{a}}' + \frac{1}{c}\,(L - v t_{\text{a}}' \cos\theta)\,, \tag{17}$$

$$t = t_{\text{b}}' + \frac{1}{c}\,(L - v t_{\text{b}}' \cos\theta + d \sin\theta)\,. \tag{18}$$

At the moment t, a signal from point a of the moving rod comes to the observer. This signal was sent at the moment t_{a}' that is defined by Eqn (17). The coordinate of point a of the rod at the moment of emission was $x_{\text{a}}' = v t_{\text{a}}'$. Therefore, the observer sees point a of the rod not at the point with the coordinates $x = vt$, $y = 0$, where it is located at the moment of observation, but at the point from which the signal came, i.e., at the point $x_{\text{a}}' = v t_{\text{a}}'$, $y = 0$. At the same moment t, a signal from point b of the moving rod also arrives at the observer. This signal was emitted at the moment t_{b}' that is determined by Eqn (18). Consequently, the observer sees point b of the rod not at the point with coordinates $x = vt$, $y = d$, where it is located at the moment of observation, but at the point from which the signal came, i.e., at the point with coordinates $x_{\text{b}}' = v t_{\text{b}}'$, $y = d$. Because $t_{\text{a}}' \neq t_{\text{b}}'$ in general, we have $x_{\text{a}}' \neq x_{\text{b}}'$. If the observer is positioned as shown in Fig. 1, we have $x_{\text{a}}' > x_{\text{b}}'$. The observer sees that the rod is not parallel to the y axis, although in reality the rod, when moving, remains parallel to the y axis. The observer sees that the rod is 'tilted': end b of the rod lags behind end a. Equations (17) and (18) can be used to determine the lag $x_{\text{a}}' - x_{\text{b}}'$:

$$x_{\text{a}}' - x_{\text{b}}' = v\,(t_{\text{a}}' - t_{\text{b}}') = \frac{v}{c}\,\frac{d \sin\theta}{1 - (v/c)\cos\theta}\,. \tag{19}$$

The observer sees that the rod ab is tilted with respect to the y axis at some angle α such that

$$\tan\alpha = \frac{x_{\text{a}}' - x_{\text{b}}'}{d} = \frac{v}{c}\,\frac{\sin\theta}{1 - (v/c)\cos\theta}\,. \tag{20}$$

However, in the picture that the observer sees, the rod ab is not only 'tilted' by the angle α, as is asserted in some studies (see, e.g., [67, 68]), but is also stretched. Indeed, one end of the rod is always located on the line $y = 0$, while the other end is located on the line $y = d$. Therefore, any change in the orientation of the rod implies a change in its length.

It was assumed in the example above that the observer is located sufficiently far from the region where the body moves, and the dimensions of the rod are rather small. A more detailed analysis [66] shows that the observer sees the rod oriented perpendicular to the velocity as bent. However, this bending can be disregarded if the rod is rather short.

We now consider the case where the moving rod is oriented parallel to the velocity. Let ends a and b of a straight

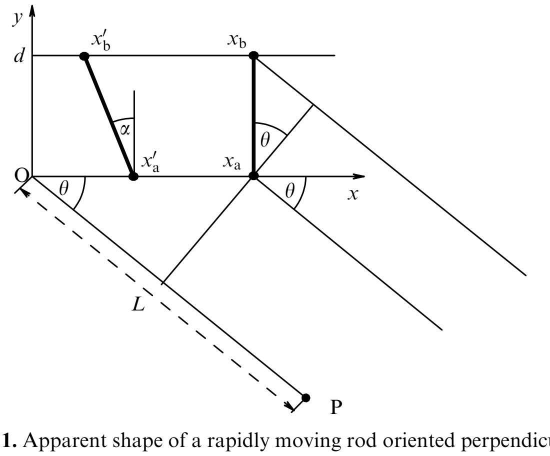

Figure 1. Apparent shape of a rapidly moving rod oriented perpendicular to its velocity.

rod of length l be placed on the x axis. The rod moves along the x axis with a velocity v. The coordinates x_{a} and x_{b} of rod ends a and b can be represented as functions of time:

$$x_{\mathrm{a}} = vt\,, \quad x_{\mathrm{b}} = vt - l\,. \tag{21}$$

Evidently, l is the rod length in the reference frame in which the observer is located.

We assume that the observer is located at a sufficiently long distance L from the region where the rod moves and watches the motion of the rod in such a way that the lines of sight make an angle θ with the x axis (and with the velocity of the rod v). The corresponding setup is shown in Fig. 2.

If the observer looks at the rod at a moment t, he/she sees the front end of the rod (point a) and its rear end (point b) at positions that correspond to respective earlier moments of time t'_{a} and t'_{b}. The values of these earlier moments of time follow from the retardation equations:

$$t = t'_{\mathrm{a}} + \frac{L - vt'_{\mathrm{a}}\cos\theta}{c}\,, \tag{22}$$

$$t = t'_{\mathrm{b}} + \frac{L - (vt'_{\mathrm{b}} - l)\cos\theta}{c}\,. \tag{23}$$

In looking at the rod in motion at a moment t, the observer sees the front end of the rod (point a) at $x'_{\mathrm{a}} = vt'_{\mathrm{a}}$, and its rear end (point b) at $x'_{\mathrm{b}} = vt'_{\mathrm{b}} - l$. This shift is caused by retardation.

Because the observer does not see the rod ends at the positions where they were located at the moment of observation, the rod length as seen by the observer is no longer l. We let l' denote the apparent length of the rod. Obviously,

$$l' = x'_{\mathrm{a}} - x'_{\mathrm{b}} = vt'_{\mathrm{a}} - vt'_{\mathrm{b}} + l\,. \tag{24}$$

Substituting the values t'_{a} and t'_{b} found from retardation equations (22) and (23) in this formula, we arrive at

$$l' = \frac{l}{1 - (v/c)\cos\theta}\,. \tag{25}$$

If retardation is taken into account, the rod length l' seen by the observer does not coincide with the real length l. If the angle θ at which observation is performed does not exceed $90°$ (just this setup is displayed in Fig. 2), then the apparent length is larger than the real one. If the observation is made at $90°$, $l' = l$. The apparent length of the rod coincides in this case with its real length.

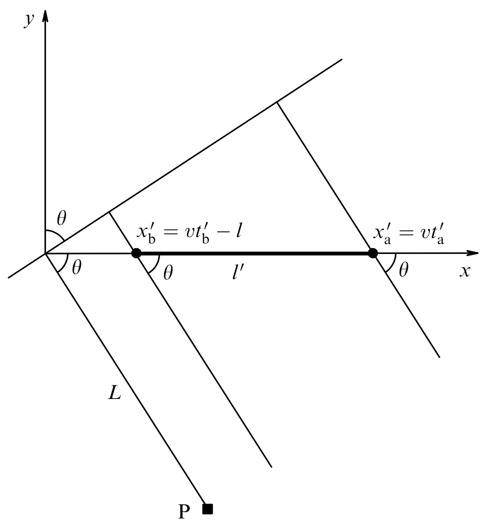

Figure 2. Apparent shape of a rapidly moving rod oriented parallel to its velocity.

It should be kept in mind that all the quantities in Eqn (9) are measured in the same reference frame, namely, in the one where the rod with the length l moves with the velocity v. The rod length l can be expressed in terms of its length l_0 in the reference frame in which it is at rest:

$$l = l_0\sqrt{1 - \frac{v^2}{c^2}} = l_0\sqrt{1 - \beta^2}\,, \tag{26}$$

where $\beta = v/c$. Substituting Eqn (26) in Eqn (25) for the apparent length of the rod, we arrive at

$$l' = \frac{l_0\sqrt{1 - \beta^2}}{1 - \beta\cos\theta}\,. \tag{27}$$

(Figure 2 only shows the length l' that is seen by the observer at rest. The rod length l is not displayed because its image partially overlaps with the image of l'.)

We now compare Eqns (26) and (27). Both formulas contain the rod length l_0 in the rest frame. Equation (26) yields the rod length in the reference frame in which the rod moves with the velocity v. Equation (27) presents the apparent length of the same rod as seen by the observer. If the observation angle is $90°$, both quantities are the same. However, for other observation angles, they are significantly different. For an observer who uses the described way of observation, the Lorentz contraction is obscured due to retardation; at some observation angles, the apparent length of the rod does not contract but, quite the opposite, increases as the speed increases. For example, let $\theta = 0$ (see Fig. 2). This implies that the observer is located on the x axis in front of the approaching rod. We can say that the measurement is made 'head-on'. Equation (27) then takes the form

$$l' = \frac{l_0\sqrt{1 + \beta}}{\sqrt{1 - \beta}}\,. \tag{28}$$

The faster the rod approaches the observer, the longer it looks (rather than shorter). Some authors refer to this result as "invisibility of the Lorentz contraction."

The reason for this difference is the finite speed of light and retardation of signals related to it, in conjunction with the selected method of observation. The length of the rod is defined in SRT as the distance between its ends a and b when the coordinates of points a and b are measured at the same time. If a remote observer watches the motion of the rod, he/she sees different points of the rod at the positions that correspond to different moments of time. For example, in the problem of motion of rod ab oriented parallel to the velocity, which we consider here, at a moment t the observer sees the front rod end, point a, at the position x'_{a} that corresponds to a moment of time t'_{a}. At the same moment t, the observer sees the rear rod end, point b, at the position x'_{b} that corresponds to the moment of time t'_{b}; in the setup displayed in Fig. 2, we have $t'_{\mathrm{b}} \neq t'_{\mathrm{a}}$.

As was noted above, if the moving rod is observed at an angle of $90°$, Eqns (26) and (27) coincide, i.e., the observer 'sees' the rod as Lorentz-contracted. In this case, $t'_{\mathrm{b}} = t'_{\mathrm{a}}$, i.e., the observer sees the rod ends at the positions that correspond to the same moment of time.

We have considered the apparent shape of the moving rod in two cases: the rod oriented parallel or perpendicular to the velocity of motion. Results of this analysis enable us to determine the apparent shape of a moving body that has edges both parallel and perpendicular to the velocity vector of motion; for example, the apparent shape of a cube in motion oriented in the corresponding way. The issue of the apparent shape of the cube is discussed in Section 3.3; here, we consider a simpler example.

Let a shape that is a square in the rest frame move along the x axis of a rectangular reference frame. Let one side of this figure be located on the x axis. We let v denote the velocity of the motion of the figure. Let the square side length in the rest frame be l_0. The two sides of the square that are parallel to the x axis, i.e., parallel to the velocity, then have the length $l = l_0(1-\beta^2)^{1/2}$. This is a consequence of the Lorentz contraction. The length of the square sides that are perpendicular to the velocity does not change. Thus, in our reference frame, it is not a square that moves but a rectangle, and the ratio of its sides depends on the velocity of motion.

We now assume that the motion of this rectangle is watched by a remote observer located in the same plane with the square and positioned such that the line of sight makes an angle θ with the x axis (as shown in Figs 1 and 2). The observer sees a moving parallelogram because the sides of the rectangle perpendicular to the x axis are 'tilted', i.e., the ends that are located farther from the observer appear to be lagging. This conclusion can be formulated differently by saying that the side parallel to the x axis, which is located farther from the observer, is perceived as lagging behind and hence the lines that connect both these sides are no longer perpendicular to the x axis. The angle α of deviation from their real direction follows from Eqn (20):

$$\tan\alpha = \frac{v}{c}\,\frac{\sin\theta}{1-(v/c)\cos\theta}\,.$$

The length of these line segments also changes (increases) because the heights of the visible shape (parallelogram) and the initial rectangle are the same. The apparent length l' of those sides that are parallel to the x axis are determined by Eqn (27):

$$l' = \frac{l_0\sqrt{1-\beta^2}}{1-\beta\cos\theta}\,, \tag{29}$$

where $\beta = v/c$.

If the observation angle is $\theta = 90°$, the last two formulas can be simplified. The apparent length l' of those sides of the square that are parallel to the velocity is nothing but their Lorentz-contracted length in the rest frame. The deviation angle α (apparent deviation angle) of those sides that are perpendicular to the velocity is given by the formula

$$\tan\alpha = \frac{v}{c}\,. \tag{30}$$

If $\beta = v/c$ is sufficiently small, we can set $\tan\alpha = \sin\alpha = \alpha = v/c$. In this very particular case, the moving square looks as if it remains a square; the length of its sides does not change, but it is 'rotated' as a whole by a small angle $\alpha = v/c$. This very particular case is a basis for the incorrect assertion that the observer sees a moving body 'rotated' by a velocity-dependent angle. There is nothing like this in the general case. The change in the apparent shape of a moving body does not reduce to its 'rotation'. We have shown that if there are edges on the outer surface, some of which are parallel to the velocity of motion and some perpendicular to it, the edges of these two types change in different ways. The length of the edges parallel to the velocity changes in an involved way that is affected by both Lorentz contraction and retardation. They do not change their direction, however, and remain parallel to the velocity of motion, while the edges perpendicular to the velocity of motion change both their length and orientation.

3.2 The case where the signal propagation speed is less than the speed of a body

The analysis presented in Section 3.1 applies when the speed of the body v does not exceed the speed c with which the signal propagates. If c is the speed of light in the vacuum, this requirement is always satisfied because the speed of a material body cannot be faster than the speed of light. But if the body moves in a refracting medium whose refractive index is greater than unity, the speed of light in such a medium is less than the speed of light in the vacuum, and the body can move with a speed that exceeds the speed of light in that medium. An electron emitting Vavilov–Cherenkov radiation [1–9] is an example of such a superluminal body. A case can be considered where a supersonic jet is observed, and the observer uses detectors of acoustic signals. The speed of the body then exceeds the speed of the signal used for observations. The observation of a moving body has in this case a number of specific features, which we now discuss.

As previously, we consider the uniform motion of a body along the x axis. The equation of motion of the body, as in Section 3.1, is $x = vt$, but this time the speed of the body v exceeds the speed of the signal c that is used for observations. Here, c can denote either the speed of light in a refracting medium or the speed of sound in air if acoustic devices are used for observations. The observer is located, similarly to the previous problem, at the point $x = 0$.

The body in motion continuously emits signals (light, radio waves, or sound). If the speed of the body exceeds the speed of the signal, the body overtakes all the waves it emitted. All the signals lag behind the body. The observer 'only' sees the body when it moves past him. The first signals arrive to the observer at $t = 0$. The observer first sees the body that flew past him/her and then starts receiving the signals that the body had emitted in approaching the observer. Signals arrive first from the nearest points on the path, followed by the signals from more distant points. The observer sees (or hears, if acoustic signals are detected) that the source of the signals moves along the x axis in the negative direction. The moment t' when the signal was emitted and the moment t when it was received are related by the formula

$$t' = -\frac{t}{v/c-1}\,, \qquad t > 0\,. \tag{31}$$

This formula shows that the signal, no matter how it was emitted, only arrives to the observer at $t > 0$, i.e., after the body moved past the observation point. We also note that as t increases, the moment t' when the signal was emitted shifts to the past. If the observer watches the sequence of events on the moving body, he/she sees those events in the reversed time sequence: from later events to increasingly earlier ones. Consequently, the body was located at the moment of emission at the point

$$x' = vt' = -\frac{vt}{v/c - 1}\,, \qquad t > 0\,. \tag{32}$$

The body moves past the observer and goes away in the positive direction of the x axis, while the observer sees that the body is located on the negative segment of the x axis and, moreover, moves in the opposite direction. The speed of motion is given by the formula

$$v' = \frac{\mathrm{d}x'}{\mathrm{d}t} = -\frac{v}{v/c - 1}\,. \tag{33}$$

This moving source of sound (in the problem of supersonic motion) or light (in the problem of superluminal motion) shapes the image of the body on the path segment where it moved toward the observer.

Concurrently with these signals, the observer receives signals from the points located on the positive semiaxis $x > 0$. The body was located at a moment $t' > 0$ at the point vt', and a signal is sent from that point to the observer. This signal arrives to the observer at a moment t, the values t and t' being related by the formula

$$t = t' + \frac{vt'}{c}\,, \tag{34}$$

whence

$$t' = \frac{t}{1 + v/c}\,. \tag{35}$$

The relation between t and t' at the receding stage, which proves to be the same for both the subluminal and superluminal cases, is described by Eqn (11).

Thus, if the observer watches the motion of an object whose speed exceeds the speed of the signal, he/she sees the following picture. Until the moment when the object moves past the observation point (moment $t = 0$), the observer receives no signals and therefore does not see the object. The reason for this situation is that the object of observation overtakes all the signals it emits. At $t = 0$, the moving body flies past the observer, and from that moment on, the observer starts receiving signals from two radiation sources. Both sources recede from the observer: one in the positive direction of the x axis and the other in the negative direction. The position of these sources is given by Eqns (32) and (33). These sources are images of the moving body. We can say that if the speed of a uniformly moving body exceeds the speed of the signal, the observer sees no image until some moment, and two images after that moment.

We note that a method closely related to the analysis presented above is used in classical electrodynamics to describe fields of a charged particle in motion. We mean the description of the moving-charge field based on the Liénard–Wiechert potentials. It is known that an electromagnetic field can be expressed in terms of potentials: a vector potential $\mathbf{A}(x, y, z, t)$ and a scalar potential $\varphi(x, y, z, t)$. The electric field $\mathbf{E}$ and the magnetic field $\mathbf{H}$ are expressed in terms of these potentials as

$$\mathbf{E} = -\operatorname{grad}\varphi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}\,, \quad \mathbf{H} = \operatorname{rot}\mathbf{A}\,. \tag{36}$$

The Liénard–Wiechert potentials provide a description of the fields created by a moving charge. We assume that the equation of motion of a moving charge is

$$\mathbf{r}_q = \mathbf{r}_q(t)\,. \tag{37}$$

The charge of the particle is denoted as q. The Liénard–Wiechert potentials can then be represented in the form

$$\mathbf{A}(x, y, z, t) = \frac{q\mathbf{v}}{|R - \mathbf{v}\mathbf{R}/c|}\bigg|_{t'}\,, \tag{38}$$

$$\varphi(x, y, z, t) = \frac{q}{|R - \mathbf{v}\mathbf{R}/c|}\bigg|_{t'}\,, \tag{39}$$

where $\mathbf{r} = (x, y, z)$ are coordinates of the observation point, $\mathbf{R} = \mathbf{r} - \mathbf{r}_q(t)$ is the vector directed from the charge to the observation point, $R = |\mathbf{R}|$ is the distance between the charge and the observation point, and $\mathbf{v}(t) = \mathrm{d}\mathbf{r}_q(t)/\mathrm{d}t$ is the velocity of the charge at a moment t. The right-hand sides of Eqns (38) and (39) are evaluated at the moment t'. The value of t' can be found from the formula

$$t = t' + \frac{1}{c}\left|\mathbf{r} - \mathbf{r}_q(t')\right|. \tag{40}$$

Equation (40) is fully equivalent to relation (1) that was our starting point in analyzing specific features of distant observation of moving bodies. A difference between Eqns (1) and (40) (of no significance for us) is that the first formula refers to the particular case of uniform motion, and the second pertains to the general case (37). If the observer determines the field of a moving charge, Eqns (37) and (38) for the Liénard–Wiechert potentials can be used; however, the time in these formulas is not the moment of observation t but an earlier moment t'. At the moment t', the charged particle at the point $\mathbf{r}_q(t')$ emits a signal that arrives at the point of observation at the moment t. The field at the moment t is determined by the position of the particle at the earlier moment t'.

Regarding the retardation equation (40), it is asserted in Landau and Lifshitz's textbook *The Classical Theory of Fields* [69] that if the speed of the charge does not exceed the speed of light, Eqn (40) has only one root. As we have seen, in the case of superluminal speed, this equation can have two (as in the observation of a superluminal body described above) or more roots. If this is the case, the sum over all roots should be taken in Eqns (38) and (39) for the potentials in the right-hand side.

3.3 Observed shape of a moving cube

We now use the apparent 'rotation' of a rapidly moving cube as an example to discuss the implications of Sections 3.1 and 3.2. This is an easily comprehensible illustration of the large number of formulas quoted in Sections 3.1 and 3.2. Following our studies [70, 71], we take the cube to be a die with unit edge. This enables us to distinguish different faces of the cube. We assume further that an observer at rest sees the die at a very small angle or, in other words, is located at a rather long distance from it. We also assume that the observer lacks stereoscopic vision and only uses one eye. The image he/she sees does not differ from that obtained using a photo camera (on a film or a charge coupled device matrix).

Figure 3a shows a die that faces the observer at rest with its 'six' face and moves past the observer with a velocity $v \ll c$. The velocity v is perpendicular to the line that connects the die and the observer, and the observer is located on this line, i.e.,

$\theta = 90°$. The image of the die for the observer at rest obviously coincides with its real location in space.

Figure 3b shows the same die that is actually oriented in space in absolutely the same way as in Fig. 3a, i.e., it faces the observer with the 'six' and moves in the same direction but this time with a relativistic speed $v = 0.786\,c$. As follows from Eqn (26), the image of the side with the 'six' face contracts for the observer by the factor $\gamma = 1/(1 - v^2/c^2)^{1/2} \simeq 1.62$, to make 0.616, but is not 'rotated'. The image of the 'three' face, which was not visible to the observer in the setup of Fig. 3a is now 'rotated', as follows from Eqn (20), by an angle α ($\tan\alpha = v/c$). The projection of the image of the 'three' face, which is parallel to the line connecting the die and the observer, is

$$\sin\alpha = \sin\left(\arctan\frac{v}{c}\right) = \frac{v/c}{1 + (v/c)^2} = 0.616\,. \tag{41}$$

Thus, if $v = 0.786\,c$, the observer sees the 'six' and 'three' faces as two identical rectangles. The ratio of the vertical and horizontal dimensions of the image of each face is 1 : 0.616. Although the image of the 'six' face is not 'rotated' with respect to the observer and the image of the 'three' face is 'rotated' by the angle $\alpha = \arctan(v/c) = \arctan 0.786 \simeq 38°$, if $\theta = 90°$, the observer does not notice that the die's shape is distorted. The reason is that the remote observer sees the die at a very small angle and cannot discern its 3D shape.

If the observer could see the upper face (with the 'one') or the lower face (with the 'five') of the die, he/she would find that they are distorted in accordance with Eqns (20), (29), and (30), and the rectangles have transformed into parallelepipeds. However, the upper and lower sides are obscured for the observer by the 'six' and 'three' faces. In a similar way, a person standing cannot see the upper and lower sides of a wardrobe that is higher than the person's height. However, if a rapidly moving empty cube with wire edges is watched instead of a die, all distortions of its shape become visible.

We now consider the maximum possible extent to which the shape of a rapidly moving die can be distorted as $v \to c$. We then have $\tan\alpha = 1$ and hence $\alpha = 45°$. Thus, the image of the 'three' face cannot 'rotate' by more than 45°. Terrell's [36] and Penrose's [37] studies show, however, that the image of the cube can 'rotate' by 90°. This error in [36, 37] is related to the fact that as is shown in Section 3.1, these studies failed to correctly incorporate the effect of different retardation of light emitted by different parts of the die (cube). As a result of this flaw, as $v \to c$, the 'rotation' of the image of the 'three' face should be $\sin\alpha = 1$, i.e., $\alpha = 90°$ according to [36, 37]. It might be expected that for $v \ll c$, $\alpha \ll 90°$ and $\sin\alpha \simeq \tan\alpha$, the results in [36, 37] and this paper should coincide. But this is so only as regards the 'rotation' by the angle α of the image of the 'three' face (Fig. 3b). As follows from [36, 37], the image of the 'six' face should also rotate by the angle α, while, according to our study, it does not 'rotate' at all and only experiences relativistic contraction. In other words, as shown in Section 3.1, the results in [36, 37] do not imply that the three-dimensional image of a die (cube) is seen by the observer at rest as deformed.

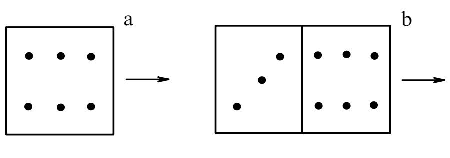

Figure 3. Apparent shape of a rapidly moving die: (a) $v \ll c$, (b) $v = 0.786\,c$.

The optical phenomena we have considered are helpful in understanding the fallacy of a paradox proposed by Gamow regarding the image of a rapidly moving streetcar (see Section 5) based on application of the "Terrell–Penrose effect."

Many unjustified assertions have been published in relation to the "Terrell–Penrose effect." For example, a conclusion was made by Weisskopf [67, 68] that a moving square (oriented in space as the square considered in Section 3.1 [see Eqns (20), (29), and (30)]) can rotate by an angle $\alpha = 180°$, which is not possible.

As regards the analysis of the rotation and deformation of the moving sphere image, this problem is significantly more complicated than determining the shape of a moving cube image, and it should be considered separately.

Material objects cannot move in the vacuum with a superluminal speed. However, superluminal (light or electron) 'spots' can be used to simulate motion of such an object. For example, superluminal 'spots' can be used to visualize the motion of eight faces of a cube with a speed $v > c$. Rather complex effects involving the distortion of dimensions, shape, and speed of the images of moving objects, which were discussed in Section 3.1, can manifest themselves in these cases. This problem requires a separate detailed analysis, which is beyond the scope of this paper.

4. Discovery and exploration of optical phenomena related to the finiteness of the speed of light: a brief history

Optical phenomena related to the finite speed of light propagation were observed for the first time almost 350 years ago, and the history of how this effect has been explored, being very exciting and instructive, involves the names of famous physicists and astronomers. Observation of these phenomena in laboratory only became feasible with the invention of lasers. Astronomers were the first to study these phenomena: the time that light takes to arrive to a terrestrial observer over immense distances is rather large. During that time, the emitting star moves a long distance away from the position where light was emitted.

4.1 Stellar aberration in classical physics

The stellar aberration phenomenon was discovered more than 400 years ago [32, 34, 72, 73]. In the last quarter of the 17th century, R Hooke (1635–1703) and astronomers J-F Picard (1620–1682) and J Flamsteed (1646–1719) discovered a deviation in the positions of stars with a yearly period [29]. An explanation for this mysterious phenomenon was found by J Bradley (1693–1772), an Oxford astronomer, who conducted observations beginning in 1725 [29]. Bradley concluded that while light travels from a star, Earth has time to displace in orbiting the Sun, and the direction of its displacement varies during a year. Bradley published his results [74] in 1728. In particular, as was noted in [29], Bradley actually showed that in a particular case where the direction of light propagation from the star is orthogonal to the direction of Earth's orbital velocity $\mathbf{v}$, the stellar aberration angle (the angle of apparent variation in the direction to the star) can be represented as

$$\sin\theta = \frac{v}{c}\,. \tag{42}$$

Apart from this, in [74] Bradley used observational data to calculate that the speed of light is 10,210 times greater than Earth's orbital speed, a value that virtually coincides with the modern value of c. The experiments in [74] have been considered in monographs on the history of physics [29, 75]. Generically, if the direction of light propagation from a star makes an angle φ with $\mathbf{v}$, the stellar aberration angle [29] is expressed as

$$\sin\theta = \frac{v}{c}\sin\varphi\,. \tag{43}$$

The terms 'aberration' and 'Bradley's new theory of aberration' were introduced in 1730 by Bologna astronomer and mathematician E Manfredi (1674–1739) [76].

It is known [28, 77] that the corpuscular (emission) theory of light proposed by Newton (1643–1727) was generally adopted in the 18th century [78], while the wave theory of light proposed by Hooke [79] and Huygens (1629–1695) [80] was not very popular due to Newton's high authority. However, in the late 18th century and early 19th century, when the wave theory of light was developed [28, 77] by Young (1773–1829) and Fresnel (1788–1827), the emission theory, which had difficulties in explaining light diffraction and interference phenomena, fell into deep crisis. Physicists concluded that a special optical medium should exist, the so-called luminiferous aether, in which light waves can propagate both in normal optical media and in a void. Because Eqn (43) is valid for the propagation of photons (light corpuscles in the terminology of that time), many expressions for stellar aberration have been proposed. The aberration angle depended on whether the radiation source or the receiver move with respect to the aether and what the properties of the aether itself are; there were many various models of the 'luminiferous aether' [81]. But because this problem is of interest only to researchers exploring the history of 'luminiferous aether', we abandon this subject.

4.2 Einstein's aberration theory

As was noted in the Introduction, in his paper "On the Electrodynamics of Moving Bodies" (Zur Electrodynamik bewegten Körper) [27], which was foundational for SRT, Einstein considered the phenomenon of relativistic aberration. The title of paragraph 7 in [27] is "Theory of Doppler's Principle and of Aberration" (Theorie des Dopplerschen Prinzips und der Aberration) [27, 82, 83]. Einstein first derives a formula for the relativistic Doppler effect:

$$\mathrm{v}' = \mathrm{v}\,\frac{1 - v/c\,\cos\varphi}{\sqrt{1 - v^2/c^2}}\,, \tag{44}$$

where v and v' are the respective optical radiation frequencies in the inertial reference frames (IRFs) at rest K and in motion K$'$.[3] Next Einstein derives a formula for relativistic aberration:[4]

$$\cos\varphi' = \frac{\cos\varphi - v/c}{1 - (v/c)\cos\varphi}\,. \tag{45}$$

As shown in [27], if $\varphi = 90^\circ$

$$\cos\varphi' = -\frac{v}{c}\,. \tag{46}$$

We note that the sign in the right-hand side of Eqn (46) depends on the direction in which the body moves, which is why signs in the corresponding formulas quoted in different publications are different. Here (see Section 3 and below), we consider the corresponding equation with the plus sign:

$$\cos\varphi' = \frac{v}{c}\,. \tag{47}$$

Consequently, the relativistic aberration angle is $\theta = \varphi - \varphi'$. Because the values of θ obtained from Eqns (43) and (45) mismatch, fierce discussions arose in relation to this issue between proponents and critics of SRT [32]. An especially irreconcilable stance was taken by R Tomaschek (1895–1966) and his research advisor P Lenard (1862–1947), who asserted that the relativistic aberration phenomenon is in contradiction to the SRT basic concepts and therefore disproves the SRT. This discussion also involved Einstein [88] and Thirring (1888–1976) [89].

Although Einstein derived formula (45) for relativistic aberration in [27], he drew no further conclusions regarding the apparent image of an object in motion. Einstein writes [27, § 4]: *We envisage a rigid sphere of radius R.... A rigid body which, measured in a state of rest, has the shape of a sphere, and therefore in a state of motion — viewed from the stationary system — has the shape of an ellipsoid of revolution with the axes* $R(1 - v^2/c^2)^{1/2}$, *R, R.... For* $v = c$ *all moving objects — viewed from the 'stationary' system — shrivel up into plane shapes.*

The physics community needed 55 years to comprehend that the observer at rest sees the image of a body in motion not flattened in the direction of motion (as follows from the Lorentz transformations) but rather heavily distorted; this issue was considered for the first time by Weisskopf [67], who indicated that the shape of the body in a state of motion is distorted. Exact solutions are obtained in this paper.

However, neither this circumstance nor the misprint specified above in any way deduct from the immense significance of Einstein's work [27] on SRT in particular and modern physics as a whole. Paper [27] is arguably the publication most quoted over the last 100 years.

4.3 Distortion of the image of a body in motion viewed by an observer at rest

Aksenov [90] (born in the second quarter of the 19th century, died after 1918; according to unconfirmed data, he graduated from the First Kharkov Gymnasium in 1866 (see more about him in [91, 92])), a little-known Russian philosopher, was

[3] We note that a regretful misprint was made in [27] in the important particular case $\varphi = 0$ (see [84]): "We see that in contrast to the universal opinion, at $v = -\infty$, $\mathrm{v} = \infty$." Apparently, the following was meant: "*at* $v = -c$, $\mathrm{v} = \infty$". This misprint was reproduced in two translations into Russian ([27] and [82]) and it was as late as 1967 that it was found by astronomer Gimmelfarb (b. 1919) [84]. Einstein's work [27] was so highly respected that even in its translation [83] that appeared five years later than publication [84], this misprint was again reproduced. However, it was delicately noted in the footnote on page 111 [83]: "The author presumably meant $v = -c$ here."

[4] We recall that Einstein defined relativistic aberration as a tilt of a light beam emitted by a point in motion and detected by an observer at rest. We note that Gerasimovich (1889–1937) [85], a second-year student of Kharkiv University and the director-to-be of the Pulkovo observatory, was the first astronomer to analyze the effect of relativistic aberration on the apparent location of stars. Gerasimovich won the Pavlovskii prize for his study [85]. The problem of rotation of a moving body image was considered more than 50 years later by Terrell [36] and Penrose [37].

apparently the first to have considered, albeit on a qualitative and purely intuitive level, how the shape of a body in motion is distorted for an observer at rest. The most interesting result in [90] is that Aksenov, long before the development of SRT, hypothesized that the dimensions, shape, and angular position of bodies in motion are subject to changes. It seems at first glance that Aksenov made a gross error asserting that bodies elongate in the direction of motion, since by 1896 the Fitzgerald–Lorentz contraction had already been discovered. However, as shown in [62–64, 66, 93, 94] and Section 3 of this paper, in the most general case, the body should look elongated due to retardation of light emitted by different areas of the body in motion; it is only if the body is observed at a small angle that the relativistic Lorentz–Fitzgerald contraction exactly compensates this elongation [62–64, 66, 93, 94].

We note that Aksenov was rather knowledgeable about mathematics and geometry: in 1883, he translated into Russian a textbook on geometry [95] by J Petersen (1839–1910), a Danish mathematician; however, study [90] does not contain a single formula.

The problem was quantitatively considered in 1924 by Lampa (1868–1938) [96]. [5] Lampa showed that the image of a moving rod in an IRF at rest is seen as rotated by the relativistic aberration angle [96]. We note that Einstein only indicated that a light beam emitted by a moving pointlike source must be observed in an IRF at rest rotated by the relativistic aberration angle [27]. However, Lampa did not show that the shape of the body in motion is distorted. Unfortunately, Lampa's results [96] were not noticed at that time and actually had no impact on progress in understanding the issue of the apparent image of a body in motion. We note that as shown in Fig. 3, Lampa was inaccurate in [96]: the image of the moving rod is rotated by the angle that follows not from Eqn (47) but from Eqns (20) and (27); Lampa did not take into account that the rotation angle of the rod image depends on its orientation with respect to its velocity.

Some authors argue that McCrea (1904–1995), an Irish astrophysicist, was the first to consider the phenomenon of relativistic aberration and retardation of light signals in 1952 [107] (see [108] for more about him). However, an analysis of [107] shows that it only contains an examination of whether the Lorentz–Fitzgerald contraction is real. It is of interest that in his younger years McCrea actively studied the effect of stellar aberration on astronomical observations of solar and stellar atmospheres [109–113].

Finally, this issue was addressed in 1957 by Terrell, a researcher at the Los Alamos National Laboratory [114]. This study was unpublished, and physicists only became aware of its results in 1959 [36, 115]. Terrell found that due to relativistic aberration and the differences in retardation of light emitted by different parts of an object in motion, the shape of that object remains unchanged for an observer at rest, while the object is seen as rotated by the relativistic aberration angle [36, 114, 115]. The same results were independently obtained by Penrose [37]. This phenomenon was named the Terrell effect or the Terrell–Penrose effect.

However, as shown in Section 3, Terrell's [36, 114, 115] and Penrose's [37] results are not universal and are only valid for a particular case of so-called orthogonal projection (or if the body in motion is viewed within a small solid angle). It is also shown in Section 3 that in the general case of viewing a body in motion, the results in [36, 37, 114, 115] are only valid if $\tan(v/c) \simeq \sin(v/c)$.

This issue was subsequently explored in a large number of works [61, 66–68, 93, 94, 116–159] and presented in a number of educational courses (see, e.g., [160]).

Studies [144, 161] examined the relativistic transformation of the light beam power, and [162] explored the visual observation of objects moving with a superluminal speed. Problems related to superluminal motion are discussed in [23–25] and in Section 2.

5. Gamow paradox. Relativistic streetcar

We show in this section that the limited validity of Terrell's [36, 114, 115] and Penrose's [37] results for relativistic aberration effects and the absence in [36, 37, 114, 115] of effects related to the difference between the retardation time of light emitted by different parts of an extended body give rise to a rather impressive paradox formulated in 1961 by Gamow (1904–1968) [122]. Like any other paradox, the Gamow paradox sooner or later was to be resolved in a rational way. We offer an explanation based on the results in Section 3.

In 1940 Gamow published a popular-science book [163], the main character of which, Mr. Tompkins, a minor bank clerk, similarly to Alice in Lewis Carroll's book, gets into Wonderland. However, while Alice fell asleep on a river bank, Mr. Tompkins (grown bald, judging by the illustrations by Gamow and Hookham) fell asleep at a university lecture on the theory of relativity, to which he had accidentally dropped in. Mr. Tompkins awakes at a bus stop in Wonderland, an ancient English town, where, for unknown reasons, the speed of light is very small in comparison to the regular case, owing to which relativistic effects can be observed in everyday life. Mr. Tompkins, who gained some basic knowledge of SRT at the lecture, observes and fairly correctly (in line with the understanding in 1940) interprets relativistic effects. In particular, he sees a bicycle rider passing him by and his bicycle contracted in the direction of motion, and bicycle wheels transformed into ellipses with a ratio $\sim 2:1$ (judging by a picture in [163]).

Gamow's book [163] enjoyed great popularity; it was reprinted many times and translated into a number of languages. But then the year 1959 came, when the papers by Terrell [36] and Penrose [37] were published, and Gamow came to the conclusion that there was an error in his picture rendering the bicycle and bicycle rider in [163]. Gamow then published a new and very brief article [122] in which he took the results in [36, 37] into consideration and rendered a bicycle rotated due to the Terrell–Penrose effect, but this time without a bicycle rider rendered.

Gamow, one of the major physicists of the 20th century, who exhibited a highly developed physical intuition, noticed

[5] Anton Lampa was an Austrian experimentalist physicist and Mach's student (1838–1916). Rather scant and controversial data about his life and scientific research can be found in [97–104]. An ethnic Czech, he was a champion of the Germanization of Czechs. A fact that is best known in Lampa's biography is that he, from 1909 being a secretary of the Scientific Council of German Karl-Ferdinand University in Prague, actively participated in 1910 in inviting Einstein to the position of full professor. Less known is that Lampa and well-known mathematician G Pick (1859–1942) (Mach's student and colleague) were in a sense Einstein's co-authors: they issued a joint report [105] on research and teaching activities at the University (it is quoted in [106]). The fate of Einstein's co-authors of the report [105] was tragic: Germanophile Lampa, shocked by events in Germany and Austria, died in Vienna one month prior to the Anschluss by Germany, and Pick died in the Theresienstadt concentration camp.

and qualitatively described the paradoxical implications of the Terrell–Penrose effect [36, 37], which disagree with sound sense. Considering the motion of a streetcar in Wonderland in [122], Gamow noted: *In the case of a streetcar running along a straight track, the situation is more difficult, and the fact that the rear of the car can easily be seen while all the wheels follow the track cannot be explained in any normal way except by assuming that the entire car suffered an unusual sheer deformation.* In other words, two pairs of streetcar wheels 'rotated' due to the Terrell–Penrose effect cannot be placed on rails simultaneously. The rails do not move with respect to the observer at rest and therefore their image is not deformed. However, the 'rotated' images of the streetcar wheels are at different distances from the observer. Thus, the image of either (front or rear) pair of wheels can be placed on rails, but the image of the other pair then 'hangs in the air'. This is the essence of the Gamow paradox, showing that the Terrell–Penrose effect does not comply with reality.

This paradox had arguably impressed Gamow to the extent that he referred to one of the latest poems by Nikolai Gumilev (1886–1921) [164], where the passenger (the poem's main character) watches mystical scenes of the past and future of various countries through the window of a crazily racing Petrograd streetcar. Gamow's detailed biography [165] does not contain any mention of his work on relativistic aberration [122] published during his life in the USA (1934–1968). In principle, another paradox can be suggested that is close to Gamow's: in what way can the image of a streetcar wheel 'rotated' by an angle roll along a straight rail?

A resolution of Gamow's paradox is offered by the results in Section 3. The image of the streetcar side (for example, the right-hand one) that faces an observer at rest is seen shortened as a result of the Lorentz transformations, but not rotated. Consequently, the images of both pairs of wheels (front and rear) can simultaneously 'stand on rails'. At the same time, images of wheel axes look 'rotated', i.e., for an observer at rest, images of both the right-hand side wheels are somewhat ahead of the images of both left-hand wheels. The images of the wheels themselves, although seen as contracted in the direction of motion, do not look rotated and can therefore roll on rails. Apart from this, the observer sees the streetcar rear side and the rear rims of all four wheels. Hence, it follows from the results in Section 3 that there is no need "to subject the Gamow streetcar to an unusual sheer deformation."

6. Detecting the relativistic aberration effect and differences between retardation of light emitted by different parts of an extended body

Duguay, a researcher at Bell Laboratories, in 1971 used a fast Kerr shutter to take an image of the so-called light dumbbell, two short light pulses propagating in water [166]. A rotation of the 'light dumbbell' was observed in [166]. The results in [166] were discussed in [61, 167–172] and reported in more detail by Duguay in [173]. Ugarov (1922–1977), executive editor of *Physics – Uspekhi* and an associate professor at Moscow State Pedagogical University (see publication about him [174]), proposed the following experiment [168]: *Maybe it will be possible to create a light object consisting of eight points* (*eight pulses*) *occupying all of the vertices of a moving cube. A photo of this object could then illustrate rotation of the cube as a whole.*

Still, is there any chance to make a photo of the Lorentz contraction itself?

To take an image of the body that experiences the Lorentz contraction, one should know for comparison, in addition to the distance between two points that belong to the body in motion, its proper length at rest, i.e. to know the distance between those points when they are at rest or move with a nonrelativistic speed. How can this be done? Is it possible at all to make the required image by working only with light pulses? There are no answers as yet to these questions.

7. Applying formulas for relativistic aberration and difference between retardation of light emitted by different parts of an extended body to the calculation and interpretation of some physical phenomena

7.1 Thomas precession

Thomas precession (TP) [175] is a relativistic kinematic phenomenon: the direction of the spin of an elementary particle or the rotation axis of a macroscopic mechanical gyroscope, as well as the coordinate axis of a reference frame moving along a curvilinear trajectory rotate (experience precession) with respect to the axes of the laboratory IRF. This method of calculating TP was proposed in [70, 71, 176, 177] by one of the authors of this paper. The method is based on the solid angle theorem [178, 179] proved in the early 1950s by A Yu Ishlinskii, which can be formulated as follows [180] (see also [181]): if a preferred axis in a solid body that has three degrees of freedom describes a closed conical surface in the process of motion and the projection of the body's angular velocity on that axis is zero, then, after the axis returns to its initial state, the body turns out to be rotated with respect to this axis by an angle that is numerically equal to the solid angle of the described cone. Translational motion of the axis does not matter.

We consider a body that moves along a circular trajectory in a plane orthogonal to the line connecting the object rotation center and the observer. Because the observer is located rather far from the circular trajectory, the angle at which the object is viewed at any point of its trajectory is $\theta \simeq 90^\circ$. Figure 4 shows a die that moves along a circular trajectory. According to the results in Section 3, the die image displayed in Fig. 4 corresponds to $v = 0.786\,c$ (the Lorentz factor $\gamma \simeq 1.62$). We note that because the results in Section 3 were not known at the time of writing [70, 71], the values of v and γ were calculated using the results in [36, 37] and were therefore incorrect. An observer at rest sees the image of a specific die face (the rear one in relation to the velocity of the die, for example, between the 'three' and 'two' faces) at an angle α [see Eqn (20)]. As the die moves along the circular trajectory, the direction of its velocity changes, and the angle α describes a cone with the vertex angle 2α. The solid angle contained within the cone is numerically equal to the area that is bounded on the unit-radius sphere by the generator of the cone whose vertex is located in the center of the sphere [182]. We can then easily derive the formula that relates the solid angle to the cone vertex angle:

$$\xi = 4\pi \sin^2 \frac{\alpha}{2} = 2\pi(1 - \cos\alpha) = 2\pi\left(1 - \frac{1}{\gamma}\right), \qquad (48)$$

where $\gamma = 1/(1 - v^2/c^2)^{1/2}$ is the Lorentz factor. As follows from [70, 71, 177], the body rotation angle due to TP after one

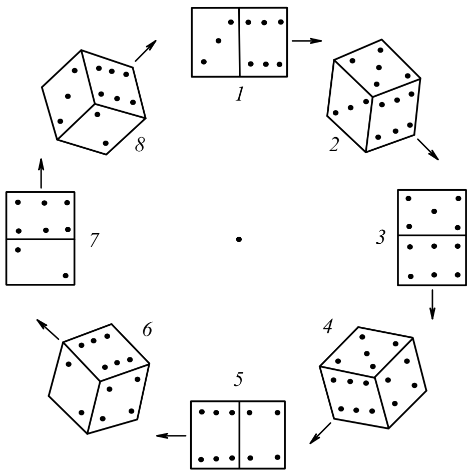

Figure 4. Apparent shape of a die rapidly moving along a circular trajectory. The orbital speed of the die is $v = 0.786\,c$.

revolution around the circle is

$$\zeta = 2\pi\,\frac{\Omega_{\mathrm{T}}}{\omega} = 2\pi\left(1 - \frac{1}{\gamma}\right). \tag{49}$$

A comparison of Eqns (48) and (49) shows that $\zeta \equiv \xi$, i.e., the angle of body rotation due to TP is equal to the solid angle that describes the image of the axis comoving with the body along the circular trajectory according to the Ishlinskii theorem. This conclusion holds when the actual change in the body orientation angle is equal to the change in the rotation angle of the image of the body moving relativistically along a curvilinear trajectory, which is viewed in the laboratory IRF. Thus, TP can be interpreted as resulting from a formal application of the Ishlinskii theorem to the solid angle that corresponds to the observed rotation of the solid body image in the process of body motion relative to an observer at rest. This seems to be the simplest and most easily comprehensible method of calculating TP [177].

7.2 Transverse Doppler effect

The transverse Doppler effect (TDE) was predicted by Einstein [27] and experimentally observed more than three decades later [183–185]. TDE has been measured by now with a very high accuracy, because due to this effect, absorbing-gas molecules in a laser containing a nonlinearly absorbing cell (NAC) whose velocity is orthogonal to the optical axis of the NAC cause a shift of the power peak frequency in the laser; despite being very small, this shift can nevertheless be measured with a very high accuracy [186].

Many studies have been published where attempts were made to derive TDE in a purely classical approach not involving Lorentz transformations (see, e.g., [187, 188]). TDE is detected when a receiver of radiation (an observer at rest) is located on the line that is perpendicular to the velocity of the source and connects the observer with the source. The main argument of SRT critics is that if a moving source is located in front of a receiver, the radiation emitted by the source at an angle of 90° to its velocity would never reach the receiver [187, 188]. SRT critics assert that radiation can reach the receiver only if it was emitted by the source not at an angle of 90° but at a somewhat larger angle (measured with respect to the direction of the source velocity). Then, in the special case $v \ll c$, a small regular Doppler effect occurs that diminishes the radiation wavelength by a factor of $\sim 0.5v^2/c^2$. An illusion occurs in this way that TDE can be explained in not only a qualitative but also a quantitative way without involving SRT. However, it should be taken into account that even in this particular case, the radiation frequency diminishes not by a factor of $\sim 0.5v^2/c^2$ but by $\sim v^2/c^2$, an observation that clearly demonstrates the fallacy of the explanation of TDE proposed in [187].

However, in what way does radiation from a moving source emitted at an angle of 90° to its velocity reach the receiver? An explanation is provided in [189]: radiation is emitted before the source is located in front of the receiver. Another, more physical explanation can also be suggested. Due to the relativistic aberration effect, the radiation emitted by the source at an angle of 90° is viewed by an observer at rest, in accordance with Eqn (46), at the angle $\arccos(-v/c) \sim 0.5v^2/c^2$, and, from the observer's standpoint, the source is located just in front of him/her.

7.3 Transverse Fizeau effect

As was shown by one of the authors [190], if an optical medium moving with a velocity $\mathbf{v}$ is hit from the vacuum by a plane wave whose wave vector is perpendicular to the interface (or, equivalently, is directed along the normal to the interface), the wave vector does not alter its direction in the moving optical medium. Therefore, radiation does not undergo a transverse shift. However, if the radiation aperture is finite, as, for example, is the case with a laser beam, the beam can be decomposed according to the Huygens principle [80] into a sum of waves whose wave vectors are oriented in different directions. Calculation of the transverse shift of the beam is in this case a challenging problem. However, if Eqn (46) is used, an observer at rest would come to the conclusion that at $v \ll c$ the beam propagates in the moving optical medium at the angle $\sim 0.5v^2/c^2$ to the normal and, if the thickness of the moving medium is d, the shift of the beam exiting the medium is $0.5dv^2/c^2$.

7.4 Observation of rapidly moving space objects (quasars)

A number of astronomical observations, if analyzed in a superficial manner, indicate that some space objects move with a superluminal speed. This applies primarily to sensational observations, dating back to the late 1960s through early 1980s, of the quasar 3C279 located at a distance of 3 bn light years from Earth [191–202]. Observations in [191–202] showed that the quasar 3C279 consists of two sources of radio waves of different brightness flying away from each other, and the speed of their separation, as estimated in [194, 195, 201, 202], exceeds the speed of light by an order of magnitude. A number of explanations have been proposed to explain this phenomenon. It was hypothesized, for example, in [193, 202] (see also review [203]) that the motion of objects consisting of real tachyons had been observed. (We note that such hypotheses are still being discussed [204–207].) Developed in [199] was even a theory of superluminal magnetic dipole radiation generated by charged particles that move with ultrarelativistic velocities in the magnetic field of a quasar. It was hypothesized recently [208] that the observed superluminal speed is a consequence of light echo.

However, already in studies [194, 195, 201, 209], conceptually correct conjectures were formulated that the observed phenomenon is an effect of a relativistic illusion rather than real superluminal motion. The phenomenon has been explained in a comprehensive way by Bolotovskii [64, 66] (see also Section 3). It was shown in [64, 66] that if an object moves with a relativistic speed relative to an observer, and its velocity is perpendicular or at least approximately perpendicular to the line connecting the object and the observer, the speed of the object is perceived by the observer in a quite realistic way. But if the velocity makes an acute angle with that line and the object approaches the observer, both longitudinal and transverse components of the velocity are perceived by the observer as superluminal; this phenomenon is referred to in [64, 66] as the 'apparent velocity'. This phenomenon results from the relativistic aberration effect and the difference between retardation times of light emitted by different parts of an extended body [64, 66, 209], which emerges due to relativistic contraction of linear dimensions of the moving object along the direction of its motion.

We note that the illusion of superluminal expansion is produced in absolutely the same way by an object with variable dimensions (a pulsar or supernova) that rapidly approaches at an acute angle to the direction of the observer.

7.5 Relativistic aberration and difference between retardation of light emitted by different parts of an extended moving object illuminated by an external source

We have considered moving self-luminous objects. However, objects are illuminated (irradiated) in some cases by an external source. A situation like this occurs, for example, if the position of an object is localized using laser ranging or spacecraft telemetry [210–212]. It was shown in [212] that if a laser beam is reflected directly from the spacecraft surface, the reflection angle changes due to the motion of the spacecraft. If a corner reflector or a 'cat's eye' is installed on the spacecraft, the beam reflection angle is the same as the incidence angle regardless of the spacecraft motion [212].

Such an aberration can be detected in the laboratory in making holograms of rapidly moving bodies [147, 148, 213–215]. If monochromatic radiation is used for this, the reconstructed image of the body is blurred. Therefore, radiation with a very small coherence length is used to make such holograms. Holograms of rotating fan blades were made in [147, 148, 213–215]. A specific feature of a holographic image as well as any other interference pattern is that the image of the largest details of the object is controlled by the narrowest interference bands. Therefore, to properly reconstruct the image of an object (in this case, fan blades), the finiteness of the speed of light [214], the relativistic aberration effect, the relativistic contraction of the length of a moving body, and TDE [147, 148, 215] have to be taken into account.

Holographic images of a light-pulse wave front were obtained in [216], and those of the probe light pulse in a plasma created by laser radiation, in [217]. Computer simulation of the visual image of a relativistic object was reported in [218].

8. Conclusion

We have shown here that the effects related to the difference between retardation of light emitted by different parts of rapidly moving extended object that comes simultaneously to an observer at rest affect the object image viewed by the observer to no less an extent and in some cases to an even greater extent than the relativistic contraction of its longitudinal dimensions. The shape and spatial orientation of the image of a rapidly moving object can only be determined in a correct way if both effects, i.e., light retardation and relativistic contraction, are taken into consideration.

We summarize the main results of this review.

1. We have shown that the well-known studies by Terrell [36] and Penrose [37] failed to correctly address the effects related to the difference between retardation of light emitted by different parts of an extended object and arriving simultaneously to an observer at rest. We have derived formulas that correctly describe the combined effect that relativistic aberration and the difference between retardation of light emitted by different parts of an extended object have on the object image. We have shown that a cube moving with a speed that is comparable to the speed of light is seen by an observer at rest not 'rotated' by some angle [36, 37] but deformed. The image of the cube front face is apparently shorter, as follows from the Lorentz transformation, but not rotated by any angle.

2. We have considered so-called superluminal motions of light and electron 'spots' whose speed is $v > c$, and motion in an optical medium with a refractive index n with a speed $v < c$ such that $v > c/n$, a motion that can be considered superluminal for this medium. We have shown that these phenomena enable simulation of the motion of an object with a superluminal speed.

3. We have presented a history of studies of relativistic aberration. The phenomenon of relativistic aberration, which has no relation whatsoever to SRT, was discovered by Hooke and astronomers Picard and Flamsteed in the late 17th century. The concept of relativistic aberration as a distortion of the image of a rapidly moving body was introduced in 1896 on a purely intuitive level by the philosopher Aksenov [90]. In 1905, Einstein defined the relativistic aberration as a tilt of the light beam emitted by a moving body as viewed by an observer at rest [27]. Lampa showed in 1924 that the image of a rapidly moving rod in an IRF at rest is rotated by the relativistic aberration angle [96]. This question was analyzed in 1957–1959 by Terrell [36] and Penrose [37], but some inaccuracies plagued papers [36, 37].

4. We have considered Gamow's so-called relativistic streetcar paradox. The 'Gamow streetcar' is shown to clearly demonstrate the fallacy of Terrell's [36] and Penrose's [37] results: if the streetcar image is rendered in accordance with the formulas in [36, 37], images of its front and rear wheels cannot simultaneously stand on the image of the rails. Our results, in contrast, enable obtaining a self-consistent image of the moving streetcar and rails.

5. We have considered the methods used to record images of fast moving objects that are based on taking photos of the motion of light pulses in a medium with a known refractive index.

6. We have analyzed how the formulas for relativistic aberration and the difference between the retardation of light emitted by different parts of an extended rapidly moving body, which are derived in this review, can be used to calculate and interpret a number of physical phenomena.

The paper had already been completed when the authors became aware of study [219], where distortion of the shape of moving bodies is considered.

The authors are grateful to V I Pozdnyakova for her great help in the work, Barbara Garea Moreda, G Mamedov, E A Romanets, F R Tangherlini, and P A Shilyagin for assistance in searches for hard-to-get publications. The authors are especially grateful to the referee for a number of helpful comments that enabled improving the presentation of material. The paper was supported by the projects included in government task Biofoton-2 No. 0035-2014-0018.

Proofreading notes. The paper was being prepared for publication when study [220] appeared. In the early hours of 17 August 2017, the LIGO and Virgo gravitation observatories detected the GW170817 gravitational wave that was generated from the merging of two objects: neutron stars with masses of about $1.1 M_{\odot}$ and $1.6 M_{\odot}$ located at a distance of about 100 mln light years from Earth. In a mere 2 seconds, the Fermi and INTEGRAL telescopes detected a bright burst of gamma-ray radiation that arrived from the same region of the sky. Astronomers also detected radiation in the optical, ultraviolet, infrared, X-ray, and radio wave bands. The intensity of radiation in the radio wave and X-ray bands kept growing during the next 150 days after merging, to rather rapidly vanish later. An array of terrestrial VLA and GBT telescopes and 10 VLBA telescopes were used to measure the observed position of the source twice, 75 and 230 days after the burst. The source center was found to be displaced during the time that elapsed between the two measurements by almost 0.003 angular seconds, a value that corresponds to motion with the apparent speed $v' = 4c$. The authors of [220] have qualitatively correctly interpreted this result as a manifestation of a nonzero tilt angle of the source velocity with respect to the line of sight; according to estimates made in [220], this angle is $\theta = 20°$. A formula that relates the apparent speed of a moving object v' and its real speed v, which can be derived from Eqns (6) and (25), is $v' = v[1 - (v/c)\cos\theta]^{-1}$. Consequently, the real speed of the object is $0.84c$, being, obviously, less than the speed of light.

References

1. Cherenkov P A *C.R. Acad. Sci. USSR* **2** 451 (1934); *Dokl. Akad. Nauk SSSR* **2** 451 (1934); *Usp. Fiz. Nauk* **93** 385 (1967)
2. Vavilov S I *C.R. Acad. Sci. USSR* **2** 457 (1934); *Dokl. Akad. Nauk SSSR* **2** 457 (1934); *Usp. Fiz. Nauk* **93** 383 (1967)
3. Tamm I E, Frank I M *C.R. Acad. Sci. USSR* **14** 107 (1937); *Dokl. Akad. Nauk SSSR* **14** 107 (1937); *Usp. Fiz. Nauk* **93** 388 (1967)
4. Bolotovskii B M *Usp. Fiz. Nauk* **62** 201 (1957)
5. Bolotovskii B M *Sov. Phys. Usp.* **2** 165 (1959); *Usp. Fiz. Nauk* **67** 163 (1959);
6. Bolotovskii B M *Sov. Phys. Usp.* **4** 781 (1962); *Usp. Fiz. Nauk* **75** 295 (1961)
7. Ginzburg V L *Phys. Usp.* **39** 973 (1996); *Usp. Fiz. Nauk* **166** 1033 (1996); *Phys. Usp.* **44** 1037 (2001); *Usp. Fiz. Nauk* **171** 1091 (2001)
8. Bolotovskii B M *Phys. Usp.* **52** 379 (2009); *Usp. Fiz. Nauk* **179** 405 (2009)
9. Bolotovskii B M *Phys. Usp.* **52** 1099 (2009); *Usp. Fiz. Nauk* **179** 1161 (2009)
10. Ginzburg V L *Sov. Phys. JETP* **35** 92 (1972); *Zh. Ekp. Teor. Fiz.* **62** 173 (1972)
11. Bolotovskii B M, Ginzburg V L *Sov. Phys. Usp.* **15** 184 (1972); *Usp. Fiz. Nauk* **106** 577 (1972)
12. Bolotovskii B M, Ginzburg V L, in *Einshteinovskii Sbornik 1972* (Einstein Almanac 1972) (Ed. V L Ginzburg) (Moscow: Nauka, 1974) p. 212
13. Ginzburg V L *Applications of Electrodynamics in Theoretical Physics and Astrophysics* (New York: Gordon and Breach Sci. Publ., 1989); Ginzburg V L *Teoreticheskaya Fizika i Astrofizika. Dopolnitel'nye Glavy* (Theoretical Physics and Astrophysics. Complementary Chapters) (Moscow: Nauka, 1981) p. 211
14. Ginzburg V L *O Teorii Otnositel'nosti* (On Theory of Relativity) (Moscow: Nauka, 1979) p. 212
15. Strum L *Z. Phys.* **20** 36 (1923); Strum L *Naukovi Zapiski Kyivskoi Doslidnoi Kafedry* **2** 81 (1924)
16. Tangherlini F R "The velocity of light in uniformly moving frame", PhD Thesis (Stanford: Stanford Univ., 1958); reprinted in: *Abraham Zelmanov J.* **2** 44 (2009)
17. Recami E *Riv. Nuovo Cimento* **9** (6) 1 (1986)
18. Fröman P O *Vestn. Ross. Univ. Druzhby Narodov* **1** (1) 160 (1993)
19. Fröman P O *Arch. Hist. Exact Sci.* **48** 373 (1994)
20. Malykin G B *Phys. Usp.* **47** 739 (2004); *Usp. Fiz. Nauk* **174** 801 (2004)
21. Malykin G B *Tech. Phys.* **50** 1383 (2005); *Zh. Tekh. Fiz.* **75** (10) 135 (2005)
22. Malykin G B *Tech. Phys.* **52** 133 (2007); *Zh. Tekh. Fiz.* **77** (1) 138 (2007)
23. Malykin G B, Shcherbak E A "Sverkhsvetovye dvizheniya" ("Superluminal motions"), Preprint No. 1/2011 (Dnepropetrovsk: Dnepropetrovsk National Univ., 2011)
24. Malykin G B, Romanets E A *Opt. Spectrosc.* **112** 920 (2012); *Opt. Spektrosk.* **112** 993 (2012)
25. Malykin G B, Savchuk V S, Romanets E A *Phys. Usp.* **55** 1134 (2012); *Usp. Fiz. Nauk* **182** 1217 (2012)
26. Mantegna M *Proc. R. Soc. Lond. A* **471** 20140541 (2016)
27. Einstein A *Ann. Physik* **17** 891 (1905); Translated into Russian: *Sobranie Sochinenii* (Collected Works) Vol. 1 (Moscow: Nauka, 1965) p. 7
28. Whittaker E A *History of the Theories of Aether and Electricity* Vol. 1 (London: Nelson and Sons, 1951); Translated into Russian: *Istoriya Teorii Efira i Elektrichestva* Vol. 1 (Moscow-Izhevsk: RKhD, 2001); Whittaker E A *History of the Theories of Aether and Electricity* Vol. 2. (London: Nelson and Sons, 1953); Translated into Russian: *Istoriya Teorii Efira i Elektrichestva* Vol. 2 (Moscow-Izhevsk: RKhD, 2004)
29. Frankfurt U I, Frenk A M *Optika Dvizhushchikhsya Tel* (Optics of Moving Bodies) (Moscow: Nauka, 1972)
30. Herassimovitch B *Bull. Astronomique* **31** 385 (1914)
31. Joos G *Phys. Z.* **27** 1 (1926); Translated into Russian: *Usp. Fiz. Nauk* **6** 21 (1926)
32. Gimmel'farb B N *Usp. Fiz. Nauk* **51** 99 (1953)
33. Luk'yanov B N *Astron. Zh.* **30** 32 (1953)
34. Gimmelfarb B *Kvant* (4) 11 (1995)
35. Mamedov G *Relyativistskie Effekty ili zhe Sveto-Vremennye sdvigi?* (Relativistic Effects of Light-Time Shifts?) (Saarbrücken, Deutschland: LAMBERT Acad. Publ., 2015)
36. Terrell J *Phys. Rev.* **116** 1041 (1959)
37. Penrose R *Proc. Cambridge Phil. Soc.* **55** 137 (1959) Pt.1
38. Kaiser T R *Nature* **247** 400 (1974)
39. Tyapkin A A *Sov. Phys. Usp.* **17** 288 (1974); *Usp. Fiz. Nauk* **112** 735 (1974)
40. Bolotovskii B M *Oliver Heaviside* (Ed. V L Ginzburg) (Moscow: Nauka, 1985); *Oliver Heaviside* 2nd extended ed. (Ed. V L Ginzburg) (Moscow: LIBROKOM, 2012)
41. Heaviside O *The Electrician* **22** 23 (1888 – 1889); Heaviside O *The Electrician* **22** 83 (1888 – 1889); Heaviside O *The Electrician* **22** 147 (1888 – 1889); Heaviside O *The Electrician* **22** 458 (1888 – 1889); Heaviside O *Electrical Papers* Vol. 2 (New York: Macmillan and Co., 1894) p. 490
42. Schott G *Electromagnetic Radiation and the Mechanical Reactions Arising from It* (Cambridge: Univ. Press, 1912)
43. Ginzburg V L *Zh. Eksp. Teor. Fiz.* **10** 589 (1940)
44. Heaviside O *Electromagnetic Theory* Vol. 3 (New York: Dover Publ., 1950) p. 13
45. Frank I M *Izv. Akad. Nauk SSSR Ser. Fiz.* **6** 3 (1942)
46. Bolotovskii B M *Phys. Usp.* **48** 903 (2005); *Usp. Fiz. Nauk* **175** 943 (2005)
47. Arkhipov R M et al. *Quantum Electron.* **45** 590 (2015); *Kvantovaya Elektron.* **45** 590 (2015)
48. Neiman M S 'Electron beam generator', Inventor's Certificate 67770. Application dated 20.05.1940 No.308187 to the USSR People's Commissariat of Electric Engineering Industry USSR (1940)
49. Krokhin O N, Sigov A S *Quantum Electron.* **43** 502 (2013); *Kvantovaya Elektron.* **43** 502 (2013)

50. Rivlin L A, in *Trudy Nauchno-Issledovatel'skogo Instituta Ministerstva Radiotekhnicheskoi Promyshlennosti SSSR* (Proc. of the Research Institute of the USSR Ministry of Radio Engineering Industry) Issue 4 (33) (Moscow: Ministerstvo Radiotekhnicheskoi Promyshlennosti SSSR, 1956) p. 3
51. Budker G I et al. *Sov. At. Energy* **44** 459 (1978); *Atom. Energ.* **44** 397 (1978)
52. Budker G I et al. *Part. Accel.* **10** 41 (1979)
53. Zapryagaev I A et al., in *XVI Soveshchanie po Uskoritelyam Zaryazhennykh Chastits, GNTs RF Inst. Fiziki Vysokikh Energii, Protvino, 20–22 Oktyabrya 1998 g., Sb. Dokladov* (Report at the 16th Meeting on Accelerators of Charged Particles. Institute of High-Energy Physics. Protvino. 20–22 October 1998. Collection of Reports) Vol. 1 (Protvino: GNTs RF Inst. Fiziki Vysokhikh Energii, 1998) p. 72
54. Mikhailichenko A A "Super-high-frequency generator", Preprint No. 88-88 (Novosibirsk: Institute of Nuclear Physics, 1988)
55. McRae J W "Harmonic generation system", US Patent 2 408 437 (1946)
56. Kaufman I, Oltman G *IEEE Trans. Electron Devices* **12** 31 (1965)
57. Tallerico P J "The gyrocon: a deflection-modulated, high-power microwave amplifier", LA-6907 (Los Alamos: Scientific Laboratory of the Univ. of Los Alamos, 1977)
58. Tallerico P J, Wilson M T, US Patent 431-3072 (1982)
59. Mandelstam L I "Lektsii po fizicheskim osnovam teorii otnositel'nosti" ("Lectures on physical basis of the theory of relativity") (1933-1934), in *Polnoe Sobranie Trudov* (Complete Collection of Works) Vol. 5 (Ed. M A Leontovich) (Moscow: Izd. AN SSSR, 1950) p. 181. 8th Lecture. 10.03.1934; in *Akademik L I Mandelstam. Lektsii po Optike, Teorii Otnositel'nosti i Kvantovoi Mekhanike* (Academician L I Mandelstam. Lectures on Optics, Theory of Relativity, and Quantum Mechanics) (Ed. S M Rytov) (Moscow: Nauka, 1972) p. 168
60. Bolotovskii B M, Mikhal'chi E D *Kratk. Soobshch. Fiz.* (5) 35 (1976)
61. Bolotovskii B M, in *Einshteinovskii Sbornik 1980–1981* (Einstein Almanac 1980–1981) (Ed. I Yu Kobzarev) (Moscow: Nauka, 1985) p. 142
62. Moreda G B "O vidimoi forme sverkhsvetovykh tel" ("On the apparent shape of superluminal bodies"), Diploma Thesis (Scientific advisor B M Bolotovskii) (Moscow: Moscow State Pedagogical Univ., Department of Physics, 1979)
63. Moreda G B, Curbelo A, Volotovski B M *Revista Cubana Física* **6** (3) 3 (1986), in this publication the author's last name 'Bolotovskii' is misspelled as 'Volotovski'
64. Bolotovskii B M, in *Problemy Fizki i Astrofiziki. K 70-letiyu V L Ginzburga* (Problems of Physics and Astrophysics. On the 70th Anniversary of V L Ginzburg) (Eds L V Keldysh, V Ya Fainberg) (Moscow: Nauka, 1989) p. 24
65. Bolotovskii B M, Bykov V P *Sov. Phys. Usp.* **33** 477 (1989); *Usp. Fiz. Nauk* **160** (6) 141 (1989)
66. Bolotovskii B M, in *Einshteinovskii Sbornik 1986–1990* (Einstein Almanac 1986–1990) (Ed. I Yu Kobzarev) (Moscow: Nauka, 1990) p. 279
67. Weisskopf V F *Phys. Today* **13** (9) 24 (1960); Translated into Russian: *Usp. Fiz. Nauk* **84** 183 (1964)
68. Weisskopf V F *Physics in Twentieth Century* (Cambridge: MIT Press, 1972); Translated into Russian: *Fizika v Dvadtsatom Stoletii* (Moscow: Atomizdat, 1977)
69. Landau L D, Lifshitz E M *Teoriya Polya* (Theory of Fields) (Moscow: Nauka, 1973) Ch. VIII, § 68; Translated into English: *The Classical Theory of Fields* (Oxford: Pergamon Press, 1980)
70. Malykin G B *Phys. Usp.* **42** 505 (1999); *Usp. Fiz. Nauk* **169** 585 (1999)
71. Malykin G B *J. Appl. Math. Mech.* **63** 731 (1999); *Prikl. Matem. Mekh.* **63** 775 (1999)
72. Gimmelfarb B N *Priroda* (8) 28 (1951)
73. Gimmelfarb B *Zemlya Vselennaya* (1) 22 (1992)
74. Bradley J *Philos. Trans.* **35** 637 (1728)
75. Rosenberger F *Die Geschichte der Physik in grundzügen* Tl. 2 (Braunschweig: F. Vieweg und Sohn, 1884); Translated into Russian: *Istoriya Fiziki* (History of Physics) Pt. 2 (Moscow-Leningrad: ONTI, 1937) p. 181
76. Manfredi E "De Bononiensi scientiuarum et artium Instituto atque Academia Commentarii" *Bologna* **1** 599 (1730)
77. Kudryavtsev P S *Kurs Istorii Fiziki* (History of Physics: a Course) (Moscow: Prosveshchenie, 1982)
78. Newton I *Optics* 3rd ed., corr. (London: William and John Innys, 1721); Translated into Russian: *Optika* 2nd ed. (Translated into Russian and commented S I Vavilov) (Moscow: Gostekhizdat, 1954)
79. Hooke R *Micrographia* (London.: Martyn and Allestry, 1665)
80. Huygens Ch *Traité de la lumierè* (Leyde, 1690); Translated into Russian: *Traktat o Svete* (Treatise on Light) (Ed. V Frederiks) (Moscow-Leningrad: ONTI, 1935)
81. Lorentz H A *Lectures on Theoretical Physics, Delivered at the University of Leiden* I. *Aether Theories and Aether Models* (Ed. H Bremekamp) (London: Macmillan and Co., 1901 – 1902); Translated into Russian: *Teorii i Modeli Efira* (Eds A K Timiryazev, Z A Tseitlin) (Moscow: ONTI, 1936)
82. Einstein A *Ann. Physik* **17** 891 (1905); Translated into Russian: *Printsip Otnositel'nosti* (Principle of Relativity) (Eds V K Frederiks, D D Ivanenko) (Leningrad: ONTI, 1935) p. 133
83. Einstein A, in *Printsip Otnositel'nosti* (Principle of Relativity) (Compiled A A Tyapkin) (Moscow: Atomizdat, 1972) p. 97
84. Gimmelfarb B N *Usp. Fiz. Nauk* **91** 567 (1967)
85. Herassimovitch B *Bull. Astronomique* **31** 385 (1914)
86. Tomaschek R *Ann. Physik* **74** 136 (1924)
87. Lenard P, Schmidt F *Z. Tech. Phys.* **6** 81 (1925)
88. Einstein A *Über die spezielle und die allgemeine Relativitätstheorie* (Braunschweig: F. Vieweg, 1919); Translated into Russian: *Sobranie Sochinenii* (Collected Works) Vol. 1 (Moscow: Nauka, 1965) p. 530
89. Thirring H *Z. Tech. Phys.* **6** 561 (1925)
90. Aksenov M S *Transtsedental'no-kineticheskaya Teoriya Vremeni* (Transcendental Kinetic Theory of Time) (Kharkov: Parovaya Tipografiya i Litografiya Zilberberg, 1896); *Transtsedental'no-kineticheskaya Teoriya Vremeni* (Transcendental Kinetic Theory of Time) (Compiled, ed., commented S A Zhigalkin) (Moscow: Yazyki Slavyanskikh Kul'tur, 2011) p. 49
91. Zhigalkin S A, in Aksenov M S *Transtsedental'no-kineticheskaya Teoriya Vremeni* (Transcendental Kinetic Theory of Time) (Compiled, ed., commented S A Zhigalkin) (Moscow: Yazyki Slavyanskikh Kul'tur, 2011) p. 5
92. Malykin G B "Russkii filosof Mitrofan Aksenov i teoriya otnositel'nosti" ("Russian philosopher Mitrofan Aksenov and theory of relativity"), Tribuna UFN No. 115, http://ufn.ru/tribune/trib117.pdf
93. Field J, in *Fundamental Problems of High Energy Physics and Field Theory. Proc. of the XX Workshop of High Energy Physics and Field Theory, Protvino, Russia, 24–26 June 1997* (Eds I V Filimonova, V A Petrov) (Protvino: IHEP, 1997) p. 214
94. Field J H *Am. J. Phys.* **68** 367 (2000)
95. Petersen J "Methods and theories for the solution of problems of geometrical construction", in *String Figures, and Other Monographs* (New York: Chelsea Publ. Co., 1960); Translated into Russian: *Metody i Resheniya Geometricheskikh Zadach s Postroeniem bolee 400 Zadach* (Translated into Russian M S Aksenov, ed. A P Gruzintsev, foreword K A Andreev) (Kharkov: Tipografiya Zilberberga, 1883)
96. Lampa A *Z. Phys.* **27** 138 (1924)
97. "Mr. A. E. Munby" *Nature* **141** 586 (1938) Obituare
98. Holton G *Daedalus* **98** 636 (1968)
99. Kleitner A *Gesnerus* **32** 285 (1975)
100. Illy J *Isis* **70** 76 (1979)
101. Pais A *'Subtle is the Lord...' The Science and the Life of Albert Einstein* (Oxford: Oxford Univ. Press, 1982); Translated into Russian: *Nauchnaya Deyatel'nost' i Zhizn' Alberta Einsteina* (Albert Einstein's Research and Life) (Ed. A A Logunov) (Moscow: Nauka, 1989)
102. Wolters G *Mach I, Mach II. Einstein und die Relativitätstheorie. Eine Fläschung und Ihre Folgen* (Berlin: De Gruyter, 1987)
103. Goenner H et al. (Eds) *Einstein Studies* Vol. 7 *The Expanding Worlds of General Relativity* (Boston: Birkhäuser, 1999)

104. Chertanov M *Einstein* (Ser., The Life of Outstanding People, Issues 1718 (1518)) (Moscow: Molodaya Gvardiya, 2015)
105. Einstein A, Lampa A, Pick G, Report of College of Professors of the Faculty of Philosophy of the German Univ. in Prague
106. Klein M J, Kox A J, Schulmann R, in *The Collected Papers of Albert Einstein* Vol. 5 *Swiss Years. Correspondence, 1902–1914* (Princeton, NY: Princeton Univ. Press, 1994) doc. 400
107. McCrea W H *Sci. Proc. R. Dublin Soc.* **26** 27 (1952)
108. Mestel H, Pagel B E J *Biograph. Memoirs Fellows R. Soc.* **53** 223 (2007)
109. McCrea W H *Mon. Not. R. Astron. Soc.* **89** 423 (1929)
110. McCrea W H *Mon. Not. R. Astron. Soc.* **89** 718 (1929)
111. McCrea W H *Mon. Not. R. Astron. Soc.* **91** 836 (1931)
112. McCrea W H *Mon. Not. R. Astron. Soc.* **95** 80 (1934)
113. McCrea W H *Mon. Not. R. Astron. Soc.* **95** 509 (1935)
114. Terrell J "The clock 'paradox'", Los Alamos Document LADC-2842 (1957) unpublished paper
115. Terrell J *Bull. Am. Phys. Soc.* **4** 294 (1959); *Phys. Rev. Lett.* **116** 1014 (1959)
116. Rytov S M *Priroda* (4) 64 (1960)
117. Weinstein R *Am. J. Phys.* **28** 607 (1960)
118. Sherwin C W *Am. J. Phys.* **29** 67 (1961)
119. Ranninger J *Acta Phys. Austriaca* **14** 50 (1961)
120. Crowell A D *Am. J. Phys.* **29** 370 (1961)
121. Boas M L *Am. J. Phys.* **29** 283 (1961)
122. Gamow G *Proc. Natl. Acad. Sci. USA* **47** 728 (1961)
123. Sänger J *J. Br. Interplanet Soc.* **18** 273 (1961)
124. Atwater M A *J. Opt. Soc. Am.* **52** 184 (1962)
125. Yngström *Arkiv Fysik* **23** 367 (1962)
126. Jones R T *J. Franklin Inst.* **275** 1 (1963)
127. Durbin K A *Am. J. Phys.* **32** 639 (1964)
128. Oliver B M *IEEE Spectrum* **1** 87 (1964)
129. Kichenassamy S *Comptes Rendus* **260** 1581 (1965)
130. Sears F V *Am. J. Phys.* **32** 266 (1965)
131. Scott G D, Viner M R *Am. J. Phys.* **33** 534 (1965)
132. Guess A W *Phys. Rev.* **161** 1295 (1967)
133. Shirer P R, Bartel T W *Am. J. Phys.* **35** 971 (1967)
134. Moskowitz S *Sky Telescope.* **33** 290 (1967)
135. McGill N C *Contemp. Phys.* **9** 33 (1968)
136. Scott G D, van Driel H J *Am. J. Phys.* **38** 971 (1970)
137. Lang D W *Am. J. Phys.* **38** 1181 (1970)
138. Bhandari R *Am. J. Phys.* **38** 1200 (1970)
139. Mathews P M, Lakshmanan M *Nuovo Cimento B* **12** 168 (1972)
140. Hollenbach D *Am. J. Phys.* **44** 91 (1976)
141. Syrovatskii S I *Sov. Phys. Usp.* **19** 273 (1976); *Usp. Fiz. Nauk* **118** 545 (1976)
142. Bhandari R *Am. J. Phys.* **46** 760 (1978)
143. Hickey F R *Am. J. Phys.* **47** 711 (1979)
144. McKinley J M, Doherty P *Am. J. Phys.* **47** 309 (1979)
145. McKinley J M *Am. J. Phys.* **47** 760 (1979)
146. McKinley J M *Am. J. Phys.* **48** 612 (1980)
147. Abramson N *Appl. Opt.* **23** 4007 (1984)
148. Abramson N *Appl. Opt.* **24** 3323 (1985)
149. Sastry G P *Am. J. Phys.* **55** 943 (1987)
150. Sheldon E *Am. J. Phys.* **56** 199 (1988)
151. Suffern K *Am. J. Phys.* **56** 729 (1988)
152. Terrell J *Am. J. Phys.* **57** 9 (1989)
153. Sheldon E *Am. J. Phys.* **57** 487 (1989)
154. Phips T E *Am. J. Phys.* **57** 549 (1989)
155. Burke J R, Strode F J *Am. J. Phys.* **59** 912 (1991)
156. Hannay J H, Walters G D *J. Phys. A* **24** L1333 (1991)
157. Wolf K B *J. Opt. Soc. Am. A* **10** 1925 (1993)
158. Sorensen R A *Am. J. Phys.* **63** 413 (1995)
159. Hamdan N *Proc. Pakistan Acad. Sci.* **44** 121 (2007)
160. Ugarov V A *Spetsial'naya Teoriya Otnositel'nosti* (Special Theory of Relativity) (Moscow: Nauka, 1977)
161. McKinley J M *Am. J. Phys.* **47** 602 (1979)
162. Gron Ø *Lett. Nuovo Cimento* **23** 97 (1979)
163. Gamow G *Mr. Tompkins in Wonderland* (Cambridge: Cambridge Univ. Press, 1940); Translated into Russian: "Mister Tompkins v strane chudes", in Gamow G *Priklyuchenia Mistera Tompkinsa* (Mr.Tompkins's Adventures) (Moscow: Byuro Kvantum, 1993)
164. Gumilev N "Zabludivshiisya tramwai" ("Crazy streetcar"), in *Ognennyi Stolp* (Pillar of Fire) (Peterburg-Berlin: Petropolis, 1922) p. 36
165. Chernin A D *Phys. Usp.* **37** 791 (1994); *Usp. Fiz. Nauk* **164** 867 (1994)
166. Duguay M A *Am. Scientist* **59** 551 (1971); Translated into Russian: Duguay M *Usp. Fiz. Nauk* **109** 157 (1973)
167. Smorodinskii Ya A, Ugarov V A *Sov. Phys. Usp.* **15** 340 (1972); *Usp. Fiz. Nauk* **107** 141 (1972)
168. Ugarov V *Nauka Zhizn* (6) 49 (1973)
169. Ugarov V A, in *Einshteinovskii Sbornik 1973* (Einstein Almanac 1973) (Ed. V L Ginzburg) (Moscow: Nauka, 1974) p. 201
170. Malov N N *Kvant* (4) 10 (1974)
171. Pesin M A, Fabelinskii I L *Sov. Phys. Usp.* **19** 844 (1976); *Usp. Fiz. Nauk* **120** 273 (1976)
172. Sala K L *Phys. Rev. A* **19** 2377 (1979)
173. Duguay M A *Progress in Optics* Vol. 14 (Ed. E Wolf) (Amsterdam: North-Holland Publ. Co., 1976) p. 161
174. Aronov R A et al. *Sov. Phys. Usp.* **21** 180 (1978); *Usp. Fiz. Nauk* **124** 359 (1978)
175. Malykin G B, Permitin G V 'Tomas precession' in *Fizicheskaya Entsiklopediya* (Encyclopedia of Physics) Vol. 5 (Moscow: Rossiiskaya Entsiklopediya, 1998) p. 123
176. Malykin G B *Izv. Ross. Akad. Nauk. Mekh. Tverd. Tela* (4) 187 (2000)
177. Malykin G B *Phys. Usp.* **49** 837 (2006); *Usp. Fiz. Nauk* **176** 865 (2006)
178. Ishlinskii A Yu *Mekhanika Spetsial'nykh Giroskopicheskikh Sistem* (Mechanics of Special Gyroscopic Systems) (Kiev: Izd. AN USSR, 1952)
179. Ishlinskii A Yu *Mechanics of Gyroscopic Systems* (Jerusalem: Israel Program for Scientific Translations, 1965); Translated from Russian: *Mekhanika Giroskopicheskikh Sistem* (Mechanics of Gyroscopic Systems) (Moscow: Izd. AN SSSR, 1963)
180. Zhuravlev V F *Osnovy Teoreticheskoi Mechaniki* (Primer of Theoretical Mechanics) (Moscow: Fizmatlit, 1997)
181. Malykin G B, Kharlamov S A *Phys. Usp.* **46** 957 (2003); *Usp. Fiz. Nauk* **173** 985 (2003)
182. Vygodskii M Ya *Spravochnik po Elementarnoi Matematike* (Reference Book on Elementary Mathematics) (Moscow: Nauka, 1982)
183. Ives H E, Stilwell G R *J. Opt. Soc. Am.* **28** 215 (1938)
184. Otting G *Phys. Z.* **40** 681 (1939)
185. Ives H E, Stilwell G R *J. Opt. Soc. Am.* **31** 369 (1941)
186. Letokhov V S, Chebotaev V P *Nelineinaya Lazernaya Spektroskopiya Sverkhvysokogo Razresheniya* (Nonlinear Laser Super-High Resolution Spectroscopy) (Moscow: Nauka, 1990)
187. Semikov S A *Ballisticheskaya Teoriya Ritsa i Kartina Mirozdaniya* (The Ritz Ballistic Theory and Image of Universe) (Nizhny Novgorod, 2009); *Ballisticheskaya Teoriya Ritsa i Kartina Mirozdaniya* (The Ritz Ballistic Theory and Image of Universe) 2nd ed. (Nizhny Novgorod: Stimul ST, 2010); *Ballisticheskaya Teoriya Ritsa i Kartina Mirozdaniya* (The Ritz Ballistic Theory and Image of Universe) 3rd ed. (Nizhny Novgorod: Perspektiva, 2013)
188. Semikov S A *Metody Eksperimental'noi Proverki Ballisticheskoi Teorii Ritsa* (Methods of Experimental Verification of the Ritz Ballistic Theory) (Nizhny Novgorod: Kirillitsa, 2017)
189. Kupryaev N V *Phys. Usp.* **48** 971 (2005); *Usp. Fiz. Nauk* **175** 1013 (2005)
190. Bolotovskii B M, Stolyarov S N *Sov. Phys. Usp.* **32** 813 (1989); *Usp. Fiz. Nauk* **159** 155 (1989)
191. Kellermann K I, Pauliny-Toth I I K *Annu. Rev. Astron. Astrophys.* **6** 417 (1968); *Annu. Rev. Astron. Astrophys.* **6** 448 (1968)
192. Knight C A et. al. *Science* **172** 52 (1971)
193. Whitney J J et al. *Science* **173** 225 (1971)
194. Cavalierri A, Sartori L, Morrison P *Science* **173** 525 (1971)
195. Dent W A *Science* **175** 1105 (1972)
196. Cotton W D et al. *Astrophys. J.* **229** (6) 115 (1979)
197. Cohen M H et al. *Astrophys. J.* **231** 293 (1979)
198. Blandford W K, Königl A *Astrophys. J.* **232** 34 (1979)
199. Bahcall J N, Milgrom M *Astron. J.* **236** 24 (1980)
200. Cohen M H et al. *Astrophys. J.* **247** 774 (1981)

201. Kellermann K I, Pauliny-Toth I I K *Annu. Rev. Astron. Astrophys.* **19** 373 (1981)
202. Pearson T J, Unwin S C, Cohen M H *Nature* **290** 365 (1981)
203. Barashenkov V S *Sov. Phys. Usp.* **18** 774 (1975); *Usp. Fiz. Nauk* **114** 133 (1974)
204. Tomaschitz R *Astropart. Phys.* **23** 117 (2005)
205. Tomaschitz R *Eur. Phys. J. C* **45** 493 (2006)
206. Tomaschitz R *Astropart. Phys.* **27** 92 (2007)
207. Tomaschitz R *Europhys. Lett.* **84** (1) 19001 (2008)
208. Ekers R D *ASP Conf. Ser. E* **300** 301 (2003)
209. Rees M J *Nature* **211** 468 (1966)
210. Nugent L J, Condon R J *Appl. Opt.* **5** 1832 (1966)
211. Peres A *Am. J. Phys.* **55** 516 (1987)
212. Kochkin B M, Kutaev Yu F, Mankevich S L"Lazerno-opticheskie sistemy i tekhnologii" ("Laser optical systems and technologies"), in *Yubileinyi Nauchno-Tekhnicheskii Sbornik* (Anniversary Scientific and Engineering Almanac) Issue 110 (Moscow: NPO Astrofizika, 2009)
213. Abramson N *Opt. Lett.* **3** 121 (1978)
214. Abramson N *Appl. Opt.* **22** 215 (1983)
215. Abramson N *Appl. Opt.* **23** 1481 (1984)
216. Bartelt H O, Case S K, Lohmann A W *Opt. Commun.* **30** 13 (1979)
217. Attwood D T, Coleman L W, Sweeney D W *Appl. Phys. Lett.* **26** 616 (1975)
218. Gekelman W, Xu L *Comput. Phys.* **5** (4) 1 (1991)
219. Müller T, Boblest S *Eur. J. Phys.* **35** 065025 (2014); arXiv:1410.4583
220. Mooley K P et al. *Nature* **561** 355 (2018)