Physics – Uspekhi **62** (1) 32 – 45 (2019)

REVIEWS OF TOPICAL PROBLEMS PACS numbers: 92.70.Gt, 92.70.Np, 92.70.Qr

Earth's insolation variation and its incorporation into physical and mathematical climate models

V M Fedorov

DOI: https://doi.org/10.3367/UFNe.2017.12.038267

Contents

Abstract. **We review research on long-term variations in Earth's insolation due to celestial mechanics processes. Based on an analytical survey of Earth's insolation calculations, general problems encountered in the physical and mathematical modeling of climate are outlined.**

Keywords: Earth's insolation, orbital motion, variations, Eart's climate, climate models

1. Introduction

Climate is a state of the natural environment (system) that is characterized for a specific region (or Earth) by time-averaged hydrometeorological, soil–biological, and other parameters. The problem of the change in the current global climate and especially in its temperature characteristics is very important for modern science [1]. This problem is defined by the need to predict climate change effects on the natural environment and society. The most important question in the research on and prediction of climate change is related to the causes of these changes [2–4].

Earth's radiation and thermal balance, as well as climate, depend on solar radiation [4, 5–10]. Solar radiant energy is the main source of energy for hydrometeorological and many other processes that take place in the atmosphere, in the hydrosphere, and on Earth's surface. Therefore, the study of spatial and temporal changes in the amount of radiation that reaches Earth is of great importance for the study of the cases of climate change and for the prediction of its future evolution.

The annual integral of the solar radiation incident on Earth's atmospheric boundary layer (ABL) averages to 5.49×10^{24} J [11–13]. This value is not constant in time and demonstrates interannual and long-term (secular and periodic) variations. The variations in the incoming radiant energy are mainly determined by two factors of different physical natures. One is the variation of solar activity. The other is the celestial mechanics processes that influence Earth's orbit parameters [4, 14–16] and the rotation axis tilt. In this paper, we give a historical overview of the research on variations caused by celestial mechanical processes, analyze the current state of the art, and consider the main challenges that appear when trying to take insolation into account in climate change models.

We can distinguish two stages in the history of studies of Earth's insolation variations related to celestial mechanics processes. The first is the earlier and the longer one. It corresponds to investigations of low-frequency (secular) variations and is motivated by the need to explain the causes of paleoclimate changes. The other stage began relatively recently. It is related to studies of Earth's high-frequency (periodic) insolation variations and is motivated by the search for causes for the current climate change.

V M Fedorov Lomonosov Moscow State University,
Leninskie gory 1, 119991 Moscow, Russian Federation
E-mail: fedorov.msu@mail.ru

Received 14 September 2017, revised 21 November 2017
Uspekhi Fizicheskikh Nauk **189** (1) 33 – 36 (2019)
DOI: https://doi.org/10.3367/UFNr.2017.12.038267

2. Secular (low-frequency) insolation variations (historical development of the astronomical theory of climate)

Solar radiation variations caused by celestial mechanics processes are obtained using analytic methods. Earth's solar climate is defined as the theoretically calculated influx and distribution of solar radiation over the ABL or over Earth's surface without taking the atmosphere into account [12, 14]. When studying secular (low-frequency) solar flux variations, the astronomical parameters of Earth's orbit that undergo secular perturbations are taken into account. These parameters include the longitude of the perihelion and the eccentricity, as well as the obliquity of the ecliptic, which have quite long variation periods [14, 16–22].

The history of studies of secular (low-frequency) solar radiation variations caused by celestial mechanics processes reflects the progress in the astronomical theory of climate [4, 11, 16, 23–30]. The appearance of the astronomical theory of climate was preceded by the development of ideas regarding the evolution of ice sheets in the course of Earth's history. The astronomical theory of climate appeared as a physical explanation and justification of the glaciation theory.

The history of the astronomical theory of climate started in the middle of 19th century with the work by Adhémar [31], where he formulates ideas that the main cause of ancient glaciers could be the changes in the regular revolutions of Earth around the Sun. According to Adhémar's theory, the glacial climates depended on a 22,000-year precession cycle, and the ice sheets formed every 11,000 years alternately in one of the hemispheres, the one where the winter seasons were longer (the winter season coincided with Earth's position close to the aphelion). The fundamental idea of Adhémar that the northern and southern hemispheres are heated and cooled alternately was doubted by German geographer A Humboldt and English astronomer J Herschel. Humboldt, for example, rightly noted that the average temperature of each of the hemispheres is determined not by the number of hours (days) of warm and cold seasons of the year but by the solar-energy calories it acquires over the year [32]. Nevertheless, Adhémar's hypothesis about the connection between possible climate changes and specific astronomical phenomena (anticipation of equinoxes) was the foundation for the further development of theories where astronomical factors influence climate changes [14, 20, 21, 25, 26, 33–35].

The ideas of a connection between the ancient glacial climate periods and astronomical processes were developed in the work of Scottish scientist Croll [14, 36, 37]. Croll considered the changes in Earth's orbit eccentricity and the precession cycle to be the main astronomical factors. Using the expressions due to Leverrier [38], Croll calculated Earth's orbit eccentricity for a number of moments throughout the last three million years (seven planets known by that time were taken into account). This resulted in the derivation of the cyclic eccentricity variation. After analyzing the obtained data, Croll made an assumption that the glacial epochs can be correlated with the occurrence of a maximally elongated orbit. One of the conclusions of Leverrier's work was that the overall amount of energy acquired by Earth over a year is almost independent of the changes in its orbit eccentricity. However, Croll showed that if Earth's reflectivity (albedo) changes are taken into account, then the intensity of the incoming radiation during the astronomical semester depends quite strongly on the eccentricity changes. As a result, Croll proposed a theory that explained global climate changes (glacial and interglacial epochs) based on the seasonal effects of Earth's orbit eccentricity changes, together with the albedo and positive feedback influence [14, 25, 26, 33-35, 39, 40]. Croll's ideas were based on two astronomical factors: the precession cycle and periodic variations in the shape of Earth's orbit, and they were published in the *Philosophical Magazine* in 1864.

Secular variations of the incoming solar radiation were considered as a mathematical problem by Meech [41]. He obtained expressions that connected the amount of incoming radiation (for any latitude) with the secular variation of the eccentricity, longitude of the perihelion, and obliquity of the ecliptic. Ball [42] provided a mathematical form for Croll's theory with two parameters taken into account: eccentricity and precession. At the same time, the total energy received by two different hemispheres was influenced by a third astronomical parameter, the obliquity of the ecliptic. Astronomical theory was also developed by Culverwell [43, 44], Hargreaves [45], and Ekholm [46], but the most detailed mathematical investigation of the problem was performed by Milankovitch [14, 16, 24–26, 33–35, 39, 40, 47].

By the end of the 19th century, it was known that the insolation distribution over Earth's surface (more precisely, Earth's globe with the atmosphere not taken into account) depends on three orbital characteristics: the orbit eccentricity, the obliquity of the ecliptic, and the positions of the equinoctial points in the precession cycle (variations determined by the ellipsoidal shape of Earth were not taken into account). At the beginning of the 20th century, German mathematician Pilgrim published the results of new calculations of these astronomical characteristics over the time period of 1010 millennia prior to 1850 AD [48]. After that, the calculation of the incoming solar radiation became possible in principle.

Calculations of secular variations of the astronomical parameters made by Pilgrim (over a period of 600,000 years) were later repeated by V Miskovic (director of the Astronomical Observatory in Belgrade). These data were later used by Milankovitch to calculate secular variations in Earth's insolation. In his calculations, Miskovic used the calculations of orbital characteristics and masses of the planets done by Leverrier. Pilgrim used the data obtained by Stockwell [49], which had a lower accuracy than Leverrier's calculations. The accuracy of Miskovic's calculations was as follows: up to four decimal places for the eccentricity, up to several seconds for the obliquity of the ecliptic, and up to several minutes for the longitude of the perihelion over the time of 100,000 years before and after the epoch corresponding to the beginning of 1800.

We note that at that time, Newcomb [50] compiled an extremely precise handbook of planet motion (taking Uranus, Neptune, and some satellites of the planets into account), which was used in astronomy until the middle of the 20th century. However, Milankovitch, for some reason, did not use this data in his calculations. Leverrier's calculations were based on the secular perturbations derived by Lagrange [51]. The values of the planet masses and the numerical data that Lagrange used to estimate the initial conditions were not accurate enough (Milankovitch notes that too). Out of the nine known planets, Lagrange could take only six into account. Uranus was discovered (by W Herschel in 1781) when Lagrange was performing his calculations, and the parameters of this new planet were not determined, even approximately. Nothing was known about its satellites. Neptune was discovered (based on Leverrier's calculations related to the perturbations in the orbital motion of Uranus) only in 1846. Mars was assumed to have no satellites. Therefore, for the masses of two planets (Mars and Uranus), Lagrange could only use rough estimates. Nevertheless, he succeeded in finding the approximate bounds within which the eccentricity of the planet orbits and the inclination of the orbital planes to the ecliptic plane could vary. One of the conclusions made by Lagrange is of fundamental importance in the context of this review: the major semiaxis of an orbit does not demonstrate secular variations. This statement was later included by Laplace into his Solar System Stability Theory [52–55].

Milankovitch defined the climate effects of secular variations of three astronomical parameters: the obliquity of the ecliptic ε, the eccentricity e, and the longitude of the perihelion Π. The increase in the tilt of Earth's rotation axis (with respect to the normal to the ecliptic plane) leads to a decrease in the annually received radiation in the equatorial region and to an increase in the polar regions, such that the difference between the equatorial and polar regions becomes smaller. Reducing the tilt has the opposite effect and enhances the latitude contrasts in the solar radiation distribution over Earths' surface (without taking the atmosphere into account). For $\varepsilon = 0$, when both poles remain nonilluminated during the whole year, the latitude contrasts are maximal [14, 39]. The changes in the rotation axis tilt result in a relative regular oscillatory character with an average period of 40,000 years. At one moment during this period, the latitude contrast reaches its maximum (for the smallest obliquity of the ecliptic) with the seasonal difference reaching its minimum, and at another moment, 20,000 years later, the latitude contrast reaches its minimum, while the seasonal contrast is maximally pronounced. These phenomena would be periodically repeated if they were not superimposed by the influence of other astronomical elements [14, 39].

The annual influx of solar radiation is also determined by the duration of the summer and winter seasons (semesters), which is a function of two of Earth's orbit elements: the eccentricity e and the longitude of the perihelion Π. Due to variations in e and Π, the difference between the incoming radiation during summer and winter seasons (semesters) is a function of time. The longitude of the perihelion increases almost gradually by 360° every 21,000 years, while the perihelion makes a full circle at an almost constant speed. The eccentricity e also oscillates with a period of 92,000 years (with a much smaller amplitude) and remains between the values of 0 and 0.0677. A change in the difference between durations of the summer and winter seasons depends on both e and Π, but the influence of the first quantity is much stronger. This results in oscillations of the difference (between the incoming radiation during summer and winter time) around some mean value with a period of 21,000 years, and the amplitude of these oscillations is modulated with a period of approximately 46,000 years [14, 39].

The dependence of the secular radiation influx on the longitude of the perihelion (the longitude of the perihelion of an orbit is the angle between the directions from the Sun to the perihelion and to the vernal equinoctial point; currently it equals $102°08'$) is expressed as follows. The difference between the duration of summer and winter semesters equals zero only if the longitude of the perihelion equals 0° (the perihelion coincides with the vernal equinoctial point) or 180° (the perihelion coincides with the autumnal equinoctial point; equinoctial points are those where Earth's orbit intersects the plane of the celestial equator). Under this condition, the semester totals of the incoming solar energy are equal for both semesters and both hemispheres. Latitude contrasts in the radiation arriving at Earth's ABL are maximal in this case (Fig. 1).

When the longitude of the perihelion increases from 0° to 90°, the duration of the summer semester in the northern hemisphere increases, while that of the winter semester decreases. Accordingly, the Sun's radiation intensity increases for the summer period and decreases for the winter

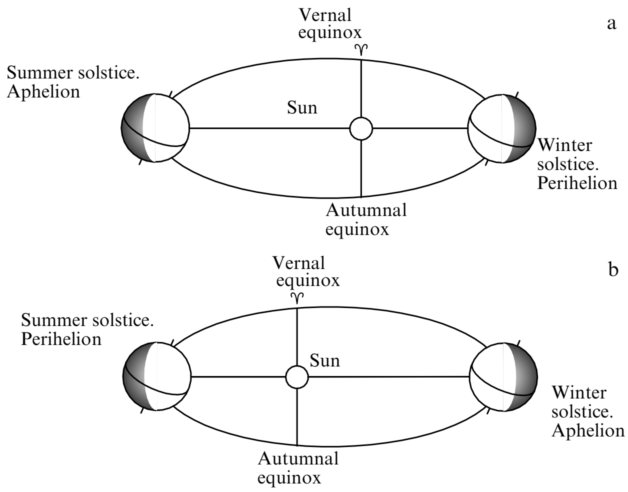

Figure 1. Configuration of Earth and the Sun: (a) when the perihelion coincides with the winter solsticial point (longitude of perihelion: 90°), (b) when the perihelion coincides with the summer solsticial point (longitude of perihelion: 70°).

period. For the 90° longitude of the perihelion (the perihelion coincides with the winter solsticial point), the duration of the summer semester in the northern hemisphere reaches its maximum, while the mean radiation intensity decreases to a minimum in summer and reaches its maximum in winter. According to our calculations, such an event took place around 1250. This means that the seasonal contrasts are minimal during this period. However, this is true only for the northern hemisphere. In the southern hemisphere, the summer semester duration (equal to the winter semester in the northern hemisphere) decreases, while the amount of radiation supplied to the southern hemisphere during such a short summer equals the amount that the northern hemisphere receives during its longer summer. Therefore, in the southern hemisphere, a shorter summer semester during this period is characterized by a larger amount of incoming solar radiation, while a longer winter semester corresponds to a smaller value. That is why the seasonal differences in the southern hemisphere are maximal at this moment (unlike the northern hemisphere).

For a 180° longitude of the perihelion (the perihelion coincides with the autumnal solsticial point), the summer and winter semesters have the same duration in both hemispheres. The sum of the incoming solar energy over the semester is equal for both semesters and both hemispheres. Latitudinal differences in the radiation falling on Earth's ABL are maximal in this case. If the longitude of the perihelion equals 270° (the perihelion coincides with the summer solsticial point), the summer semester duration is minimal in the northern hemisphere, and the difference between the summer and winter seasons (semesters) is pronounced most strongly. This value of the perihelion longitude for the southern hemisphere results in the opposite situation: a longer summer season (with a lower intensity of the incoming radiation) and shorter winter season (with a higher intensity of the incoming radiation). Seasonal contrasts smooth out in this case.

This means that the latitudinal contrasts are maximal when the perihelion is located at the equinoctial points, and smooth out when the perihelion coincides with the solsticial points. If the perihelion is located at the winter solstice, the seasonal contrasts are most strongly pronounced in the southern hemisphere and are smoothed out in the northern one. On the other hand, if the perihelion coincides with the summer solstice, the situation reverses; seasonal contrasts in this case are maximal in the northern hemisphere and are smoothed out in the southern one [14, 16, 24, 25, 33, 34].

Based on the calculations performed by Miskovic of the secular variations in the astronomical elements, Milankovitch calculated the values of the summer insolation for the 65° parallel of the northern hemisphere over the last 650,000 years. The insolation dependence graph obtained by him (in units of latitudinal equivalents) was first published in 1924 in the paper by Köppen and Wegener "The Climates of the Geological Past" [56] (Fig. 2). By latitudes equivalent to the latitude of 65° N, one assumes those latitudes which at the current moment acquire the same amount of solar energy as the latitude of 65° N was receiving previously. A decrease in the equivalent latitude means a decrease in the incoming radiation and vice versa (for example, the amount of solar radiation that was incident on the latitude 65° N 590,000 years ago is characteristic of the latitude of 72° N during the epoch of 1800).

Instead of calculating the overall solar radiation during the summer and winter semesters, Milankovitch used caloric semesters. Caloric semesters are defined as the semesters with the same duration ($T_0/2$), when at a given latitude any value of the daily insolation during the summer semester is larger than any value of the daily insolation during the winter semester. The tropical year duration (the period between two consecutive Earth's positions at the vernal equinoctial point) was assumed to be constant. The calculations were performed using canonical units (the solar constant value of 2 cal min^{-1} cm^{-2} or 1395.6 W m^{-2} corresponded to 1 canonical unit; the tropical year duration corresponded to 100,000 canonical units). Later, Milankovitch calculated the insolation variations for eight parallels located between 5° and 75° north latitude. The main results of his investigations are presented in [14].

The calculations done by Milankovitch were further improved by a number of authors. Their calculations were based on new solutions of the secular perturbation theory that were obtained for the whole Solar System in 1950 by Brouwer and Van Woerkom [57]. The calculations were based on the latest data concerning the mass and the motion of planets and included second-order effects caused, for example, by long-period variations in the motions of Jupiter and Saturn.

Detailed calculations of the solar radiation incident on Earth's ABL were performed by Soviet astronomers Sharaf and Budnikova [17–19]. They found errors in the initial values of the longitude of the Venus and Earth nodes used by Brouwer and Van Woerkom. Based on the corrected values, Sharaf and Budnikova recalculated the integration constants and derived trigonometric expressions for the precession and the rotation axis tilt, which included second-order terms for the eccentricity and the tilt. As a result, they calculated the insolation variations for the time period of 30 million years in the past and 1 million years into the future. It turned out that the eccentricity values oscillated in the range from 0.0007 to 0.0658 (the current value is 0.01675) mainly with periods of approximately 0.1, 0.425, and 1.2 million years. The variations of the rotation axis tilt occurred with the periods of 41,000 and 200,000 years, and its values were in the range between 22.068° and 24.568°. The deviation of the parameter $e \sin \Pi$ from its value in 1950 oscillated in the range from +0.03 to −0.07 with a mean period of about 21,000 years. Variations in the equivalent latitudes were found in the range from 58° to 79° (quite a broad one) with dominating periods of 41,000 years and 1.2 million years. The variations in the equivalent latitudes obtained by Sharaf and Budnikova are plotted in Fig. 3.

The values obtained by Sharaf and Budnikova for variations in the axis tilt, eccentricity, and longitude of the perihelion were used at the Institute of Oceanology of the Russian Academy of Sciences to calculate the insolation values for a million years in the past and into the future from the modern epoch (the early 1950s) with a time step of 5,000 years and latitude step of 10° [4, 16].

The integral of the incoming radiation over the caloric semesters was calculated based on the relation

$$Q_{S,W} = \frac{I_0 T_0}{2\pi}\left[S(\varphi, \varepsilon) \pm \sin\varphi \sin\varepsilon \pm \frac{4}{\pi}\, e \sin \Pi \cos\varphi\right], \quad (1)$$

where I_0 is the solar constant (equal to 2 cal min^{-1}cm^{-2} or 1395.6 W m^{-2}), T_0 the tropical year duration (assumed to be constant), S the function that describes the annual insolation over the meridian, φ the geographical latitude, Π the longitude of the perihelion, e the eccentricity, and ε the obliquity of the ecliptic. We can see from (1) that $Q_{S,W}$ depends on the axis tilt ε and on Earth's orbit elements r_0 (via I_0 and T_0), e, and Π [16]. Calculations based on expression (1) have shown that the insolation anomalies during the caloric semesters are maximal in the summer polar regions, where they reach ±250 MJ m^{-2}, and decrease for the winter polar regions. It

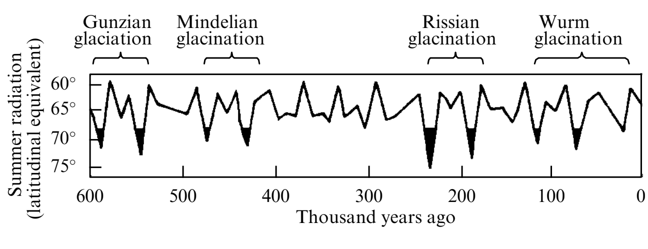

Figure 2. Amplitudes of the secular deviations of the summer radiation integrals for 65° north latitude in the units of latitudinal equivalents [14].

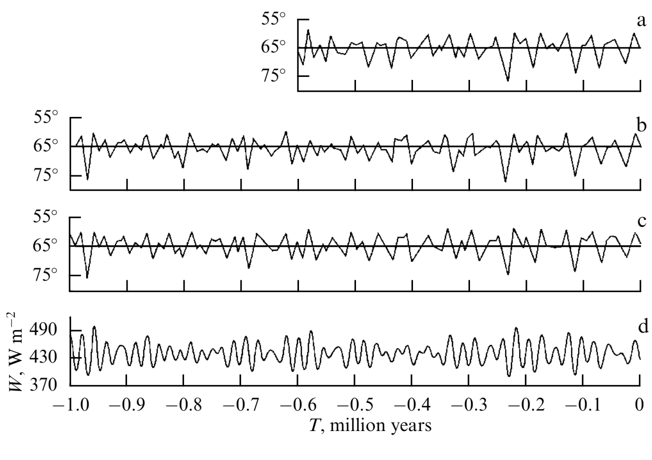

Figure 3. Variations in the insolation over the summer caloric semester for the latitude of 65° N according to data from various researchers [28]: (a) [14], (b) [57], (c) [17], and (d) [58]. The x axis shows time in units of thousands with respect to 1950. The y axis (a–c) is insolation in the equivalent latitudes over a summer semester. (d) Monthly insolation in July W [W m^{-2}].

is noted that the anomalies change in a quasiperiodic way with a period of about 40,000 years [4, 15, 16].

Calculations of secular variations of Earth's orbit elements and insolation were repeated by Vernekar [59]. Later, Berger [20] suggested an improvement to the solution by Brouwer and Van Woerkom (by including third-order terms in the eccentricity and axis tilt) and calculated the variations of both the orbit elements and the insolation. Calculated variations of the incoming solar radiation are compared in Fig. 3. It can be seen that the last maximum in the incoming solar radiation for the northern hemisphere was reached approximately 10,000 years ago (which agrees with general estimates of the degradation period for the inland ice in Europe and North America, as well as with the positioning of Earth's orbit perihelion at the vernal equinoctial point). After this moment, the amount of the incoming solar radiation gradually decreases and, as indicated by Sharaf and Budnikova's calculations, this process will continue for approximately 10,000 years [17–19]. By that time, Earth's orbit perihelion will reach the vernal equinoctial point [60].

Recently, Smulsky and Krotov [61] suggested a new method for the insolation calculation based on an exact solution of the two-body problem (Earth and the Sun). At specific Earth latitudes, the authors calculated the insolation for the time period of 200,000 years before the current epoch (1950). The deviation of the calculation results from Milankovitch's results [14] is not more than 0.1% in the specified time interval [61]. The results obtained by Smulsky and Krotov, as noted by the authors, are in good agreement with Sharaf and Budnikova's calculations, but there are significant divergences at the extremum points with the results by Laskar and colleagues. This inconsistency is explained by the difference in the initial conditions of the insolation calculation [17–19, 61]. The authors of [20, 21] note a good agreement between the calculated insolation values obtained by Berger and the results by Sharaf and Budnikova [17–19].

We note that approximate analytic solutions of problems related to orbital movement are based on physical and mathematical simplifications (for example, the interacting bodies are regarded as point masses or as bodies with some specific shapes, etc.). This leads to higher possible errors in the calculations for longer time periods (which is why Milankovitch assumed that reliable results can be obtained only for a time period of 600,000 years). Sharaf and Budnikova improved the initial data together with the precession solutions and calculated the insolation for a time period of 30 million years. Berger and Loutre performed the calculations for time periods of 5 and 3 million years [58, 62], while Quinn and colleagues calculated the insolation variations for a 3 million year period [63]. Laskar and colleagues improved the secular perturbation theory and calculated the insolation variations for a 200 million year period, but due to the chaotic behavior of the main characteristics they came to the conclusion that the results can be reliable only in the range from 20 million years in the past to 10 million years in the future [64–66]. These methods, calculation programs [58, 62, 67], and insolation data are used for numerical experiments in paleoclimate modeling.

Generally, the historical development of the astronomical theory of climate went by a series of calculations for secular (low-frequency) variations of the incoming solar radiation (Earth's solar climate) that are determined by secular variations of Earth's orbit elements (eccentricity, longitude of perihelion) and the rotation axis tilt. The calculation results obtained by various scientists for the secular variations of solar radiation are slightly different due to the difference in the initial conditions and the calculation methods (see Fig. 3). There is still no unique solution to the problem of global climate changes (glaciation development) in the framework of the astronomical theory of climate. One may search for correlations between periods when the incoming radiation reached its extremes and periods when the glaciation was maximal or minimal [37, 68]. However, without taking into account how the atmosphere, ocean, and other factors affect global climate formation, these investigations turned out to be inefficient and did not reveal the causes of climate change. Such an approach is additionally complicated by a number of phenomena such as the metachronism of glaciation [40, 69, 70] or the absence of glaciation during long geological periods (for example, during the Cretaceous). This means that the amount of solar radiation incident on Earth's ABL is not the only critical parameter on the geological time scale. Other such factors can be Earth's rotation speed, the motion of the poles, continental drift, the topography and shape of the continents and oceans, the trajectories of ocean currents, the atmospheric composition, the character of atmospheric circulations, volcanic activity, and others [71–78]. Solar climate variation is assumed to be the primary climate-formation factor when analyzing climate changes on short time scales, where the influence of the above-mentioned factors (for example, the motion of continents and poles or the change in the shape of continents and oceans) can be ignored (or can be assumed to be constant).

The time span of investigations related to the astronomical theory of climate is set by the study of secular (low-frequency) variations in the incoming solar radiation, which is connected with the main problem: explaining global climate events on geological time scales. In the astronomical theory of climate, the secular solar radiation variations are calculated through secular perturbations of two orbital elements: the orbit eccentricity and the longitude of the perihelion together with the rotation axis tilt (or the obliquity of the ecliptic). It is assumed that the "perturbations can be of two types: periodical with very small amplitude and secular. The first ones have almost no influence on the Earth's illumination and are therefore of no interest for us" [14, p. 37]. The calculations performed by Milankovitch show that the "amounts of radiation obtained during the astronomical spring and summer are the same, just as for the amount of radiation obtained during autumn and winter." It also follows that "every latitude in the southern hemisphere during its summer semester receives the same amount of radiation as the same latitude in the northern hemisphere during its summer semester; the same applies to winter semesters" [14, p. 33]. In studying secular variations, it is assumed that both the major semiaxis of Earth's orbit (Laplace's stability theorem) and the time of Earth's revolution around the Sun (Kepler's third law) are constant [14]. However, we note that the Kepler laws are valid for nonperturbed motion. It is also important that the Laplace theorem is valid only when there is no commensurability in the mean motion of the large planets [53, 54, 79]. However, the mean motions of Earth and the closest planets — Mars, Venus, and Jupiter — demonstrate commensurabilities (2/1, 2/5, and 12/1, respectively) or the orbital resonance effect [80]. This means that neither Earth's orbit major semiaxis nor the period of Earth's revolution around the Sun is precisely constant in reality. Periodic variations in the major semiaxis, revolution period, and rotation axis tilt due to orbital resonances result in weak periodic variations in Earth's solar climate [81–87].

All calculations of the incoming solar radiation described above share one property: they were performed for a range of low-frequency variations. As correctly noted by Lagrange, Laplace, and Milankovitch, when considering Earth's solar climate on geological time scales, the periodic (high-frequency) perturbations are negligible with respect to secular (low-frequency) perturbations. For short time periods (several tens to hundreds of years), high-frequency variations in Earth's solar climate can play an important role together with other factors influencing today's global climate change.

Earth's insolation should be calculated in the range of high-frequency variations due to several reasons. The first is the constant update of initial astronomical data for insolation calculations (that take advances in the perturbation theory into account). The second reason is that the response of Earth's climate system is not determined precisely over the time of secular (low-frequency) variations, while in the range of periodic (high-frequency) variations, it is very poorly studied.

Studies of Earth's solar climate variations in the range of high-frequency variations on the modern short time scale is considered to be promising due to a number of other reasons. First, this time period is supplied with astronomical data that are currently the most precise ones. Second, it is also backed by thorough climatological information, which allows performing detailed investigations of the connection between climate element variations and the amount of incoming solar radiation. Such connections could be used in paleogeographic reconstructions (based on the actualism method, which is well known in geology) or in paleoclimate modeling. Such a time scale is also convenient for comparing solar radiation variations of various natures (due to celestial mechanics processes or the Sun's activity). It provides an opportunity to determine the character of Earth's climate system response to solar radiation variations of various natures.

3. Periodic (high-frequency) insolation variations

Calculations that take periodic perturbations of Earth's orbit elements and the resulting high-frequency variations of the incoming solar radiation into account were started at the Voeikov Main Geophysical Observatory in Russia [88, 89]. However, no further steps in that direction followed. High-frequency insolation variations are currently being studied at the Institute of Astronomy and Geophysics Georges Lemaître, Belgium [90, 91]. The insolation calculations (based on the solution of the two-body problem) were performed for this range at the Institute of Earth's Cryosphere (Tyumen) by Smulsky and Krotov [61].

We have also performed insolation calculations in the high-frequency variation range [92–94]. The incoming solar radiation was calculated using data on high-precision astronomical ephemerides [95, 96] for all of Earth's surface (without taking the atmosphere into account) within the interval from 3000 BC to 2999 AD. As the initial astronomical data for the insolation calculations, we used the Sun's declination and ecliptic longitude, the distance from Earth to the Sun, and the difference between the local mean solar time and the local apparent solar time. Earth's surface was approximated by an ellipsoid (Geodetic Reference System, 1980, GRS80) with semi-axes 6,378,137 m (major) and 6,356,752 m (minor). Generally, the calculation algorithm can be described by the expression

$$I_{nm}(\varphi_1, \varphi_2) = \int_{t_1}^{t_2} \left(\int_{\varphi_1}^{\varphi_2} \sigma(H, \varphi) \left(\int_{-\pi}^{\pi} \Lambda(H, t, \varphi, \alpha)\, \mathrm{d}\alpha \right) \mathrm{d}\varphi \right) \mathrm{d}t\,, \tag{2}$$

where I_{nm} [J] is the incoming solar radiation per elementary nth fraction of the mth tropical year, σ [m^{-2}] is the area factor used to calculate the area differential $\sigma(H, \varphi)\, \mathrm{d}\alpha\, \mathrm{d}\varphi$ (of an infinitely small trapezoid) of the ellipsoid section, α [rad] is the hour angle, φ [rad] is the geographic latitude, H [m] is the height of the ellipsoid surface above Earth's surface, $\Lambda(H, \varphi, t, \alpha)$ [W m^{-1}] is the insolation at the specific moment and in the specific region of the ellipsoid, and t [s] is time. Integration steps were 1° for longitude, 1° for latitude, and 1/360 of the tropical year duration for time [82]. The value of the total solar irradiance was assumed to be 1361 W m^{-2} [97]. Solar activity variation was not taken into account. A detailed description of the performed calculations for the solar energy incident on Earth's ellipsoid (without taking the atmosphere into account) is given in [93, 94].

The main differences between our approach [92–94] (with respect to time, space and initial data) and other calculations of the low-frequency insolation variations well-known from the astronomical theory of climate are as follows.

(1) Milankovitch and his followers calculated Earth's insolation (with the atmosphere not taken into account) over long time periods (from several hundred thousand to millions of years) with only secular variations related to the changes in the eccentricity, longitude of the perihelion, and Earth's rotation axis tilt (with periods of several dozen thousand years) taken into account. The time resolution in calculations was approximately from 5000 years in the work by Milankovitch [14], Sharaf and Budnikova [17–19], and Monin [16] to 1000 years in the papers by Vernekar [59] and Berger [20–22, 58]. Milankovitch and his followers calculated daily or yearly insolation for some initial year (for example, 1850 or 1950) for a specific latitude. A time step (from 1000 to 5000 years) was then made into the past (or into the future) and the calculations repeated (taking the changes in the eccentricity, longitude of the perihelion, and axis tilt into account). Periodic variations in insolation were not taken into account (the duration of the tropical year was assumed to be constant). In our calculations, we considered both secular and periodic variations (in the Earth–Sun distance, tropical year duration, rotation axis tilt, etc.). The time resolution in integration was 1/360 of the tropical year duration (approximately one day) taking variations in this value into account [82, 84, 93].

(2) Milankovitch and all his followers performed the calculations only for specific geographic parallels (latitudes), and Earth was assumed to have a spherical shape. In our investigations, the insolation was calculated for Earth's surface approximated by an ellipsoid (also for specific latitudinal regions). Spatial resolution in integration was 1° for longitude and 1° for latitude.

(3) The calculations by Milankovitch were based on long-term astronomical ephemerides calculated by Miskovic for the eccentricity, the longitude of the perihelion, and Earth's rotation axis tilt. These values were further improved by Milankovitch's followers [17–22, 57–59, 98]. Smulsky and Krotov [61] based their calculations on the solution of the two-body problem (this is associated with a number of limitations in the initial astronomical data). Our calculations

of insolation used the parameters given in expression (2), which took secular and periodic variations of Earth's orbit elements and its rotation axis into account. As the initial data, we used high-precision astronomical ephemerides calculated at the Jet Propulsion Laboratory of the California Institute of Technology (in the time range from 3000 BC to 3000 AD) available at the NASA internet resource [96].

The differences between our approach to the analysis of high-frequency insolation variations and the methods employed by Y P Borisenkov, M-F Loutre, S Bertrand, and their colleagues are, first, the initial astronomical data used in the calculations and, second, the way the calculations treat Earth's surface. Third, these approaches deal with different time periods. As the initial data, Borisenkov and colleagues used the Ephemerides calculated at the Institute of Theoretical Astronomy of the USSR Academy of Sciences (e-mail correspondence with A V Tsvetkov, 2015). The calculations performed by Belgian researchers [90, 91] were based on the VSOP82 ephemerides [98]. Our calculations were based on the JPL Planetary and Lunar Ephemerides DE-405/406 [96, 99].

Earth's surface was assumed to be spherical in the calculations of our predecessors and only specific parallels (latitudes) were considered. Borisenkov and colleagues [88, 89] obtained data only for the latitudes of 20° N, 40° N, 60° N, and 80° N. In the investigations by Belgian scientists [90], the calculations (in mid-July or at the 120° geocentric longitude point) were performed only for the latitude of 65° N, and at the equinoctial and solsticial points they were done for the equator and latitudes of 30°, 60°, and 90° in each hemisphere. We recall that the geocentric longitude of the Sun is the angle between the directions from the center of Earth to the vernal equinoctial point and to the Sun. The vernal and autumnal equinoctial points are points where Earth's orbital plane (ecliptic) intersects the celestial equatorial plane [100].

In the paper by Bertrand and colleagues [101], the insolation was calculated, as above, for July (at the point with the geocentric longitude of 120°) in the latitudinal region from 65° N to 70° N over the time of the past millennium. The values for the latitudinal region were calculated as the mean product of the data for the limiting 65° N and 70° N parallels. Smulsky and Krotov performed their calculations for the parallels of 0°, 10°, 25°, 45°, 65°, 80°, and 90° in each hemisphere. In our calculations, Earth's surface was approximated by an ellipsoid and the incoming radiation was calculated not for specific parallels (latitudes) but for the surfaces of specific latitudinal regions (with the latitude resolution of 1°) and for the whole Earth.

The time resolution in the high-frequency variation calculations by Borisenkov and colleagues was approximately a day [88]. However, the calculations were performed only for summer and winter semesters (and only for the northern hemisphere) in the period from 1800 to 2100. In [90], Loutre and colleagues performed the calculations for the period of 5000 years (in the past) with one-year resolution and only for July (a specific point with the geocentric latitude of 120°), the equinox, and the solstice. In [101], Bertrand and colleagues performed the insolation calculations for the past millennium, but only for one month, July (with one-year resolution). Moreover, the value of the solar constant used in our calculations was 1361 W m^{-2} [97], equal to 1368 W m^{-2} in the investigations by our predecessors [91, 101], 1367 W m^{-2} in the papers by Borisenkov and colleagues (e-mail correspondence with Tsvetkov, 2015) and by Loutre and colleagues [90], and 1366 W m^{-2} in [67]. Smulsky and Krotov used the same value for the solar constant as Milankovitch did, 1395.6 W m^{-2} [14, 61].

Our calculations are based on high-precision ephemerides, use a new value for the solar constant (1361 W m^{-2}), consider the period of 5999 years in more detail, and provide data for the whole surface of Earth. In our investigations, Earth is not regarded as a sphere, and we approximate it by an ellipsoid. The obtained results fill the spatial and temporal 'gaps' in the insolation calculations for the period from 3000 BC to 2999 AD, which provides opportunities for a detailed analysis of Earth's insolation and its solar climate in the specified time interval.

An analysis of the calculated insolation shows that in the modern epoch there is a minor tendency toward a decrease [92, 93]. According to our data, in the period from 3000 BC to 2999 AD, the amount of incoming solar radiation decreased by 0.005% [93]. This tendency is associated with a low-frequency variation in Earth's orbit eccentricity (with a period of approximately 92,000 years) [14, 16, 19, 27]. The decrease in insolation during the modern epoch (in the summer semester at 65° N latitude) is also visible in the calculations shown in Fig. 3. However, more significant insolation changes appear according to latitude and season of the year (Fig. 4).

The overall amount of solar energy arriving at Earth's ellipsoid during the tropical year experiences a small decrease. However, in the regions below the 45° latitude of each hemisphere, we observe a more pronounced tendency toward an insolation increase, while for the latitudes above 45°, the opposite. This is another tendency for the variation in the incoming radiation in the modern epoch: an increase in the latitudinal contrast (inter-latitude gradient) of the insolation [84, 92]. Similar behavior of the insolation distribution over latitudes was noted for the modern epoch in [61]. The analysis of Earth's insolation demonstrates that during the winter semesters of the hemispheres, the amount of solar radiation incident on the ABL increases, while during the summer semesters it decreases, and we therefore observe the smoothing of seasonal contrasts in the insolation (Fig. 5).

The decrease in the amount of radiation incident on the latitudinal region from 65° N to 70° N (at the ABL) in the past 1000 years for mid-July (the point with the geocentric latitude 120°) was also noted in investigations by Belgian scientists [91]. The observed low-frequency variations (increase in the latitudinal contrast and smoothing of seasonal differences) in Earth's insolation are related to a

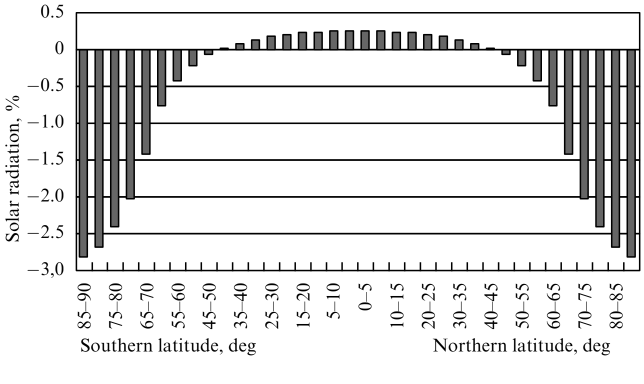

Figure 4. Earth's yearly insolation for various latitudes in the period from 3000 BC to 2999 AD [92].

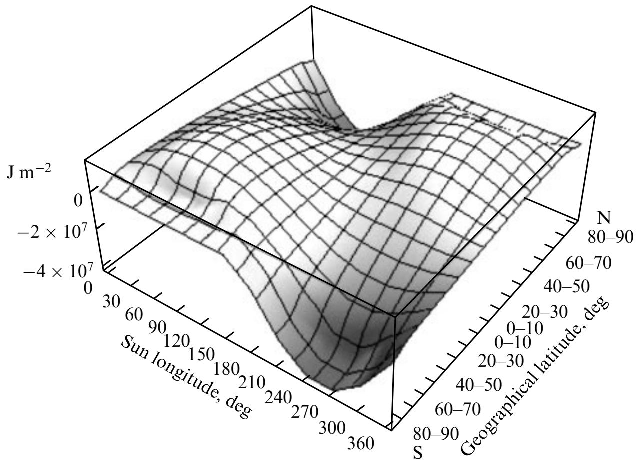

Figure 5. Spatio-temporal variations in Earth's insolation (without taking the atmosphere into account) in the period from 3000 BC to 2999 AD [92].

secular tendency of Earth's rotation axis to decrease its tilt (with respect to the normal to the ecliptic plane) due to precession (with a period of 40,000 years). It was noted above that the increase in the rotation axis tilt leads to an increase in the amount of solar radiation incident on the polar regions, which means smoothing of the latitudinal contrast in the hemispheres and an increasing seasonal difference. As the tilt angle decreases, the amount of incident radiation increases for the equatorial region and decreases for the polar region. At the same time, the latitudinal contrasts grow and the seasonal differences smooth out [4, 14, 16, 39]. According to our calculations, the decrease in Earth's rotation axis tilt during the period from 3000 BC to 3000 AD is approximately 0.7° [93].

Simultaneously with the observed tendencies, which are mostly underlain by the low-frequency oscillations of the eccentricity and rotation axis tilt, the amount of radiation incident on the ABL is modulated with high-frequency variations (Fig. 6), which appear due to periodic perturbations of Earth's orbit elements caused by the Moon and planets of the Solar System. We note that we consider high-frequency variations in Earth's insolation in units of Earth's revolution period around the Sun, that is, one year [82, 92, 93]. This period characterizes the most important oscillation in Earth's climate system.

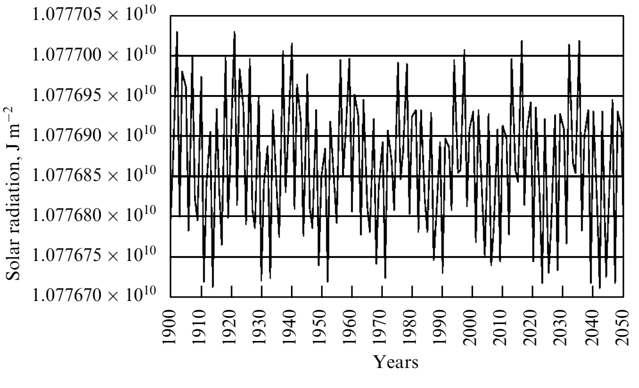

Figure 6. Many-year variations in Earth's insolation in the period from 1900 to 2050 [93].

These high-frequency oscillations, as we have noted, were not taken into account in the calculations based on the secular variations of the eccentricity, longitude of the perihelion, and Earth's rotation axis tilt i [14, 16, 17, 20, 21, 39, 59].

In [88], Borisenkov and colleagues apply spectral analysis to the calculation results for the radiation incident on the ABL during a period of 300 years (1800–2100) and distinguish the variation periods of 2.7, 4.0, 5.9, and 11.9 years. The first three frequencies, as the authors believe, correspond to the perturbations introduced by Venus and Mars. The 11.9 year variation agrees with the Jupiter perturbation. An 18.6 year period is also observed, and it is related to the lunar node period [88]. The data calculated by Smulsky and Krotov for the incoming radiation over the time period of 200,000 years in the past reveals variation periods of 2.75, 3.98, 11.86, and 18.6 years (the 18.6 year period corresponds to the perturbation of Earth's rotation axis caused by the Moon) [61]. Loutre and colleagues from the Institute of Astronomy and Geophysics Georges Lemaître also applied spectral analysis to the data on insolation calculated for the time period of 6000 years in the past [90]. The authors revealed a large number of variations with periods of 2.67, 3.98, 5.92, 8.1, 11.9, 15.7, and 18.6 years [90]. The variations with periods of 2.67, 3.98, and 8.1 years are considered to be related to Venus, while the periods of 5.93 and 11.9 years relate to Jupiter's motion. The period of 15.7 years is attributed to the motion of Mars and the 18.6 year period to the lunar node period. Bertrand and colleagues note that the insolation periods of 2.7 and 4.0 years found by them in the insolation dependence over 1000 years in the past can be attributed to the relative motion of Earth and Venus, while the periods of 5.9 and 11.9 years can be attributed to the motion of Jupiter. The period of 18.61 years is attributed to the lunar node motion [91].

Insolation calculations were recently performed by researchers at the National Technological University of Argentina and at the Harvard-Smithsonian Center for Astrophysics (USA) for the time period from 12,000 years in the past to 1000 years in the future using new astronomical ephemerides (DE-431). A spectral analysis revealed a characteristic set of high frequencies [102]. The authors associate variations with the periods of 2.69, 3.98, 7.88, and 8.1 years with the interaction between Earth and Venus. Perturbations in the mean motions of Mars and Earth are attributed to the periods of 2.42, 2.9, 3.56, 5.26, and 15.76 years. The periods of 11.86 and 6.0 years are attributed to Jupiter's motion. The periodicity of 18.6 years is assumed to correspond to the nutation cycle. The highest spectral density is observed in the range of periods from 2 to 3 years and around the period of 20 years. Spectral analysis also reveals frequencies with longer periods, which are associated by the authors with the periodic motion of the distant planets (Saturn, Uranus, Neptune) [102]. These insolation calculations were performed for specific latitudes of Earth's sphere with the daily change in the Sun longitude taken into account. At the same time, the change in the tropical year duration was not accounted for.

We believe that real periodic influences of planet motion on Earth's motion can be manifested by periods with only integer values (multiples of one year), because the primary motion of Earth has a period of one year. The same relation should be valid for the high-frequency components of Earth's insolation variation.

Among the periodicities mentioned above and found in all considered high-frequency insolation calculations, the common ones are 2.7, 4.0, 11.9, and 18.6 years. We note that the authors of [90, 102] also distinguish a periodicity of approximately 8 years. The series obtained by us also demonstrates periodicity. The observed periodicity is not hidden, and therefore spectral analysis is not needed [81, 82, 92].

The solar constant, the tropical year duration, and insolation calculated by us all clearly demonstrate variations with 2-year and 3-year cycles, which form alternating series with the durations of 8 $(2+3+3)$ and 11 $(2+3+3+3)$ years [81, 82, 84, 92]. The maximum of the spectral density is observed at the value of 2.7 years. Because the studied time period includes approximately 70% of full 3-year and 30% of full 2-year cycles, the period of 2.7 years obtained from the spectral analysis is defined precisely by the ratio between 2- and 3-year cycles in the insolation evolution [82, 84].

It is known that the parameters of planetary and satellite motion satisfy a number of peculiar relations due to the existence of commensurabilities and resonances [79, 80, 103]. The resonance conditions are defined by coincidences between frequencies of forced oscillations (those under an external action) and of natural oscillations. We consider this in more detail. The sidereal or orbital period of planet revolution is the time that it takes a planet to perform a full revolution along its orbit around the Sun. The sidereal period of Venus is 224.701 days (0.61521 tropical year), that of Mars is 686.980 days (1.88089 tropical years), and Earth's sidereal year is 365.526 days (1.00004 tropical years). The planet revolution frequencies $(\omega = 2\pi/T)$ are 0.0279624 days^{-1} for Venus, 0.0091460 days^{-1} for Mars, and 0.0171894 days^{-1} for Earth. This means that $2\omega_{\text{Mars}}$ (0.0182920 days^{-1}) $-\omega_{\text{Earth}}$ (0.0171894 days^{-1}) $=$ 0.0011026 days^{-1} and $3\omega_{\text{Venus}}$ (0.0838872 days^{-1}) $-5\omega_{\text{Earth}}$ (0.085947 days^{-1}) $= -0.0020598$. Thus, there is a commensurability between the orbital motion of Earth and the nearest planets Mars and Venus (a type of coupling between the orbital objects). The resonance is 2/1 for Earth and Mars and 5/3 for Earth and Venus [80, 103].

This means that the position of Earth and Mars with respect to the Sun is repeated every 2 years and the relative position of Earth and Venus every 3 years. These effects cause periodic resonant perturbations of Earth's motion and hence variations in the solar constant, the tropical year duration, and Earth's ABL insolation. The ratio of 2- and 3-year oscillations in the tropical year duration and insolation dependences should probably indicate the ratio of the influence of these planets. Approximately 70% of the cycles in the studied time dependence are formed by 3-year cycles, which are determined by Earth's motion perturbation by Venus (3-year periodicity). The rest of the time dependence, approximately 30%, manifests 2-year cycles, which are determined by a weaker perturbation of Earth's orbital motion by Mars [82, 84, 93].

Alternating 11-year and 8-year cycles (formed by the sums of 2- and 3-year cycles) result in a 19-year cycle, which influences the trajectory and the speed of Earth's orbital motion (the second high peak in the spectrum). Thus, the reasons for the observed periodicity (2, 3, 8, 11, and 19 years) may be in the resonant insolation perturbations caused by the commensurability between the orbital motion of Earth, Mars, and Venus, as well as in the nutation related to the lunar node period and, possibly, Metonic cycle [84, 104]. This means that the high-frequency variations in Earth's insolation demonstrate synchronization of 2- and 3-year periodicities with 8- and 11-year phases of the 19-year cycle. The 2- and 3-year cycles form 8-year $(2+3+3)$ and 11-year $(2+3+3+3)$ series corresponding to the phases of the 19-year cycle. Spectral analysis slightly distorts the real picture due to the shifts of the real harmonics and the emergence of superficial ones like 4.0, 6.0, and others [90]. Small variations in the insolation and its distribution over Earth's ABL are determined by the orbital resonances between Earth, the Moon, and the nearest planets, Venus and Mars. Correlations of the high-frequency insolation variations found by various researchers with the periodicity in the motion of distant planets [90, 103] are not justified. However, one of the reasons for occasional violations of the correct sequence of 8- and 11-year cycles could be the influence of Jupiter, because its mean motion is in a weak resonance with the mean motion of Earth (1/12).

For simplicity in determining the spatial structure of the 19-year variation, we analyzed the difference in the incoming solar energy for the tenth and the first year of the decades for the corresponding latitudinal regions. Indeed, as we have noted, the phases of the 19-year cycle are characterized by 8- and 11-year periods [81–84, 93]. The analysis demonstrates regular changes in the spatial structure of Earth's insolation in different phases of the 19-year cycle. For one of the phases, the amount of the incoming radiant energy decreases in the polar regions and increases in the equatorial region. During the other half of the cycle, the situation is reversed: incoming energy is lower for the equatorial region and higher for the polar ones (Fig. 7). Such behavior is associated with nutation. The lunar orbit slowly rotates and after 18.61 years finishes a round trip and returns to the initial position (this means that the Moon will cross the ecliptic at the same point only after 18.61 years). This period determines the effect of nutation (change in Earth's rotation axis due to irregularity in the action of the Moon) and is therefore called the nutation cycle or period [79, 100, 103].

One of the phases of the 19-year nutation cycle enhances the noted tendency of the secular variations to increase the latitudinal contrast, while the other phase weakens it. Maximal amplitudes of the high-frequency variations are detected for 19-, 11-, and 8-year cycles. They are most pronounced in the polar regions of the winter hemispheres and are equal to 0.0678% in all of the cycles. The amplitude of 2- and 3-year variations during winter seasons at these latitudes is characterized by a value of 0.0538% [83, 84, 92, 93]. This is

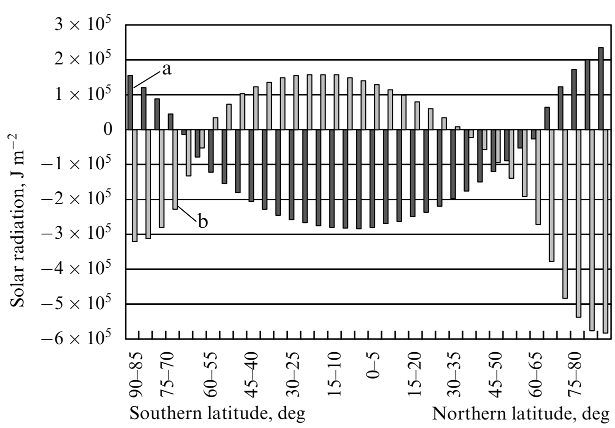

Figure 7. Latitudinal variations in Earth's insolation during the 19-year cycle (a) in the phase between 2000 and 2009, (b) in the phase between 2010 and 2019 [84].

determined, first, by large variations in the axis tilt due to nutation in polar regions and, second, by small values of winter insolation. Such variations in the polar regions have the same order of magnitude as the total solar irradiance (TSI) variations in the 11-year cycle (around 0.07%) [105]. The localization of the maximal amplitudes of the 18.6-year period insolation variation in the polar regions was also observed in [61, 89].

An interesting relation between the synodic month and the mean solar year was found in 433 BC by the Athenian astronomer Meton: 235 synodic months almost precisely correspond to 19 solar years. This means that the insolation variation can include two frequencies related to the Moon. One is determined by the periodic influence on Earth's rotation axis (nutation) and is connected with lunar orbit drift (lunar nodes). The other frequency is determined by the differences in the distance between Earth and the Sun caused by the action of the Moon on Earth's orbital motion [100].

The calculations helped to identify small and regular high-frequency insolation variations caused by periodic perturbations of Earth's orbital motion and its rotation axis tilt [61, 88, 90–92, 102]. These variations can be enhanced by the resonance response of Earth's climate system if the period of the orbital motion perturbation is a multiple of one year (natural period of the climate system). Moreover, we can expect an effect of stochastic resonance: the response of a bistable or metastable nonlinear system to a weak periodic signal in the presence of noise with specific power [106].The insolation variations discussed above can be considered weak periodic signals for Earth's climate system in the presence of noise.

As our results show, Earth's climate system (global climate) has a stronger response to variations in the latitudinal and seasonal distributions of the incoming solar radiation (related to the axis tilt variations) than to variations in its amount [60, 86, 93, 107, 108]. These variations in the solar radiation distribution over latitudes and seasons are much stronger than those in the total amount of the radiation.

Based on our calculations, we have published an open-source database on the amount of incident solar energy for all of Earth's latitudinal regions (5° wide) per astronomical month of each year in the period from 3000 BC to 2999 AD [109]. Earth's insolation values are given as data arrays in three different units, J, J m^{-2}, and W m^{-2}. These dimensions correspond to the overall radiant energy, specific energy, and intensity. The data presented in this database corresponds to the variations in solar radiation that are caused by celestial mechanics processes. These values can be used as the input energy signal in the physical–mathematical climate models. The calculated insolation data can also be used for precise calculations of Earth's radiative balance, because the ABL is a conditional reference surface for the radiant energy incident on Earth (Fig. 8) [110–115]. By taking the spatio-temporal variations in the solar radiation incident on the ABL (i.e., variations in the initial conditions) into account in the calculations of the radiative and thermal balance of Earth, its surface, and its atmosphere, one can improve both the accuracy of the calculations and the quality of predictions. This is because small deviations in the initial conditions (in the case of nonlinear processes) can lead to the difference in the calculation results increasing in time.

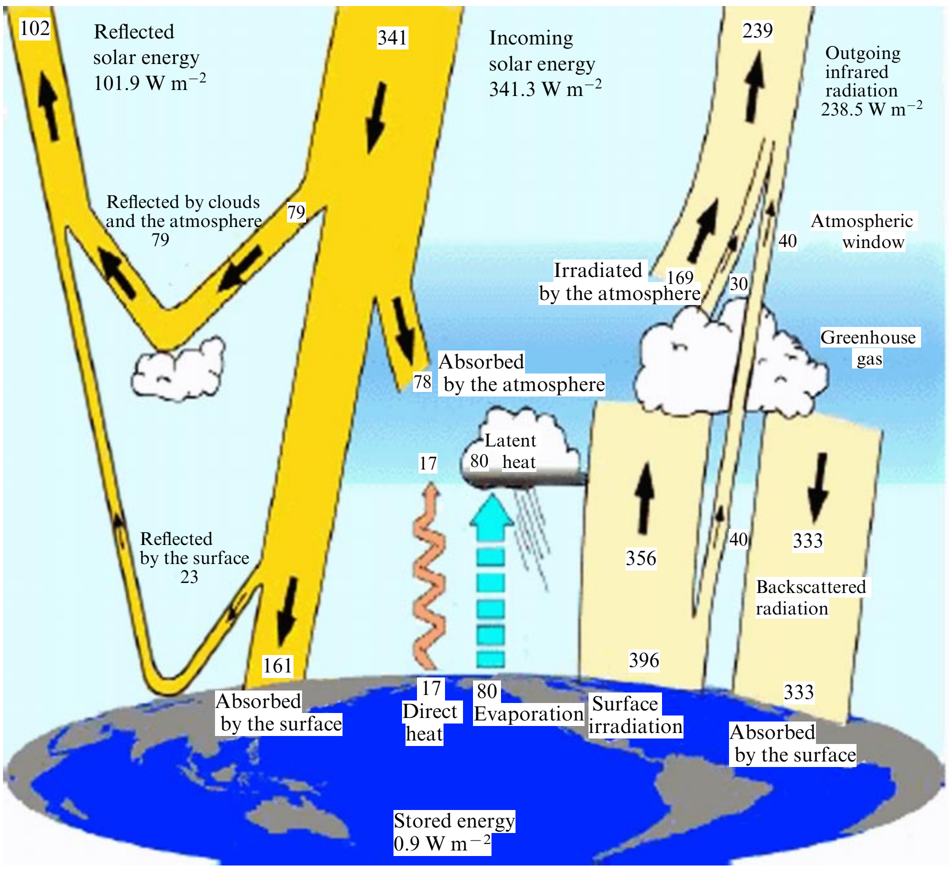

Figure 8. Components of Earth's radiation balance [113].

The value of 341 W m^{-2} at the input (at the ABL or on Earth's surface without taking the atmosphere into account) is the ratio of the solar constant (for Fig. 4, 1364 W m^{-2}) and 4. The solar constant is the radiant energy incident on a unit area of Earth's disk (for a distance of 1 a.u.), while the area of a sphere is 4 times the area of its great circle. In our database, the input radiation signal is calculated not with respect to the area of a sphere but with respect to the area of an ellipsoid [84, 92, 93, 116].

We note that in investigations of the insolation variations caused by celestial mechanics processes, the calculations are performed for a given value of the solar constant (the TSI value averaged over many years) and are not normalized to distance. When studying the TSI variations, calculations are performed with respect to a constant distance between the Sun and Earth, 1 a.u. (averaged over a year).

Because solar radiation is the main source of heat in Earth's climate system, its values are used in climate modeling. However, despite the success in the measurement and calculation of the incoming solar radiation, some significant unsolved problems can be identified in the review of the incoming radiation variations given above.

4. General problems of including incoming solar radiation variations in physical and mathematical climate models

4.1 Incoming energy signal

In agreement with the IPCC, it is recommended to use the following data for the external energy signal input into the radiation block of physical–mathematical models of climate (CLIM-5): data obtained in [117] from radiometric measurements of the total radiation flux (since 1978) and TSI reconstruction (starting in 1610 with yearly resolution and in 1882 with monthly resolution) [118]. Reconstruction of the total solar irradiance was performed based on solar activity variations (the number of spots and flares). This reconstructed data do not reflect the insolation contrast changes, which are related to the enhancement of the interlatitude thermal exchange (work performed by a "heat machine of the first kind") [6] and to the many-year variations in the surface air temperature (SAT), ocean surface temperature (OST), sea level (SL), and distribution area of sea ice [85, 86, 93]. Insolation contrast is the difference between the yearly incoming radiation in the regions from 0° to 45° (heat source) and from 45° – 90° (heat sink) in each hemisphere (for Earth, we consider the value averaged over hemispheres). Insolation contrast manifests the character of the meridional energy transfer to the ABL.

Due to several reasons, it is incorrect to use the above-mentioned measured and reconstructed TSI [118] as the input energy signal in climate models. First, this data reflects the change in the total amount of radiation incident on Earth. As Earth's insolation calculations show, such changes are negligible for the modern epoch (around 0.005% every 6000 years) [92, 93]. These variations in the incoming solar radiation can also be calculated for different latitudinal regions as the cosine of latitude (the change in Earth's rotation axis tilt is not taken into account). But because the variations in the incoming radiation are small, the variations in the latitudinal regions are also small. It is important to include not the variations of the total irradiance but the variations in its distribution over the latitudinal regions due to changes in Earth's rotation axis tilt (for a period of 6000 years, these variations have an amplitude of about 3%) (see Fig. 4).

Precisely these variations determine the trends in the changes in the global SAT of Earth (and its specific latitudinal regions) [85, 86, 93]. First, in the implementation of the incoming radiation variations in modern climate models, the process mentioned above are ignored (as are the changes in the energy transfer to the ABL and hence in the ocean–atmosphere system). Second, the models do not account for the time-resolution-dependent changes in the ratio between the TSI variations of different physical natures and their different influence on climate formation and change. We let TSI_{CMP} denote the variations related to celestial mechanics processes and TSI_{SA} denote the variations related to solar activity.

4.2 Relation between variations of different physical natures in TSI fluctuations

It follows from the calculated data on incoming radiation [93] that the relation between variations of different physical natures changes with the time resolution, even for small fluctuation amplitudes. For example, 55% of the interannual insolation fluctuations in the monthly resolution range (Fig. 9) are determined by celestial mechanics processes (TSI_{CMP}), while 45% are determined by solar activity variations (TSI_{SA}).

The differences between the components of the TSI interannual fluctuations determined by celestial mechanical processes (TSI_{CMP}) and changes in the solar activity (TSI_{SA}) have the specific character of annual variation. Interannual variations related to solar activity (TSI_{SA}) are higher than the interannual variations determined by the celestial mechanical processes (TSI_{CMP}) for 4 months (1/3 of a year): June, July, December, and January. Time periods of the TSI_{SA} variation domination are chronologically localized in the vicinity of the summer and winter solsticial points. During the remaining 8 months (2/3 of a year), the interannual TSI fluctuations are dominated by variations determined by the celestial mechanical processes (TSI_{CMP}). Maximum values of the dominant variations (TSI_{CMP}) are observed during periods in the vicinity of the equinoctial points. For the period of satellite radiometric observations from 1978 to 2008, the average ratio is characterized by the weighted values of 45.71% (TSI_{SA}) and 54.29% (TSI_{CMP}) [93].

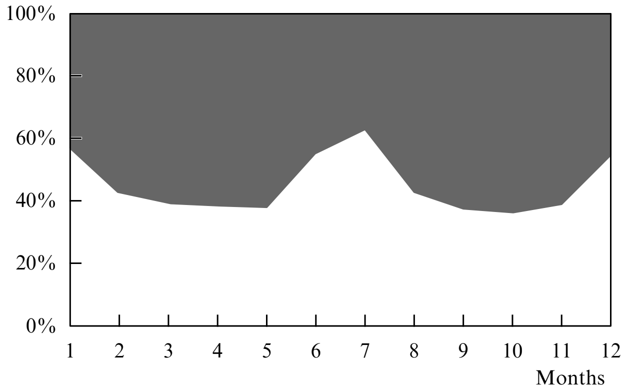

Figure 9. Ratio between the TSI_{SA} and TSI_{CMP} variations (shown with dark background) in the TSI interannual fluctuations (in %) for the period from 1882 to 2008 with monthly resolution [93].

The suggested IPCC values of the reconstructed TSI do not account for this effect. The obtained results show that for the input energy signal in climate modeling, one should take the insolation and/or the insolation contrast into account. The connection between the latter and the many-year variations in the SAT and OST anomaly was analyzed and estimated in [85, 86, 93, 94, 107, 108].

4.3 Changes in the energy transfer over the ABL

The main blocks of physical–mathematical climate models are the atmospheric general circulation model (AGCM) and the ocean general circulation model (OGCM). These blocks are described by a system of hydrodynamic equations that correspond to the principal physical laws (matter, energy, and momentum conservation laws). However, these equations describe the averaged and static atmosphere and ocean [7, 119] and, for example, do not take changes in the radiant energy transfer over the ABL (and in the ocean–atmosphere system) into account. Insolation calculations [93] show that due to the elliptical shape of Earth, the equatorial region receives more radiant energy than the polar regions. The inhomogeneous distribution of the incoming radiation on the ABL leads to the appearance of an interlatitudinal insolation gradient, which determines the energy transfer over the ABL. A gradual increase in the radiant energy transfer from the equator to the polar circles is observed in each hemisphere (Fig. 10).

The physical process of energy transfer is radiation. Maxima of the meridional radiant energy transfer are localized approximately in the region of the 65th parallel in each hemisphere (near the polar circles). A slight decrease in the radiant energy transfer is observed in the polar regions (from the polar circles to the poles). This means that in each hemisphere there is a region with an increasing (from the equator to the polar circle) and decreasing (from the polar circle to the pole) radiant energy transfer. The maximal increase (by 2.61×10^{15} W or 1.25%) is observed near the polar circles (65° latitude) in each hemisphere ("turbulent region"). The maximal decrease in the transfer (by 3.3×10^{15} W or 2.56%) is observed in the polar regions (85° latitude).

The mean energy transfer in the ocean–atmosphere system [7, 119] is linearly dependent on the mean energy transfer over the ABL [93]. The correlation coefficient is 0.98. Because the mean annual energy transfer in the ocean–atmosphere system is determined by the mean annual energy transfer over the ABL, the fluctuations observed in this transfer in the modern epoch (from 3000 BC to 2999 AD) can also manifest themselves in the ocean–atmosphere system. The energy transfer determines the interlatitudinal thermal exchange (work performed by a heat machine of the first kind), which is currently increasing due to the decreasing tilt of Earth's rotation axis [85–87, 93]. The obtained variations in the meridional energy transfer over the ABL should also be taken into account in the atmospheric system of hydrodynamic equations (equations for the mass, momentum, and energy conservation laws, and the gas state equation) [120, 121] used for numerical experiments in the physical–mathematical climate models. The hydrodynamic equations used in the AGCM and OGCM describe the static atmosphere and ocean, which is not the actual case.

Recent calculations of Earth's insolation variation can provide solutions to these general problems, which may lead to an improvement in physical–mathematical climate models and climate change predictions.

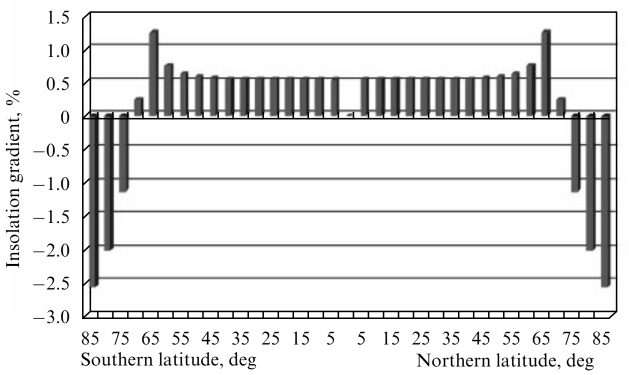

Figure 10. Variations in the meridional annual transfer of radiant energy over the ABL for 5998 years (in %) [93].

References

1. The Intergovernmental Panel on Climate Change, IPCC, http://www.ipcc.ch/
2. Kondrat'ev K Ya *Global'nyi Klimat i Ego Izmeneniya* (Global Climate and Its Changes) (Leningrad: Nauka, 1987)
3. Kondrat'ev K Ya *Global'nyi Klimat* (Global Climate) (St. Petersburg: Nauka, 1992)
4. Monin A S, Shishkov Yu A *Phys. Usp.* **43** 381 (2000); *Usp. Fiz. Nauk* **170** 419 (2000)
5. Zubov N N *Morskie Vody i L'dy* (Sea Water and Ice) (Moscow: Gidrometeoizdat, 1938)
6. Shuleikin V V *Fizika Morya* (Sea Physics) (Moscow: Izd. Akad. Nauk SSSR, 1953)
7. Lorenz E N *The Nature and Theory of the General Circulation of the Atmosphere* (Geneva: World Meteorological Organization, 1967); Translated into Russia: *Priroda i Teoriya Obshchei Tsirkulyatsii Atmosfery* (Leningrad: Gidrometeoizdat, 1970)
8. Budyko M I *Izv. Akad. Nauk SSSR. Ser. Geograficheskaya* **5** 36 (1968)
9. Budyko M I *The Earth's Climate, Past and Future* (New York: Academic Press, 1982); Translated from Russian: *Klimat v Proshlom i Budushchem* (Leningrad: Gidrometeoizdat, 1980)
10. Kondrat'ev K Ya *Changes in Global Climate, a Study of the Effect of Radiation and other Factors During the Present Century* (Rotterdam: A.A. Balkema, 1985); Translated from Russian: *Radiatsionnye Faktory Sovremennykh Izmenenii Global'nogo Klimata* (Leningrad: Gidrometeoizdat, 1980)
11. Drozdov O A et al. *Klimatologiya* (Climatology) (Leningrad: Gidrometeoizdat, 1989)
12. Khromov S P, Petrosyants M A *Meteorologiya i Klimatologiya* (Meteorology and Climatology) (Moscow: Izd. MGU, 2006)
13. Abdusamatov Kh I *Solntse Diktuet Klimat Zemli* (Sun Dictates the Earth's Climate) (St. Petersburg: Logos, 2009)
14. Milankovitch M *Mathematische Klimalehre und astronomische Theorie der Klimaschwankungen* (Handbuch der Klimatologie, Bd. 1, Teil A) (Berlin: Gebrüder Borntraeger, 1930); Translated into Russian: *Matematicheskaya Klimatologiya i Astronomicheskaya Teoriya Kolebanii Klimata* (Moscow-Leningrad: GONTI, 1939)
15. Vulis I L, Monin A S *Izv. Akad. Nauk SSSR. Fiz. Atmos. Okeana* **15** (1) 3 (1979)
16. Monin A S *An Introduction to the Theory of Climate* (Dordrecht: D. Reidel Publ. Co., 1986); Translated from Russian: *Vvedenie v Teoriyu Klimata* (Leningrad: Gidrometeoizdat, 1982)
17. Sharaf Sh G, Budnikova N A *Byull. Inst. Teor. Astron. Akad. Nauk SSSR* (11) 231 (1967)
18. Sharaf Sh G, Budnikova N A *Sov. Phys. Dokl.* **13** 835 (1969); *Dokl. Akad. Nauk SSSR* **182** 291 (1968)
19. Sharaf Sh G, Budnikova N A *Tr. Inst. Teor. Astron. Akad. Nauk SSSR* **14** 48 (1969)
20. Berger A L *Quaternary Res.* **9** 139 (1978)

21. Berger A L *J. Atmos. Sci.* **35** 2362 (1978)
22. Berger A L *Climatic Variations and Variability* (Dordrecht: D. Reidel, 1981)
23. Khrgian A Kh *Fizika Atmosfery* (Atmosphere Physics) (Moscow: Izd. MGU, 1986)
24. Monin A S, Shishkov Yu A *Istoriya Klimata* (Climate History) (Leningrad: Gidrometeoizdat, 1979)
25. Imbrie J *Icarus* **50** 408 (1982)
26. Imbrie J et al. *Paleooceanology* **8** 699 (1993)
27. Mel'nikov V P, Smul'skii I I *Kriosfera Zemli* (8) 3 (2004)
28. Mel'nikov V P, Smul'skii I I *Astronomicheskaya Teoriya Lednikovykh Periodov: Novye Priblizheniya. Reshennye i Nereshennye Problemy* (Astronomical Theory of Ice Ages: New Approximations. Solved and Unsolved Problems) (Novosibirsk: GEO, 2009)
29. Smul'skii I I *Vestn. Ross. Akad. Nauk* **83** 40 (2013)
30. Kotlyakov V M, Sonechkin D M *Led Sneg* **55** (2) 103 (2015)
31. Adhémar J *Révolutions de la Mer: Déluges Périodiques* (Paris: Carilian-Goeury et V. Dalmont, 1842)
32. Gumbol'dt A *Kosmos* (*Opyt Fizicheskogo Miroopisaniya*) (Space (An Experiment of Physical Description of the World)) (Moscow: Brat'ya Salaevy, 1866)
33. Imbrie J, Imbrie K P *Ice Ages. Solving the Myster* (Cambridge, Mass.: Harvard Univ. Press, 198); Tanslated into Russian: *Tainy Lednikovykh Epokh* (Moscow: Progress, 1988)
34. Bol'shakov V A *Novaya Kontseptsiya Orbital'noi Teorii Klimata* (New Concept of the Orbital Climate Theory) (Moscow: Moskovskii Univ., 2003)
35. Bol'shakov V A, Kapitsa A P *Vestn. Ross. Akad. Nauk* **81** 603 (2011)
36. Croll J *Climate and Time in their Geological Relations: a Theory of Secular Changes of the Earth's Climate* (London: Edward Stanford, 1875)
37. Schwarzbach M *Das Klima der Vorzeit; eine Einführung in die Paläoklimatologie* (Stuttgart: F. Enke, 1950); Tanslated into Russian: *Klimaty Proshlogo* (Moscow: IL, 1955)
38. Leverrier U J J *Recherches Astronomiques* (Paris: Annales de l'Observatoire imperial de Paris, 1855 , 1856)
39. Milankovitch M *Theorie Mathematique des Phenomenes Thermiques Produits par la Radiation Solaire* (Paris: Gauthier-Villarset Cie, 1920)
40. Markov K K *Paleogeografiya* (Paleogeography) (Moscow: Izd. MGU, 1960)
41. Meech L W *On the Relative Intensity of the Heat and Light of the Sun Upon Different Latitudes of the Earth* (Washington: Smithsonian Institute, 1856)
42. Ball R *The Cause of an Ice Age* (New York: D. Appleton and Co., 1891)
43. Culverwell E P *Nature* **51** 33 (1894)
44. Culverwell E P *Nature* **52** 53 (1895)
45. Hargreaves R *Trans. Cambridge Philos. Soc.* **16** 58 (1896)
46. Ekholm N *Quart. J. R. Meteorolog. Soc.* **27** 1 (1901)
47. Berger A L *Nature* **269** 44 (1977)
48. Pilgrim L *Naturkunde Wurtemberg* **60** 1 (1904)
49. Stockwell J N *Memoir on the Secular Variations of the Elements of the Orbits of the Eight Principal Planets, Mercury, Venus, The Earth, Mars, Jupiter, Saturn, Uranus and Neptune: with Tables of the Same: Together with the Obliquity of the Ecliptic, and the Precession of the Equinoxes in Both Longitude and Right Ascension* (Washington: Smithsonian Institute, 1872)
50. Newcomb S *The Elements of the Four Inner Planets and the Fundamental Constants of Astronomy* (Washington: Gov't. print. off., 1895)
51. Lagrange J-L *Theorie des Variations Seculaires des Elements des Planets* (Berlin: Decker, 1781)
52. Laplace P S *Traite de Mecanique Celeste* (Paris: Bachelier Libraire, 1825)
53. Arnol'd V I *Russ. Math. Surv.* **18** 85 (1963); *Usp. Mat. Nauk* **18** 91 (1963)
54. Duboshin G N *Nebesnaya Mekhanika. Osnovnye Zadachi i Metody* (Celestial Mechanics. Main Problems and Methods) (Moscow: Nauka, 1975)
55. Laplace P S *Exposition du Système du Monde* (Paris: Bachelier, 1824); Translated into Russian: *Izlozhenie Sistemy Mira* (Leningrad: Nauka, 1982)
56. Köppen W, Wegener A *Die Klimate der Geologischen Vorzeit* (Berlin: Gebr. Bornetraeger, 1924)
57. Brouwer D, Van Woerkom A J *J. Astron. Papers* **13** 81 (1950)
58. Berger A, Loutre M F *Quatern. Sci. Rev.* **10** 297 (1991)
59. Vernekar A *Long-Period Global Variations of Incoming Solar Radiation* (New York: American Meteorological Society, 1972)
60. Fedorov V M *Kriosfera Zemli* (3) 106 (1999)
61. Smulsky J, Krotov O *Appl. Phys. Res.* **6** (4) 56 (2014)
62. Berger A, Loutre M F *Earth Planet. Sci. Lett.* **111** 369 (1992)
63. Quinn T R, Scott T, Duncan M *Astron. J.* **101** 2287 (1991)
64. Laskar J *Astron. Astrophys.* **198** 341 (1988)
65. Laskar J, Joutel F, Boudin F *Astron. Astrophys.* **287** 522 (1993)
66. Laskar J et al. *Icarus* **170** 343 (2004)
67. Berger A, Loutre M-F, Yin Q *Quatern. Sci. Rev.* **29** 1968 (2010)
68. Kukla G J *Nature* **253** 600 (1975)
69. Kolosov D M *Problemy Drevnego Oledeneniya Severo-Vostoka SSSR* (Problems of the Ancient Glacination of North-East USSR) (Moscow–Leningrad: Glavsevmorput', 1947)
70. Fedorov V M *Dinamika Balansa Massy Lednikov v Svyazi s Makrotsirkulyatsionnymi Protsessami v Atmosfere* (Dynamics of the Glacier Mass Balance and Connections with the Macrocirculative Processes in the Atmosphere) (Moscow: Fizmatlit, 2011)
71. Wegener A *Die Entstehung der Kontinente und Ozeane* (Braunschweig: F. Vieweg, 1922); Trnslated into English: *The Origin of Continents and Oceans* (New York: Dover Publ., 1966); Translated into Russian: *Proiskhozhdenie Materikov i Okeanov* (Moscow–Leningrad: Gos. Izd., 1925)
72. Lichkov B L *Dvizhenie Materikov i Klimaty Proshlogo* (Continental Drift and the Climates of the Past) (Leningrad: Izd. Akad. Nauk SSSR, 1935)
73. Takeuchi H, Uyeda S, Kanamori H *Debate about the Earth; Approach to Geophysics Through Analysis of Continental Drif* (San Francisco: Freeman, 1967); Translated into Russian: *Dvizhutsya li Materiki*? (Moscow: Mir, 1970)
74. Le Pichon X, Francheteau J, Bonnin J *Plate Tectonics* (Amsterdam: Elsevier, 1973); Translated into Russian: *Tektonika Plit* (Moscow: Mir, 1977)
75. Sinitsin V M *Vvedenie v Paleoklimatologiyu* (Introduction to Paleoclimatology) (Leningrad: Nedra, 1980)
76. Uyeda S *The New View of the Earth. Moving Continents and Moving Oceans* (San Francisco: W.H. Freeman, 1978); Translated into Russian: *Novyi Vzglyad na Zemlyu* (Moscow: Mir, 1980)
77. Ushakov S A, Yasamanov N A *Dreif Materikov i Klimaty Zemli* (Continental Drift and the Earth's Climate) (Moscow: Mysl', 1984)
78. Sidorenkov N S *Fizika Nestabil'nostei Vrashcheniya Zemli* (Physics of Instabilities in the Earth's Rotation) (Moscow: Fizmatlit, 2002)
79. Duboshin G N *Nebesnaya Mekhanika. Analiticheskie i Kachestvennye Metody* (Celestial Mechanics. Analytical and Quantative Methods) (Moscow: Nauka, 1978)
80. Grebenikov E A, Ryabov Yu A *Rezonansy i Malye Znamenateli v Nebesnoi Mekhanike* (Resonances and Small Denominators in Celestial Mechanics) (Moscow: Nauka, 1978)
81. Fedorov V M *Sol. Syst. Res.* **46** 170 (2012); *Astron. Vestn.* **46** 184 (2012)
82. Fedorov V M *Dokl. Earth Sci.* **451** 750 (2013); *Dokl. Ross. Akad. Nauk* **451** 95 (2013)
83. Fedorov V M *Dokl. Earth Sci.* **457** 869 (2014); *Dokl. Ross. Akad. Nauk* **457** 222 (2014)
84. Fedorov V M *Dokl. Earth Sci.* **460** 96 (2015); *Dokl. Ross. Akad. Nauk* **460** 339 (2015)
85. Fedorov V M *Kriosfera Zemli* (3) 46 (2015)
86. Fedorov V M *Uchenye Zapiski Ross. Gos. Gidrometeorologicheskogo Univ.* **45** 151 (2016)
87. Fedorov V M, Grebennikov P B *Izv. Atmos. Ocean. Phys.* **53** 757 (2017); *Geofiz. Protsessy Biosfera* **16** (1) 5 (2017)
88. Borisenkov Ye , Tsvetkov A V, Agaponov S V *Climatic Change* **5** 237 (1983)
89. Borisenkov E P, Tsvetkov A V, Eddy J A *J. Atmosph. Sci.* **42** 933 (1985)
90. Loutre M F et al. *Climate Dynamics* **7** 181 (1992)
91. Bertrand C, Loutre M F, Berger A *Geophys. Res. Lett.* **29** 40 (2002)
92. Fedorov V M *Izv. Atmos. Ocean. Phys.* **51** 779 (2015); *Geofiz. Protsessy Biosfera* **14** (1) 5 (2015)

93. Fedorov V M *Insolyatsiya Zemli i Sovremennye Izmeneniya Klimata* (Earth's Insolation and Modern Climate Changes) (Moscow: Fizmatlit, 2018)
94. Kostin A A, Fedorov V, http://www.solar-climate.com/sc/mtd.htm
95. Giorgini J D et al. *Bull. Am. Astron. Soc.* **28** 1158 (1996)
96. Solar System Dynamics, http://ssd.jpl.nasa.gov
97. Kopp G, Lean J *Geophys. Res. Lett.* **37** L01706 (2011)
98. Bretagnon P *Astron. Asrtrophys.* **114** 278 (1982)
99. Standish E M *Astron. Asrtrophys.* **114** 297 (1982)
100. Bakulin P I, Kononovich E V, Moroz V I *Kurs Obshchei Astronomii* (General Course of Astronomy) (Moscow: Nauka, 1983)
101. Bertrand C, Van Ypersele J-P *Climatic Change* **43** 387 (1999)
102. Cionco R G, Soon W W-H *Earth-Sci. Rev.* **166** 206 (2017)
103. Roy A E *Orbital Motion* (Bristol: Hilger, 1978); Translated into Russian: *Dvizhenie po Orbitam* (Moscow: Mir, 1981)
104. Fedorov V M *Solar Syst. Res.* **50** 220 (2016); *Astron. Vestn.* **50** 233 (2016)
105. The Physikalisch-Meteorologisches Observatorium Davos/World Radiation Cente, PMOD/WRC, http://www.pmodwrc.ch/
106. Anishchenko V S et al. *Phys. Usp.* **42** 7 (1999); *Usp. Fiz. Nauk* **169** 7 (1999)
107. Fedorov V M *Zhizn Zemli* (3) 245 (2017)
108. Fedorov V M *Zhizn Zemli* (4) 381 (2017)
109. Solar Radiation and Climate of the Earth. Radiation Databases, http://www.solar-climate.com/en/ensc/bazard.htm
110. Budyko M I *Climatic Changes* (Washington, DC: American Geophysical Union, 1977); Translated from Russian: *Izmenenie Klimata* (Leningrad: Gidrometeoizdat, 1974)
111. Hansen J et al. *Atmos. Chem. Phys.* **11** 13421 (2011)
112. The Atmosphere s Energy Budget, http://earthobservatory.nasa.gov/Features/EnergyBalance/page6.php
113. Trenberth K E, Fasullo J T, Kiehl J *Bull. Am. Meteorolog. Soc.* **3** 311 (2009)
114. Trenberth K E, Fasullo J T *Meteorol. Z.* **18** 369 (2009)
115. Trenberth K E, Fasullo J T *Surv. Geophys.* **33** 413 (2011)
116. Solar radiation and climate of the Earth. Insolation databases, http://www.solar-climate.com/sc/bd01.htm
117. Lean J, Beer J, Bradley R *Geophys. Res. Lett.* **22** 3195 (1995)
118. SOLARISHEPPA. Recommendations for CMIP5 solar forcing data, http://solarisheppa.geomar.de/cmip5
119. Palmén E, Newton C W *Atmospheric Circulation Systems. Their Structure and Physical Interpretation* (New York: Academic Press, 1969); Translated into Russian: *Tsirkulyatsionnye Sistemy Atmosfery* (Leningrad: Gidrometeoizdat, 1973)
120. Gill A E *Atmosphere-Ocean Dynamic* (New York: Academic Press, 1982); Translated into Russian: *Dinamika Atmosfery i Okeana* (Moscow: Mir, 1986)
121. Pedlosky J *Geophysical Fluid Dynamics* (New York: Springer-Verlag, 1979); Translated into Russian: *Geofizicheskaya Gidrodinamika* (Moscow: Mir, 1984)