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On the properties of the 'potential' neutron dispersion law in a refractive medium

A I Frank

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Abstract. It is known that the so-called potential law of neutron-wave dispersion in matter is distinct from all other dispersion laws in that the normal component of the wave vector in a medium depends only on its vacuum value. There is also another feature of the potential dispersion law, formulated as the statement that the neutron group velocity in a medium is equal to the vacuum velocity times the refractive index. Although generally accepted as obvious, this statement turns out to be valid only for the potential dispersion law, for which the neutron effective mass in a medium is equal to the inertial one.

Keywords: neutron waves, refractive medium, dispersion law, effective mass

For waves of any type, the refractive index is known to be defined as the vacuum-to-medium ratio of wave numbers. Foldy obtained the relation for the refractive index of scalar waves of any origin by solving the problem of multiple wave scattering [1],

$$k^2 = k_0^2 + 4\pi\rho f(0)\,, \tag{1}$$

where k_0 is the wave number of an incident wave, ρ is the volume density of scattering centers, and f is the zero-angle scattering amplitude. Foldy's formula is obviously valid to a high accuracy for the slowest, so-called ultra-cold, neutrons [2, 3], for which it takes the form

$$k^2 = k_0^2 - 4\pi\rho b\,, \quad n^2 = 1 - \frac{4\pi\rho}{k_0^2}\, b\,, \quad n = \frac{k}{k_0}\,, \tag{2}$$

where b is the scattering length of neutrons by nuclei. Because the scattering length is a constant quantity, it follows from (2) that the wave number squared becomes a constant quantity at the substance boundary. This allows describing neutron–matter interaction by introducing the concept of an effective potential,[1]

$$U_{\text{eff}} = \frac{2\pi\hbar^2}{m}\,\rho b\,. \tag{3}$$

For this reason, dispersion law (2) is often referred to as a potential law. For cold and thermal neutrons, dispersion law (2) is not quite applicable. In this case, the Lax formula [4, 5]

$$n^2 = 1 + \frac{4\pi\rho}{k_0^2}\, f(k_0)\, c(k_0) \tag{4}$$

is applicable, which includes both the dependence of the scattering amplitude on the wave number

$$f(k_0) \approx -b + \mathrm{i}k_0 b^2 \tag{5}$$

and the correction to the coherent field c determined by correlations between the positions of scatterers [6–8]. Furthermore, in the case of close resonances, the scattering amplitude $f(k_0)$ defined by the Breit–Wigner formula can be very strongly k_0-dependent. We do not aim here to review the current state of the neutron dispersion theory (see, e.g., [6, 9]) but only note that in the general case, the neutron wave dispersion law in a refracting medium can differ from dispersion law (2).

It is known that the potential dispersion law has a very important property that distinguishes it from all other dispersion laws [10, 11]. If it is valid, the change in the wave number at the vacuum–matter interface is only due to the change in the wave-vector component $k_{0\perp}$ normal to the interface. Hence, all optical phenomena can in this case be described using the wave vector component normal to the interface and ignoring the parallel component. Moreover, the fact that the medium is moving parallel to the interface in no way affects the properties of the refracted wave. On the contrary, this testifies to the invalidity of (2) and (3) [12–14]. It is shown below that the potential dispersion law has one more important property.

In defining the refractive index as the vacuum-to-medium ratio of wave numbers, one normally means that the refractive index determines the velocity ratio in the same way (see, e.g., [10, 15]). This assumption is not obvious, however.

We consider a neutron passing through a refracting sample of length L with the refractive index $n(k_0)$. We define the dispersion law of the medium as $k = F(k_0^2)$ and calculate the neutron velocity in the medium $v = L/\tau$, where τ is the time of passage through the sample. This time can be written as the group time[2] [16, 17]

$$\tau = \hbar\, \frac{\mathrm{d}(\Delta\Phi)}{\mathrm{d}E}\,, \tag{6}$$

A I Frank Joint Institute for Nuclear Research,
Frank Neutron Physics Laboratory,
ul. Joliot-Curie 6, 141980 Dubna, Moscow region, Russian Federation
E-mail: frank@nf.jinr.ru

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[1] In recent years, potential (3) has been frequently called the Fermi potential. We avoid this term and prefer the term 'effective potential', because Fermi himself used the notion of the refractive index in a medium and introduced a model point-like quasipotential only to describe neutron scattering by an isolated nucleus in the Born approximation.

[2] The group delay time (6) was previously frequently referred to as the phase time.

where $E = [\hbar^2/(2m)]\, k_0^2$ is the neutron energy and, taking into account that the phase increment $\Delta\Phi$ is defined in an obvious manner as $\Delta\Phi = kL$, we obtain the velocity in the medium

$$v = \frac{1}{\hbar}\left(\frac{\mathrm{d}k}{\mathrm{d}E}\right)^{-1} . \qquad (7)$$

This implies that in general the velocity in a medium is

$$v = \frac{\hbar}{2m}\,(F')^{-1} , \qquad (8)$$

where m is the neutron mass and $F' = \mathrm{d}F/\mathrm{d}k_0^2$. Thus, the velocity in a medium depends not only on the refractive index n but also on the dispersion law of the medium. From (8), it follows that the validity of the relation

$$v = nv_0 \qquad (9)$$

immediately implies that $k^2 = k_0^2 + \chi^2$, where χ^2 is an arbitrary constant. Therefore, relation (9), which was taken as obvious, is only valid for the potential dispersion law.

We can readily account for the result obtained. Because the origin of the refractive index for any wave is associated with the interference of primary and secondary waves generated by scatterers in the medium, it is natural to believe that the neutron in the medium is not a true particle but a quasiparticle with an effective mass m^*. The concept of effective mass has been introduced previously for a neutron passing through a crystal under conditions of diffraction [18, 19]. Writing $v = \hbar k/m^*$ for the neutron velocity, we use the formula (8)

$$m^* = 2mkF' \qquad (10)$$

to obtain the equality $m^* = m$, valid for the potential dispersion law $F(k_0^2) = k_0^2 + \chi^2$. Consequently, an important property of potential dispersion law (2), (3) that distinguishes it from all the rest is the equality of the neutron effective mass in a medium to its inertial mass.

We also note that the proportionality of the effective mass to the derivative of the dispersion function implies that the mass can be negative. Deflection of a neutron beam by a magnetic field under diffraction was reported in [19]. In particular, the deflection of a beam was demonstrated in the direction opposite to the direction of the acting force, which corresponds to the case of a negative effective mass.

In the case of passage through a refracting sample, a negative effective mass can in principle appear, with the scattering amplitude exhibiting a resonance behavior. This possibility probably needs a more comprehensive discussion.

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