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METHODOLOGICAL NOTES

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Antigravitation in the Universe

I D Novikov

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Contents

Abstract. We discuss how pressure affects the matter–matter gravitational interaction. We show the frequently used claim that gravitation is produced in General Relativity not only by the mass density ρ but also by the pressure p in the combination $\rho + 3p/c^2$ to be incorrect. The way pressure actually influences gravitation is discussed together with some related problems.

Keywords: Einstein's general theory of relativity, antigravitation

1. Introduction

Two concepts prevail in modern physical cosmology: the theory of inflation at the beginning of the expansion of the Universe [1] and the observed accelerating expansion of the Universe at the present time [2]. Both phenomena are interpreted in the framework of General Relativity (GR) as gravitational repulsion caused by matter with a special equation of state.

The very existence of gravitational repulsion in the Universe arouses great interest in the problem of antigravitation in general. The theoretical possibility of antigravitation in GR has been known for a long time. Unlike the Newtonian theory, where gravitation is a particular field, in GR gravitation is described by the curvature of space–time. However, the appearance of antigravitation relates not to this feature of GR but to the parameters of the ordinary matter that creates gravitation. This feature is usually described as follows. We quote the specialists.

I D Novikov Astro Space Center, Lebedev Physical Institute, Russian Academy of Sciences,
ul. Profsoyuznaya 84/32, 117997 Moscow, Russian Federation;
Niels Bohr International Academy, Niels Bohr Institute,
Blegdamsvej 17, DK-2100, Copenhagen, Denmark;
National Research Center 'Kurchatov Institute',
pl. Akademika Kurchatova 1, 123182 Moscow, Russian Federation
E-mail: novikov@asc.rssi.ru

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"According to GR, the gravitational field is created not only by the mass of matter but also by all kinds of energy, pressure and stresses that are present in the matter" [1].

In a similar way, the authors of [3] write "... the pressure participates on equal footing with the density ρ in producing the gravitational field...," or, more quantitatively [2], "According to GR, gravitation is produced not only by the density of a medium but also by the pressure in the combination

$$\rho + \frac{3p}{c^2}\,,\text{"}$$

where ρ is the density of matter, p is the pressure, and c is the speed of light. Finally, as stated in [3], "taking pressure into account, as was shown by Tolman (1930), gravitational acceleration in GR for a body at rest is

$$g = -G\,\frac{4\pi}{3}\,\frac{R^3}{R^2}\left(\rho + 3\,\frac{p}{c^2}\right).\text{"} \tag{1}$$

Here, g is the free-fall acceleration on the surface of a ball with radius R, G is Newton's gravity constant, $4\pi R^3/3$ is the volume of the ball, and the density ρ and pressure p are assumed to be constant inside the ball.

According to the quotations given above, it is very easy to take the GR effects into account: to calculate the gravitational mass, the density ρ of Newtonian theory should be substituted by $\rho + 3p/c^2$.[1]

It is now clear that to obtain antigravitation for a positive ρ, it is necessary to assume that the pressure is negative and has a sufficiently high absolute value:

$$p < -\frac{1}{3}\,\rho c^2\,. \tag{2}$$

Inequality (2) is satisfied, in particular, for the vacuum equation of state, a vacuum-like 'quintessence' [4], and some other possible forms of matter [5–7].

Most frequently, questions about gravitational repulsion are considered in models of a homogeneous Universe. Here,

[1] We stress that so far we ignore the GR space–time curvature effects, i.e., we consider sufficiently small noncompact masses (see below for more details).

the usual way of taking the pressure effect on the evolution of the Universe into account is as follows. In a homogeneous universe, a small spherical volume is considered, and gravitational forces created by matter inside the volume are analyzed. The size of the sphere is small, the expansion velocity is low compared to the speed of light, and matter can be considered to be at rest. According to both the Newtonian theory and GR, for a spherical distribution of matter, matter outside a spherical volume does not produce any gravitational field in the interior of that volume. Therefore, knowing the gravitation of matter inside an isolated sphere is sufficient for determining the acceleration and dynamics of matter. One usually considers a sufficiently small volume in order to ignore the space–time curvature inside it. Then the entire difference between GR and the Newtonian theory reduces to different laws of gravity produced by matter.

As noted above, it has been recognized that to calculate gravitational acceleration at the boundary of an isolated ball in GR, it is sufficient to replace ρ in the Newtonian theory with the quantity $\rho + 3p/c^2$. We turn again to the specialists.

"The active gravitational mass density in an almost homogeneous distribution is [8]

$$\rho_{\mathrm{grav}} = \rho + 3p\,;\text{"} \tag{3}$$

here, $c = 1$.

In the case of a vacuum, the equation of state is

$$p_{\mathrm{v}} = -c^2 \rho_{\mathrm{v}}\,. \tag{4}$$

Substituting (4) in (3) yields

$$\rho_{\mathrm{grav}} = -2\rho_{\mathrm{v}}\,. \tag{5}$$

The quantity ρ_{grav} is negative for a positive density ρ_{v}. Hence, we find

$$g = -G\,\frac{4\pi R^3}{3R^2}(-2\rho_{\mathrm{v}}) = G\,\frac{8}{3}\,\pi R \rho_{\mathrm{v}}\,. \tag{6}$$

The acceleration is positive, which means gravitational repulsion, and is proportional to R. Hence, one usually deduces that if we select a sphere of radius R with the vacuum matter of the Universe, and place it into empty space, we obtain an object creating antigravitation in the ambient empty space. The same must hold for a ball made of any matter with the equation of state satisfying (2). The goal of the present methodological notes is (1) to show that the above statements are incorrect and (2) to consider some related issues.

2. Einstein equations for spherically symmetric distribution and motion of matter

We write the Einstein equations for a spherically symmetric distribution of matter. The interval is given by[2]

$$\mathrm{d}s^2 = \exp\nu\, c^2\,\mathrm{d}t^2 - \exp\lambda\,\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\,. \tag{7}$$

[2] Equation (7) is written in the curvature coordinates r. For the simplest topology of three-dimensional space applicable, for example, to stars or planets, $0 \leqslant r < \infty$. For a more complex case of wormholes, more complicated coordinates should be used (see, e.g., [10]).

Here and below, we let r denote the current radial coordinate and R denote the radius of the sphere. For a body instantly at rest, i.e., when the radial velocity is $v_{\mathrm{r}} = 0$, $\dot{\lambda} = 0$ at a given time, the GR equations for the nonzero components take the form [9]

$$8\pi G p = \exp(-\lambda)\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}\,, \tag{8}$$

$$8\pi G p = \frac{1}{2}\exp(-\lambda)\left(\nu'' + \frac{\nu'^2}{2} + \frac{\nu' - \lambda'}{r} - \frac{\nu'\lambda'}{2}\right) - \frac{1}{2}\exp(-\nu)\ddot{\lambda}\,, \tag{9}$$

$$8\pi G \rho = -\exp(-\lambda)\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) + \frac{1}{r^2}\,. \tag{10}$$

Here and below, $c = 1$, G is Newton's constant, ρ is the density of the matter, p is its pressure, the prime denotes differentiation with respect to r, and the dot denotes differentiation with respect to time.

The equilibrium equation is [9]

$$\nu' = -\frac{2p'}{p + \rho}\,. \tag{11}$$

The covariant vector of gravitational–inertial acceleration at a given point in static reference frame (7) is expressed as [11][3]

$$F_r = -\frac{1}{2}\,\nu'\,. \tag{12}$$

Equation (10) can easily be integrated to give

$$\exp\lambda = \left(1 - \frac{2GM}{r}\right)^{-1}, \tag{13}$$

where

$$M = 4\pi\int_0^r \rho r^2\,\mathrm{d}r\,. \tag{14}$$

Combining Eqns (8), (13), and (14), we obtain

$$-F_r = \frac{\nu'}{2} = G\,\frac{M + 4\pi r^3 p}{r^2(1 - 2GM/r)}\,. \tag{15}$$

If the reference frame is static, F_r is the gravitational acceleration (see Section 5 for clarification). The sign of ν' defines the direction of action of gravitation (see expression (12)). The plus sign means attraction and the minus sign means antigravitation.

3. Comparison of matter density and pressure contributions to gravitational acceleration

If at some moment of time reference frame (7) has no deformation accelerations (i.e., $\ddot{\lambda} = 0$ in formula (9)), the vector F_r describes gravitational acceleration only. To separate gravitational accelerations, equilibrium distribu-

[3] To avoid misunderstanding, we recall that F_r is a coordinate vector. The physical acceleration is expressed by the scalar $F = \sqrt{F_r F^r}$. In the cases considered below, where the gravitational radius of all objects is much smaller than their sizes, F_r almost coincides with the physical acceleration.

tions can be considered, with Eqn (9) satisfied. We consider such a case and ignore distribution edges, which cannot be static.

Gravitational acceleration in reference frame (7), in which the body is at rest at a given time, is defined by covariant vector (15) (see [12]). We note that the matter density ρ and pressure p enter the definition of v' in significantly different ways. The quantity v' depends on two parameters: the mass M and the pressure p. The mass M depends on ρ only and is determined by formula (14), similar to the Newtonian one but differing from the Newtonian value (as the sum of the masses of the volume elements) by the gravitational mass defect (see [12]). For small R with fixed ρ, this difference is insignificant. The pressure p does not enter expression (14) at all. Under the same conditions, the quantity in parentheses in the denominator of (15) is little different from unity. Ignoring the second term in the numerator of (15), we then recover the Newtonian expression F_r^{N} for the free-fall acceleration g^{N}:

$$F_r^{\mathrm{N}} \approx g^{\mathrm{N}} = -\frac{GM}{r^2}\,. \tag{16}$$

The second term in the numerator of (15) that takes p into account has quite a different meaning. Here, p is taken not as an integral over the entire gravitating mass distribution (as was the case for ρ) but at the point of measurement of v'. Therefore, the effect of the pressure p on the value of the gravitational acceleration v' is essentially different from the effect of density ρ!

The fundamentally important consequences of this fact are considered in Sections 4 and 5.

4. Mass of an object and the gravitational acceleration it produces

What acceleration is produced by a spherical matter distribution? It is expressed by formula (15) both inside matter and outside it, in empty space. In empty space, $\exp\lambda$ is given by the Schwarzschild solution [11]

$$-g_{11} = \exp\lambda = \left(1 - \frac{2GM_{\mathrm{S}}}{r}\right)^{-1}, \tag{17}$$

where M_{S} is the Schwarzschild mass.

Solution (17) coincides with formulas (13) and (14) outside the sphere $r > R$ if $M(R) = M_{\mathrm{S}}$, where $M(R)$ is the value of M on the sphere at $r = R$. Hence, the acceleration produced by a spherical mass in empty space outside the sphere is

$$-F_r = \frac{v'}{2} = G\,\frac{M(R)}{r^2(1 - 2GM(R)/r)}\,, \tag{18}$$

$$M(R) = 4\pi \int_0^R \rho r^2\,\mathrm{d}r\,. \tag{19}$$

These expressions do not contain the pressure p at all (cf. the argument in Section 3). This, in particular, implies that if we take a ball made of vacuum matter (say, 'quintessence') with the vacuum equation of state (4) and $\rho > 0$, then outside the ball, for $r > R$, we have not repulsion but attraction! The hasty conclusion that a ball made of vacuum matter creates antigravitation in empty space follows from a careless application of the statements quoted in the Introduction that, to calculate gravitational forces in GR, it is sufficient (with the space–time curvature ignored) to make the substitution $\rho \to \rho + 3p$. The example given above shows that this is not the case. What is then the reason for the appearance and broad use of such statements?

The point is as follows. When considering the cosmological problem with a homogeneous matter distribution $\rho =$ const and $p =$ const at a time t and selecting a sufficiently small sphere such that (a) the body can be considered at rest and (b) the space–time curvature can be disregarded, we have

$$-F_r = \frac{v'(R)}{2} = G\,\frac{M(R) + 4\pi R^3 p}{R^2}\,, \tag{20}$$

$$M(R) = \frac{4}{3}\,\pi R^3 \rho\,, \tag{21}$$

where R is the radius of the sphere. These expressions coincide with (1). In addition, we note that, as mentioned in the Introduction, Tolman showed in [13] (see also [9, 14]) that in the case of a stationary equilibrium matter distribution in an asymptotically flat space–time, the total mass is

$$M = \int (\rho + 3p)\sqrt{-g}\,\mathrm{d}V\,, \tag{22}$$

where $\mathrm{d}V$ is the element of the physical three-dimensional space, g is the determinant of the fundamental metric tensor, and the integral is taken over the entire physical static three-dimensional space. For $g = -1$, Eqn (22) coincides with Eqn (1) and (20), (21).

However, Eqn (22) is valid only for matter in equilibrium and when the integration is performed over the entire space in order to determine the total mass M and not the current mass depending on the radial coordinate. The calculation of the total mass, of course, is not suitable for the cosmological problem. Here is the root of misunderstandings arising from recipes like 'use $\rho + 3p$ instead of ρ' for nonuniform distributions, whereas for uniform cosmological models these statements are correct. (See also the discussion of the total mass in [14, p. 295].)

5. Variations in the gravitational acceleration for different spherical distributions of matter

We consider an isolated ball with radius R made of vacuum-like matter in empty space. The equation of state of matter inside the ball is given by Eqn (4). The density of matter inside the ball is constant, $\rho_{\mathrm{v}} = \mathrm{const} > 0$. The value of v' determining the acceleration can be calculated from (15). Figure 1 shows the qualitative dependence of $F_r = -v'/2$ on t. In this figure, as well as in Figs 2 and 3, we disregard the boundary effects of the distributions. As discussed in Section 4, the gravitational field in the vacuum describes the attraction by the mass M in (21), whereas inside the ball at $r < R$, there is antigravitation, and for a ball with a small radius R,

$$-F_r = \frac{v'}{2} = G\,\frac{4\pi r^3}{3r^2}\,(\rho_{\mathrm{v}} + 3p_{\mathrm{v}}) = -G\,\frac{8\pi}{3}\,\rho_{\mathrm{v}} r\,. \tag{23}$$

We now assume that around the radius-R ball under consideration, at some distance from its surface, there is a spherical shell made of vacuum matter with a density ρ_{v}, the inner boundary at $r_1 > R$, and the thickness $\Delta r = r_2 - r_1$. Variations in gravitational acceleration $F_r = -v'/2$ are shown in Fig. 2.

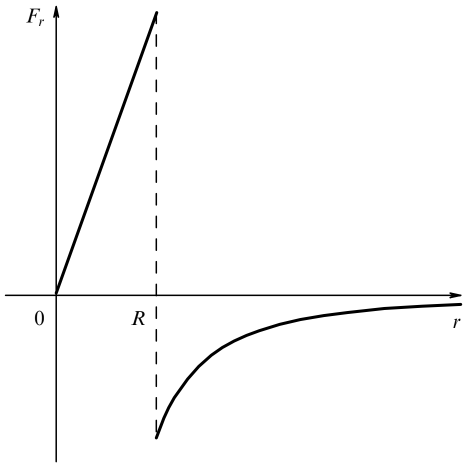

Figure 1. Change in $F_r = -\nu'/2$ in the case of a homogeneous ball with radius R and $\rho_v = -p_v$, $\rho_v > 0$.

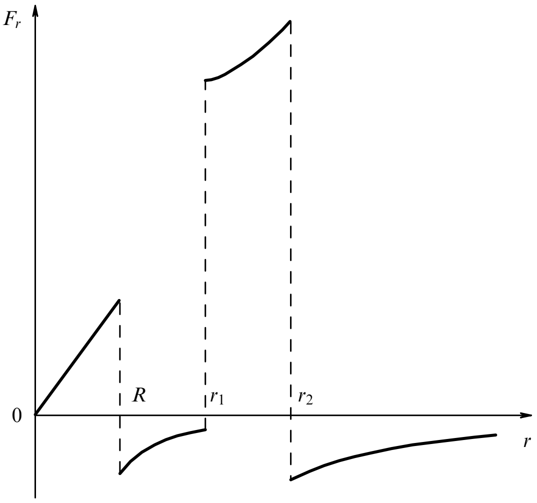

Figure 2. Change in $F_r = -\nu'/2$ in the case of a homogeneous ball with radius R and $\rho_v = -p_v$, $\rho_v > 0$ surrounded by a spherical shell with $\rho = \rho_v = -p_v$ located between r_1 and r_2.

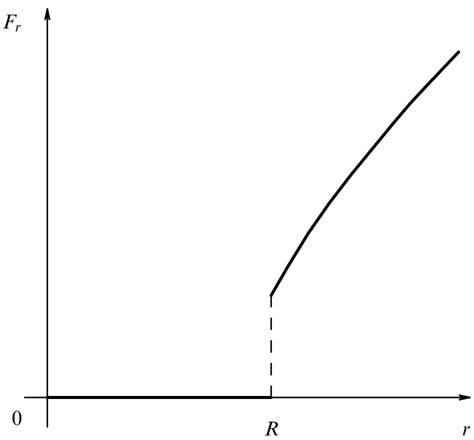

Figure 3. Change in $F_r = -\nu'/2$ in the case of a hollow sphere with radius R surrounded by homogeneous vacuum matter with $\rho_v = -p_v$, $\rho_v > 0$.

Inside the spherical layer with the thickness Δr, there is antigravitation, and in the empty space both inside and outside the sphere, there is gravitation. Of course, if we enclose a ball made of ordinary matter with $\rho > 0$ (say, a planet) by a spherical envelope of vacuum matter, then inside the envelope antigravitation is summed with the attraction force from the central ball.

Finally, we consider an empty sphere with radius R surrounded by vacuum matter with a constant density $\rho_v =$ const > 0, $p_v =$ const. The change in $F_r = -\nu'/2$ with radius is determined by Eqns (14) and (15) (see Fig. 3).

We make the following remark. If we consider an instantly static but nonequilibrium distribution of matter, Eqn (9) contains the term with $\ddot{\lambda} \neq 0$ in general. This means that the vector F_r that we determine also depends in part on the initial accelerations in the deformation of this system.[4] For example, when calculating antigravitation in empty space, we are interested in the acceleration F_r with respect to a static frame asymptotically flat at infinity. In the examples that we discuss, we do exactly this, bearing Birkhoff's theorem in mind.

The form of $\exp\lambda$ for M defined by (14) and the expression for ν' in (15) determine the solution uniquely. We note that in the expressions from Section 5, homogeneous distributions of ρ_v are equilibrium except at their edges.

6. Conclusion

We draw attention to Fig. 2. At the inner edge of the shell, the repulsion starts immediately, jump-wise. This could result in unusual physical processes. However, it should be remembered that the construction shown in Fig. 2 is nonequilibrium at the edges.

So far, we have considered only the case of isotropic, 'Pascal' pressure. We now make important remarks about the case of anisotropic pressure where the radial pressure p_1 does not coincide with the transverse pressure p_2. Everywhere above, we have considered the value of ν' and derived the corresponding equations using the value of p_1 only. It is the radial pressure p_1 that determines the amplitude of gravitation or antigravitation. The transverse pressure p_2 is used when we consider the problem of equilibrium.

We note that if we find λ and ν from (8) and (10), then in the case $p_1 \neq p_2$ we can set $\ddot{\lambda} = 0$ in (9) and determine p_2 that makes the system equilibrium. We do not consider the possible properties of p_2 or how physically realistic they could be, but the very possibility of such a mathematical procedure suggests that formally the system defined by (8) and (10) can always be made static without changing its other properties.

In these notes, we have also ignored the possibility of the existence (at least in principle) of objects made of matter with $\rho < 0$. This possibility and related issues will be considered in detail elsewhere. Here, we only note that such a possibility actually exists (see, e.g., [6, 10]). In this article, besides the principal issues, we focus on the methodological aspect of the problem, stressing that a superficial interpretation of formulas can sometimes lead to erroneous statements in quite serious papers and monographs, including in those written by the author.

We emphasize that the first ideas on possible antigravitation in the Universe were put forward by E B Gliner (see reviews [15, 16]).

[4] In the general case, F_r does not reduce to (12) (see [3]), but we are interested in the cases where Eqn (12) holds.

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