

METHODOLOGICAL NOTES

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On laws of conservation in the electrodynamics of continuous media
(on the occasion of the 100th anniversary
of the S I Vavilov State Optical Institute)

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Abstract. This article discusses conservation laws in the electrodynamics of dissipative continuous media, with particular emphasis on the electric area conservation rule for electromagnetic pulses and its application to short pulse propagation problems in resonant media.

Keywords: electrodynamics of continuous media, conservation laws, extremely short pulses, unipolar pulses, electric field area, coherent effects

From the Editorial Board. 15 December 2018 will mark the centenary of the establishment of the State Optical Institute (SOI), which has borne the name of Sergei Ivanovich Vavilov since 1952. Its founder and visionary Dmitrii Sergeevich Rozhdestvenskii, a talented scientist and distinguished science administrator, set the then unusual task of combining basic and applied optics in one research team to embrace all the main branches of this science and span an extremely broad spectral range: from the infrared to the ultraviolet. Considering that the optics of that time stood by the cradle of revolutionary changes caused primarily by the formation of quantum theory, and that only few discrete scientists were engaged in optical research and the optical industry was still in its infancy and far from meeting the demands of society (primarily of the Army), there is no escape from calling this task a mammoth one.

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Voluminous books and numerous papers, including those published in the journal *Uspekhi Fizicheskikh Nauk* [1–4], describe what has been possible to achieve at the SOI in a relatively short period of time. Namely, it was not long before the SOI reached the frontline in the main areas of 'academic' optics, developed optical glass, and made a decisive contribution to domestic optical instrument engineering and the formation of the optical industry for scientific, special, and civilian purposes. Experiencing continual pressure from above to broaden the scope of applied research at the expense of basic research, the SOI still managed to uphold the latter. An excellent example is provided by V A Fock's discovery of 'exchange forces' [5] and the development of the 'Hartree–Fock method' [6], which became a workhorse in solving many-body quantum-mechanical problems (a short time before, V A Fock had been a student and laboratory assistant at the SOI, later to become the head of SOI's Theoretical Sector). The SOI's early experimental and theoretical investigations in atomic, and later molecular, spectroscopy, glass and crystal spectroscopy, and many others are in the treasury of the physics of our country.

The post-war period also saw a rapid growth of research and staff at the SOI, which numbered 12,000 staff members by the 1990s. Some areas of research moved to the back-ground, partly in connection with the establishment of new problem-oriented optical institutes. But new areas also emerged and made rapid strides, including space optics, where SOI instruments prevailed, computer optics, holography (three-dimensional optical holography was proposed by Yu N Denisyuk at the SOI [7]), quantum optics (the effects of interference of atomic states, which permit implementing new techniques of ultrahigh-resolution spectroscopy and precision magnetometry and were discovered by E B Aleksandrov [8, 9]), and laser and nonlinear optics.

SOI's applied research was more 'bulky', but it is significant that basic and pilot investigations accounted for 30% of the impressive total financing, and the institute allocated these resources independently. It is this circumstance, along with the presence of highly skilled specialists in practically all areas of optics and of relatively modern equipment at the SOI, which enabled the SOI to quickly and validly solve intricate complex problems. This accounts for the fact that the first domestic laser was made at the SOI in June of 1961, only several months later than the American one [10, 11].

Interestingly, under development even in the early 1980s was a large-scale LIGA project (laser interferometric gravitational antenna) on the territory of the Baksan Neutrino Observatory, which was intended to detect gravitational radiation from cosmic sources [12].

Unfortunately, the 'pressure' from above to more towards applied research at the SOI has gained momentum in recent years, when the institute has been deprived of state budget support. Basic and pilot investigations at the SOI are nevertheless being continued: without them, applied research has no prospects and is doomed to stagnation. We confirm this point by publishing below a paper by one of the oldest SOI staff members and his colleagues.

The Editorial Board of the journal *Uspekhi Fizicheskikh Nauk* congratulates the staff members of the renowned SOI — both present-day staff members and those who left the SOI buildings on Vasil'ev Island of St. Petersburg — on the centenary and wishes them new success in advancing, even under the difficult present conditions, optical research, which is in demand by our country and global science.

Contents

1. Introduction

Conservation laws, whose origins are traced back even to the work of ancient philosophers, occupy a special place in the modern natural sciences and especially in physics. Not only do they play a 'prohibitive' role, but they also are an aid in simplifying the dynamics analysis of a system, predicting its qualitative character, and in some cases finding the results of the evolution of various physical systems. In physics, the conservation laws are usually applied to conservative systems isolated from the environment; here, important examples are provided by the momentum, angular momentum, and energy conservation laws in mechanics, gravitational theory, and electrodynamics in a vacuum [13, 14].

For open systems with dissipation, the possibility of the existence of conservation laws is in doubt. Restricting ourselves to the domain of the electrodynamics of continuous media [15], we note that the initial electromagnetic field cluster disappears in the course of a long evolution in dissipative media with energy transfer to the absorbing medium. One would therefore think that the conservation laws in dissipative media cannot be formulated exclusively in terms of the electromagnetic field.

In Refs [16–19], several exclusively electrodynamic conserved integral characteristics were nevertheless indicated for a rather general class of media with dissipation: isotropic and anisotropic, linear and nonlinear, with an arbitrary form of frequency and spatial dispersion, with absorption, and with laser amplification. The aim of this report is a more systematic exposition of the derivation of these conservation rules and an illustration of their usefulness in the analysis of the interaction of extremely short laser pulses with different media.

2. Derivation of conservation laws

The 'evolutional' Maxwell equations of the electrodynamics of continuous media have the form [15]

$$\operatorname{rot}\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}\,\mathbf{j}\,, \tag{1}$$

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}\,. \tag{2}$$

Here, $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field strength, $\mathbf{D}$ and $\mathbf{B}$ are the electric and magnetic inductions, $\mathbf{j}$ is the electric current density, c is the speed of light in a vacuum, and t is the time. The difference in structure of the right-hand sides of Eqns (1) and (2) is due to the absence of magnetic currents in nature; the influence of hypothetical magnetic charges and currents [20] on the form of conservation laws is discussed in Ref. [19].

The field is assumed to be localized in a finite spatial and temporal domain. We integrate Eqn (2) over an infinite volume. Since the field intensity for localized structures must vanish at the periphery (asymptotically), we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\mathbf{V}_B = 0\,, \tag{3}$$

where we introduced the 'volume of the magnetic field'

$$\mathbf{V}_B = \iiint_V \mathbf{B}\,\mathrm{d}\mathbf{r}\,. \tag{4}$$

As regards the 'volume of electric induction'

$$\mathbf{V}_D = \iiint_V \mathbf{D}\,\mathrm{d}\mathbf{r}\,, \tag{5}$$

it should be borne in mind that the integral of current density over the volume of a unit cell is equal to zero under thermodynamic equilibrium not only in insulators ($\mathbf{j} = 0$), but even in ferromagnetic crystals [15]:

$$\int_{V_c} \mathbf{j}\,\mathrm{d}\mathbf{r} = 0\,. \tag{6}$$

Similarly, from Eqn (1) it therefore follows that

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\mathbf{V}_D = 0\,. \tag{7}$$

We integrate Eqn (2) in time, also with infinite limits, to obtain

$$\operatorname{rot}\mathbf{S}_E = 0\,, \tag{8}$$

where the 'electric pulse area' appears:

$$\mathbf{S}_E = \int_{t=-\infty}^{+\infty} \mathbf{E}\,\mathrm{d}t\,. \tag{9}$$

As pointed out in Ref. [21], the electric pulse area has the meaning of the moment of the electric part of the Lorentz force, which acts on a unit electric charge and changes its mechanical momentum according to Newton's second law.

On integration of Eqn (1) over time, for 'ordinary' media we find

$$\operatorname{rot}\mathbf{S}_H = \frac{4\pi}{c}\,\mathbf{q}(\mathbf{r})\,. \tag{10}$$

Introduced here is the 'magnetic pulse area'

$$\mathbf{S}_H = \int_{t=-\infty}^{+\infty} \mathbf{H}\,\mathrm{d}t\,, \tag{11}$$

and

$$\mathbf{q}(\mathbf{r}) = \int_{t=-\infty}^{+\infty} \mathbf{j}(\mathbf{r}, t)\,\mathrm{d}t \tag{12}$$

is a vector whose modulus coincides with the integral charge density passed through a point with coordinates $\mathbf{r}$ over the pulse time, and the direction shows the time-integrated direction of charge flux. For 'extraordinary' media — pyroelectrics, which possess at least two states with different spontaneous electric induction $\mathbf{D}$ — an additional constant term $(\mathbf{D}|_{t=+\infty} - \mathbf{D}_{t=-\infty})/c$ may emerge on the right-hand side of Eqn (10). In the absence of electric current, Eqn (10) takes on a form similar to that of Eqn (8):

$$\operatorname{rot}\mathbf{S}_H = 0\,. \tag{13}$$

Equations (3) and (7) are the rules of conservation (invariability in time) of the corresponding integral quantities, while Eqns (8) and (13) testify to the vortex-free nature of the electric and magnetic pulse areas and the consequential possibility of introducing potentials [18]. Nevertheless, both Eqns (8) and (13) also become analogous to the conservation rules in the plane-wave approximation, in which the field strength (1) and (2) depend only on one Cartesian coordinate z (the longitudinal coordinate along the prevailing direction of radiation propagation). In this case, the form of Eqns (8) and (13) becomes simpler:

$$\frac{\mathrm{d}}{\mathrm{d}z}\,\mathbf{S}_E = 0\,, \tag{14}$$

$$\frac{\mathrm{d}}{\mathrm{d}z}\,\mathbf{S}_H = 0\,. \tag{15}$$

As for Eqns (3) and (7), their form persists for plane waves when the volume integration in Eqns (4) and (5) is replaced by integration with respect to the longitudinal coordinate z.

Let us define more precisely the limits of validity of the assumption made in the derivation of Eqn (8) that the light structures are localized. We assume that initially the wave packet of radiation is located in a vacuum and there is no field in the medium, and that after a sufficiently long time interval after the packet passage the field vanishes in any fixed point in space (due to dissipation in the medium and natural radiation recession at the speed of light). In practice this implies that we have a laser system capable of generating a single short pulse, which begins traveling in the vacuum towards the medium, in which there is no macroscopic electromagnetic field prior to the arrival of the pulse. One more limitation: either soft or hard excitation of lasing by the initial pulse should be impossible in the system (the latter alternative is realized, for instance, in a laser with saturable absorption [18, 22–25]. Last, here, we invoke only the classical representation of an electromagnetic field in the absence of noise and fluctuations.

Since the action of an electric field on the medium is usually more significant, in what follows we restrict ourselves to the application of rule (14) to the electric field area.

3. 'Electric area' and 'envelope area' of a pulse

It is noteworthy that, in nonlinear optics, by the pulse area is commonly meant another quantity, which is referred to as the pulse envelope. In 1969, McCall and Hahn discovered and theoretically described the effect of self-induced transparency (SIT) [26], which constitutes the fact that a short pulse of sufficiently high energy may propagate in a resonantly absorbing medium with practically no loss. The theory of the effect is applicable to the radiation close to a monochromatic plane wave of frequency ω_0 and wavenumber k_0. Then, in a one-dimensional geometry, it is possible to introduce a slowly varying envelope $\varepsilon(z,t)$: $\mathbf{E} = \mathbf{e}\,\mathrm{Re}\,[\varepsilon(z,t)\exp(\mathrm{i}k_0 z - \mathrm{i}\omega_0 t)]$, where $\mathbf{e}$ is the unit vector of radiation polarization. For a two-level medium with the dipole moment d_{12} of the working transition, use is made of the area of the pulse envelope [26–29]:

$$\theta(z) = \frac{d_{12}}{\hbar}\int_{-\infty}^{\infty} \varepsilon(z,t)\,\mathrm{d}t\,, \tag{16}$$

where $\hbar$ is the reduced Planck constant. Here, we emphasize the difference between the electric pulse area (9) and the area of envelope (16) $\theta(z)$, which is proportional to the integral of a slowly varying field amplitude. The evolution of the pulse envelope area in the coherent propagation of many-cycle pulses in a two-level resonance medium is described by the area theorem of McCall and Hahn [26–29]:

$$\frac{\mathrm{d}}{\mathrm{d}z}\,\theta(z) = -\frac{\alpha_0}{2}\sin\theta(z)\,. \tag{17}$$

Here, α_0 is the weak-signal absorption coefficient (per unit length). From the area theorem, it follows, in particular, that when the pulse envelope area is not a multiple of an even number of π, it increases or decreases and tends to the nearest multiple of an even number of π. Considered in Ref. [30], which appeared after the work by McCall and Hahn, was the solution to the SIT problem for a two-level medium without resorting to the slow envelope approximation. The authors of Ref. [30] demonstrated, in particular, the existence of a soliton solution in the form of a unipolar pulse in the form of a hyperbolic secant. They applied expression (16) to determine the area of this soliton and used the electric field strength rather than the envelope, i.e., the electric pulse area, to within a constant factor. This area of the unipolar SIT soliton turned out to be equal to 2π. Below, we will use this definition for the electric pulse area. We note that in the theoretical investigation of the interaction of extremely short soliton-like pulses, a number of simplifications lead to sine-Gordon-type equations, which describes the time and space evolution of precisely the electric pulse area [31–33]. Simplified models yield incorrect values of the electric and magnetic areas. This points to the limited applicability of such models and happens due to disregarding of the formation of long pulse 'tails' and the generation of a counter-propagating wave, which make their contribution to the electric area. Furthermore, if relaxation is ignored, the oscillators of a medium excited even by a short pulse may oscillate indefinitely long and continuously radiate electromagnetic waves, which would additionally violate the energy conservation law.

Consider now the application of the above rules to problems that arise in nonlinear optics and laser physics. Considered most often in this case are one-dimensional problems, which relations (14) and (15) apply to.

4. Unipolar pulse in an absorbing medium

Although the electric area conservation rule is general in nature, its nontrivial character is most amply manifested as applied to extremely short, and especially so to unipolar, pulses, whose electric field strength of fixed polarization retains its sign throughout the pulse duration. Recent years have seen a manifestation of interest in these pulses (see review Ref. [34] and references therein).

The problems of interaction between extremely short pulses with a duration shorter than the resonance transition period and a two-level system are solved with the use of the one-dimensional wave equation,

$$\frac{\partial^2 E(z,t)}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E(z,t)}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 P(z,t)}{\partial t^2}\,, \tag{18}$$

which is supplemented with material equations for the two-level medium:

$$\frac{\partial \rho_{12}(z,t)}{\partial t} = -\frac{\rho_{12}(z,t)}{T_2} + \mathrm{i}\omega_0 \rho_{12}(z,t) - \frac{\mathrm{i}}{\hbar}\, d_{12} E(z,t)\, n(z,t)\,, \tag{19}$$

$$\frac{\partial n(z,t)}{\partial t} = -\frac{n(z,t)-n_0}{T_1} + \frac{4}{\hbar}\, d_{12} E(z,t)\, \mathrm{Im}\, \rho_{12}(z,t)\,, \tag{20}$$

$$P(z,t) = 2N_0 d_{12}\, \mathrm{Re}\, \rho_{12}\,. \tag{21}$$

System of Eqns (18)–(21) contains the following quantities: E is the electric field strength of fixed linear polarization, P is the polarization of the medium, N_0 is the density of active centers, ω_0 is the resonance transition frequency ($\lambda_0 = 2\pi c/\omega_0$ is the wavelength of the resonance transition), n_0 is the stationary difference of the populations of the two working levels in the absence of an electromagnetic field, ρ_{12} is the nondiagonal element of the density matrix, and $n \equiv \rho_{11} - \rho_{22}$ is the population difference between the ground (1) and excited (2) states of the two-level system. The polarization of the medium is associated with the nondiagonal element ρ_{12} of the density matrix, according to expression (21). The polarization relaxation time T_2 and the population difference relaxation time T_1 of the two-level resonance medium appear in the equations. The medium may be considered to be absorbing or amplifying, depending on the sign of the population difference.

Prior to demonstrating the validity and usefulness of the electric area conservation rule, we draw attention to the following fact: in the solution to the problems of the passage of few-cycle pulses of zero electric area through dissipative media, the question of exact fulfillment of the area rule has not been posed. That is why the results arrived at in the calculation of unipolar pulse propagation do not correspond to notions reliant on the intuition elaborated. It is seemingly impossible to expect the conservation of electric area of a unipolar pulse after passage through a strongly absorbing medium, for the medium should absorb the radiation. But this is a seeming contradiction. For an interaction with the resonantly absorbing medium to occur and lead to absorption, the unipolar radiation must have a spectrum, a significant portion of which is in the domain of the resonance transition. In this case, the unipolar pulse has to be sufficiently short and its duration comparable with the reciprocal of the resonance transition frequency. A portion of the spectrum about the resonance frequency will vanish due to absorption, but the constant component will not vanish. Even if the absorption cancels irreversibly the entire high-frequency portion of the spectrum, a narrow portion of the spectrum about the zero frequency will persist. The electric field amplitude in the pulse will therefore become lower and the pulse will become longer. In this case, the electric area will remain constant. In fact, by no means does the electric area come to the energy which is proportional to the square of the field strength. Pulses of equal electric area possess different energies. For instance, when the pulse amplitude decreases 10-fold and its duration increases 10-fold, which leaves the electric area invariable, the pulse energy decreases 10-fold. The area rule may therefore hold good in a dissipative system with a loss in energy.

Figure 1 shows results of numerical simulations of the passage through an absorbing medium for a pulse with the initial shape of a hyperbolic secant and an electric area of $3\pi/2$. If the electric area had obeyed the 'area theorem' of McCall and Hahn, the pulse area at the output of the medium would have increased and approached 2π. The data of numerical simulations show that this is not the case. In all instances, the electric pulse area remains invariable throughout its propagation path, and a unipolar pulse transforms into a bipolar one to acquire oscillatory components. Although the electric area is conserved, the degree of unipolarity, which is defined by the relation [34]

$$\xi = \frac{\left|\int_{-\infty}^{\infty} E\,\mathrm{d}t\right|}{\int_{-\infty}^{\infty} |E|\,\mathrm{d}t}\,, \tag{22}$$

decreases with propagation in the medium.

To complete the example with absorption, we note that a two-level system is inapplicable for describing real systems exposed to extremely short high-power pulses. And yet, when the relaxation of the system is taken into account, the electric pulse area is conserved both for a two-level system and for multilevel models. We emphasize that no significant unipolar counterpropagating wave emerges in the reflection from the boundaries of the medium or in the medium itself (as is clear

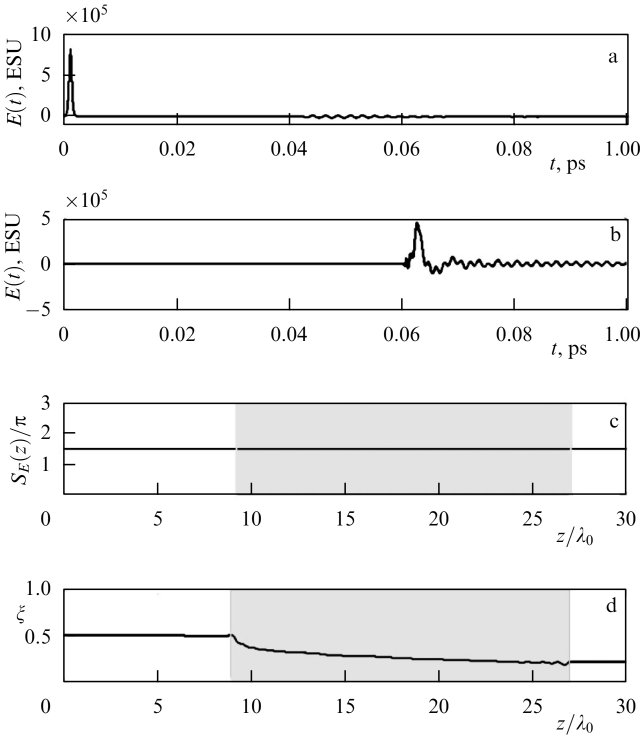

Figure 1. Example of the passage of a pulse with an electric area of $3\pi/2$ through an absorbing medium. The medium begins at a point $z = 9\lambda_0$ and ends at $z = 27\lambda_0$. (a) Time dependence of the electric field strength at a distance of $3\lambda_0$ from the left edge of the medium, (b) time dependence of the electric field strength at the output of the medium, (c) electric area of the pulse in the path of pulse propagation, (d) degree of pulse unipolarity in this path. The domain of the medium is shown in grey in (c) and (d). Parameters used in the simulation: resonance transition wavelength $\lambda_0 = 700$ nm, transition dipole moment $d_{12} = 5$ D, population difference relaxation time $T_1 = 1$ ps, polarization relaxation time $T_2 = 0.1$ ps, two-level particle density $N_0 = 10^{21}$ cm^{-3}, unipolar pulse amplitude $E_0 = 8 \times 10^5$ ESU, electric input-pulse area $S_E = 3\pi/2$ (for the absorber), pulse duration $\tau_p = 190$ as ($T_0/12$), shape of the pulse incident on the medium is hyperbolic secant, and $n(z, t = 0) = n_0 = 1$.

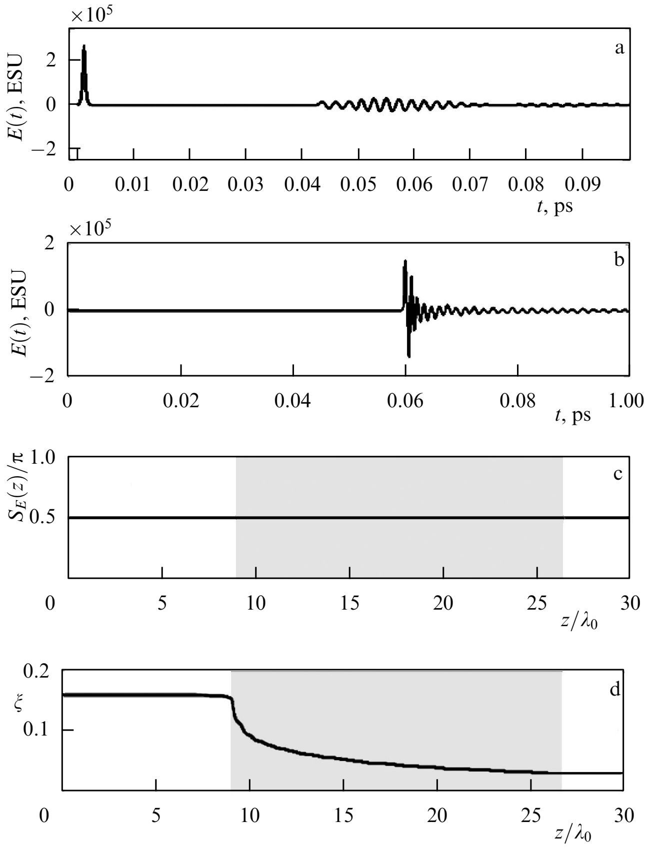

Figure 2. Unipolar pulse propagation in an amplifying medium. (a) Time dependence of the electric field strength at a distance of $3\lambda_0$ from the left edge of the medium, (b) time dependence of the electric field strength at the output of the medium, (c) electric pulse area in the path of propagation, (d) degree of pulse unipolarity in the path of propagation. The domain of the medium is shown in grey in (c) and (d). The simulation was performed for the same parameters as in Fig. 1 with the exception of unipolar pulse amplitude $E_0 = 2.7 \times 10^5$ ESU, $S_E = \pi/2$, $n(z, t = 0) = -1$, $n_0 = 1$.

in Fig. 1a, the amplitude of the counterpropagating wave is bipolar in character). Therefore, the electric pulse area at each point in space is equal to the area of the pulse incident on the medium. And the degree of unipolarity to the left of the medium is lower than unity due to the reflected wave.

5. Unipolar pulse in an amplifying medium

Let us now consider the case of unipolar pulse amplification. The conservation of electric pulse area in an amplifying medium is also at variance with physical intuition. How can the electric area — after passing through the amplifying medium — be equal to the electric pulse area prior to amplification? This will be the case if the peak amplitude increases and the duration becomes shorter. Then, the pulse energy will increase with retention of the area. A bipolar tail of zero electric area would be expected to emerge behind a unipolar pulse. A simulation of the amplification of a pulse with an electric area $\pi/2$ is exemplified in Fig. 2. The amplification is provided by the initial population inversion of the medium, $n(z, t = 0) = -1$.

One can see in Figs 2a and 2b that the unipolar pulse acquires an appreciable bipolar component in the course of amplification, which leads to a lowering of the degree of unipolarity (Fig. 2d). And, again, the electric area remains invariable throughout the simulation range (Fig. 2c).

We draw attention to the fact that the degree of unipolarity to the left of the medium is smaller than unity and smaller than in the example with absorption. The wave reflected from the medium is responsible for this, which is seen at the center in Fig. 2a. Since the medium is amplifying, the amplitude of the counterpropagating wave is higher than in the case of an absorbing medium. It also is of the form of a bipolar 'ringing' of zero electric area caused by the radiation of the medium at the resonance transition frequency. We also note that the area conservation in the amplification of a unipolar pulse was pointed out in Ref. [35].

According to the 'area theorem', in the case of many-cycle bipolar pulse amplification, the area of the pulse envelope under amplification should vary and tend to π. In the course of pulse propagation in an amplifying medium, the pulse amplitude will grow and the duration will shorten. It will tend to a pulse consisting of one field oscillation cycle. Note that this bipolar pulse cannot transform — on the strength of the requirement of zero electric area conservation — into a single unipolar pulse (due to amplification of primarily the first half-wave of the field), which would behave like a π pulse in the amplifying medium.

6. Some implications of the area rule

Generally speaking, the electric area conservation rule says nothing about what may happen with the area of the initial pulse after its passage through one dissipative medium or another. However, an important inference can be made when the reflected waves may be ignored. When a pulse of zero electric area is incident on the medium, under no conditions in the medium or behind it will the transmitted radiation be of the form of a single pulse of nonzero electric area — the initial pulse may only divide into subpulses of opposite polarity. An example of such a division into codirectional subpulses is provided in Ref. [18]. A similar division into subpulses is clearly seen in the substantial reflection of light. Namely, such an example is provided in Ref. [36], which theoretically considered the reflection problem of a single-cycle pulse of zero electric area from a thin metallic layer and showed the emergence of a reflected pulse with a constant component. Unipolar pulses were earlier discovered in the reflection from a nonlinear medium in simulations [37]. Unipolar pulse amplification is possible in a dynamic resonator with an oscillating mirror [38–40]; its quantum analogue is known as the dynamic Casimir effect [41, 42].

7. Conclusions

A theoretical analysis of the topical problems of ultrashort pulse propagation and unipolar pulse generation has led to the discovery of conservation rules in the electrodynamics of dissipative media, whose existence was hard to assume *a priori*. In solving problems in the electrodynamics of dissipative media, it turns out that there are electromagnetic field quantities which are conserved in the entire space, although, it seems, no conserved quantities for an electromagnetic field may exist in the presence of electromagnetic energy loss in the system. These rules assume their simplest form in one-dimensional radiation–matter interaction problems. In our opinion, this permits speaking about the conservation rules for the electric and magnetic pulse areas.

In the numerical simulations outlined in this report, these rules are shown to hold well in the propagation of light in both absorbing and amplifying media. As noted above, the area conservation rule possesses predictive power. Unipolar pulses may be produced from a bipolar one in its division into subpulses of opposite polarity or, in the one-dimensional geometry, in the emergence of a counterpropagating wave, for instance, in the reflection of a pulse from the boundaries or bulk inhomogeneities of a medium. We believe that the conservation rules under discussion are also significant in the elucidation of the prospect of dissipative soliton compression in fiber lasers [43].

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