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METHODOLOGICAL NOTES

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On an estimate of turbulent transport in a magnetized plasma

(on the 90th anniversary of the birth of B B Kadomtsev)

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Abstract. In 1964, B B Kadomtsev proposed that the results of a linear plasma instability analysis alone can be used to estimate the coefficients of turbulent transport across the magnetic field in a plasma. In this paper, we discuss examples of experimental support for this proposal that concern, specifically, the properties of a turbulent positive discharge column in a strong magnetic field, the ionization turbulence of a weakly ionized plasma in crossed electric and magnetic fields, and turbulence in a tokamak wall plasma.

Keywords: plasma, instability, turbulent transport, magnetic field

1. Turbulence of a positive column

The problem of turbulent plasma transport arose in relation to research in controlled thermonuclear fusion, which revealed numerous instabilities of plasma in a magnetic field. Theories of instabilities for small perturbations were created relatively fast. The first steps in theoretical research of plasma turbulence were made in studies by Kadomtsev [1–3].

As a result of developing helical instability in a strong magnetic field, plasma turns out to be turbulent in a discharge, with the transverse diffusion coefficient of the same order as for ambipolar diffusion in discharges without a magnetic field [4–6]. The nonconducting walls of a discharge tube do not constrain plasma motion in crossed electric and magnetic fields under the action of a pulsating electric field, which allowed Kadomtsev to resort to an analogy with the gasdynamic Prandtl submerged jet. The analogy was based on the fact that in convective motion, plasma flows in separated twisted tubes onto a nonconducting wall and recombines there. ‘Bubbles’ that are free of plasma penetrate into the tube from the outside, and smaller-scale oblique perturbations develop on their boundaries. As a result, helical turbulent pulsations, stretched along the current, evolve in the plasma at various transverse scales and angles to the longitudinal axis. For such pulsations, Kadomtsev introduced an effective mixing length l, entering the expressions for the pulsations of plasma density n' and velocity v' as

$$n' = l\,\frac{\mathrm{d}n}{\mathrm{d}r}\,, \quad v' = U\,\frac{n'}{n}\,,$$

where

$$U = \frac{1}{2}\,b_{\mathrm{e}}E\sqrt{\frac{b_{\mathrm{i}}}{b_{\mathrm{e}}}}\,,$$

E the longitudinal electric field, and b_{e} and b_{i} the mobilities of electrons and ions [1]. The averaged diffusive flux in this case becomes

$$q = n'v' = Ul^2 n^{-1}\left(\frac{\mathrm{d}n}{\mathrm{d}r}\right)\left|\frac{\mathrm{d}n}{\mathrm{d}r}\right| = -D_{\mathrm{T}}\,\frac{\mathrm{d}n}{\mathrm{d}r}\,.$$

The scale l was estimated by comparing data on the electric field strength in a turbulent column and in discharges without a magnetic field, giving the ratio of l to the discharge tube radius $l/a \approx 0.15$ [5–11]. Close values of l/a were confirmed in experiments [12]. Another experiment confirming the theory in [1] was the measurement of the plasma density profile in a turbulent positive column [13].

Kadomtsev considered the nonlinear interaction of waves in weak and strong turbulence, compared theory with available experimental data, and proposed an estimate for the mean turbulent transport of magnetized plasma transverse to the magnetic field [2]. In drift instabilities, ions move transversely to the magnetic field, mainly because of the drift in crossed electric and magnetic fields. In this case, the motion of plasma as an incompressible medium bears the character of turbulent convection. Harmonic oscillations of plasma with equilibrated amplitudes do not create a mean particle flux in and of themselves. However, if the amplitude of oscillations is growing with time, then for each subsequent half period the plasma experiences a slightly larger displacement than in the

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preceding half period. In the presence of a density gradient, a mean plasma flow is generated perpendicularly to the magnetic field. The mean plasma displacement, depending on the instability increment, increases. The amplitude of a nonlinear wave stops growing when the density gradient in the wave is comparable to the ambient gradient. Based on this reasoning, Kadomtsev used the linear approximation to derive the expression for the mean plasma transport:

$$q \approx -\left\langle \frac{\gamma}{k_\perp^2} \right\rangle \frac{\mathrm{d}n}{\mathrm{d}x}\,. \tag{1}$$

Here, γ is the instability increment, $k_\perp$ is the wave number, the angular brackets denote averaging over all oscillations, and the factor $\langle \gamma / k_\perp^2 \rangle$ corresponds to the effective coefficient of turbulent diffusion [2].

Formula (1) was questioned by many prominent theoreticians, who believed that no reliable estimate for strong plasma turbulence can be obtained based only on the linear instability analysis.

Derivation of the effective coefficients of heat conductivity χ and diffusion D in strongly turbulent plasma was proposed in Ref. [3]: *Suppose that from the very beginning we take all the appropriate effects into account in the equations of motion and then select χ and D such that perturbations with minimum localization be at the instability boundary and all stronger localized perturbations be decaying. In this way the effect from all perturbations of smaller scales is taken into account in large-scale perturbations.* The coefficients D and χ found in this way are determined by virtually all perturbations and differ from the true ones only by a small value due to the contribution of large-scale perturbations. The validity of ideas put forward in Ref. [2, 3] on the transport coefficients in turbulent plasmas was tested experimentally in gas discharges and in near-wall regions in tokamaks.

2. Ionized turbulence in crossed fields

In discharges in crossed electric and magnetic fields, nonisothermal weakly ionized plasma is unstable if the product of the electron cyclotron frequency and the electron mean free path is larger than some critical value of the order of unity ($\omega_H \tau_e > 1$). Such an instability, having an ionization nature, was thoroughly discussed in reviews [14, 15].

In a homogeneous plasma, prior to the onset of instability, the current perpendicular to the magnetic field obeys the generalized Ohm's law with the Hall parameter $\beta \cong \omega_H \tau_e$,

$$\mathbf{j} + \mathbf{j} \times \boldsymbol{\omega}_H \tau_e = \sigma(\mathbf{E} + \mathbf{u} \times \mathbf{H})\,.$$

Here, $\mathbf{j} = -e_0 n \mathbf{v}_e$, $\boldsymbol{\omega}_H = e_0 \mathbf{H} / m_e c$, $\sigma = e_0^2 n \tau_e / m_e$, $\mathbf{u}$ is the plasma velocity, $\mathbf{H}$ is the magnetic field, and $\mathbf{E}$ is the electric field.

In a turbulent regime, the effective Hall parameter $\beta_{\mathrm{eff}} = \langle E_x \rangle / \langle E_y \rangle$ ceases to increase as the magnetic field is increased. An example of the experimental dependence of $\langle E_x \rangle / \langle E_y \rangle$ on the product $\omega_H \tau_e$ is given in Fig. 1. Short-circuiting of the Hall EMF by fluctuating currents limited the effective Hall parameter, which in a turbulent plasma is close to the value of the critical Hall number β_c estimated from the linear theory of ionization instability. These results confirmed the correspondence between the level of developed turbulence and the stability boundary for the plasma as a whole [14, 15]. When the quantity $\beta_{\mathrm{eff}} = \langle E_x \rangle / \langle E_y \rangle$ is saturated, the effective conductivity does not depend on the electron-gas collision frequency and reduces with an increase in the magnetic field:

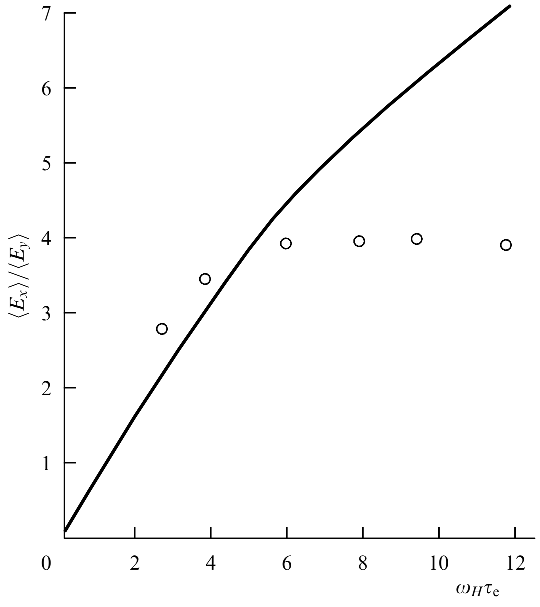

Figure 1. Dependence of $\langle E_x \rangle / \langle E_y \rangle$ on the product $\omega_H \tau_e$.

$$\sigma_{\mathrm{eff}} = \frac{\langle j_y \rangle}{\langle E_y \rangle} = \beta_{\mathrm{eff}}\, e_0^2 \, \frac{\langle n \rangle}{m_e \omega_H}\,.$$

3. Turbulence in the near-wall region in tokamaks

In tokamaks, there is a periphery scrap-off-layer (SOL) between closed magnetic surfaces and the walls where magnetic field lines cross metallic surfaces of the limiter or the divertor walls. In the SOL, plasma flows out along the magnetic field on a metallic surface, closing the longitudinal current of equilibrium [16].

There is a jump in the electric potential φ of the order of the electron temperature between the plasma and the surface. The ratio of the potential difference at the ends of a field line to this potential jump is determined by a dimensionless parameter proposed by Kadomtsev [17],

$$\sqrt{\frac{m_e}{m_i}}\,\frac{L}{\lambda_e}\,.$$

Here, L is the length of the field line and λ_e is the electron mean free path.

Plasma motion transverse to the magnetic field is governed largely by turbulent processes, whose nature is explained in Refs [18–23]. On the outer torus radius, a positive perturbation of the plasma density perturbs the longitudinal electric current density and the poloidal electric field, creating plasma drift to the side of the negative pressure gradient. Such plasma (flute) instability in an inhomogeneous magnetic field leads to turbulence with a characteristic diffusivity coefficient that is close to the expression proposed by Bohm:

$$D_{\mathrm{T}} = \frac{cT}{e_0 B}\,.$$

Detailed studies of near-wall turbulence in tokamaks and stellarators lent support to the existence of the two-dimensional convective cells described above. In these cells, the correlation length for density and potential oscillations along the magnetic field by far exceeds the scale of transverse perturbations. In the near-wall turbulence, the scale of transverse pulsations is determined by their wave number times the ion cyclotron radius $k_\theta \rho_\mathrm{i} \approx 0.1$. The turbulent pulsation frequency is mainly determined by plasma convection in crossed electric and magnetic fields.

The comparison of experimental and computed profiles of plasma concentration and pressure confirmed the estimate (1) proposed by Kadomtsev, which describes turbulence in magnetized plasmas with cardinally different physical parameters [24, 25].

4. Conclusions

In modern toroidal magnetic systems, plasma is observed to rotate at a speed close to the speed of sound, which was not discussed in the 1960–1970s. This called for the modification of most of the results obtained for a resting equilibrium plasma [26]. With this goal, numerical simulations are being carried out for complex nonlinear processes and experiments are being conducted aimed at measuring turbulence in plasmas of tokamaks and stellarators.

At present, in exploring physical phenomena in toroidal systems with magnetic confinement, the main focus is on geodesic acoustical modes (GAMs) [27], occurring as a result of the evolving drift turbulence. There is a large number of computational and observational studies of GAMs [28–33]. It is supposed that GAMs limit the drift instabilities, leading to regimes with improved plasma confinement.

An essential role in explaining the anomalous transfer of energy and particles may be played by ultrashort wave oscillations related to the electron temperature-gradient mode (ETG mode) [34]. Results of explorations of electrostatic turbulence excited by the ETG mode indicated the presence of large transport flows [35]. For shortwave instability, estimate (1) gives small values of the transport coefficient.

GAMs cannot be considered in the framework of linear theory either, because this theory disregards effects related to the influence of nonlinear terms that contribute essentially to the structure of GAMs.

References

1. Kadomtsev B B *Zh. Tekh. Fiz.* **31** 1273 (1961)
2. Kadomtsev B B *Plasma Turbulence* (London: Academic Press, 1965); Translated from Russian: in *Voprosy Teorii Plazmy* Issue 4 (Ed. M A Leontovich) (Moscow: Gosatomizdat, 1964) p. 188
3. Kadomtsev B B, Pogutse O P, in *Reviews of Plasma Physics* Vol. 5 (Ed. M A Leontovich) (New York: Consultants Bureau, 1970) p. 249; Translated from Russian: in *Voprosy Teorii Plazmy* Issue 5 (Ed. M A Leontovich) (Moscow: Gasatomizdat, 1967) p. 209
4. Lehnert B, in *Proc. of the Second United Nations Intern. Conf. on the Peaceful Uses of Atomic Energy, Geneva, 1–13 September 1958* Vol. 32 (Geneva: United Nations, 1958) p. 349; Report P/146
5. Hoh F C, Lehnert B *Phys. Fluids* **3** 600 (1960)
6. Kadomtsev B B, Nedospasov A V *J. Nucl. Energy C* **1** 230 (1960)
7. Kadomtsev B B *Zh. Tekh. Fiz.* **31** 1209 (1961)
8. Hoh F C *Phys. Fluids* **5** 22 (1962)
9. Lehnert B *Sov. Atom. Energ.* **14** 72 (1963); *Atom. Energ.* **14** 82 (1963)
10. Furth H P, Killeen J, Rosenbluth M N *Phys. Fluids* **6** 459 (1963)
11. Zaitsev A A, Vasil'eva M Ya *Sov. Phys. JETP* **11** 1180 (1960); *Zh. Eksp. Teor. Fiz.* **38** 1639 (1960)
12. Paulikas G A, Pyle R V *Phys. Fluids* **5** 348 (1962)
13. Artsimovich L L, Nedospasov A V *Sov. Phys. Dokl.* **7** 717 (1963); *Dokl. Akad. Nauk SSSR* **145** 1002 (1962)
14. Nedospasov A V *Sov. Phys. Usp.* **11** 174 (1968); *Usp. Fiz. Nauk* **94** 439 (1968)
15. Velikhov E P, Golubev V S, Dykhne A M *Atom. Energy Rev.* **14** 325 (1976)
16. Nedospasov A V, Petrov V G, Fidel'man G N *Nucl. Fusion* **25** 21 (1985)
17. Kadomtsev B B, in *Proc. of the 7th Conf. on Phenomena in Ionized Gases, Belgrade 1965* Vol. 1
18. Endler M *J. Nucl. Mater.* **266–269** 84 (1999)
19. Zweben S J et al. *Plasma Phys. Control. Fusion* **49** S1 (2007)
20. Nedospasov A V *Sov. J. Plasma Phys.* **15** 659 (1989); *Fiz. Plazmy* **15** 1139 (1989)
21. Garbet X et al. *Nucl. Fusion* **31** 967 (1991)
22. Berk H L, Ryutov D D, Tsidulko Yu A *Phys. Fluids B* **3** 1346 (1991)
23. Nedospasov A V, Uzdensky D A *Contrib. Plasma Phys.* **34** 277 (1994)
24. Endler M et al. *Phys. Scr.* **51** 610 (1995)
25. Endler M et al. *Nucl. Fusion* **35** 1307 (1995)
26. Ilgisonis V I *Phys. Usp.* **52** 746 (2009); *Usp. Fiz. Nauk* **179** 790 (2009)
27. Diamond P H et al. *Plasma Phys. Control. Fusion* **47** R35 (2005)
28. Ilgisonis V I et al. *Plasma Phys. Rep.* **40** 843 (2014); *Fiz. Plazmy* **40** 955 (2014)
29. Fujisawa A *Nucl. Fusion* **49** 013001 (2009)
30. McKee G R et al. *Plasma Phys. Control. Fusion* **48** S123 (2006)
31. Conway G D et al. *Phys. Rev. Lett.* **106** 065001 (2011)
32. Bulanin V V et al. *Tech. Phys. Lett.* **40** 375 (2014); *Pis'ma Zh. Tekh. Fiz.* **40** (9) 24 (2014)
33. Melnikov A V et al. *Nucl. Fusion* **55** 063001 (2015)
34. Jenko F, Dorland W *Phys. Rev. Lett.* **89** 225001 (2002)
35. Gusakov E Z et al. *Plasma Phys. Control. Fusion* **48** A371 (2006)