REVIEWS OF TOPICAL PROBLEMS

Hybrid nanophotonics

S I Lepeshov, A E Krasnok, P A Belov, A E Miroshnichenko

DOI: https://doi.org/10.3367/UFNe.2017.12.038275

Contents

1.	Introduction	1035
2.	Optical properties of plasmonic and dielectric nanoparticles	1036
3.	Hybrid nanostructures for controlling radiation properties	1039
4.	Hybrid nanostructures for controlling near-field characteristics	1040
	4.1 General theory of the Purcell effect; 4.2 Hybrid nanostructures for controlling near-field characteristics	
5.	Nonlinear and tunable hybrid nanoantennas	1045
6.	Conclusions	1048
	References	1048

Abstract. Advances in the field of plasmonics, i.e., nanophotonics exploiting the optical properties of metal nanostructures, paved the way for the development of ultrasensitive devices, including biological and various other sensors whose operation is based on the nanoscale localization of the electromagnetic field. However, the high dissipation loss of metal nanostructures limits their use in many current applications, including metasurfaces, metamaterials, and nanowaveguides, thus requiring the development of new devices that combine metal nanostructures with highly refractive dielectric nanoparticles. The resulting metal-dielectric (hybrid) nanostructures have demonstrated many interesting properties from the practical application standpoint (moderate dissipation loss, resonant magnetooptical response, strong nonlinear optical properties, etc.), thus placing this field at the vanguard of the modern science of light. We review the current state of theoretical and experimental research into hybrid metal-dielectric nanoantennas and their derivative nanostructures capable of selectively scattering light waves, directionally amplifying and transmitting optical signals, controlling the propagation of such signals, and generating optical harmonics.

S I Lepeshov, P A Belov St. Petersburg National Research University of Information Technologies, Mechanics and Optics, Faculty of Physics and Engineering, Kronverkskii prosp. 49, 197101 St. Petersburg, Russian Federation E-mail: s.lepeshov@gmail.com A E Krasnok St. Petersburg National Research University of Information Technologies, Mechanics and Optics, Faculty of Physics and Engineering, Kronverkskii prosp. 49, 197101 St. Petersburg, Russian Federation; Photonics Initiative, Advanced Science Research Center, City University of New York, New York 10031, USA E-mail: akrasnok@gc.cuny.edu A E Miroshnichenko School of Engineering and Information Technology, University of New South Wales, Canberra, ACT, 2600, Australia E-mail: andrey.miroshnichenko@unsw.edu.au Received 17 July 2017, revised 14 December 2017 Uspekhi Fizicheskikh Nauk 188 (11) 1137-1154 (2018)

DOI: https://doi.org/10.3367/UFNr.2017.12.038275 Translated by A L Chekhov; edited by A M Semikhatov **Keywords:** nanophotonics, plasmonics, high-index dielectric nanoparticles, optical magnetic response, hybrid nanostructures, nanoantennas

1. Introduction

Light manipulation at the nanoscale is a priority research avenue in photonics and the information industry, as well as in the life sciences. It has been shown in many laboratories around the world that the electric and magnetic components of optical waves can be controlled on the nanoscale by using resonant nanostructures, which form the basis of modern nanophotonics and are known as optical nanoantennas [1–12]. They hold great potential for future applications in various areas: from photovoltaics [13-17] and optical data processing [2, 5, 18-20] to super-resolution microscopy in biology [21-25] and the treatment of oncological diseases in medicine [26-30]. Recent investigations have shown that nanoantennas allow creating single-photon sources [31-35], nanolasers [36-40], and efficient converters of optical-range and near-infrared (IR) radiation into the medium and far-IR radiation [41, 42].

This broad variety of practical applications of nanoantennas and nanostructures based on them has been achieved due to their capability of effectively converting free propagating waves into localized electric and magnetic fields and increasing the field strength in subwavelength regions by many times [43–45]. Moreover, it has been shown that optical nanoantennas can increase the spontaneous emission rate of optical radiation from a quantum emitter (Purcell effect) [46– 53] and concentrate and redirect this radiation into a specified spatial region [54, 55].

Generally, nanoantennas can be separated into several types depending on the material they are made from and on the corresponding operation principles: metallic, dielectric, and metal-dielectric (hybrid). Metallic nanoantennas are often referred to as plasmonic. This type of nanoantennas is characterized by a strong localization of the exciting electromagnetic field on scales of the order of 100 nm and by large Purcell factors due to the excitation of a plasmon resonance [56]. Traditionally, chemically resistant noble metals—gold

or silver—are used for their fabrication. The dissipative losses for these materials in the optical range are lowest among the metals but are still relatively high [57, 58]. On the one hand, this limits the application of such structures in systems where an efficient transfer of optical energy is required; on the other hand, this opens new opportunities for application in thermoplasmonics and medicine [59].

The plasmonic nanoantenna limitations can be overcome by using dielectric nanoantennas [60, 61]. Dielectric nanoantennas are fabricated from materials with a high refractive index and the lowest possible absorption coefficient in the optical range [62]. An electromagnetic field can freely penetrate dielectrics; therefore, these structures support both electric and magnetic Mie resonances [8, 52, 63–79], which allows nanoantennas to equally interact with the electric and magnetic components of the wave. Therefore, dielectric nanoantennas are a viable alternative to plasmonic ones, which resonantly interact only with the electric component of light [80]. At the same time, the field localization in dielectric nanoantennas is not high enough and their Purcell factor is lower than in plasmonic nanoantennas.

Hybrid nanostructures have been under development over the last few years. These nanostructures combine the advantages of dielectric and metallic nanoantennas, effectively transform the incident field into a near field and vice versa, and control the magnetic component. It has been shown that such hybrid nanoantennas have many properties that are interesting for future applications: low thermal losses, an optical magnetic resonant response, and strong nonlinear optical effects. These factors pushed developments in this field to the leading positions in the modern physics of light.

In this review, we focus on the state of the art of theoretical and experimental research on hybrid metaldielectric nanoantennas and nanostructures based on them. Such structures can selectively scatter light waves, enhance and transmit optical signals in a specific direction, control the propagations of such signals, and generate high-order harmonics. We note that currently there are no reviews on the topic of hybrid nanophotonics, in either Russian or international journals, which makes this study an important contribution to the research area. The review outline is as follows. In Section 2, we discuss optical properties of plasmonic and dielectric nanoparticles. In Section 3, we review achievements in the applications of hybrid nanostructures for controlling the optical emission of quantum emitters. In Section 4, we generally introduce the theoretical aspects of the Purcell effect and discuss the results of investigations on the influence of hybrid nanostructures on the spontaneous emission rate of quantum emitters. Finally, in Section 5 we review nonlinear and tunable hybrid nanostructures.

2. Optical properties of plasmonic and dielectric nanoparticles

The optical response of plasmonic (metallic) and dielectric spherical nanoparticles can be considered using the formalism based on the exact solution of the corresponding diffraction problem, first obtained by Gustav Mie in 1908 [63]. Here, we present several general conclusions that follow from this theoretical model. The field of a plane wave scattered on a spherical particle can be written as a sum of partial spherical waves with coefficients a_l and b_l . Using this approach, we can obtain expressions for the normalized total scattering cross

section (Q_{sc}) , the absorption cross section (Q_{abs}) , and the extinction cross section (Q_{ext}) for a spherical particle with radius *R* located inside a nonmagnetic dielectric medium with the dielectric permittivity $\varepsilon = n^2$ [81, 82]:

$$Q_{\rm sc} = \frac{2}{(kR)^2} \sum_{l=1}^{\infty} (2l+1) \left(|a_l|^2 + |b_l|^2 \right),$$

$$Q_{\rm ext} = \frac{2}{(kR)^2} \sum_{l=1}^{\infty} (2l+1) \operatorname{Re}(a_l + b_l), \qquad (1)$$

 $Q_{\rm abs} = Q_{\rm ext} - Q_{\rm sc} \,,$

where the electric (a_l) and magnetic (b_l) scattering amplitudes for a nonmagnetic material with the dielectric permittivity $\varepsilon = n^2$ (*n* is the refractive index of the nanoparticle) are

$$a_{l} = \frac{R_{l}^{(a)}}{R_{l}^{(a)} + iT_{l}^{(a)}}, \quad b_{l} = \frac{R_{l}^{(b)}}{R_{l}^{(b)} + iT_{l}^{(b)}}.$$
 (2)

The functions R_l and T_l can be expressed as

$$R_{l}^{(a)} = n\psi_{l}'(kR)\psi_{l}(nkR) - \psi_{l}(kR)\psi_{l}'(nkR),$$

$$T_{l}^{(a)} = n\chi_{l}'(kR)\psi_{l}(nkR) - \chi_{l}(kR)\psi_{l}'(nkR),$$
(3)

$$R_l^{(b)} = n\psi_l'(nkR)\psi_l(kR) - \psi_l(nkR)\psi_l'(kR),$$

$$T_l^{(b)} = n\chi_l(kR)\psi_l'(nkR) - \chi_l'(kR)\psi_l(nkR),$$
(4)

where

$$\psi_l(x) = \sqrt{\frac{\pi x}{2}} J_{l+1/2}(x), \quad \chi_l(x) = \sqrt{\frac{\pi x}{2}} N_{l+1/2}(x)$$

 $J_{l+1/2}(x)$ and $N_{l+1/2}(x)$ are the Bessel and Neumann functions, and the prime stands for the derivative with respect to the corresponding argument.

Using expressions (1)–(4), we compare the scattering and absorption cross sections for a plasmonic and a high-refractive-index nanoparticle. As the plasmonic nanoparticle material, we choose silver (Ag), which has the lowest losses in the optical spectrum among all noble metals [83]. For the dielectric nanoparticle, we use single-crystal silicon (c-Si), which also has the best characteristics in the optical range [62, 84] among all available dielectric materials.

The calculation results are shown in Fig. 1a, c, where the full normalized scattering cross sections (Q_{sc}) of a silver nanoparticle (a) and a silicon nanoparticle (c) are plotted versus the particle radius and the wavelength. It is clear that the silver nanoparticle supports only electric-type resonances, including the electric dipole (ed), electric quadrupole (eq), and electric octupole (eo) resonances. White dashed lines schematically show the change in the positions of these resonances as the radius increases. On the other hand, the spectrum of the resonant modes of a dielectric nanoparticle is much wider and includes the magnetic dipole moment (md), which is fundamental in this case, together with the magnetic quadrupole moment (mq) and high-order moments. Figure 1b, d demonstrates the calculation results for normalized absorption cross sections of a silver nanoparticle (b) and a silicon nanoparticle (d) versus their radius and the wavelength. Comparing these results shows that the silicon nanoparticle has much lower dissipative



Figure 1. Optical properties of a plasmonic (Ag) and dielectric (Si) nanoparticle in free space. Full normalized scattering cross section of a (a) silver and (c) silicon nanoparticle versus the radius and the wavelength. Normalized absorption cross section of a (b) silver and (d) silicon nanoparticle versus the radius and the wavelength. Schematic illustrations show the fundamental modes of a plasmonic nanoparticle (plasmon dipole moment) and a dielectric nanoparticle (Mie magnetic moment).

losses in the whole range of the parameter values. Moreover, the dissipative losses of a silicon nanoparticle rapidly decrease as the wavelength decreases, because this material has low losses far from the main absorption line. Importantly, gold nanoparticles have even higher dissipative losses.

This means that plasmonic and dielectric nanostructures have both advantages and disadvantages. The disadvantage of plasmonic nanostructures is the high dissipative losses and, consequently, low destruction thresholds. The disadvantage of dielectric nanostructures with Mie resonances is their low local field enhancement factor. Moreover, in dielectric nanoparticles, the field is enhanced mostly inside the particles, which makes it difficult to use them for Raman spectroscopy and biological sensors. This was the reason for the development of hybrid metal-dielectric nanostructures, which would combine the advantages of metal and dielectric nanoantennas (Fig. 2a) and effectively perform direct and inverse transformations between the incident field and the local field, also controlling the magnetic component of light. Currently, such hybrid nanostructures are used for linear control of radiation in the far field (Fig. 2b), for control of the decay rate of the excited state in a quantum emitter (Fig. 2c), for nonlinear control of light scattering using light (Fig. 2d), and for the generation of new optical harmonics (Fig. 2e). These applications of hybrid nanostructures are discussed in detail in the following sections.

It follows from the foregoing that hybrid nanoparticles have a larger variety of resonant modes. Studies of the mode spectrum of hybrid nanostructures of various shapes (including nontrivial ones) are performed by multipole expansion of the near and far fields calculated using various methods [85, 86]. One of these methods is based on the spherical function (Y_{lm}) expansion of the scattered electric field \mathbf{E}_{sc} projected on a spherical surface surrounding the hybrid nanostructure [87, 88]. The multipole coefficients a_E and a_M are then calculated as

$$a_{\rm E}(l,m) = \frac{(-{\rm i})^{l+1}kR}{h_l^{(1)}(kR) E_0 \sqrt{\pi(2l+1)(l+1)l}} \times \int_0^{2\pi} \int_0^{\pi} Y_{lm}^*(\theta,\varphi) \, \mathbf{r} \mathbf{E}_{\rm sc}(\mathbf{r}) \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi \,, \tag{5}$$

$$a_{\mathbf{M}}(l,m) = \frac{(-\mathbf{i})^{l} k R}{h_{l}^{(1)}(kR) E_{0} \sqrt{\pi(2l+1)}} \times \int_{0}^{2\pi} \int_{0}^{\pi} \mathbf{X}_{lm}^{*}(\theta,\varphi) \mathbf{E}_{\mathrm{sc}}(\mathbf{r}) \sin \theta \, \mathrm{d}\theta \, \mathrm{d}\varphi \,, \tag{6}$$

where *R* is the nanoparticle radius, *k* the wave number, $h_l^{(1)}$ the Hankel function, E_0 the incident wave amplitude, Y_{lm} and \mathbf{X}_{lm} are the scalar and vector spherical functions, **r** is the radius vector, and θ and φ are spherical coordinates.



Figure 2. (a) The idea of hybrid nanophotonics: to combine plasmonic and dielectric nanostructures in a single system in order to achieve unique optical properties. (b–e) Various practical applications of hybrid nanophotonics devices: (b) linear control of far-field radiation, (c) control of the decay rate of an excited state in a quantum emitter (QE), (d) nonlinear control of light scattering using light, (e) generation of new optical harmonics.

A typical example of a hybrid nanoparticle is a combined system of dielectric and plasmonic nanoparticles of different shapes. For example, the authors of [87] studied a hybrid nanoparticle consisting of a silicon nanocone and a gold nanodisk placed on the smaller base of the nanocone (Fig. 3). It was shown that the laser heating of the gold nanoparticle allows controllably changing its shape from a nanodisk (Fig. 3b) to a nanocup (Fig. 3c) and a nanosphere (Fig. 3d). Examples of the multipole expansion of near fields are shown for a silicon nanocone and a hybrid nanoparticle in Fig. 3a and Fig. 3b–d. In this case, the multipole expansions allow estimating the contributions from different multipoles (md, ed, eq) to the scattering spectrum and determining the type of each resonance.

An analysis of optical resonances in hybrid nanoantennas shows that such nanoantennas support electric and magnetic resonances of the dielectric part and the localized surface plasmon resonance of the metallic part, while the spectral position of the latter can be changed using laser ablation. The overlapping of the magnetic dipole mode of a dielectric nanoparticle with the plasmonic mode of the metal nanoparticle leads to the hybridization and results in the formation of an additional electric quadrupole response of the system. The near-field distributions in the corresponding resonances are shown in insets A–H in Fig. 3. It is seen that the modification of the gold nanoparticle leads to a change in both the spectral properties of scattered light and the near-field distribution.

In the next sections, we systematically discuss various application fields of hybrid nanostructures. At the end, we draw conclusions from this review and make a forecast on the future development of this fascinating and promising field of modern science dealing with light–matter coupling on the nanoscale.



Figure 3. (Color online.) Spectral dependences of the scattering cross section of (a) dielectric and hybrid metal-dielectric nanostructures with plasmonic nanoparticles in the shape of a (b) disk, (c) cup, and (d) sphere. Dashed lines show the contributions of multipole moments [electric dipole — red (1), magnetic dipole — blue (2), electric quadrupole — green (3)]. Insets A–H correspond to the electric field distribution at resonances. The electric field strength is given in units of V m⁻¹ [87].

3. Hybrid nanostructures for controlling radiation properties

Hybrid metal-dielectric nanostructures with dielectric and metal components open new possibilities for the design of devices with specified scattering properties and directional pattern of radiation. The coupling of the plasmon resonance and the Mie resonances of a dielectric nanoparticle gives rise to a unique mode structure and creates scattering channels each of which is characterized by its own directional pattern and spectral range. This allows spatially separating the incoming optical signals at different frequencies and concentrating and directing the radiation from the quantum emitter to a given far-field region. These properties of hybrid nanoantennas can be used to build Huygens sources [89], superscatterers of incident radiation [90], superlenses [91], sensors [92, 93], and optical routers [94].

Nanoantennas and nanostructures operating as opticalsignal receivers and routers play the key role in all-optical data processing [2, 95–97]. On the one hand, these devices require quite strong signal amplification and focusing; on the other hand, they need spatial and spectral filtering of the signal. Hybrid nanoantennas can combine these properties: the metallic part of a hybrid nanoantenna localizes the optical signal energy at the subwavelength scale due to surface plasmon excitations [98, 99], while the dielectric part directionally scatters the incident light waves [100].

Figure 4a shows a hybrid metal-dielectric nanoantenna consisting of a gold nanorod and a silicon nanosphere on top of it [101]. The gold nanorod supports two fundamental modes: a longitudinal mode and a plasmonic mode localized in the gap between the rod and the sphere. The nature of these two modes is purely electric, while the silicon nanosphere supports both electric and magnetic modes. Because the distance between these nanoparticles is much smaller than the wavelength in the optical range, the induced electric and magnetic dipoles are coupled. This leads to the hybridization



Figure 4. (a) A hybrid nanoantenna consisting of a silicon (Si) nanosphere and gold (Au) nanorod. (b) Scattering channel diagram of a rod-sphere hybrid nanoantenna. (c) Illustration of a hybrid dimer nanoantenna on a glass (SiO₂) substrate. (d) Directional diagrams of a dimer nanoantenna in the *E*-plane corresponding to the symmetric (on the left) and antisymmetric (on the right) collective whispering gallery modes. The insets show the electric field distributions in a dimer nanoantenna for symmetric and antisymmetric whispering gallery modes [94, 100].

of the nanorod plasmon mode localized in the gap with the magnetic and electric modes of the nanosphere. The hybrid modes shift toward the lower-frequency part of the spectrum (Fig. 4b). At the same time, the longitudinal plasmon mode remains unchanged, because its excitation depends only on the aspect ratio of the refractive indices of the nanorod and the ambient medium. This means that the hybrid nanostructure has four scattering channels, each corresponding to the resonant frequency of the mode. Further investigations should focus on the increase in mode quality factors in this system, which would improve the selectivity of spectrally dense optical signals and would allow using this system in photonic integrated circuits.

A dimer hybrid nanostructure that spatially separates optical signals in the near-IR range is shown in Fig. 4c [94]. This nanostructure consists of two hybrid disks with a silicon bottom, silver top, and intermediate aluminum oxide (Al_2O_3) layer. Unlike the nanorod-sphere system with predominantly dipole resonances considered above, these disks support socalled whispering gallery modes in the operating wavelength range. These modes are mainly localized inside the Al₂O₃ layer and are enhanced due to field localization by silver and silicon parts of the nanoantenna. Strong coupling between the disks in the dimer nanostructure leads to splitting of the whispering gallery mode into two pairs. One pair corresponds to the symmetric distribution of the mode maxima and minima inside the disks with respect to the dimer symmetry plane (see insets in Fig. 4d). The second pair corresponds to the antisymmetric field distribution. Interference between the modes at different wavelengths provides different directional patterns in the far field, which can be used for spatial separation of the signals (Fig. 4d). Due to the high quality factor of the whispering gallery modes, such a nanostructure has high frequency selectivity in a narrow wavelength range, which makes these hybrid dimers a good solution for signal separation in dense wavelength division multiplexing (DWDM) systems.

In some practical applications like optical information processing [31, 54, 102], directional excitation of waveguide modes [103], and photovoltaics [104], the unidirectional scattering of the incident radiation is very important. It is difficult to implement this property in fully dielectric or metallic nanostructures without using specially designed reflectors [105-108] or a complicated device like a Yagi-Uda antenna. This is because standard nanostructures have only one type of multipole resonance dominating at a specific wavelength. Because the directional pattern of a multipole is symmetric, it has at least two maxima, as in the case of a dipole in the *E*-plane. Therefore, these structures scatter both forward and backward. However, combining, for example, magnetic and electric dipole resonances in a hybrid core-shell nanoparticle (Fig. 5a) allows arranging their constructive interference in the forward direction and destructive interference in the backward direction [90, 109]. Moreover, the appearance of additional maxima in the directional pattern has been shown in compound structures (oligomers of coreshell nanoparticles), which leads to optical signal branching (Fig. 5b).

The authors of [110] studied asymmetric hybrid dimer nanostructures consisting of spherical gold and silicon nanoparticles (Fig. 5c) known as *Janus dimers*. Similarly to the core-shell nanoantenna considered above, the suppression of the backscattering lobe of the directional pattern is achieved by destructive interference between the optical



Figure 5. Illustration of (a) a hybrid core–shell nanoantenna and (b) the nanostructure formed with these nanoantennas. (c) Illustration of a chain of spherical dimer hybrid nanoantennas. Underneath the figures, directional patterns of such nanoantennas are shown [90, 110].

waves scattered on the magnetic dipole moment of the dielectric nanosphere and the waves scattered on the electric dipole moment of the metal nanosphere. Due to the relatively narrow spectral width of the magnetic dipole mode, the unidirectional scattering has a narrow band character. Moreover, the field in the dimer gap is strongly enhanced, which can be used to induce energy transitions in fluorescent molecules and other quantum emitters. It was shown that the unidirectional scattering is enhanced for chains consisting of a large number of Janus dimers. This is explained by the fact that the additional interference of waves scattered on the neighboring dimers suppresses the side- and back-scattering and enhances radiation in the forward direction. Mode structure studies of such dimers using the generalized dipole-dipole interaction model, as well as coupling with quantum emitters, are discussed in detail in [111].

The efficient transformation of a freely propagating optical wave into a unidirectional waveguide mode is an important problem for all-optical data transfer between two logical elements. Many various devices have been suggested as the solution to this problem (including optical nanoantennas) based on a plasmonic waveguide [2, 5, 112, 113], which can localize light at subwavelength scales and transmit it over



Figure 6. (a) Diagram and (b) operating principle of a hybrid nanoantenna consisting of a dielectric nanodisk on a silver film. (c) Directional pattern of a surface plasmon excited by this nanoantenna [114].

micrometer distances with reasonable damping. Nanoantennas that unidirectionally launch a surface plasmon should provide the phase matching condition between the exciting light wave and the waveguide mode, as well as manyfold enhancement and concentration of the wave in a given direction. Figure 6a shows a metal-dielectric nanoantenna based on a silicon nanoparticle located above a silver film surface [114]. A TM-polarized optical wave incident on the structure at a grazing angle induces hybrid modes in the gap between the nanoparticle and the film, with the quality factor reaching 10^3 . This results in a huge enhancement of the electric field strength (up to a 60-fold increase) with respect to the incident wave strength (Fig. 6b). At the same time, the hybrid mode supplies the energy to the plasmonic waveguide and transforms into a predominantly unidirectional surface plasmon-polariton (Fig. 6c).

4. Hybrid nanostructures for controlling near-field characteristics

Light energy localization in a near field and the corresponding effects, such as quantum emitter efficiency enhancement [102], photoluminescence enhancement [115], and the Purcell effect [56], are the most important properties of metallic nanoantennas. These effects are caused by the excitation of surface plasmons in nanoparticles, which, on the one hand, concentrate the incident electromagnetic field on the metal surface and, on the other hand, accumulate and reemit the energy of a quantum emitter located near the nanoantenna. In what follows, we discuss the general theory of the Purcell effect, which amounts to a change in the spontaneous emission rate of a quantum emitter due to its coupling to a nearby nanoantenna.

At the same time, dielectric nanoantennas, which have high directionality of radiation and small dissipative losses, cannot provide the same light localization efficiency as those for metal nanoantennas. Therefore, a new approach is being developed in fields such as surface-enhanced Raman spectroscopy (SERS), photoluminescence spectroscopy, and alloptical data processing. The new method is based on combining dielectric components fabricated from semiconductors with a high refractive index and low absorption coefficient and metallic components into a single hybrid nanostructure. This approach provided solutions to a number of problems starting with control over the spontaneous emission of a quantum emitter [116] and the development of efficient devices transforming the excited level energy into an optical signal [117] for applications in photonic calculations, and ending with the local excitation [118] of fluorescent tags for super-resolution microscopy and optical energy concentration in various systems [119–121], including solar cells [118].

4.1 General theory of the Purcell effect

As mentioned above, the Purcell effect is one of the key effects in quantum optics and nanophotonics. Large Purcell effects are usually observed when the source is placed inside microcavities [122–124], hotspots of resonant nanoantennas [9, 53, 125–133], photonic crystals [134–140], or metamaterials [49, 130, 141–145]. The spontaneous emission rate is usually measured in the optical range by recording the temporary photoluminescence signal from the source under pulsed excitation [146–148]. In contrast, for the microwave and terahertz range, the Purcell effect can be observed as an increase in the power emitted by the antenna in the stationary regime [48]. From the practical standpoint, the increased spontaneous emission rate can be used in lasers, singlephoton sources, fluorescence microscopy, biological investigations, and spectroscopy. There are several detailed reviews [48, 53, 142, 149] and books [150] on the Purcell effect and spontaneous emission modification, and we therefore discuss only the fundamental aspects that are needed to understand further considerations.

Until recently, the research on the Purcell effect was focused on the spontaneous emission rate of electric dipole transitions because the electric dipole transitions are usually much stronger than the magnetic ones in quantum emitters. For the same reason, the magnetic permeability of most materials is 1 in the optical range. Therefore, the magnetic response of emitters is much more difficult to control than the electric one. However, there are a number of quantum emitters, like rare-earth ions or semiconducting quantum dots, with magnetic transitions comparable to or even higher than the electric ones [151–155]. A broad overview of possible emitters with dominating magnetic transitions is given in [53].

The simplest model of quantum sources that describes most of the properties of real quantum optical emitters is the two-level system with the ground state $|g\rangle$, the excited state $|e\rangle$, and the energy difference $\hbar\omega_0$. Transition processes are characterized by the dipole matrix element $\mathbf{d}_{eg} = \langle e|q\mathbf{r}|g\rangle$. The main property of this system is that the relaxation of the excited state to the ground state is accompanied by the emission of a photon with the energy $\hbar\omega_0$. To calculate such processes, we need to take the two-level system interaction with the state continuum of the free space into account. These calculations were performed by Weisskopf and Winger in [156]. They showed that the excited state of the two-level system located in free space decays at the rate

$$y_0 = \frac{\omega_0^3}{3\pi\hbar\varepsilon_0 c^3} \left| \mathbf{d}_{\rm eg} \right|^2,\tag{7}$$

where ε_0 is the dielectric permeability of the vacuum. When the two-level system is placed in a medium, the spontaneous emission rate changes. Here and hereafter, we use the International System of Units because it is most often used in this field. In order to change to the CGS system, ε_0 should just be replaced with $1/(4\pi)$.

By using the Fermi golden rule, the decay rate can be rewritten as

$$\gamma = \frac{\pi\omega_0}{\hbar\varepsilon_0} \left| \mathbf{d}_{\rm eg} \right|^2 \rho_{\mathbf{n}}(\mathbf{r}_0, \omega_0) \,, \tag{8}$$

where $\rho_{\mathbf{n}}(\mathbf{r}_0, \omega_0)$ is the local density of states of the electromagnetic field at the system position \mathbf{r}_0 :

$$\rho_{\mathbf{n}}(\mathbf{r}_{0},\omega_{0}) = \sum_{\mathbf{k}} \left[\mathbf{n} \, \mathbf{e}_{k}(\mathbf{r}_{0}) \otimes \mathbf{e}_{k}^{*}(\mathbf{r}_{0}) \, \mathbf{n} \right] \delta(\omega_{k} - \omega_{0}) \,, \qquad (9)$$

where the sum is taken over all eigenmodes of the system \mathbf{e}_k with eigenfrequencies ω_k . The eigenstates \mathbf{e}_k are solutions of the homogeneous wave equation and are normalized by the condition $\int_V \varepsilon(\mathbf{r}) \mathbf{e}_i(\mathbf{r}) \mathbf{e}_j(\mathbf{r}) d^3 \mathbf{r} = \delta_{ij}$, where $\varepsilon(\mathbf{r})$ is the dielectric permittivity of the medium. The unit vector **n** is oriented in the direction of the dipole moment \mathbf{d}_{eg} . By changing the local density of states, we can significantly modify the spontaneous emission rate.

Expression (8) gives the correct emission rate in the case of the weak-coupling regime, when the coupling constant between the emitter and the electromagnetic states is less than the decay rate of the latter. This regime corresponds to the Markovian dynamics, when the system has no memory of its time evolution and decays exponentially. In the opposite case, strong coupling between the emitter and the electromagnetic states leads to non-Markovian dynamics, and Eqn (8) cannot be used to calculate the emission rate. In this regime, the source may not decay exponentially and can undergo Rabi oscillations between the states [157–159].

The electromagnetic local density of states in Eqn (9) can be calculated by using the dyadic tensor of the electric source Green's function, which can be expressed in terms of the eigenstates (eigenmodes) of the system [160]. For a cavity with zero losses, the dyadic Green's function has the form

$$\mathbf{G}(\mathbf{r},\mathbf{r}',\omega) = \sum_{\mathbf{k}} c^2 \, \frac{\mathbf{e}_k^*(\mathbf{r}) \otimes \mathbf{e}_k(\mathbf{r}')}{\omega_k^2 - \omega_0^2} \,, \tag{10}$$

where $\mathbf{G}(\mathbf{r}, \mathbf{r}', \omega)$ is the tensor Green's function, defined as the solution of the inhomogeneous wave equation

$$\nabla \times \nabla \times \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) - \varepsilon(\mathbf{r}) \, \frac{\omega_0^2}{c^2} \, \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = \mathbf{I} \delta(\mathbf{r} - \mathbf{r}') \,, \, (11)$$

where \mathbf{I} is the unit tensor. The eigenstates in (9) can be used to express the local density of states as

$$\rho_{\mathbf{n}}(\mathbf{r}_0,\omega_0) = \frac{2\omega_0}{\pi c^2} \,\mathbf{n} \,\mathrm{Im} \,\mathbf{G}(\mathbf{r}_0,\mathbf{r}_0,\omega_0) \,\mathbf{n} \,. \tag{12}$$

Relation (12) implies an expression for the spontaneous emission rate

$$\gamma = \frac{2\omega_0^2}{\hbar\epsilon_0 c^2} \left| \mathbf{d}_{eg} \right|^2 \mathbf{n} \operatorname{Im} \mathbf{G}(\mathbf{r}_0, \mathbf{r}_0, \omega_0) \mathbf{n} \,. \tag{13}$$

A convenient variable for describing the spontaneous emission rate variations is the dimensionless quantity known as the Purcell factor F_P . It is defined as the ratio between the emission rate γ of a two-level system located in an inhomogeneous medium (for example, near a nanoantenna) and in free space, γ_0 :

$$F_{\mathbf{P}} = \frac{\gamma}{\gamma_0} = \frac{6\pi c}{\omega_0} \,\mathbf{n} \,\mathrm{Im} \,\mathbf{G}(\mathbf{r}_0, \mathbf{r}_0, \omega_0) \,\mathbf{n} \,. \tag{14}$$

We note that in the weak coupling regime, F_P is independent of the two-level system dipole moment and is determined only by the electromagnetic properties of the ambient medium.

For open systems or in the case of nonzero losses, expression (10) cannot be used because the set of eigenstates \mathbf{e}_k is not defined. However, the Green's function method used in Eqn (14) gives the correct result for the emission rate if the source with an electric dipole transition is located inside an open cavity.

Expression (14) gives the quantum source emission rate if a classical characteristic, the tensor Green's function, is known. Moreover, this expression can also be used for interpreting the Purcell effect in the classical case. The increase in the emission rate can be represented as an increase in the work performed by the electric field. The work performed by the electric field of an oscillating dipole $\mathbf{d} \exp(-i\omega_0 t)$ is

$$P = \frac{\omega_0}{2} \operatorname{Im} \left[\mathbf{d} \mathbf{E}(\mathbf{r}_0) \right] = \mu_0 \frac{\omega_0^3}{2} \left| \mathbf{d} \right|^2 \mathbf{n} \operatorname{Im} \mathbf{G}(\mathbf{r}_0, \mathbf{r}_0, \omega_0) \, \mathbf{n} \,,$$
(15)

where μ_0 is the magnetic constant. After dividing this quantity by the power emitted by the same source into free space,

$$P_0 = \frac{\omega_0^4}{12\pi\varepsilon_0 c^3} \left|\mathbf{d}\right|^2,$$

we obtain an expression identical to (14).

The full decay rate γ in Eqn (13) includes two terms: radiative and nonradiative contributions. In most practical cases, the increase in the radiative power of the source in the far field is of greater interest. The radiative Purcell factor is then defined as $F_{\rm P}^{(r)} = \gamma_{\rm r}/\gamma_0$, where $\gamma_{\rm r}$ is the radiative decay rate, which can be calculated by integrating the Poynting vector over the surface surrounding the source and the inhomogeneous environment. To describe the part of the energy emitted in the form of photons, the quantity known as the *quantum yield* is used, which is defined as

$$\eta_0 = \frac{\gamma_r}{\gamma_r + \gamma_{nr} + \gamma_{int}} , \qquad (16)$$

where γ_{nr} is the nonradiative decay rate corresponding to the electromagnetic losses of the medium and γ_{int} is the intrinsic nonradiative decay rate that exists even when the environment is homogeneous (in free space). The quantum yield describes the part of the energy that is emitted by an isolated two-level system. When the system is located inside some medium, the expression for the quantum yield takes the form

$$\eta = \frac{F_{\rm P}^{(r)}}{\eta_0 F_{\rm P} + (1 - \eta_0)} \,. \tag{17}$$

Expression (14) for the Purcell factor can be simplified if the leading contribution to the tensor Green's function is given by only one mode. In this special case, we can use the single-mode approximation, wherein the tensor Green's function is expressed as

$$\mathbf{G}(\mathbf{r},\mathbf{r}',\omega) \approx c^2 \frac{\mathbf{e}_k^*(\mathbf{r}) \otimes \mathbf{e}_k(\mathbf{r}')}{\omega_k^2 - \omega_0^2 - 2i\gamma_k\omega}, \qquad (18)$$

where γ_k is the pump rate for the corresponding eigenmode $\mathbf{e}_k(\mathbf{r})$. Assuming the source rate to be the same as the cavity mode frequency and the source to be parallel to the emitted field, we derive the well-known expression for the single-mode Purcell factor

$$F_{\rm P} = \frac{3}{4\pi^2} \,\lambda^3 \,\frac{Q}{V} \,, \tag{19}$$

where $Q = \omega_k/2\gamma_k$ is the mode quality factor, λ is the wavelength in the vacuum, and V is the effective mode volume, which can be expressed in the low-loss approximation as [161, 162]

$$V = \frac{\int \varepsilon(r) |E(r)|^2 \,\mathrm{d}V}{\max\left(\varepsilon(r)|E(r)|^2\right)} \,. \tag{20}$$

Usually, the effective mode volume is much smaller than the real volume of the cavity. For example, plasmonic nanocavities can support modes with extremely small volumes down to $\lambda^3/10^4$ [163]. We note that expression (20) is not always applicable to real optical cavities with losses, especially to plasmonic nanoparticles, because these have high dissipative losses [164]. Therefore, the single-mode approximation does not give the correct result for some systems, for

example, those located near plasmonic nanoparticles. This problem is discussed in a number of papers [52, 163–168]. Notably, a detailed analysis of the effective mode volume in plasmonic nanocavities was performed in [163], where it was shown that the so-called complex mode volume has to be introduced.

We note that the resonant states are not normalized in the traditional sense, which places some restrictions on expression (20). This is because the field of the resonant electromagnetic eigenstates of an open cavity diverges exponentially with the distance from the system. The resonant state normalization problem has been studied and eventually solved by many authors, starting with the works by Zel'dovich [169]. This was also achieved recently in electrodynamics in [170, 171].

4.2 Hybrid nanostructures for controlling near-field characteristics

The simplest realization of the hybrid nanoantenna concept consists of a gold nanosphere and a planar dielectric antenna, as shown in Fig. 7a. The quantum emitter in this configuration is placed inside the surface layer of the dielectric nanoantenna, which performs two functions. First, it increases the spontaneous emission rate of the source by a factor of n (n is the dielectric refractive index) [116, 172]. Second, the dielectric nanoantenna localizes the field of the surface plasmon supported by the gold nanoparticle in the gap between the two elements, which additionally increases the Purcell factor. This system serves as an emitting nanoantenna, with the metallic nanoparticle playing the role of a standard radio-antenna director and the planar dielectric part the role of the guiding element (reflector).

The hybrid nanoantenna shown in Fig. 7b has a better design, which results not only in a greater Purcell factor but also in the possibility of concentrating and transmitting radiation in a specified direction using a dielectric (Si) nanoparticle [89, 173, 174]. The optical elements of the antenna are a silicon disk and a gold nanorod located above the disk at an optimal distance and oriented parallel to its surface. To fix the nanorod at a certain height, the free space is filled with glass. The metallic nanorod in this structure plays the role of an optical signal amplifier and increases the spontaneous emission rate of the quantum source (indicated by a red arrow) by transforming the energy of its excited state into a surface plasmon. The plasmon then reemits optical waves and pumps the electric quadrupole mode of the silicon nanodisk with unidirectional emission. As a result, the output signal is greatly increased and concentrated in one direction. Such a hybrid nanoantenna is fabricated by two-stage electron-beam lithography, where first the silicon nanodisk is formed on the substrate, then the glass layer is applied to the structure, and finally the metallic nanorod is spin-coated on the glass [173].

Together with metallic nanoantennas, microcavities are often used in order to increase the coupling between the light wave and the quantum emitter due to a high Purcell effect in such structures. Dielectric disk cavities provide high Purcell factors due to both the high quality factor Q of the whispering gallery modes excited in them and the large mode volume V. To achieve a higher Purcell factor, a disk microcavity and a metallic nanoantenna with a small V were combined into one hybrid system, as we have mentioned above (Fig. 7c) [117]. The operation principle and the design are similar to those of the device considered above: a gold ellipsoidal nanorod is



Figure 7. (a) A hybrid nanoantenna consisting of a gold nanoparticle and a planar substrate with a high refractive index. (b) Illustration of a nanodisk– nanorod hybrid nanoantenna. Red arrows indicate the quantum light sources. (c) Hybrid system based on a disk microcavity and a metallic nanoantenna [89, 116, 117].

located above a microscale dielectric disk made from silicon nitride (Si₃N₄) with n = 1.997 and the quantum emitter is placed near one of the nanorod ends. The coupling between the localized plasmon mode of the metallic nanoantenna and the high-quality whispering gallery mode leads to the emission enhancement by a factor of more than 1000.

The concept of a 'bowtie' hybrid metal-dielectric antenna was suggested theoretically in [175] (Fig. 8a). This nanoantenna is a conventional bowtie plasmonic nanoantenna with diamond tips containing nitrogen vacancy centers (NV centers) [176, 177]. The NV centers in diamond are promising candidates for the role of quantum emitters because their energy spectrum is quite simple and the electron spins of the centers can be manipulated by microwave radiation [178], which, in turn, provides control over the absorption and luminescence spectra. The authors of [175] have shown that the mode volume in such nanoantennas is very small and the electric field is concentrated in the center of the bowtie (Fig. 8b), where the NV centers are located. This increases the Purcell factor to values of the order of 110 and also increases the collection efficiency of the emitted photons by a factor of 1.77.

Among various resonant optical phenomena in strongly coupled ensembles of nanostructures, of particular interest is the Fano resonance, first suggested by Fano in 1961 [179] to describe a strongly pronounced asymmetry of absorption lines in noble-gas spectra. Later, these considerations were extended to other physical systems, including optical nanostructures [81, 180-187]. In particular, it was shown that in optics this phenomenon leads to narrow features in the scattering, reflection, or absorption spectra, which is of fundamental importance for chemical and biological sensors. Usually, Fano resonance appears in structures due to the interference of two modes, one of which is wide and nonresonant and the other is narrow and resonant [68, 179, 181, 183, 187-193]. This can also result in antiphase oscillations of modes excited in the nanostructures with the corresponding suppression of energy reemission into the far field and the formation of the so-called Fano minimum in the spectrum.

Figure 8c illustrates a hybrid nanoantenna consisting of a silicon spherical particle placed inside a gold ring. The whole structure has a subwavelength size. The structural feature of this nanoantenna is the existence of a gap between the inner



Figure 8. (a) A bowtie hybrid nanoantenna consisting of gold and diamond with NV-enters. (b) Horizontal cross section of the field distribution of the electric dipole mode localized inside the hybrid nanoantenna (white lines — the air region, black lines — the gold trapezia). (c) Illustration of a metal ring and a dielectric sphere combined into a hybrid nanoantenna. (d) Electric and magnetic field distributions in such a nanoantenna at the Fano resonance wavelength [118, 175].

walls of the ring and the sphere. Due to low-quality plasmonic resonances, the gold ring provides optical response in a broad wavelength range. At the same time, the magnetic-dipole Mie resonance of a silicon sphere is relatively narrow. Strong coupling between the magnetic dipole and plasmon modes leads to the emergence of a magneto–electric Fano resonance with strong enhancement of both electric and magnetic fields in the gap between the ring and the sphere (Fig. 8d). Such enhancement can be used to increase the photoluminescence power of quantum emitters with either electric or magnetic transitions [53].

For applications in optoelectronics and especially in photodetection, all-optical modulation, and photovoltaics, one often needs to achieve high absorption in the near-surface region of a semiconducting substrate. Surface nanostructuring or implementation of semiconducting nanoinclusions and nanoantennas cannot always provide sufficient absorption of the optical radiation due to low field localization in the bulk of the semiconductor. This problem can be solved by using surface plasmon resonances of metallic nanostructures because, as was repeatedly noted above, they enhance the optical field near the metallic surface by an order of magnitude.

The use of metallic coatings, along with one-dimensional semiconductor gratings (Fig. 9a) and nanoantennas (Fig. 9b, gave birth to a new type of metasurface formed by metaldielectric nanostructures, which fully absorb the incident optical radiation (so-called *perfect absorbers*) [194, 195]. Active semiconducting components with Mie resonances typical of dielectric nanoparticles (in this case, electric dipole ones) absorb the light energy, which results in the formation of a large number of electron-hole pairs inside the semiconductor. In turn, the metallic coatings provide field localization and enhancement inside the semiconducting components (Fig. 9c, d). Moreover, nonradiative losses in the metal-semiconductor interface into the semiconductor conduction band, which forms an additional photocurrent. The last process also occurs for photon energies lower than the semiconductor bandgap, which is especially important for the development of broadband photodetectors.

Both nanostructures shown in Fig. 9a, b have their advantages and disadvantages. The one-dimensional grating can absorb up to 95% of the incident radiation in the near-IR range but is highly sensitive to the polarization of the radiation. On the other hand, the two-dimensional grating or the metasurface formed by hybrid nanoantennas has a low absorption coefficient (80–90%), but no polarization dependence. Such structures are fabricated in two steps. First, electron-beam lithography and reactive ion etching are used to form semiconductor gratings. Then, a 15 nm gold layer is deposited on top of the 1 nm titan layer for greater adhesion.

The metal-semiconductor nanostructured surfaces are promising candidates for solar-cell coatings because their large absorption can lead to an increase in the internal quantum efficiency. The maximal efficiency of optical energy to photocurrent conversion should be equal to the absorption coefficient of the structure. However, the process of photoinduced charge carrier collection by the electrodes is inefficient, and the overall efficiency of the solar cell considerably decreases. A possible solution to this problem was suggested in [196]. The idea is to replace the surface photoelectric elements with hybrid core-shell nanorods formed by metallic (e.g., silver) electrodes covered with a semiconducting layer (Fig. 10a). Due to the small thickness of the semiconducting coating and the existence of two interfaces, semiconductor-air and metal-semiconductor, the polarization-dependent Mie resonances typical of dielectric wires and rods degenerate into polarization-independent Fabry-Perot resonances, which, as is clearly seen from the field distributions in Fig. 10b, c, can be interpreted as the magnetic (TE₀₁) and electric (TE₁₁) dipole resonances. The



Figure 9. Schematic and scanning electron microscopy images of (a) a one-dimensional grating and (b) a metasurface formed by hybrid nanoantennas. (c) Electric field distributions in the structural unit of a one-dimensional grating, and (d) a hybrid nanoantenna [194, 195].



Figure 10. (a) A hybrid core–shell nanostructure consisting of a silver rod covered with an absorbing semiconducting shell. The electric field distributions in the hybrid nanostructure at the wavelengths of the (b) TE_{01} and (c) TE_{11} resonances [196].

possibility of flexibly tuning these resonances provides broadband enhancement of the absorption efficiency in the optical range, reaching values of 1.9.

5. Nonlinear and tunable hybrid nanoantennas

High-order harmonic generation has broad application prospects in visualization of biological molecules [197, 198] and inner cell processes [199, 200] because it allows pumping photoluminescent dyes at near-IR frequencies in the transparency window of the cell walls. This kind of application requires nanoparticles that efficiently transform the optical signal at the fundamental frequency into signals at doubled or tripled frequency; these nanoparticles must be placed inside or close to the investigated object. Materials that can generate a second harmonic include nonlinear dielectric crystals with a noncentrosymmetric crystal lattice (LiNbO₃, ZnS, and others). Because the conversion efficiency depends on the length of the optical path inside the crystal, the nanoparticles cannot provide high-efficiency generation due to their small dimensions, which significantly lowers the signal level and therefore decreases the probability of interaction with dye molecules. Therefore, new approaches are being developed in order to increase the conversion efficiency. One such approach consists in placing resonant nanoantennas near the nonlinear particles or in implementing nanostructured elements directly inside the particle.

Figure 11a shows a hybrid metal-dielectric nanoantenna consisting of an orthogonally oriented gold dipole antenna and a dielectric (ZnS) dipole antenna, together with a ZnS nanoparticle, which are responsible for the optical signal conversion into the second harmonic [201]. This nanoantenna is fabricated in two steps. First, electron-beam lithography is used to form gold dipoles and special positioning tags. At the second step, these tags are detected and electron-beam lithography is used to form ZnS dipoles at their positions. The gold dipole localizes the external light field in the gap, and the field enhancement is maximal when the electric field vector is directed parallel to the symmetry axis of the metallic dipole. A dielectric antenna with a resonance at the second harmonic frequency is used to increase the spontaneous emission rate of the dipoles oscillating at the second harmonic frequency. The polarization of the output radiation is directed parallel to the axis of the dielectric dipole antenna. These two factors, field enhancement at the fundamental frequency and the Purcell factor enhancement at the second harmonic frequency, lead



Figure 11. (a) A hybrid nanoantenna for efficient second-harmonic generation and its operation principle. (b) A hybrid nanodimer consisting of Au and BaTiO₃ spherical nanoparticles. (c) A hybrid core(Au)–shell(Cu₂O) nanoantenna. (d) A hybrid dipole nanoantenna with an active ITO-nanoparticle. (e) Operation diagram of a hybrid ring–disk nanostructure on a sapphire (Al₂O₃) substrate; the inset shows the image of this structure obtained by scanning electron microscopy [201, 203–205].

to the amplification of the second harmonic signal by 500,000 times with respect to the array of ZnS nanoparticles.

The authors of [202] demonstrate enhanced second harmonic generation in a hybrid JaJanus-type metal-dielectric dimer nanoantenna (Fig. 11b). The dimer consists of two elements: a spherical plasmonic nanoparticle 70 nm in diameter, which is commonly used in such nanoantennas, and a barium titanate (BaTiO₃) nanoparticle 100 nm in diameter. This nanoantenna was fabricated using the method of sequential capillarity-assisted particle assembly for Au and BaTiO₃ nanoparticles in the cavities formed inside a polymer resist by electron-beam lithography. BaTiO₃ has a noncentrosymmetric crystal structure and generates the second harmonic signal, while the Au nanoparticle enhances the Purcell effect in the system. As a result, the efficiency of the second harmonic generation from a hybrid dimer is 15 times higher than that for single BaTiO₃ nanoparticles. Despite such a small value of the enhancement compared with the 500,000 in the previous nanoantenna, this dimer has the advantages of a relatively simple fabrication method and the capacity to use commercially available Au and BaTiO₃ nanoparticles.

Unlike the second harmonic, the third harmonic can be generated in materials with a centrosymmetric crystal lattice. One of these materials is copper oxide Cu₂O, a dielectric with a high refractive index. However, as in the case of second harmonic generation, efficient conversion into the third harmonic requires the particles to be optically resonant in the IR range, which makes their dimensions very large, such that the particles become inappropriate for the localization, for example, of single intracellular processes. To overcome these disadvantages, a new hybrid core-shell nanostructure was suggested with a gold rod core coated with a Cu₂O shell (Fig. 11c) [203]. Tuned to the plasmon resonance in the optical range, the nanorod provides strong electric field localization in the near field. The high-index Cu₂O shell in this structure redshifts the plasmon resonance, absorbs the localized plasmon energy, and emits the third harmonic signal.

The hybrid nanoantenna shown in Fig. 11d has a similar operation principle. This nanoantenna combines a conventional metallic dipole nanoantenna, which enhances the electric field in the gap more than 30-fold, and a nonlinear indium tin oxide (ITO) nanoparticle with the dielectric permittivity $\varepsilon = 2.9$ at the operating wavelength, which converts the signal at the fundamental frequency into the third harmonic [204]. The fabrication of this nanoantenna proceeds in several steps: first, the usual electron-beam lithography is used to form the metallic part of the nanoantenna on the glass, which is then covered with a resist layer illuminated by the electron beam in the dipole gap region. After subsequent etching, holes are formed in the gaps of the metallic dipole antenna. A drop of highly concentrated solution of monodisperse ITO nanocrystals suspended in hexane is deposited onto the resist and swept across the sample surface by a cut piece of polydimethylsiloxane (PDMS). The particles are dragged along with the meniscus of the slowly evaporating hexane solution and deposited into the resist openings. The obtained nanostructure allows the nonlinear response of the particle to be increased by 10⁶ times.

In [205], the third-harmonic generation was studied in a hybrid metal-dielectric structure that included a gold ring and a silicon disk placed in its center (Fig. 11e). The source of the optical signal at the third harmonic in this construction

was the silicon nanoparticle. The third-harmonic generation enhancement in the ring-disk structure is caused by strong localization of the electromagnetic field inside the silicon nanoparticle and on the surface of the gold disk, as well as by the suppression of far-field radiation at the fundamental frequency. These effects are achieved by exciting the so-called anapole mode in the structure [36, 206-213]. An anapole is a combination of an electric dipole and toroidal moments oscillating in antiphase with respect to each other. Because the directional patterns of each of these two moments alone are almost the same, the destructive interference leads to the suppression of the waves emitted by the anapole, and the farfield signal decreases. At the same time, the electromagnetic energy is localized in the near field of the anapole structure. In the hybrid nanoantenna shown in Fig. 11e, the toroidal moment appears in the gold ring, while the electric dipole moment is supported by the silicon disk. Suppression of the secondary waves at the fundamental frequency, together with strong near-field localization, provide a third-harmonic conversion efficiency up to 0.007%.

The hybrid nanostructure was fabricated using two-stage electron-beam lithography combined with reactive ion etching. At the first stage, a chrome mask is formed for ion etching of the silicon disks by using electron-beam lithography with a PMMA resist, followed by chrome deposition and a lift-off procedure. At the second stage, the resist is selectively deposited near the disks marked with cross-like tags. After the electron-beam lithography and resist etching, the gold layer is deposited and the gold rings are obtained after the liftoff procedure [203].

The nonlinear hybrid metal-dielectric nanostructure is a promising candidate for the role of optical switches and logical elements in all-optical integrated circuits, because the material parameters of these nanostructures (dielectric permittivity and refractive index) can be controlled by modulation of the incident radiation intensity [214–217]. Because the mode structure of the nanoantenna is directly dependent on the dielectric permittivity of its elements, the change in the radiation intensity leads to a shift of the nanoantenna resonances and to a change in the directional pattern.

A theoretical description of a nonlinear dimer nanoantenna consisting of silver (Ag) and silicon (Si) nanospheres was given in [218]. Third-order nonlinearity in the silver nanoparticle is sufficiently high to influence its dielectric permittivity. Figure 12a schematically shows such a hybrid nanoantenna. The wave vector of the incident radiation is directed along the dimer symmetry axis. The silver nanosphere size corresponds to the excitation of a surface plasmon resonance at the photon energy of 3.14 eV under low intensities of optical radiation. In the nonresonant regime, the direction of the induced electric dipole moment inside the silver nanoparticle is opposite to the direction of the incident electric field and to the induced electric moment inside the silicon nanoparticle. This leads to a destructive interference of the waves scattered by the electric dipoles of two particles, and the directional pattern becomes oriented oppositely to the incident wave vector (Fig. 12b).

In the plasmon resonance regime, all dipole moments are parallel and the scattered waves are added constructively, and therefore the directional pattern becomes omnidirectional. Because the spectral position of the plasmon resonance depends on the dielectric permittivity of the nanoparticle material, changing it leads to a shift of the dipole resonance



Figure 12. (Color online.) (a) A nonlinear hybrid dimer Si–Ag nanoantenna. (b) Directional patterns of such a nanoantenna under illumination with lowintensity (top) and high-intensity (botton) light waves; red arrows indicate the direction of the wave vector. (c) Operation diagram of a nonlinear chain of core–shell hybrid nanoparticles; yellow (1) shows the directional patterns of a quantum light source placed in the center of the chain [6, 218].

and to a transition of the system from the resonant regime to the nonresonant one and vice versa. This results in the switching of the directional pattern and the propagation direction of the scattered light. The switching speed is 40 fs for a relatively low intensity of 6 MV cm⁻², which can be used in ultrafast photonic devices and circuits. In practice, the hybrid dimers can be obtained using a recently suggested combination of top–down fabrication and template-guided self-assembly [219], which allows a precise and controlled positioning of the plasmonic elements with respect to the dielectric spheres in the vertical and horizontal directions. Moreover, such systems can be fabricated using atomic force nanomanipulation.

The Yagi-Uda nonlinear hybrid nanoantenna shown in Fig. 12c is based on a chain of core-shell nanoparticles (Ag/Ge) [6]. As we have mentioned, such nanoparticles support both the magnetic dipole mode of the dielectric shell and the electric dipole mode of the metallic core at the operating frequency. This is why the hybrid nanoparticles are a unidirectional source of optical waves. At the same time, the chains of all-dielectric spheres tuned to magnetic resonances [8, 70, 220] demonstrate a highly directional regime, which is connected with dark mode excitation and the appearance of the Van Hove singularity. A chain of hybrid nanoparticles combines the highly directional properties of its components, which increases the directionality of such an antenna by many times. If a dipole source were placed in the center of this chain, the radiation pattern of such a source would have two oppositely directed narrow lobes. It is known that if a semiconductor is exposed to intense optical radiation with the photon energy higher than its bandgap, the generation of electron-hole plasma occurs. This process greatly increases the conductivity and decreases the real part of the dielectric permittivity by several units [70, 220-222]. It was demonstrated for the chain shown in Fig. 12c that under the illumination of edge core-shell particles with a femtosecond laser, the symmetry of the directional pattern is violated. This happens due to a decrease in the dielectric permittivity of the germanium (Ge) shells, which causes a blue shift of their magnetic resonance. The breaking of the directional pattern symmetry manifests itself in the suppression of scattering in the direction of nonmodified particles and the enhancement of scattering in the direction of modified nanoparticles. This data demonstrates the potential of nonlinear Yagi-Uda hybrid nanoantennas for the development of unidirectional emitters, which could be reversibly reconfigured at ultrafast scales.

Another approach to controlling the optical properties of hybrid nanostructures using intense light pulses is connected with an irreversible modification of their shape after fabrication [223]. A design for asymmetric metal-dielectric nanostructures was proposed in [224] together with a fabrication method that allows subsequent laser processing that changes the shape of the metallic part and alters its spectral characteristics accordingly. Such a hybrid nanostructure consists of a silicon cone with a gold disk on top of it. The lithography stages provide the specific shape of the dielectric part in order to achieve a controlled modification of the metallic component without disturbing the magnetic and electric Mie resonances. As a result, femtosecond laser melting allows changing the shape of the gold nanoparticle from a disk to a cup or a sphere without influencing the dielectric nanoparticle (Fig. 13a). The main mechanism causing the change in the gold disk shape can be described in terms of the thermal deformation process [225]. During this process, the shape of the surface of a solid under heating tends to change the shape in order to minimize the surface energy. Thermal deformation is determined by the temperature and the width-to-thickness ratio of the heated nanoparticle. In the case of asymmetric hybrid nanostructures, the shape of the metallic part after the femtosecond modification depends on the absorbed energy and the ratio between the diameter and the thickness of the gold disk.

Because the plasmon resonance strongly depends on the shape and dimensions of the metallic nanoparticle in which it is excited, the controlled modification of the metallic component of a hybrid nanoantenna (Fig. 13a) leads to the tuning of the resonance observed in this case as a spectral shift [224, 226, 227]. Figure 13b shows the change in the scattering spectrum and the directional pattern after the laser melting of a gold disk on top of the silicon cone. The shape of the disk gradually transforms into a sphere, and this process is accompanied by a blue shift of the resonance and an increase in the backscattering at the 600 nm wavelength. It was also shown that using asymmetric hybrid nanoantennas as a structural unit in oligomer nanostructures allows modifying their collective resonant response, including the Fano resonance [87]. Figure 13c shows the scattering spectra of hybrid oligomers consisting of tunable nanoantennas with different degrees of modification of the metallic components. The Fano resonance is observed in the spectral range from 640 to 670 nm. It was shown in [228, 229] that this resonance has a magnetic nature and is caused by the interference between two modes: a narrow magnetic dipole mode of the



Figure 13. (Color online.) (a) The concept of tunable asymmetric hybrid nanoantennas. (b) Scattering spectra of hybrid nanoantennas with a metallic nanoparticle in the shape of a disk (blue curve 1) and a sphere (green curve 2). (c) Scanning electron microscopy images of hybrid oligomers with different degrees of modification of the metallic nanoparticles and their scattering spectra [87, 224].

central silicon cone and a broad collective magnetic mode of the ring formed by the silicon cones. The coupling of the Fano resonance of the dielectric components to the plasmon resonances of the metallic components results in a shift and profile change of the Fano resonance under femtosecond laser modification of the whole oligomer.

6. Conclusions

Large thermal losses of metallic nanostructures limit their applications in many modern fields of nanophotonics, including metasurfaces, metamaterials, and nanowaveguides. This limitation led to the development of new devices using dielectric nanoparticles with a high refractive index. Such metal-dielectric or hybrid nanostructures are currently one of the most modern topics in the science of light. Hybrid nanostructures demonstrate many properties useful for practical applications, some of which were discussed in this review: low thermal losses, optical magnetic resonant response, and enhanced nonlinear optical properties. We focused our attention on the state of the art of theoretical and experimental research on hybrid metal-dielectric nanoantennas and nanostructures based on them. The review shows how such structures can selectively scatter light waves, enhance and transmit optical signals in a specific direction, control the propagation of these signals, and generate optical harmonics. Moreover, we have briefly discussed fabrication methods for these nanostructures using specific examples.

Acknowledgements

The authors are grateful to their colleagues, including Yuri Kivshar (Australian National University), Andrea Alù (University of Texas at Austin), and Denis Baranov for the fruitful discussions and useful comments. The work was performed with support from the Russian Science Foundation grant No. 17-19-01731, the government assignment 3.4424.2017/NM, and the Australian Research Council.

References

- 1 Krasnok A E et al. Phys. Usp. 56 539 (2013); Usp. Fiz. Nauk 183 561 (2013)
- Alù A, Engheta N Phys. Rev. Lett. 104 213902 (2010) 2
- 3. Monticone F, Alù A Chinese Phys. B 23 47809 (2014)
- 4. Chen P-Y et al. Nanophotonics 1 221 (2012)
- 5. Alù A, Engheta N Phys. Rev. Lett. 101 43901 (2008)
- 6. Savelev R S et al. Phys. Rev. B 95 235409 (2017)
- Krasnok A E et al. Appl. Phys. Lett. 104 133502 (2014) 7
- Makarov S et al. Nano Lett. 15 6187 (2015) 8.
- 9 Klimov V V, Istomin Ya N, Kosevich Yu A Phys. Usp. 51 839 (2008); Usp. Fiz. Nauk 178 875 (2008)
- 10 Klimov V Nanoplasmonics (Singapore: Pan Stanford Publ., 2014)
- Klimov V V, Guzatov D V Appl. Phys. A 89 305 (2007) 11.
- 12. Shalin A S, Sukhov S V, Krasnok A E, Nikitov S A Photon. Nanostruct. Fundam. Appl. 12 2 (2014)
- 13. Ferry V E, Munday J N, Atwater H A Adv. Mater. 22 4794 (2010)
- Robak E, Grześkiewicz B, Kotkowiak M A Opt. Mater. 37 104 14. (2014)
- 15. Simovski C et al. Opt. Express 21 A714 (2013)
- Milichko V A et al. Phys. Usp. 59 727 (2016); Usp. Fiz. Nauk 186 801 16. (2016)
- 17. Omelyanovich M, Ra'di Y, Simovski C J. Opt. 17 125901 (2015)
- 18. Alù A, Engheta N Nature Photon. 2 307 (2008)
- Engheta N Science 317 1698 (2007) 19
- Guo R et al. Nano Lett. 15 3324 (2015) 20.
- Kim K et al. Small 8 892 (2012) 21.
- 22. Wertz E et al. Nano Lett. 15 2662 (2015)
- 23. Acuna G P et al. Science 338 506 (2012)
- 24 Garcia-Parajo M F Nature Photon. 2 201 (2008)
- 25 Stranahan S M, Willets K A Nano Lett. 10 3777 (2010)
- 26.
- West J L, Halas N J Annu. Rev. Biomed. Eng. 5 285 (2003)
- Tam F et al. Nano Lett. 7 496 (2007) 27.
- 28. Hirsch L R et al. Proc. Natl. Acad. Sci. USA 100 13549 (2003)
- 29. Loo C et al. Nano Lett. 5 709 (2005)
- 30. O'Neal D P et al. Cancer Lett. 209 171 (2004)
- 31. Curto A G et al. Science 329 930 (2010)
- Esteban R, Teperik T V, Greffet J J Phys. Rev. Lett. 104 026802 32. (2010)
- 33. Busson M P et al. Nature Commun. 3 962 (2012)
- 34. Schietinger S et al. Nano Lett. 9 1694 (2009)
- 35. Lee K et al., in 2011 Conf. on Lasers and Electro-Optics Europe and 12th European Quantum Electronics Conf., CLEO EUROPE/ EQEC, 22-26 May 2011

- Totero Gongora J S et al. Nature Commun. 8 15535 (2017) 36.
- 37. Sarychev A K, Tartakovsky G Phys. Rev. B 75 085436 (2007)
- 38. Zhang C et al. Nano Lett. 15 1382 (2015)
- 39 Suh J Y et al. Nano Lett. 12 5769 (2012)
- 40 Noginov M A et al. Nature 460 1110 (2009)
- Lepeshov S et al. Laser Photon. Rev. 11 1600199 (2017) 41.
- Chettiar U K, Engheta N Phys. Rev. B 86 075405 (2012) 42.
- Liu Z et al. Metamaterials 2 45 (2008) 43.
- 44. Hatab N A et al. Nano Lett. 10 4952 (2010) 45.
- Nien L-W et al. J. Phys. Chem. C 117 25004 (2013) Akselrod G M et al. Nature Photon. 8 835 (2014) 46
- 47
- Cang H et al. Nano Lett. 13 5949 (2013) 48.
- Krasnok A E et al. Sci. Rep. 5 12956 (2015) Slobozhanyuk A P et al. Appl. Phys. Lett. 104 161105 (2014) 49
- Gu Q et al. Opt. Express 21 15603 (2013) 50.
- 51.
- Eggleston M S et al. Proc. Natl. Acad. Sci. USA 112 1704 (2015) 52. Zambrana-Puyalto X, Bonod N Phys. Rev. B 91 195422 (2015)
- Baranov D G et al. Laser Photon. Rev. 11 1600268 (2017) 53
- 54. Krasnok A E, Simovski C R, Belov P A, Kivshar Yu S Nanoscale 6 7354 (2014)
- Krasnok A E, Miroshnichenko A E, Belov P A, Kivshar Yu S JETP 55. Lett. 94 593 (2011); Pis'ma Zh. Eksp. Teor. Fiz. 94 635 (2011)
- 56 Giannini V et al. Chem. Rev. 111 3888 (2011)
- 57. Boltasseva A, Atwater H A Science 331 290 (2011)
- 58. Boriskina S V et al. Adv. Opt. Photon. 9 775 (2017)
- 59. Baffou G, Quidant R Laser Photon. Rev. 7 171 (2013)
- 60. Krasnok A E, Miroshnichenko A E, Belov P A, Kivshar Yu S Opt. Express 20 20599 (2012)
- 61. Krasnok A et al. Adv. Opt. Mater. 6 1701094 (2018)
- Baranov D G et al. Optica 4 814 (2017) 62
- 63. Mie G Ann. Physik 330 377 (1908)
- Zywietz U et al. Nature Commun. 5 3402 (2014) 64.
- Evlyukhin A B et al. Nano Lett. 12 3749 (2012) 65.
- Gómez-Medina R J. Nanophoton. 5 53512 (2011) 66.
- 67. Liu W, Miroshnichenko A E, Kivshar Y S Chin. Phys. B 23 047806 (2014)
- Hopkins B et al. ACS Photon. 2 724 (2015) 68
- 69. Minovich A E et al. Laser Photon. Rev. 9 195 (2015)
- Baranov D G et al. ACS Photon. 3 1546 (2016) 70
- Proust J et al. Adv. Opt. Mater. 3 1280 (2015) 71.
- Wang Z Y et al. Sci. Rep. 5 7810 (2015) 72.
- 73. Proust J et al. ACS Nano 10 7761 (2016)
- 74. Zhu L, Dong L J. Opt. 16 125105 (2014)
- 75 Lewi T et al. Nano Lett. 17 3940 (2017)
- 76. Ma C et al. J. Mater. Chem. C 5 4810 (2017)
- 77 Kim S J et al. Nano Lett. 14 1446 (2014)
- Shi L et al. ACS Photon. 1 408 (2014) 78.
- Savelev R S, Yulin A V, Krasnok A E, Kivshar Yu S ACS Photon. 3 79. 1869 (2016)
- 80. Kuznetsov A I et al. Sci. Rep. 2 492 (2012)
- Luk'yanchuk B et al. Nature Mater. 9 707 (2010) 81.
- 82. Bohren C F, Huffman D R, in Absorption and Scattering of Light by Small Particles (Eds C F Bohren, D R Huffman) (Weinheim: Wiley-VCH, 1998) p. 544
- Johnson P B, Christy R W Phys. Rev. B 6 4370 (1972) 83.
- Vuye G et al. Thin Solid Films 233 166 (1993) 84.
- Kaelberer T et al. Science 330 1510 (2010) 85.
- 86. Bruning J H, Lo Y IEEE Trans. Antennas Propag. 19 378 (1971)
- 87. Lepeshov S et al. ACS Photon. 4 536 (2017)
- 88 Grahn P, Shevchenko A, Kaivola M New J. Phys. 14 93033 (2012)
- Rusak E et al. Appl. Phys. Lett. 105 221109 (2014) 89.
- 90. Liu W, Miroshnichenko A E, Neshev D N, Kivshar Yu S ACS Nano 6 5489 (2012)
- Boriskina S V, Reinhard B M Proc. Natl. Acad. Sci. USA 108 3147 91. (2011)
- 92. Santiago-Cordoba M A et al. Appl. Phys. Lett. 99 073701 (2011)
- 93. Santiago-Cordoba M A et al. J. Biophoton. 5 629 (2012)
- 94. Wang Q et al. Opt. Express 21 11037 (2013)
- 95. Keil R et al. Sci. Rep. 1 94 (2011)
- Hu Q et al. Sci. Rep. 3 3095 (2013) 96.
- Devilez A, Stout B, Bonod N ACS Nano 4 3390 (2010) 97.
- 98. Ono A, Kato J, Kawata S Phys. Rev. Lett. 95 267407 (2005)
- 99 Kawata S, Ono A, Verma P Nature Photon. 2 438 (2008)

- 100. Fu Y H et al. Nature Commun. 4 1527 (2013)
- 101. Yan J H et al. J. Appl. Phys. 116 154307 (2014)
- Novotny L, van Hulst N Nature Photon. 5 83 (2011) 102.
- 103. Krasnok A et al. Phys. Rev. Appl. 9 014015 (2018)
- 104. Atwater H A, Polman A Nature Mater. 9 865 (2010)
- 105. Kosako T, Kadoya Y, Hofmann H F Nature Photon. 4 312 (2010)

1049

- 106. Pakizeh T, Käll M Nano Lett. 9 2343 (2009)
- 107. Li Z et al. Nano Lett. 9 4383 (2009)
- 108. Aouani H et al. Nano Lett. 11 637 (2011)
- 109. Liu W et al. Opt. Lett. 38 2621 (2013)
- 110. Wang H et al. ACS Nano 9 436 (2015)
- 111. Sun S et al. J. Phys. Chem. C 121 12871 (2017)
- 112 Boardman A D, Zayats A V Handbook Surf. Sci. 4 329 (2014)
- 113 Schuller J A et al. Nature Mater. 9 368 (2010)
- 114. Yang Y et al. Nano Lett. 17 3238 (2017)
- 115. Muskens O L et al. Nano Lett. 7 2871 (2007)
- 116. Chen X-W, Agio M, Sandoghdar V Phys. Rev. Lett. 108 233001 (2012)
- 117. Doeleman H M, Verhagen E, Koenderink A F ACS Photon. 3 1943 (2016)
- 118 Ci X et al. Opt. Express 22 23749 (2014)
- 119. Hong Y et al. Adv. Mater. 25 115 (2013)
- 120. Ahn W et al. ACS Nano 7 4470 (2013)
- 121. Boriskina S V, Reinhard B M Opt. Express 19 22305 (2011)
- 122. Vahala K J EQEC'05. European Quantum Electron. Conf. 352 (2005) https://doi.org/10.1109/EQEC.2005.1567517
- 123. Robinson J T et al. Phys. Rev. Lett. 95 143901 (2005)
- 124. Vollmer F, Yang L Nanophotonics 1 267 (2012)
- 125. Akselrod G M et al. ACS Photon. 3 1741 (2016)
- 126. Krasnok A E et al. Laser Photon. Rev. 9 385 (2015)
- 127. Busson M P, Bidault S Nano Lett. 14 284 (2014)
- 128. Carminati R et al. Opt. Commun. 261 368 (2006)
- Klimov V V, Ducloy M, Letokhov V S Quantum Electron. 31 569 129. (2001); Kvantovaya Elektron. 31 569 (2001)

Yao P, Manga Rao V S C, Hughes S Laser Photon. Rev. 4 499 (2010)

130. Klimov V V Opt. Commun. 211 183 (2002)

135. Hu S, Weiss S M ACS Photon. 3 1647 (2016)

137. Pyatkov F et al. Nature Photon. 10 420 (2016)

Li T, Khurgin J B Optica 3 1388 (2016)

Li L et al. ACS Photon. 4 501 (2017)

149. Pelton M Nature Photon. 9 427 (2015)

Oxford Univ. Press, 2008)

bridge Univ. Press, 2006)

148. Ma R-M et al. Nature Mater. 10 110 (2011)

153. Li D et al. Phys. Rev. B 89 161409(R) (2014) 154. Hussain R et al. Opt. Lett. 40 1659 (2015)

Karaveli S et al. ACS Nano 7 7165 (2013)

Weisskopf V, Wigner E Z. Phys. 63 54 (1930)

Müller K et al. Phys. Rev. X 5 031006 (2015)

163. Sauvan C et al. Phys. Rev. Lett. 110 237401 (2013)

164. Koenderink A F Opt. Lett. 35 4208 (2010)

139. Ellis B et al. Nature Photon. 5 297 (2011)

138. Wu S et al. Nature 520 69 (2015)

142. Ginzburg P Rev. Phys. 1 120 (2016)

136. Kristensen PT, Hughes S ACS Photon. 1 2 (2014)

141. Tanaka K et al. Phys. Rev. Lett. 105 227403 (2010)

Jakubczyk T et al. ACS Photon. 3 2461 (2016)

Poddubny A N et al. Phys. Rev. B 86 035148 (2012)

Dodson C M et al. Opt. Mater. Express 4 2441 (2014)

Noginova N et al. Opt. Express 21 23087 (2013)

145. Poddubny A N, Belov P A, Kivshar Yu S Phys. Rev. B 87 035136

Kavokin A V et al. Microcavities (Ser. on Semiconductor Science

and Technology, Eds R J Nicholas, H Kamimura, Vol. 16) (Oxford:

Stete F, Koopman W, Bargheer M ACS Photon. 4 1669 (2017)

160. Novotny L, Hecht B Principles of Nano-Optics (Cambridge: Cam-

Piatkowski L, Accanto N, van Hulst N F ACS Photon. 31401 (2016)

Choi H, Heuck M, Englund D Phys. Rev. Lett. 118 223605 (2017)

Marquier F, Sauvan C, Greffet J-J ACS Photon. 4 2091 (2017)

134

140.

143

144.

146.

147

150.

151.

152.

155.

156.

157.

158.

159

161.

162.

(2013)

- 131. Klimov V V, Ducloy M Phys. Rev. A 62 43818 (2000)
- 132. Klimov V V, Guzatov D V Phys. Rev. B 75 24303 (2007)
- 133. Guzatov D V, Klimov V V Chem. Phys. Lett. 412 341 (2005) Noda S. Fujita M. Asano T Nature Photon. 1 449 (2007)

- 165. Derom S et al. Europhys. Lett. 98 47008 (2012)
- 166. Kristensen P T, Hughes S ACS Photon. 1 2 (2014)
- 167. Muljarov E A, Langbein W Phys. Rev. B 94 235438 (2016)
- 168. Shahbazyan T V ACS Photon. **4** 1003 (2017)
- Zel'dovich Y B Sov. Phys. JETP 11 594 (1960); Zh. Eksp. Teor. Fiz. 38 819 (1960)
- Muljarov E A, Langbein W, Zimmermann R Europhys. Lett. 92 50010 (2011)
- 171. Weiss T et al. Phys. Rev. Lett. 116 237401 (2016)
- 172. Glauber R J, Lewenstein M Phys. Rev. A 43 467 (1991)
- 173. Guo R et al. ACS Photon. **3** 349 (2016)
- 174. Decker M, Pertsch T, Staude I *Philos. Trans. R. Soc. A* **375** 2016031 (2017)
- 175. Karamlou A, Trusheim M E, Englund D Opt. Express 26 3341 (2018)
- 176. Kurtsiefer C et al. Phys. Rev. Lett. 85 290 (2000)
- 177. Manson N B, Harrison J P, Sellars M J Phys. Rev. B 74 104303 (2006)
- 178. Charnock F T, Kennedy T A Phys. Rev. B 64 041201(R) (2001)
- 179. Fano U Phys. Rev. 124 1866 (1961)
- 180. Fano U J. Opt. Soc. Am. 31 213 (1941)
- Miroshnichenko A E, Flach S, Kivshar Yu S Rev. Mod. Phys. 82 2257 (2010)
- 182. Shafiei F et al. Nature Photon. 7 367 (2013)
- 183. Lassiter J B et al. Nano Lett. 12 1058 (2012)
- 184. Rahmani M et al. Nano Lett. 12 2101 (2012)
- 185. Limonov M F et al. Nature Photon. 11 543 (2017)
- 186. Ho J F, Luk'yanchuk B, Zhang J B Appl. Phys. A 107 133 (2012)
- Luk'yanchuk B S, Miroshnichenko A E, Kivshar Yu S J. Opt. 15 073001 (2013)
- 188. Wang W et al. Appl. Phys. Lett. 106 181104 (2015)
- 189. Yang Y et al. Nano Lett. 15 7388 (2015)
- Savelev R S et al., in Proc. of the Intern. Conf. Days on Diffraction 2015 (Piscataway, N.J.: IEEE, 2015) p. 281
- 191. Shafiei F et al. Nature Nanotechnol. 8 95 (2013)
- 192. Rybin M V et al. Phys. Rev. Lett. 119 243901 (2017)
- 193. Metzger B, Hentschel M, Giessen H ACS Photon. 3 1336 (2016)
- 194. Li W, Valentine J Nano Lett. 14 3510 (2014)
- 195. Narasimhan V K et al. ACS Nano 9 10590 (2015)
- 196. Mann S A, Garnett E C Nano Lett. 13 3173 (2013)
- 197. Zipfel W R et al. Proc. Natl. Acad. Sci. USA 100 7075 (2003)
- 198. Campagnola P J, Loew L M Nature Biotechnol. 21 1356 (2003)
- 199. Zoumi A, Yeh A, Tromberg B J *Proc. Natl. Acad. Sci. USA* **99** 11014 (2002)
- 200. Moreaux L, Sandre O, Mertz J J. Opt. Soc. Am. B 17 1685 (2000)
- 201. Linnenbank H et al. Light Sci. Appl. 5 e16013 (2016)
- 202. Timpu F et al. Nano Lett. 17 5381 (2017)
- 203. Bar-Elli O et al. ACS Nano 9 8064 (2015)
- 204. Metzger B et al. Nano Lett. 14 2867 (2014)
- 205. Shibanuma T et al. Nano Lett. 17 2647 (2017)
- 206. Miroshnichenko A E et al. Nature Commun. 6 8069 (2015)
- 207. Grinblat G et al. Nano Lett. 16 4635 (2016)
- 208. Wei L et al. Optica 3 799 (2016)
- 209. Liu W et al. J. Nanomater. 2015 1 (2015)
- 210. Nemkov N A, Basharin A A, Fedotov V A *Phys. Rev. B* 95 165134 (2017)
- 211. Grinblat G et al. ACS Nano 11 953 (2017)
- 212. Liu W, Kivshar Yu S Philos. Trans. R. Soc. A 375 20160317 (2016)
- 213. Feng T, Xu Y, Zhang W, Miroshnichenko A E *Phys. Rev. Lett.* **118** 173901 (2017)
- 214. Chen P-Y, Alù A Phys. Rev. B 82 235405 (2010)
- 215. Abb M et al. Nano Lett. 11 2457 (2011)
- Maksymov I S, Miroshnichenko A E, Kivshar Yu S Opt. Express 20 8929 (2012)
- 217. Large N et al. Nano Lett. 10 1741 (2010)
- 218. Noskov R E, Krasnok A E, Kivshar Yu S *New J. Phys.* **14** 93005 (2012)
- 219. Ahn W et al. ACS Nano 6 951 (2012)
- 220. Baranov D G et al. Laser Photon. Rev. 10 1009 (2016)
- 221. Sokolowski-Tinten K, von der Linde D Phys. Rev. B 61 2643 (2000)
- 222. Fischer M P et al. Phys. Rev. Lett. 117 047401 (2016)
- 223. Makarov S V et al. Laser Photon. Rev. 11 1700108 (2017)
- 224. Zuev D A et al. Adv. Mater. 28 3087 (2016)

- 225. Ye J, Thompson C V Adv. Mater. 23 1567 (2011)
- 226. Sun Y et al. Ann. Physik **529** 1600272 (2017)
- 227. Lepeshov S I et al. J. Phys. Conf. Ser. 741 012140 (2016)
- 228. Miroshnichenko A E, Kivshar Yu S Nano Lett. 12 6459 (2012)
- 229. Chong K E et al. Small 10 1985 (2014)