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The critique of quantum mind: measurement, consciousness, delayed choice, and lost coherence

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Contents

Abstract. **The formal logic of many celebrated paradoxes of quantum measurements does not rule out the effect of the observer's mind on the outcome of a measurement or even on the prehistory of a quantum system. The modern methodology of quantum mechanics offers a consistent explanation of the role of an observer's consciousness in physical measurements in universal terms of the interaction between a quantum system and the environment, loss of quantum coherence, and the unitary evolution of the state vector. In this picture, the observer's consciousness is no longer an agent that changes the prehistory of a quantum system, but is rather a subject of physical study. Instruments and methods based on quantum physics open a new phase in such studies.**

Keywords: measurements in quantum mechanics, observer in quantum measurement, quantum decoherence

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1. Introduction

The problems of measurement and their related problems of defining the boundary between quantum and classical worlds are at the heart of many recent developments in quantum physics, driving the ongoing quantum revolution [1–5]. The formal logic of many paradoxes of quantum measurements put forward in the early era of quantum mechanics does not rule out the effect of an observer's mind on the outcome of a measurement and even on the prehistory of a quantum system. This possibility has been pointed out by the founding fathers of quantum theory. In its most complete form, this solution to quantum measurement paradoxes has been discussed by Wigner [6, 7] and Wheeler [8, 9].

As a typical approach, many prominent modern areas of epistemology of quantum mechanics tend to postpone the solution to the fundamental question regarding preferred quantum states selected in the process of measurement [10]. In particular, the many-worlds interpretation (MWI) of quantum mechanics [11, 12], which is gaining an increasingly broad acceptance in this context, leaves the question as to how quantum evolution yields experimentally observed physical statistics open. This interpretation of quantum mechanics avoids the inconsistency of the Copenhagen interpretation concerning the role of an observer as it treats an isolated quantum system consisting of a quantum object and an observer and extends the quantum description to this system. For Everett, the originator of the MWI, the observer's role is reduced to the role of a macroscopic measuring instrument — a photoelement, a photoplate, or similar detecting systems [11]. For Everett, the problem of an observer cannot be ruled out as purely psychological [11]. However, for a researcher concerned with epistemological problems, such an interpretation implies that the borderline between quantum and classical is shifted more and more toward the observer.

Because of the remarkable, unprecedented predictive accuracy of quantum mechanics, its epistemological difficulties were viewed over many decades as local. On the other hand, it is these difficulties that in many ways stimulated the development of quantum information technologies [13–17]. However, as the physical methods of brain research are gaining more and more prominence, providing new tools to study the most important brain functions, including consciousness [18–20], questions related to the role of an observer in physical measurements have ceased to be purely epistemological. These questions have become central for a new field of interdisciplinary research [21, 22] aimed at attempting to solve one of the key problems of modern natural sciences: understanding the principles of work and functions of the brain.

Such a state of affairs inevitably erodes the comfort zone that serves as a shelter for a large group of physicists (including, by and large, the author of these notes), who prefer to avoid philosophical arguments, following in their attitude to quantum theory the Copenhagen-school principle, 'shut up and calculate'. Increasingly urgent questions regarding the role of consciousness in quantum measurements can no longer be ignored in the rapidly growing area of research in which measuring instruments based on the principles of quantum physics are used as tools for experimental studies of the brain, offering new approaches to the exploration of consciousness [18–20, 23–27].

In this paper, based on a brief review of informational and thermodynamic aspects of the measurement process, we argue that the modern methodology of quantum mechanics offers a consistent explanation of measurement paradoxes in universal terms of the interaction between a quantum system and the environment. We will provide a brief discussion of the universal principles and concepts of thermodynamics and information theory that serve as cornerstones for both models describing the interaction of a physical system with the environment and becoming widespread unifying physical models of the brain and consciousness, such as the Bayesian brain model and brain models based on the hypothesis of free-energy minimization by consciousness. We will show that, although the information perceived by the brain and processes in the microworld are both quantum in nature, their quantumness is fundamentally different. We will consider examples of how the analysis of the interaction between quantum and classical systems can help resolve or avoid paradoxes of quantum measurement without assuming a special role for the observer or allowing the observer's consciousness to influence the outcome of a physical measurement.

2. Delayed choice and the game of twenty questions

The problem of the role of an observer and their mind in a measurement performed on a quantum system plays a central role in paradoxes of quantum mechanics. Many of the well-known thought experiments revealing the difficulties of the main quantum postulates suggest that the observer's mind not only plays a role in the interpretation of a measurement but also influences the prehistory of a quantum system. Wheeler's delayed-choice though experiment is one of the most celebrated paradoxes of this class. In the original version of this paradox, as formulated by Einstein [28, 29], a photon can propagate from a source to a detector along one of the two optical paths (Fig. 1a). As long as there is no beam splitter at point C, the photon ends up, being detected by one of the detectors — D1 or D2. However, in the presence of a beam splitter at point C, the photon is not registered by detector D1 because of the destructive interference of the waves representing two possible paths of the photon, but it is registered by detector D2, which is placed in the arm where the two waves interfere constructively.

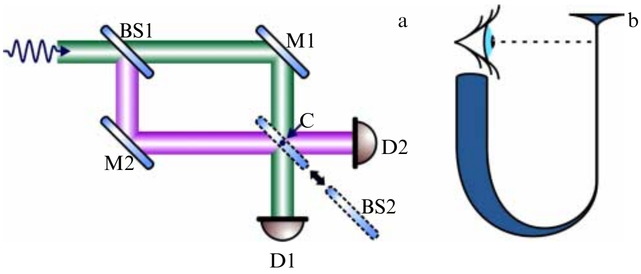

Figure 1. (Color online.) (a) Delayed-choice thought experiment: BS1, BS2 — beam splitters; M1, M2 — mirrors, and D1, D2 — photodetectors. Two possible quantum paths, representing two different histories of a photon, are shown in different colors. (b) Wheeler's delayed-choice paradox, in which a delayed choice of an observer defines the prehistory of the Universe.

Einstein proposed this thought experiment to illustrate the difficulties of the basic quantum postulates. Bohr responded to this argument by noting that two different arrangements — with and without a beam splitter (Fig. 1a) — correspond to two different experiments. To illustrate the difficulties of interpreting the results of measurements in this experiment, Wheeler proposed to decide on whether the beam splitter BS2 is inserted into the experimental scheme 'at the last picosecond' before the photon reaches point C [29]. Depending on whether the beam splitter is inserted into the experimental scheme or not, the photon reaches point C either along one or along the other possible path. In other words, insertion of the beam splitter changes the prehistory of the photon. Similar to other famous quantum paradoxes, the delayed-choice thought experiment has played a significant role in the development of quantum theory and quantum technologies. Implemented experimentally, the delayed-choice phenomenon has allowed putting into practice one of the important tests of the fundamentals of quantum mechanics, enabling new approaches in information processing [30–34].

Extrapolating this paradox to the quantum evolution of the Universe, Wheeler came up with an argument [29] that quantum measurement at a certain moment in the present can have an effect on the state of the Universe in the distant past (Fig. 1b). Moreover, the whole history of a quantum system — a photon or the Universe — becomes dependent on how an observer performs a measurement. Wheeler refers to a striking analogy with the game of twenty questions [29]. In this game, one of the players needs to guess a secret word chosen by the others by asking twenty questions and receiving only 'yes' and 'no' answers.

Trying to make a joke, the other players decide not to choose any secret word. However, the logic of questions asked by the questioner may lead the jokers to admit that the questions indeed converge on a word that can serve as an answer. Remarkably, the word which becomes that answer depends on the questions asked by the questioner during the game. The role of an observer performing an experiment on a quantum system is in many ways similar to the role of the questioner trying to guess a secret word. The outcome of a physical experiment depends on how the physical experiment is conducted. These difficulties in the interpretation of quantum measurements have not been fully overcome. However, for many specific experimental arrangements, the modern machinery of quantum mechanics offers satisfactory explanations of the results of measurements in terms of the universal properties of interaction of a quantum system with environment, decoherence, and loss of the unitary character of evolution [10, 35]. The elimination of the observer's mind from the process of quantum measurement by physical instruments based on the principles of quantum physics paves the way for applying these instruments to the exploration of the brain along with its most complex functions, including consciousness.

3. Wigner's friend paradox

Wigner's friend paradox [6, 7] is one of the most celebrated thought experiments illustrating the difficulties in understanding the role of an observer's consciousness in the process of measurement. We will start by explaining this paradox in a form close to its original formulation, keeping also Wigner's original notation [6, 7, 36]. We consider a quantum system (Schrödinger's cat [37]) that has two eigenstates — ψ_1 and ψ_2. An observer (Wigner) detects the state of this quantum object indirectly, by receiving information from his friend (Wigner's friend). If Wigner's friend finds the quantum object in state ψ_1 (ψ_2), then the friend himself is in the state χ_1 (χ_2). It is in one of these states that Wigner finds his friend in by asking his friend about the results of the measurement. The 'cat + Wigner's friend' system is thus in the $\psi_1 \otimes \chi_1$ ($\psi_2 \otimes \chi_2$) state.

If the initial state of the quantum object is $\alpha\psi_1 + \beta\psi_2$, then, according to the superposition principle, following the detection of its state by Wigner's friend, the state of the 'cat + Wigner's friend' system is $\alpha(\psi_1 \otimes \chi_1) + \beta(\psi_2 \otimes \chi_2)$. Asking his friend about the result of the measurement, Wigner gets an answer that the cat is in the $\psi_1 \otimes \chi_1$ ($\psi_2 \otimes \chi_2$) state with a probability of $|\alpha|^2$ ($|\beta|^2$).

The main difficulty arises when Wigner asks his friend about the state of his mind after the measurement, but before Wigner's question about the result of the measurement. In response to this question, Wigner's friend has to tell Wigner that, even before this question, his mind has already stored the results of the measurement, ψ_1 or ψ_2. In view of such an answer, we have to admit that, right after the measurement, the 'cat + Wigner's friend' system is either in the $\psi_1 \otimes \chi_1$ or $\psi_2 \otimes \chi_2$ state, rather than in the $\alpha(\psi_1 \otimes \chi_1)+\beta(\psi_2 \otimes \chi_2)$ superposition state. Thus, trying to describe the state of the 'cat + Wigner's friend' system, we arrive at two contradictory results.

Wigner then notes that there would have been no paradox had a simple physical instrument, 'such as an atom', detecting the presence or the absence of a flash of light reporting the state of the quantum object, served as a measuring instrument instead of his friend. In Wigner's opinion, such a measuring instrument would have given the correct result: $\alpha(\psi_1 \otimes \chi_1) + \beta(\psi_2 \otimes \chi_2)$. Wigner explains this paradox by assigning a special role to the observer's consciousness in quantum measurements and concluding that "the role of a conscious subject in quantum mechanics is inevitably different from the role of unconscious instrument."

Below, we will re-examine Wigner's friend paradox and show that the methodology of quantum mechanics provides a means for consistently resolving this paradox without assuming a special role of consciousness in a quantum measurement. Before proceeding with such a re-examination, it would be helpful to gain deeper insights into the information aspects of quantum measurements and quantum studies, including experiments aimed directly at the analysis of the brain and consciousness with the use of quantum instruments.

4. Maxwell's demon: 'Information is physical'

We start with a brief examination of the fundamental properties of information, needed to understand the universal rules of information perception by the brain and the information aspects of quantum measurements. Entropy is the main quantitative parameter of information. Introduced by Clausius, Boltzmann, and Gibbs in the context of thermodynamics and statistical mechanics [38, 39], the concept of entropy was reinterpreted and used as a measure of information by Hartley, Nyquist, and Shannon [40]. Understanding the fundamental unity between thermodynamic and information definitions of entropy is central for resolving one of the most celebrated prequantum paradoxes concerned with the role of an observer, as well as the observer's memory and consciousness in a physical measurement — Maxwell's demon paradox [41]. Maxwell's demon paradox is a thought experiment that considers a supernatural creature (demon) placed in a gas-filled chamber consisting of two compartments (Fig. 2a). Being capable of determining the parameters of each gas molecule, the demon selects the fastest molecules and lets them pass into one of the compartments, seemingly without doing any work, thus violating the second law of thermodynamics.

As shown by Szilard [42], Landauer [43], and Bennett [44], Maxwell's demon paradox can be consistently resolved with no contradictions with the second law of thermodynamics, with a realization of direct thermodynamic consequences of

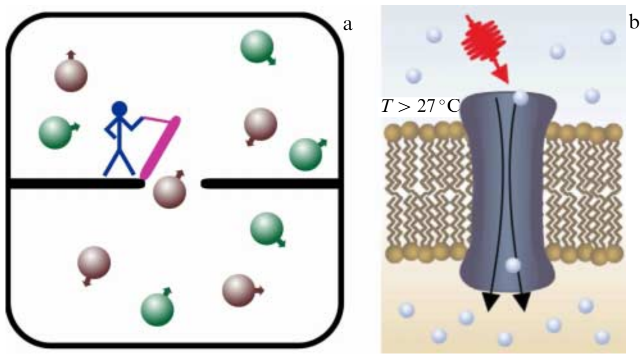

Figure 2. (a) Maxwell's demon: possessing a supernatural ability to determine the parameters of each molecule in a gas, the demon selects only fast molecules and lets them pass from one compartment of a gas chamber to the other, apparently without doing any work, thus violating the second law of thermodynamics. (b) Ion channel on a cell membrane, activated by a change in temperature (TRP channels) or directly by a laser light (channelrhodopsin, halorhodopsin, etc.).

information storage and erasure in a system playing the part of Maxwell's demon. Numerous mechanical realizations of Maxwell's-demon type devices [45, 46] convincingly demonstrate that the consciousness of a demon making a measurement is unimportant and unnecessary for the purpose of this measurement in a thought or real experiment. One of the main takeaways from this explanation of Maxwell's demon paradox is expressed by Landauer's aphorism 'information is physical' [47]. The realization that information is indeed physical is central to resolving a broad class of quantum measurement paradoxes, providing a key to understanding the role of an observer in quantum measurement.

5. Maxwell's demon and the thermodynamics of cell-membrane ion channels

Realization of an essential, physical relationship between the information–metrological and thermodynamic aspects of the Maxwell's demon paradox helps us to understand an important class of universal functions of biological cellular systems, including the action-potential generation by neurons, which is central to higher nervous activity. Such functions can be executed by biological cellular systems due to the unique properties of ion channels in cell membranes [48]. The way cell-membrane ion channels operate is strikingly similar to how Maxwell's demon operates. Very much like Maxwell's demon, cell-membrane ion channels are capable of responding to variations in ambient conditions by switching between open and closed states (Fig. 2b). This gating helps regulate the density of ions on either side of the cell membrane, giving rise to a potential difference across the membrane. Cutting-edge biotechnologies enable a genetic encoding of membrane channels that can be activated by light [19, 20, 49, 50] or temperature variations [51–54]. Neurons expressing such channels can be controlled by an optical signal or a local temperature change, providing unique methods for brain research.

Information–thermodynamic analysis offers important insights into the function of cell-membrane ion channels, allowing a quantitative description of such systems from basic informational–thermodynamic principles. In particular, for heat-sensitive transient receptor potential (TRP) cation channels, widely used in experiments on laser thermogenetics [51–54], the channel gating probability can be written out as $P_g = \left[1 + \exp\left(\Delta G_g/\theta\right)\right]^{-1}$ [55], where ΔG_g is the change in the Gibbs free energy, and θ is the temperature. The general expression for the change in the Gibbs free energy, $\Delta G_g = \Delta H_g - \theta\Delta S_g - \varepsilon$, includes changes in the enthalpy (ΔH_g) and entropy (ΔS_g), as well as the ε term that accounts for all the other energy changes. Typical estimates for TRPV1 channels, widely used in experiments, are $\Delta H_g \approx 185$ kJ/mol and $\Delta S_g \approx 590$ kJ/(mol K) [55]. On the other hand, for cold-activated TRPA1 channels, the ΔH_g and ΔS_g parameters have the opposite sign: $\Delta H_g \approx -125$ kJ/mol and $\Delta S_g \approx -440$ kJ/(mol K). Thermodynamic principles offer a key to understanding the fundamental limitations on the stability of individual heat-sensitive ion channels as membrane gates [56], defining the minimum number of ion channels needed for thermodynamically stable cell gating.

6. The brain as a quantum information system?

Quantum processes, such as tunneling and proton transfer, appear to play an important role in the function of cell-membrane channels, thus contributing to the operation of the central nervous system and brain. However, the hypothesis of the 'quantum mind', widely discussed in the contemporary literature [57–59], goes much further. This hypothesis treats the brain as a quantum computational system. It argues that quantum superposition states play a significant role in the operation of the brain (Fig. 3), allowing it to implement quantum computations. In view of successful experimental demonstrations of wave properties and quantum superposition states for even very large molecules [60], such a hypothesis cannot be rejected outright as totally noncredible.

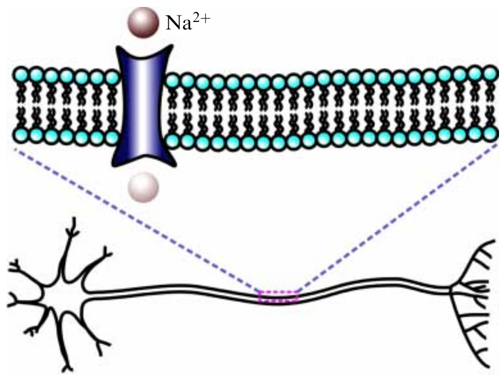

Figure 3. Hypothetical quantum superposition states of ions residing outside and inside a cell membrane, needed for quantum information processing by neurons in quantum-brain models.

If quantum superposition states could indeed play a significant role in the operation of the brain, one could not rule out direct detection of quantum superposition states in the microworld by hypothetical superposition-state neurons with no quantum decoherence and without a loss of the unitary character of evolution.

However, thus far, no convincing experimental evidence has ever been provided that would suggest the significance of quantum superposition states for the operation of the brain. Moreover, credible calculations based on realistic estimates of thermodynamic parameters of the brain and quantifiers of the key physical processes behind the cell-membrane function and cell-membrane ion-channel gating [61] clearly suggest that, when coupled to the environment with typical parameters of the brain, quantum superposition states would rapidly lose their coherence. Because of this strong decoherence, quantum superposition states playing a central role in quantum technologies cannot have any noticeable effect on brain operation. The current status, thus, offers no grounds to treat at least some brain functions or at least some elements of consciousness as quantum computation operations. This being said, there is no doubt, however, that quantum processes do play an important role in brain functions. Quantum aspects of the brain and consciousness, however, do not connect to quantum superposition states employed in quantum information science and technologies, but rather to the universal quantumness of classical information perceived and processed by the brain. We discuss this important question in the following section.

7. Quantum of information

The key message of this section is this: not only is information physical, but it also has a quantum character regardless of whether the information signal is classical or quantum in its nature. Indeed, according to the similarity theorem of Fourier analysis, the $x \to ax$ stretching or compression transformation of the argument of a function $f(x)$ leads, respectively, to a compression or stretching of the Fourier transform $F(k)$ of this function: $f(ax) \to |a|^{-1} F(k/a)$. As a consequence, the information diagram of a signal, defined, following Gabor [62], in the plane of frequency and time variables, is quantized. To prove this, we consider the variances

$$(\Delta x)^2 = \frac{\int_{-\infty}^{\infty} f(x) f^*(x)(x - x_0)^2 \, dx}{\int_{-\infty}^{\infty} f(x) f^*(x) \, dx}, \quad (1)$$

$$(\Delta k)^2 = \frac{\int_{-\infty}^{\infty} F(k) F^*(k)(k - k_0)^2 \, dk}{\int_{-\infty}^{\infty} F(k) F^*(k) \, dk}, \quad (2)$$

where x_0 and k_0 are the central values of x and k. Applying the Cauchy–Bunyakovsky–Schwarz inequality, we find that, regardless of the specific shape of a signal $f(x)$, the variances (1) and (2) have to satisfy the $\Delta x \, \Delta k \geqslant 1/(4\pi)$ inequality, referred to as the Gabor–Heisenberg–Weyl relation. Mathematically, the Gabor–Heisenberg–Weyl relation is fully equivalent to the uncertainty relation in quantum mechanics. It is straightforward to see now that, as a universal property of information, the area of the signal, defined in the information diagram as $S_f = \Delta x \, \Delta k$, is bounded from below. The information diagram of any signal thus contains a finite and numerable number of information 'quanta'. In signal analysis, these information quanta are sometimes referred to as Gabor logons [62]. The Gabor–Heisenberg–Weyl uncertainty relation can be generalized to two-dimensional information signals $f(x, y)$ [63], setting a lower-bound limit for the area of this signal, equal to $1/(16\pi^2)$, in the information diagram defined in the four-dimensional space of variables x, y, k_x, and k_y.

These universal properties of signals lead to important consequences in information processing. One such consequence is that, for a signal with a finite bandwidth Ω and duration T, the number of degrees of freedom never exceeds $2\Omega T$. Combining this relation with the Nyquist–Shannon theorem, one finds that a signal $f(x)$, whose spectrum has a cutoff at $k_{\max}$, such that $F(k) = 0$ for $|k| > k_{\max}$, can be exactly recovered from a discrete sample $f_n(\pi n / k_{\max})$. Moreover, within a broad class of signals $f(x)$, whose spectrum does not exceed one octave, knowing the set of zeros of $f(x)$ is sufficient for signal recovery (Logan theorem) [65].

8. 'Eigenfunctions' of the field of visual perception

The exact lower bound in the Gabor–Heisenberg–Weyl relation is reached with a special choice of the signal waveform. This class of special signals includes those described by the Gabor functions, whose one-dimensional version is written out as [62]

$$f_{\mathrm{G}}(x) = \exp\left[-\frac{(x - x_0)^2}{\alpha^2}\right] \exp\left[-\mathrm{i}k_0(x - x_0)\right]. \quad (3)$$

We now recall that, in quantum mechanics, the exact lower bound of quantum uncertainty for the coordinate x and momentum p is achieved in the ground state of a harmonic oscillator. The wave function of this state is similar to the Gabor functions. We also recall at this time that creation and annihilation operators $\hat{a}^\dagger$ and $\hat{a}$ in quantum optics can be defined, via harmonic-oscillator quantization, in terms of quantum-mechanical coordinate and momentum operators $\hat{x}$ and $\hat{p}$ as $\hat{a} = (m\omega\hat{x} + \mathrm{i}\hat{p})/(2\hbar m\omega)^{1/2}$ and $\hat{a}^\dagger = (m\omega\hat{x} - \mathrm{i}\hat{p})/(2\hbar m\omega)^{1/2}$. In this framework, the states of a light field with a minimum uncertainty of x and p, $\langle 0|\alpha\rangle = \exp(-|\alpha|^2/2)$, referred to as coherent states, can be generated, as shown by Glauber [66], by applying the transition amplitude $|\alpha\rangle = \exp(-|\alpha|^2/2) \sum_n (\alpha\hat{a}^\dagger)^n |0\rangle / n!$ to the field in the vacuum state.

It is remarkable that many properties of visual perception in mammals relate to neurons in their visual cortex, and their two-dimensional perception field can in many cases be adequately described [67, 68] in terms of two-dimensional generalizations of the Gabor functions [63, 69]

$$G(x, y; \xi, \eta, \gamma, \theta) = \exp\left[-\frac{(x - \xi)^2}{a^2} - \frac{(y - \eta)^2}{a^2}\right] \times \exp\left[-\mathrm{i}\gamma(x - \xi) - \mathrm{i}\theta(y - \eta)\right], \quad (4)$$

or in terms of their related functions [70] minimizing the perceived signal uncertainty.

Processing an image $S(x, y)$ by such cells can be formalized in terms of the expansion $S(x, y) = \sum_{jklm} S_{jklm} G(x, y; \xi_j, \eta_k, \gamma_l, \theta_m)$ in two-dimensional functions G, describing the perception field of neurons with spatial coordinates ξ_j and η_k and spatial spectra centered at γ_l and θ_m. In the space of $|\sigma\rangle$ kets, the expansion of the image-transmitting signal $S(x, y)$ can be written in a more compact form: $|\sigma\rangle = \sum_n s_n |n\rangle$. When written out in this form, this expansion shows that the functions G serve as eigenvectors of the perception field of neurons. Since the Gabor functions minimize the Gabor–Heisenberg–Weyl uncertainties, the eigenkets $|n\rangle$ of the perception field of neurons are similar in their properties and behavior to the eigenfunction of the ground state of a harmonic oscillator and, for that matter, coherent states of the photon field.

9. Don't surprise the demon: thermodynamic principles and the Bayesian model of the brain

The ability of the visual perception system in mammals to minimize the uncertainty of the incoming information flow in the phase space of spatial coordinates and spatial frequencies is only one of the properties characterizing the brain as an information-processing system and allowing the informational aspects of consciousness to be described in the same terms and categories as quantum information systems.

Universal principles and concepts of thermodynamics and information theory are at the heart of recent attempts to develop a unifying theory of the brain and consciousness [71]. As their starting point, such attempts assume that the basic thermodynamic principles understood in the context of information theory can be extended to self-organizing biological systems, including such complex systems as the brain. The optimal state of consciousness is defined in these models as a state corresponding to the minimum of entropy (interpreted as surprise) of sensory perception [72].

Calculations of thermodynamic potentials in such theories are performed within the framework of the Bayesian model of the brain [73, 74], which treats the formation of the picture of the outer world by consciousness as a Bayesian hypothesis optimization based on the information coming to the brain from sensory organs. Within the framework of this model, processing the incoming information flow, the brain's learning process, and forming a specific picture of the outer world are described in terms of the probabilities of hypotheses developed and updated by the brain, as well as conditional probabilities of the outcomes predicted with the use of these hypotheses. Such probabilities are assumed to obey the Bayes theorem [75]:

$$P(X|Y) = \frac{P(Y|X)\,P(X)}{P(Y)}\,, \tag{5}$$

where $P(X)$ is the *a priori* probability of hypothesis X, $P(X|Y)$ is the *a posteriori* probability of observing X occurring given that Y is true, $P(Y|X)$ is the probability of observing event Y occurring given that X is true, and $P(Y)$ is the total probability of observing event Y. Within the considered model of the brain, the Bayes theorem is used to quantify the confidence of the brain in the correctness or incorrectness of a specific hypothesis X. The Shannon entropy [40]

$$H(X) = -\sum_X P(X)\,\log P(X) \tag{6}$$

is employed as a measure of the incoming information flow.

At the $(t-1)$th step of the highest-likelihood hypothesis finding procedure, the brain processes the incoming data array $y = (y_1, \ldots, y_{t-1})$ to come up with the *a posteriori* probability to be used as an *a priori* probability at the next, tth, step of the procedure. The sequence of such steps is described by a chain of probabilities composed of products of the following form:

$$P(X_t|y_1, \ldots, y_t) \propto P(y_t|X_t)\,P(X_t|X_{t-1})\,P(X_{t-1}|y_1, \ldots, y_{t-1})\,. \tag{7}$$

Products of such a form are found in transition-amplitude path integrals in the Feynman interpretation of quantum dynamics. A fundamental difference, however, is that quantum path integrals are given by the products of probability amplitudes rather than the probabilities themselves.

The Fisher information [76]

$$I_{\mathrm{F}} = \sum_Y P(Y|\chi)\left[\frac{\partial \log P(Y|\chi)}{\partial \chi}\right]^2 \tag{8}$$

provides an important quantitative measure of the likelihood of the estimate that the brain finds for a specific parameter χ characterizing a certain class of stimuli.

The quantity inverse of I_{F} satisfies the Cramér–Rao inequality [77], $\mathrm{var}(\chi) \geqslant 1/I_{\mathrm{F}}(\theta)$, which sets a universal limit for the uncertainty of an estimate for the parameter χ found from the sensory information about the outer world. Both the Fisher information and the Cramér–Rao limit are broadly used in quantum information science and technologies [78]. However, neither this similarity nor the fact that the Gabor–Heisenberg–Weyl uncertainty proves to be applicable to the incoming flow information in the Bayesian brain can be viewed as evidence of the quantum nature of information processes occurring in the brain. The uncertainty principle in quantum mechanics means that it is fundamentally impossible to simultaneously measure two variables entering an inequality of the form of the Gabor–Heisenberg–Weyl uncertainty relation.

The Gabor–Heisenberg–Weyl uncertainty relation in signal analysis merely expresses a natural relation between the properties of a function and its Fourier transform. In processes defining brain functions, the information transfer, readout, and storage are purely classical. As an important manifestation of the classicality of these processes, the relevant Fisher information is a linear function of the number N of samplings $y = (y_1, \ldots, y_t)$ used for hypothesis generation. The Cramér–Rao bound for uncertainty in this regime decreases as $N^{-1/2}$, indicating that the information process is purely classical. According to universal principles of thermodynamics, hypothesis generation and the resulting state of mind should minimize surprise, i.e., minimize the entropy of sensory perception [71].

10. Lost coherence

To illustrate one possibility of how the problem of quantum measurement can be addressed without imposing a special role on the observer's consciousness, we examine the following instructive example [79] of interaction between a quantum object and a classical measuring device.[1] We consider a quantum particle with mass m in a pure superposition state with an uncertain momentum k described by the wave function

$$\psi(x) = \psi_1(x) + \psi_2(x) = \alpha \exp(\mathrm{i}kx) + \beta \exp(-\mathrm{i}kx)\,. \tag{9}$$

We are going to show now that the measurement of the sign of the momentum of a quantum particle, i.e., the detection of the Schrödinger-cat state described by the wave function (9), can be consistently described in terms of quantum evolution equations.

As a pointer, we consider a sphere of mass M put on top of a very high truncated cone (Fig. 4a) in such a way that the

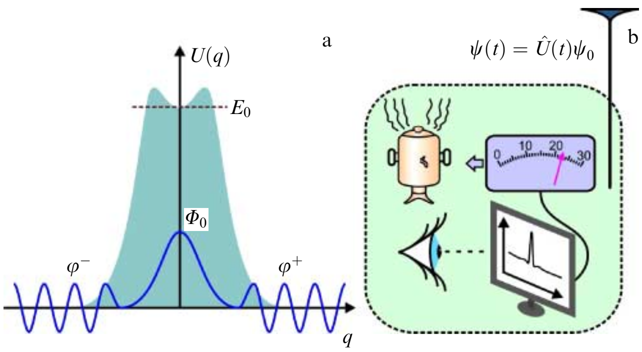

Figure 4. Interaction of a quantum particle with a pointer: $U(q)$ — potential energy of a sphere serving as a pointer; E_0 — energy of the sphere at the minimum of the potential well; $\Phi_0(q)$ — wave function of the initial state of the sphere, and φ^+ and φ^- — wave functions of the sphere rolling to the right and to the left down the cone. (b) Interaction of a quantum particle with a pointer which is a part of an open system. Interaction with the environment (shown to the left of the pointer) induces quantum decoherence and leads to a loss of unitarity in state-ket evolution. The irreversibility of this process prevents the effect of the observer's consciousness on the outcome of the experiment or the prehistory of the quantum object.

[1] The example discussed in this section was known in the methodological literature on quantum mechanics [79] long before the introduction of the general concept of decoherence as an integral part of measurement made by a classical instrument over a quantum system [35].

dependence of the potential energy U of this sphere on its coordinate q has a form shown in Fig. 4a. The minimum of energy of the sphere at $q = 0$ defines the point of equilibrium. The wave function of the pointer sphere is written as

$$\Phi_0(q) = \frac{1}{(\pi a)^{1/4}} \exp\left(-\frac{q^2}{a^2}\right). \tag{10}$$

We assume now that the energy required to push the pointer sphere from the top of the cone is so low that even the energy of the quantum particle we are measuring would suffice. Once it has been pushed out of its equilibrium position, the sphere rolls down the cone, picking up kinetic energy. At the initial moment of time, $t = 0$, the wave function that describes the system consisting of the quantum particle and the pointer sphere is given by

$$\Psi_0(x, q) = \Phi_0(q)\,\psi(x). \tag{11}$$

For $t > 0$, the evolution of the wave function $\Psi(x, q, t)$ describing the 'quantum particle + pointer' system is governed by the Schrödinger equation

$$\mathrm{i}\hbar \frac{\partial \Psi(x, q, t)}{\partial t} = \left[\hat{H}_{\mathrm{a}}(x) + \hat{H}_{\mathrm{b}}(q) + \hat{W}_{\mathrm{ab}}(x, q)\right] \Psi(x, q, t), \tag{12}$$

where $\hat{H}_{\mathrm{a}}(x)$ is the Hamiltonian of the quantum particle, $\hat{H}_{\mathrm{b}}(x)$ is the Hamiltonian of the pointer sphere, and $\hat{W}_{\mathrm{ab}}(x, q)$ is the pointer-sphere–quantum-particle interaction energy, defined as

$$\hat{W}_{\mathrm{ab}}(x, q) = \begin{cases} W_0 \delta(q - x), & t > 0, \\ 0, & t < 0, \end{cases} \tag{13}$$

with W_0 being the interaction constant.

Following the general treatment of the quantum scattering problem, we represent the solution for the wave function as [79]

$$\Psi(x, q, t) = \Psi_0(x, q) \exp\left[-\mathrm{i}(E_0 + \varepsilon_k)\,t\right] + \varphi(x, q, t), \tag{14}$$

where $\varphi(x, q, t)$ is the wave function of the scattered wave, E_0 is the energy of the pointer sphere at the minimum of the potential well at $q = 0$, and $\varepsilon_k = \hbar^2 k^2/(2m)$.

A perturbative treatment of the Schrödinger equation (12) in a small parameter W_0 gives two expressions for the function $\varphi(x, q, t)$. One of them, φ^+, is nonzero for $0 < q < vt$, where $v = \hbar k/M$ is the speed of a sphere. The second solution, φ^-, is nonzero when $-vt < q < 0$. The solutions φ^+ and φ^- correspond to quantum states with wave functions ψ_1 and ψ_2, respectively. Due to the properties of the functions φ^+ and φ^-, their product, $\varphi^+\varphi^-$, tends to zero (Fig. 4a), indicating the blurring of interference fringes of ψ_1 and ψ_2 due to the coupling of the quantum particle and a macroscopic pointer. A sphere rolling down the cone in the range of positive q corresponds to readings of a macroscopic pointer detecting the quantum particle in the state ψ_1. A sphere rolling toward negative q indicates that the quantum particle resides in the state ψ_2. Thus, the 'quantum particle + pointer', or, for that matter, the 'Schrödinger cat + Wigner's friend' system is found in the states $|\alpha|^2$ and $|\beta|^2$ with probabilities $\psi_1 \otimes \chi_1$ and $\psi_2 \otimes \chi_2$, respectively. It is this way of resolving his own paradox that Wigner identified as the most consistent in his later work. 'Wigner's friend' turned out to be a pointer with a memory and the ability to erase what had been stored in this memory.

The above example is instructive, as it illustrates the key aspects of the interaction between a quantum system and a classical measuring instrument, leading to quantum decoherence and a loss of unitarity in the state-ket evolution as the key factors behind the transition from quantum to classical descriptions (Fig. 4b). In the example we considered, the initial state of a 'quantum object + classical pointer' system is purely quantum and is described in terms of a wave function. Before it becomes coupled to the pointer, the quantum system is in a pure superposition state undergoing unitary evolution: $\psi(x, t) = \hat{U}(t)\,\psi(x, t = 0)$. For a broad class of systems, the unitary operator $\hat{U}(t)$ can be written out as $\hat{U}(t) = \exp(-\mathrm{i}\hat{H}t/\hbar)$, where $\hat{H}$ is the Hamiltonian.

However, as the state ket of the pointer is allowed to interact with the environment, the evolution of the system loses its unitarity through quantum decoherence and environment-induced selection (einselection) [10]. As mentioned already by Wheeler [29], enhancement of this interaction is an important factor behind the transition from an initial quantum state to a final classical state. In the example considered above, such an enhancement is provided by the potential defining the dynamics of the pointer sphere. The potential $U(q)$ in the considered example enhances the weak coupling between the quantum system and the sphere, leading to a classically observable outcome: the pointer sphere rolling down along one of the slopes of the cone.

The example considered above is also instructive, as it highlights the significance of equilibrium instability in the initial state of the system. With even small fluctuations enhanced by the aforementioned interaction-enhancement mechanism, this equilibrium instability can now be appreciated as yet another important factor leading to decoherence and the loss of unitarity in a macroscopic-scale measurement [79–82]. An adequate and accurate — albeit not always physically transparent — description of this process may be provided by reduced density matrix analysis. This density matrix describes a quantum system as a part of a larger system, which may include, along with the quantum system under study, a classical measuring device, treated as an open system coupled to the environment (Fig. 4b). A quantum is expelled in the example considered above, as the interaction with a classical pointer denies access to some of the state-kets of the quantum system formally existing in its Hilbert space.

11. Quantum pointers for neuroscience

Quantum technologies provide a unique resource for neuroscience. As one recent development, fiber-optic quantum sensing has been shown in Refs [53, 54, 83, 84] to enable highly sensitive thermometry of individual biological cells, offering a powerful tool for thermogenetic studies of neurons activated by gating thermosensitive TRP channels [85, 86] considered in Section 5 of this paper (Fig. 2b). Single-neuron temperature measurements in this experimental setting are enabled by the optical detection of electron spin resonance in nitrogen–vacancy (NV) centers [87–91] in diamond micro- or nanocrystals (Figs 5b, 5c). As a physical entity, spin has no classical analog. The spin dynamics of an NV center driven by an external microwave field (Fig. 5b) can be represented as a state-ket evolution on the Poincaré sphere (Fig. 5d). This evolution can be calculated as a sum over the quantum paths

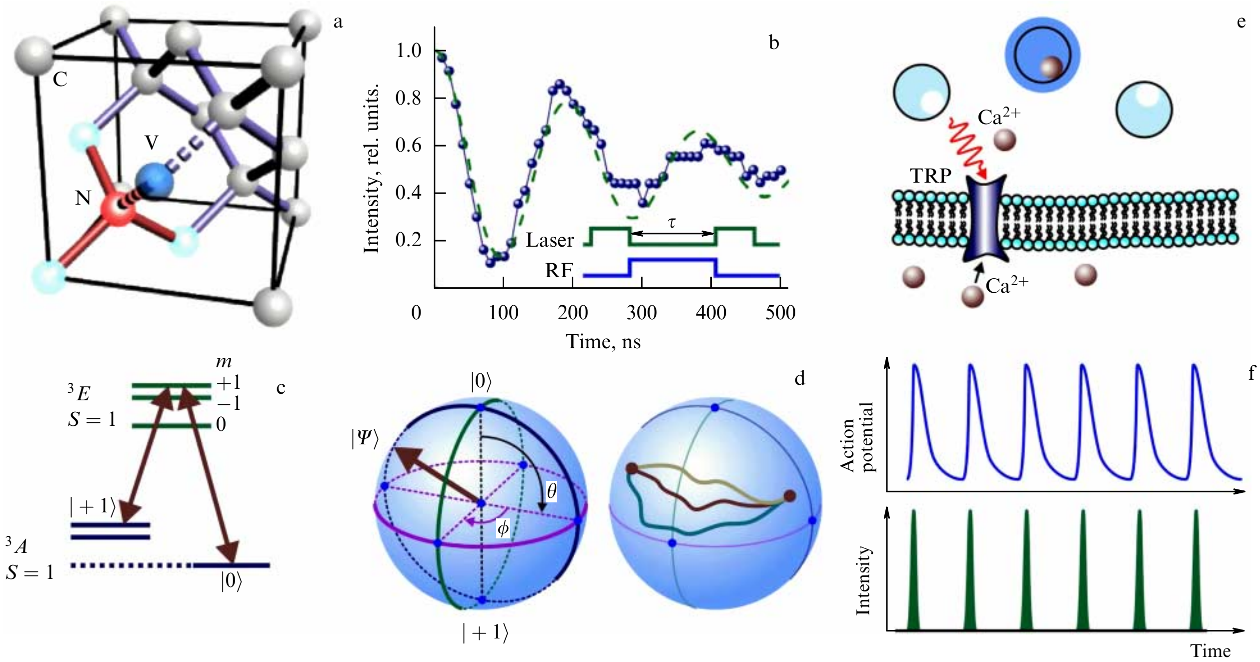

Figure 5. Quantum sensing systems for brain research: (a) NV center in the crystal lattice of diamond; (b) time dependence of an optical readout, reflecting the spin dynamics in the NV center driven by 532-nm laser pulses and optimally shaped microwave waveforms; (c) simplified diagram of spin states in the NV center; (d) evolution of the spin state-ket in the NV center shown on the Poincaré sphere (left) and individual quantum paths of the spin state-ket (right); (e) thermosensitive ion channel on a cell membrane of a neuron activated by laser-induced heating, and (f) actively shaped spike sequences of an action potential.

mapping individual histories of the spin system on the Poincaré sphere (Fig. 5d). Detection of individual quantum paths contributing to spin dynamics is forbidden by fundamental quantum postulates. Had such detection been possible, we would have been back in the realm of quantum measurement paradoxes.

Interaction with the classical microwave field couples the spin dynamics to numerous classical-field degrees of freedom, leading to a loss of coherence between individual spin paths and environment-induced selection of a preferred spin trajectory. Spin dynamics, however, can be controlled with suitably shaped microwave pulses (Fig. 5b). The spectrum of the electron spin resonance induced by the microwave field and detected with the application of optical methods is sensitive to weak magnetic fields and small temperature variations. Due to this property, spins of single NV centers or ensembles of NV centers can be utilized for a highly sensitive magnetometry or subcellular-resolution thermometry [92–102].

Accurate temperature measurement is central to careful thermodynamic control of single neurons by finely adjusted laser-induced heating (Fig. 5e). Local heating of a neuron with single laser pulses or with suitably optimized pulse trains enables the generation of isolated spikes of action potential or carefully controlled sequences of such spikes (Fig. 5f). Online detection of such spikes in experiments indicates precise control over the free energy and the entropy of laser-irradiated neurons [54, 84].

12. Conclusion

Over its century-long history, quantum mechanics has withstood a myriad of the trickiest tests in the most complex physical experiments. The results have always been in remarkable agreement with the predictions of quantum theory. Yet, despite all this success, problems related to the interpretation of the wave function and its collapse in the process of measurement have remained through all this time among the most serious difficulties in the natural sciences. The formal logic of the most celebrated paradoxes of quantum measurements does not rule out the effect of the observer's mind on the outcome of measurements or even on the prehistory of a quantum system. However, through all the times of heated debates, this possibility has always been viewed as a difficulty of the system of quantum postulates rather than a true mechanism whereby the observer's mind can change the prehistory of a quantum system.

The modern methodology of quantum mechanics offers a consistent explanation of the role of the observer's consciousness in physical measurements in universal terms of the interaction between a quantum system and the environment and environment-induced loss of quantum coherence. The evolution of a closed quantum system is unitary and deterministic. The dynamics of such a system are governed by a wave function found as a solution of a pertinent wave equation. However, a quantum system coupled to the environment, e.g., through a classical measuring device, can no longer be treated as closed. The evolution of such a system ceases to be unitary. The quantum system is now coupled to an open classical system that includes the measuring instrument. This coupling gives rise to quantum decoherence and environment-induced selection of a stable final state.

The fundamentally nonunitary, irreversible nature of this process disables pathways whereby the observer's consciousness can influence the outcome of an experiment or the prehistory of a quantum object. The observer's mind thus ceases to be an agent that can change the prehistory of a quantum system, as it does in Wheeler's thought experiment.

Instead, the brain and consciousness themselves become objects of physical study. Instruments and methods based on quantum physics open a new phase in this field of research.

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