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Quantum description of a field in macroscopic electrodynamics and photon properties in transparent media

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Abstract. Applying a quantum mechanical treatment to a high-frequency macroscopic electromagnetic field and radiative phenomena in a medium, we construct quantum operators for energy–momentum tensor components in dispersive media and find their eigenvalues, which are different in the Minkowski and Abraham representations. It is shown that the photon momentum in a medium resulting from the quantization of the vector potential differs from that defined from Abraham's symmetric energy–momentum tensor but is equal to the momentum defined from the Minkowski tensor. A similar result is obtained by calculating the intrinsic angular momentum (spin) of an electromagnetic field in the medium. Only the Minkowski tensor leads to the experimentally confirmed spin values that are multiples of $\hbar$, providing the grounds for choosing the Minkowski representation as the proper form for the momentum density of a transverse electromagnetic field in a transparent medium, in both classical and quantum descriptions of the field. The Abraham representation is unsuitable for this purpose and leads to contradictions. The conclusion drawn does not apply to quasi-static and static fields.

Keywords: Minkowski energy–momentum tensor, quantum theory, radiation phenomena, spin and mass of a photon in matter

1. Introduction

Electromagnetic fields in a medium are standardly described by the classical macroscopic Maxwell equations (see [1–5]). Quantum notions are most frequently used in macroscopic electrodynamics to calculate the linear response of matter in an electromagnetic field, the electric and magnetic permittivity. But an alternating electromagnetic field in a transparent medium (i.e., in a medium with negligible absorption), as in a vacuum, preserves the quantum structure. It consists of photons — discrete excitations with properties that are different from those of photons in the vacuum and are largely determined by the reaction of the electron component of the matter on the field. These properties turn out to be more complicated than those of vacuum photons and are specific for different media. These excitations are often referred to as polaritons, excitons, plasmons, and so on. Along with this detalization, of certain interest is the general quantum approach to describing the macroscopic electromagnetic field and quantum interpretation of basic fundamental field characteristics: the energy density, the momentum density, etc. This approach was first applied by Ginzburg [6] to Vavilov–Cherenkov radiation.

A quantum description of the electromagnetic field enables an improved treatment of some problems that have been discussed for a long time and that apparently cannot be solved classically. In this paper, we analyze one such problem related to the construction of the energy–momentum tensor in classical electrodynamics. It is well known that in the first decade of the 20th century, two different expressions for the momentum density of the electromagnetic field in a medium were proposed. When describing the field using four vectors, the field strengths **E** and **H** and inductions **D** and **B**, these expressions are

$$\mathbf{g}^{\mathrm{M}} = \frac{1}{4\pi c}\,\mathbf{D}\times\mathbf{B}\,, \qquad \mathbf{g}^{\mathrm{A}} = \frac{1}{4\pi c}\,\mathbf{E}\times\mathbf{H}\,. \qquad (1)$$

The first expression was proposed by Minkowski [7, 8] but was criticized by Abraham [9] and other researchers, mainly because it led to a nonsymmetric structure of the energy–momentum tensor. Abraham symmetrized this tensor, which

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gave rise to the appearance of an additional term in the momentum balance equation, the Abraham force. The Abraham force density is determined by the time derivative of the difference between quantities (1):

$$\mathbf{f}^{\mathrm{A}} = \frac{\partial}{\partial t}(\mathbf{g}^{\mathrm{M}} - \mathbf{g}^{\mathrm{A}}) = \frac{1}{4\pi c}\frac{\partial}{\partial t}(\mathbf{D}\times\mathbf{B} - \mathbf{E}\times\mathbf{H}) \qquad (2)$$

(see textbooks and monographs [2, 10–13], as well as reviews [14–16]). In general, without the Abraham force, the momentum conservation law involving Abraham's representation (1) is violated. However, in our recent paper [17], we considered the field as a collection of transverse eigenmodes (a wavepacket) of a transparent dielectric. It turned out that in this frequently occurring situation, a separate Abraham force is not required: it is incorporated into the structure of the Maxwell stress tensor. It was also found that both momentum densities (1) satisfy the continuity equation and preserve the total momentum of the wavepacket. In these conditions, there is no criterion for choosing the correct alternative in classical theory. To derive such a criterion, an analysis of all possible consequences and the comparison with all available experimental data are needed.

For this, we use the canonical quantum description of a macroscopic field, which enables a deeper insight into physical phenomena than the classical treatment. Our approach includes the following stages:

(a) construction of a Lagrangian for the macroscopic electromagnetic field in a dispersive but dissipationless medium and derivation of the Maxwell equations for this case;

(b) transition to the Hamiltonian form of the field equations in dispersive media;

(c) canonical quantization and construction of quantum operators for the field vectors and energy–momentum tensor components in Minkowski's and Abraham's representations;

(d) calculation of eigenvalues of the corresponding operators and their comparison in the case under consideration;

(e) quantum description of radiation processes and the proper angular momentum (spin) of photons in a transparent medium.

Stage (a) is included for completeness and because this problem has not been considered in available textbooks.

2. Derivation of macroscopic Maxwell equations from the Lagrangian formalism

This problem is of certain interest because the relation between vectors describing a field in a dispersive medium is nonlocal [see Eqn (3) below]. In vacuum electrodynamics, the Lagrangian is local, i.e., components of the vector potential $A_k(x)$ and their derivatives relate to the same point in 4-space [18], which simplifies the Lagrangian structure and the derivation of the field equation from it.

We describe the electromagnetic field in a homogeneous isotropic dissipationless medium by four vectors **E**, **B** and **D**, **H**. The first pair of vectors in the four-dimensional notation forms the antisymmetric field tensor $F_{ik}(x)$, and the second pair forms the antisymmetric induction tensor $H^{ik}(x)$ (see [2]), depending on coordinates and time. We assume a linear integral relation between the components of these tensors:

$$H^{ik}(x) = \int \epsilon^{ikmn}(x - x')F_{mn}(x')\,\mathrm{d}^4x'\,. \qquad (3)$$

The 4-tensor $\epsilon^{ikmn}(x - x')$ describes the electromagnetic properties of the medium. In a vacuum, the tensors F_{ik} and H_{ik} are identical, which corresponds to the condition

$$\epsilon^{ikmn}(x - x') = g^{im}g^{kn}\delta^{(4)}(x - x')\,, \qquad (4)$$

where g^{ik} is the metric tensor.

In an immobile dispersionless medium, the linear response ϵ^{ikmn} is expressed in terms of constant dielectric and magnetic permittivities ε and μ:

$$\epsilon^{ikmn}(x - x') = \mu^{-1}(g^{im} + \kappa\delta_0^i\delta_0^m)(g^{kn} + \kappa\delta_0^k\delta_0^n)\delta^{(4)}(x - x')\,,$$
$$\kappa = \varepsilon\mu - 1 \qquad (5)$$

(see, e.g., review [19]). This tensor yields the usual relations

$$\mathbf{D} = \varepsilon\mathbf{E}\,, \qquad \mathbf{B} = \mu\mathbf{H} \qquad (6)$$

for a quasistatic field. Writing the relation equation in integral form (3) allows us to take space and time dispersion into account.

Below, we assume that the four-dimensional 'influence domain' $\Delta\Omega$ over which integration is performed in (3) at a fixed x and which is defined by the properties of the tensor $\epsilon^{ikmn}(x - x')$ is small compared to the region in which the field is considered. This assumption is fully consistent with the macroscopic field description. The size of the influence domain is determined by macroscopic parameters — the relaxation time of the electronic system of matter and the size of its structural components (atoms, inclusions, mean free paths, etc.). The tensor response function $\epsilon^{ikmn}(x - x')$ must have all symmetry properties ensuing from the symmetry of the medium and the antisymmetry of the tensors F_{ik} and H_{ik}, in particular,

$$\epsilon^{ikmn} = \epsilon^{mnik} = -\epsilon^{kimn} = -\epsilon^{iknm}\,. \qquad (7)$$

In addition, in a nongyrotropic medium, this tensor is an even function of its argument:

$$\epsilon^{ikmn}(x - x') = \epsilon^{ikmn}(x' - x) \qquad (8)$$

[see expression (14) below]. The field tensor F_{ik} is expressed in terms of the vector potential $A_l(x)$,

$$F_{ik} = \partial_i A_k(x) - \partial_k A_i(x)\,, \qquad (9)$$

which enables writing the Maxwell equation for the field tensor:

$$\partial_l F_{ik} + \partial_i F_{kl} + \partial_k F_{li} = 0\,. \qquad (10)$$

The equation for the induction tensor H_{ik} can be obtained from the stationarity condition for the action functional

$$S = \int \mathcal{L}(A_k, \partial_l A_k, x^m)\,\mathrm{d}^4x\,, \qquad (11)$$

which should be written in terms of the Lagrangian of the electromagnetic field in a medium:

$$\mathcal{L} = -\frac{1}{16\pi}H^{ik}F_{ik} - \frac{1}{c}\,j^k_{\mathrm{ext}}(x)A_k(x)\,. \qquad (12)$$

Here, the external current $j^k_{\text{ext}}(x)$ represents a specified field source. The Lagrangian is a relativistic invariant, and under the electric charge conservation, $\partial_k j^k_{\text{ext}} = 0$, the variation of the action is invariant under gauge transformations of the vector potential $A_k \to A_k + \partial_k f(x)$, where $f(x)$ is an arbitrary scalar function.

Using connection equation (3), we represent the action in the form of a double integral over 4-space:

$$S = -\frac{1}{16\pi}\int_\Omega \mathrm{d}^4x \int_\Omega \mathrm{d}^4x'\,\epsilon^{ikmn}(x-x')F_{ik}(x)F_{mn}(x') - \frac{1}{c}\int_\Omega j^k_{\text{ext}}(x)A_k(x)\,\mathrm{d}^4x\,. \tag{13}$$

The integration domain can be taken as an arbitrarily large ('macroscopic') region at the boundary of which all field functions $A_k(x)$ and their variations $\delta A_k(x)$ vanish.

The parity of the tensor ϵ^{ikmn} as a function of coordinates, which is expressed by equality (8), is due to the symmetry of the double integral in Eqn (13). Making the substitution $x \leftrightarrow x'$ in this integral and using symmetry (7), we can represent the integral in the form

$$\frac{1}{32\pi}\int_\Omega \mathrm{d}^4x \int_\Omega \mathrm{d}^4x'\left[\epsilon^{ikmn}(x-x') + \epsilon^{ikmn}(x'-x)\right]F_{ik}(x)F_{mn}(x')\,, \tag{14}$$

which enables us to write equality (8).

By calculating variations, we find

$$H^{ik}\,\delta F_{ik} = -2H^{ik}\partial_k\delta A_i\,,\qquad \delta H^{ik} = 2\int_\Omega \epsilon^{ikmn}(x-x')\partial'_m\delta A_n(x')\,\mathrm{d}^4x'\,. \tag{15}$$

The derivative of the vector potential variation can be eliminated by integration by parts over the coordinate x'^m. The term arising in the integration vanishes at the boundary of the domain Ω. As a result, the action variation becomes

$$\delta S = \frac{1}{4\pi}\int_\Omega\left(\partial_i H^{ik} - \frac{4\pi}{c}\,j^k_{\text{ext}}\right)\delta A_k(x)\,\mathrm{d}^4x\,. \tag{16}$$

The action variation must vanish for any $\delta A_k(x)$, and this requirement yields the Maxwell equation in the 4-form:

$$\partial_i H^{ki}(x) = -\frac{4\pi}{c}\,j^k_{\text{ext}}(x)\,. \tag{17}$$

It should be complemented with connection equation (3) and Maxwell equation (10).

The structure of action functional (13) suggests that the Lagrangian of interaction of a macroscopic field with an external source retains the form that it has in a vacuum:

$$\mathcal{L}_{\text{int}} = -\frac{1}{c}\,j^k_{\text{ext}}(x)A_k(x)\,. \tag{18}$$

Here, clearly, the vector potential $A_k(x)$ depends on the properties of matter. The results in this section can be applied to transparent media with time and space dispersion.

3. Hamiltonian form of electromagnetic field equations in a transparent medium

Canonical quantization of an electromagnetic field as a continuous oscillating system in a vacuum includes the following main stages: (a) choice of generalized coordinates in the form of amplitudes of Fourier harmonics of the vector potential that depend on the frequency and wave vector; (b) construction of the classical Hamiltonian function depending on canonical variables — the generalized coordinates and momenta (the Hamiltonian function is constructed from the expression for the total energy of the field); (c) transition from classical variables to quantum operators satisfying the Heisenberg commutation relations and construction of the operators of main physical quantities — components of the energy–momentum tensor of the electromagnetic field in a medium; (d) calculation of eigenvalues of the energy, momentum, and other physical quantities, as well as of the transition probability between quantum field states.

When pursuing this plan for a field in a medium, we take into account that the physical system under consideration is made of two coupled subsystems. Therefore, the description of such a system by methods of classical electrodynamics is much more complicated than the field description in a vacuum. It suffices to recall that the energy density in a medium in classical electrodynamics was written in the approximate form obtained by Brillouin (1921):

$$w = \frac{1}{16\pi}\left[\frac{\partial}{\partial\omega}\big(\omega\varepsilon(\omega)\big)\mathbf{E}^*\mathbf{E} + \frac{\partial}{\partial\omega}\big(\omega\mu(\omega)\big)\mathbf{H}^*\mathbf{H}\right], \tag{19}$$

where $\varepsilon(\omega)$ and $\mu(\omega)$ are linear responses of the medium in the field that is described by macroscopic vectors $\mathbf{E}$ and $\mathbf{H}$ (i.e., those averaged over the states of the medium) (see [2], pp. 381–382). The main conditions restricting the application of formula (19) are as follows.

(1) The external field is small compared to internal fields in the medium.

(2) The electromagnetic energy dissipation, described by imaginary parts of the dielectric and magnetic permittivity, is low:

$$\varepsilon''(\omega) \ll \varepsilon'(\omega)\,,\qquad \mu''(\omega) \ll \mu'(\omega)\,. \tag{20}$$

Conditions (20) are satisfied only in certain frequency ranges representing 'transparency windows' of the dielectric (see [2]), but cannot be satisfied in an unbounded frequency range due to the Kramers–Kronig relations.

(3) Formula (19) results from averaging over the period $T = 2\pi/\omega$ of the energy of a wavepacket with the central frequency ω, whose field represents a plane wave with a slowly changing amplitude. Electromagnetic field vectors for such a packet can be written as the Fourier integral over the small frequency range $\alpha \leqslant \Delta\omega \ll \omega$ and wave vector range $q \leqslant \Delta k \ll k$:

$$\mathbf{E}_\omega(\mathbf{r},t) = \int \mathcal{E}(\mathbf{k}+\mathbf{q}, \omega+\alpha) \times \exp\left[\mathrm{i}(\mathbf{k}+\mathbf{q})\,\mathbf{r} - \mathrm{i}(\omega+\alpha)t\right]\frac{\mathrm{d}^3q\,\mathrm{d}\alpha}{(2\pi)^4} = \mathcal{E}(\mathbf{r},t)\exp(\mathrm{i}\varphi)\,,\qquad \varphi(\mathbf{r},t) = \mathbf{kr} - \omega t\,. \tag{21}$$

The amplitude $\mathcal{E}(\mathbf{r},t)$ is determined by the external field sources and properties of the medium. It changes in space and time slowly compared to the phase factor $\exp(\mathrm{i}\varphi)$. In particular, coordinate and time derivatives of $\mathcal{E}(\mathbf{r},t)$ are of the respective order of $\Delta k/k$ and $\Delta\omega/\omega$, unlike derivatives of the phase factor. A similar expression can be written for the field

$\mathbf{H}_\omega(\mathbf{r}, t)$. The frequency interval is chosen in the transparency window of the dielectric.

In accordance with the adopted approximations, we use the following formulas for the electric and magnetic inductions in what follows:

$$\mathbf{D}_\omega(\mathbf{r}, t) = \varepsilon(\omega)\mathbf{E}_\omega(\mathbf{r}, t), \qquad \mathbf{B}_\omega(\mathbf{r}, t) = \mu(\omega)\mathbf{H}_\omega(\mathbf{r}, t). \quad (22)$$

Thus, our approach is based on considering wavepackets, or 'quasimonochromatic waves,' for which only classical expressions for the electromagnetic field energy and momentum are known. Other components of the energy–momentum tensor in a medium are calculated in the same approximation and are analyzed in our paper [17]. Wavepackets, unlike monochromatic waves, have a finite extent in space and time, although their sizes significantly exceed the wavelength and the central harmonic period.

The need for condition 2 (smallness of the absorption) is briefly explained in [2, p. 386]: for strong absorption, $\varepsilon'' \approx \varepsilon'$ and $\mu'' \approx \mu'$, the field decays at distances of the order of the wavelength λ and does not penetrate into the medium; the quantized field (photons) are also absorbed over the same wavelength. The interaction of the field with matter in this case cannot be described using the linear responses ε and μ solely. The heat capacity and other macroscopic parameters of the medium, which are used in nonequilibrium thermodynamics, should be involved. Brillouin energy density (19) (as well as other components of the energy–momentum tensor, to be explored below) is defined in the zeroth order in the small parameters $\Delta\omega/\omega$, $\Delta k/k$, $\varepsilon''/\varepsilon'$, and μ''/μ'.

Several decades after Ginzburg's pioneering paper mentioned above [6], interest in a quantum description of the macroscopic field in a medium has been revived [20–23]. In these and other papers, the authors consider media with arbitrarily high dissipation of the electromagnetic field. To do this, in the action functional of the system, variables of the medium ('reservoir') and unbounded imaginary parts of the electric and magnetic permittivity were included in addition to the field variables. The same quantities enter the quantum mechanical Hamiltonian derived from the original Lagrangian. This approach as the basis for a quantum theory is questionable. The inclusion of dissipative variables into the Hamiltonian function violates the canonical scheme of classical mechanics. Dissipative processes cannot be described in classical mechanics by a Hamiltonian function and canonical Hamiltonian equations derived from the least action principle. Dissipation is additionally introduced into equations with the help of a dissipative function (see, e.g., classical textbook [24]). Here, at the microscopic level, the description of the interaction with the 'reservoir' does not assume the initial use of the macroscopic quantities ε'' and μ''. The equivalent parameters should result from microscopic calculations, which apparently should be carried out using the density matrix for the electromagnetic field in a medium.

For these reasons, the approach to quantum electrodynamics in media that has been developed in [20–23] cannot be considered correct. The efficiency of these methods is also unclear. In these papers, eigenvalues of the electromagnetic field energy in a medium and the field momentum are not calculated, properties of quantum field excitations (photons in the medium) are not analyzed, and comparisons with experimental data are not made. Therefore, despite the title "Canonical quantization..." (see [20–22]), these authors' approach to quantization proves to be noncanonical and requires additional justification, while the applicability limits of the method are unclear and raise doubts.

In this paper, we use the systematic canonical approach to field quantization inside the transparency windows of a dielectric. The unavoidable cost for this is the refusal to describe dissipative processes. The field in a medium is treated as a classical linear oscillating system without dissipation. Our purpose is to construct quantum operators for components of the energy–momentum tensor in a medium and to calculate their eigenvalues within the applicability domain of macroscopic electrodynamics. This condition implies that the effect of an isotropic medium on the field can be described using the dielectric $\varepsilon(\omega)$ and magnetic $\mu(\omega)$ permittivities with negligible imaginary parts without invoking other macroscopic parameters of the medium. Calculation of the linear responses ε and μ is a separate problem, which we do not consider here. For most media, it should be solved (and in fact has been solved for many simple models of matter) using quantum theory.

In this problem setting, the procedure of passing to the quantum description repeats all stages worked out by many authors [25–30] to quantize the electromagnetic field in a vacuum. Here, we repeat only the initial assumptions and the specific features for a dispersive medium. We use the Coulomb gauge for the vector potential:

$$\nabla\mathbf{A}_\omega(\mathbf{r}, t) = 0. \qquad (23)$$

The d'Alembert equation involves the medium parameters $\varepsilon(\omega)$ and $\mu(\omega)$:

$$\Delta\mathbf{A}_\omega - \frac{\varepsilon\mu}{c^2}\frac{\partial^2\mathbf{A}_\omega}{\partial t^2} = 0. \qquad (24)$$

We consider a field in a macroscopic (large) volume V with periodic boundary conditions and decompose the vector potential into plane waves. The wave vector projections k_x, k_y, k_z form a quasidiscrete countable set. Their values, along with the polarization vectors $\mathbf{e}_{\mathbf{k}\sigma}$, $\sigma = 1, 2$, orthogonal to $\mathbf{k}$, determine the oscillation type (eigenmode) and the eigenfrequency ω_k:

$$\mathbf{A}_{\mathbf{k}\sigma}(\mathbf{r}) = \mathbf{e}_{\mathbf{k}\sigma}A_k^0 \exp(\mathrm{i}\mathbf{kr}). \qquad (25)$$

The normalization constant A_k^0 is to be specified later from the canonical form of the energy operator.

The decomposition of the vector potential $\mathbf{A}(\mathbf{r}, t)$, which is considered a real function of coordinates and time, has the form

$$\mathbf{A}(\mathbf{r}, t) = \sum_{\mathbf{k},\sigma}\left[b_{\mathbf{k}\sigma}(t)\mathbf{A}_{\mathbf{k}\sigma}(\mathbf{r}) + b^*_{\mathbf{k}\sigma}(t)\mathbf{A}^*_{\mathbf{k}\sigma}(\mathbf{r})\right]. \qquad (26)$$

The complex amplitudes $b_{\mathbf{k}\sigma}(t)$ satisfy the classical linear harmonic oscillator equations:

$$\ddot{b}_{\mathbf{k}\sigma}(t) + \omega_k^2 b_{\mathbf{k}\sigma}(t) = 0, \qquad b_{\mathbf{k}\sigma}(t) = b_{\mathbf{k}\sigma}\exp(-\mathrm{i}\omega_k t),$$
$$\omega_k^2 = \frac{c^2k^2}{\varepsilon(\omega_k)\mu(\omega_k)}, \qquad (27)$$

where the last equation defines the relation between the frequency and the wavenumber, which is important in what follows. The field vectors can also be decomposed into plane waves:

$$\mathbf{E}(\mathbf{r},t) = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} = \mathrm{i}\sum_{\mathbf{k},\sigma}\frac{\omega_k}{c}\left[b_{\mathbf{k}\sigma}(t)\mathbf{A}_{\mathbf{k}\sigma}(\mathbf{r}) - b^*_{\mathbf{k}\sigma}(t)\mathbf{A}^*_{\mathbf{k}\sigma}(\mathbf{r})\right], \tag{28}$$

$$\mathbf{B}(\mathbf{r},t) = \nabla\times\mathbf{A} = \mathrm{i}\sum_{\mathbf{k},\sigma}\mathbf{k}\times\left[b_{\mathbf{k}\sigma}(t)\mathbf{A}_{\mathbf{k}\sigma}(\mathbf{r}) - b^*_{\mathbf{k}\sigma}(t)\mathbf{A}^*_{\mathbf{k}\sigma}(\mathbf{r})\right]. \tag{29}$$

Expansions (26)–(29) are valid in the entire frequency range, including the strong-dissipation domain. However, when passing to the Hamiltonian description, we should restrict ourselves to considering only transparency windows and use formulas for the energy density $w_{\mathbf{k}\sigma}$ and momentum density $\mathbf{g}^{\mathrm{A}}_{\mathbf{k}\sigma}$ in a dispersive medium, which are known from macroscopic electrodynamics:

$$w_{\mathbf{k}\sigma} = \frac{1}{8\pi}\left[\frac{\partial}{\partial\omega_k}\big(\omega_k\varepsilon(\omega_k)\big)\mathbf{E}^2_{\mathbf{k}\sigma} + \frac{\partial}{\partial\omega_k}\big(\omega_k\mu(\omega_k)\big)\mathbf{H}^2_{\mathbf{k}\sigma}\right], \tag{30}$$

$$\mathbf{g}^{\mathrm{A}}_{\mathbf{k}\sigma} = \frac{1}{4\pi c\mu(\omega_k)}\left[\mathbf{E}_{\mathbf{k}\sigma}\times\mathbf{B}_{\mathbf{k}\sigma}\right]. \tag{31}$$

In contrast to formula (19), in Eqns (30) and (31) we use the mode index $\mathbf{k},\sigma$ and real quasimonochromatic fields (28) and (29) not averaged over time.

The densities $w_{\mathbf{k}\sigma}$ and $\mathbf{g}^{\mathrm{A}}_{\mathbf{k}\sigma}$ are related by the energy and momentum conservation laws in a dissipationless medium. In terms of the group velocity

$$\mathbf{u}_k = \frac{\mathrm{d}\omega_k}{\mathrm{d}\mathbf{k}} = \frac{c\mathbf{k}}{k}\frac{\mathrm{d}\omega_k}{\mathrm{d}\left(\omega_k\sqrt{\varepsilon(\omega_k)\mu(\omega_k)}\right)}, \tag{32}$$

this relation is expressed simply as

$$\mathbf{g}^{\mathrm{A}}_{\mathbf{k}\sigma} = w_{\mathbf{k}\sigma}\frac{\mathbf{u}_k}{c^2}. \tag{33}$$

Here, we represent the momentum density according to Abraham [9], which corresponds to the symmetric energy–momentum tensor. We distinguish between Abraham's and Minkowski's notations by the superscripts A and M. The classical quantities written above and their derivation were discussed in detail in our paper [17]. Using (33), we find a useful representation for the energy density in terms of the group velocity and momentum density:

$$w_{\mathbf{k}\sigma} = \frac{c^2}{u_k^2}\mathbf{u}_k\,\mathbf{g}^{\mathrm{A}}_{\mathbf{k}\sigma}. \tag{34}$$

The total momentum $\mathbf{G}^{\mathrm{A}}_{\mathbf{k}\sigma}$ of the wavepacket (quasimonochromatic mode) can be obtained using formulas (29)–(31) after integrating (34) over the entire domain of periodicity V:

$$\mathbf{G}^{\mathrm{A}}_{\mathbf{k}\sigma} = \mathbf{k}\frac{(A^0_k)^2 V\omega_k}{2\pi c^2\mu(\omega_k)}\,b^*_{\mathbf{k}\sigma}(t)b_{\mathbf{k}\sigma}(t). \tag{35}$$

The field periodicity and the normalization of polarization vectors $\mathbf{e}^*_{\mathbf{k}\sigma'}\mathbf{e}_{\mathbf{k}\sigma} = \delta_{\sigma\sigma'}$ have been used here. The mode energy

$$W_{\mathbf{k}\sigma} = \frac{c^2}{u_k^2}\mathbf{u}_k\mathbf{G}^{\mathrm{A}}_{\mathbf{k}\sigma} \tag{36}$$

is the same in both Abraham's and Minkowski's representations and is always nonnegative, because the momentum and the group velocity vector $\mathbf{u}_k$ have the same direction.

The total field energy in a volume V is obtained by summing (36) over all wavepackets within the 'transparency window' of the dielectric. By expressing the wave number in terms of the frequency, $k = \omega_k\sqrt{\varepsilon\mu}/c$, we obtain the field energy expressed via complex amplitudes:

$$W = \sum_{\mathbf{k},\sigma}\frac{(A^0_k)^2 V\omega_k^2}{2\pi c|u_k|}\sqrt{\frac{\varepsilon(\omega_k)}{\mu(\omega_k)}}\,b^*_{\mathbf{k}\sigma}(t)b_{\mathbf{k}\sigma}(t). \tag{37}$$

We also note that the total Abraham force (2),

$$\mathbf{F}^{\mathrm{A}} = \int\mathbf{f}^{\mathrm{A}}\,\mathrm{d}V, \tag{38}$$

vanishes, $\mathbf{F}^{\mathrm{A}} = 0$, if the integral is extended to the entire volume containing the wavepacket. This follows from the fact that Abraham's force density (2) can be written as the divergence of some tensor:

$$f^{\mathrm{A}}_\alpha = \frac{\partial\sigma^f_{\alpha\beta}}{\partial x_\beta}, \qquad \sigma^f_{\alpha\beta} = -\frac{k_\alpha k_\beta}{4\pi k^2}\left(\varepsilon - \frac{1}{\mu}\right)\mathbf{E}^2_{\mathbf{k}\sigma}, \tag{39}$$

which represents a part of the Maxwell stress tensor expressed in terms of the electric field strength (see [17]). The Abraham force is interpreted as a result of the field effect on the medium (see [2], p. 361). The vanishing of this force is quite natural for a wavepacket propagating without dissipation with energy and momentum conservation.

Passing in (37) to real canonical variables $Q_{\mathbf{k}\sigma}(t)$ and $P_{\mathbf{k}\sigma}(t)$,

$$\begin{aligned} b_{\mathbf{k}\sigma}(t) &= \frac{1}{2}\left(Q_{\mathbf{k}\sigma}(t) + \frac{\mathrm{i}P_{\mathbf{k}\sigma}(t)}{\omega_k}\right), \\ b^*_{\mathbf{k}\sigma}(t) &= \frac{1}{2}\left(Q_{\mathbf{k}\sigma}(t) - \frac{\mathrm{i}P_{\mathbf{k}\sigma}(t)}{\omega_k}\right), \end{aligned} \tag{40}$$

we find the classical Hamiltonian function of the electromagnetic field in a medium as the sum of energies of independent oscillators,

$$H(Q,P) = \frac{1}{2}\sum_{\mathbf{k},\sigma}\left(P^2_{\mathbf{k}\sigma} + \omega_k^2 Q^2_{\mathbf{k}\sigma}\right), \tag{41}$$

and find the normalization constant by requiring the canonical form of Hamilton function (41):

$$A^0_k = \sqrt{\frac{4\pi c|u_k|}{V}\left(\frac{\mu(\omega_k)}{\varepsilon(\omega_k)}\right)^{1/2}}. \tag{42}$$

The obtained relations differ from similar formulas of vacuum electrodynamics mainly by coefficients that now depend on the linear response functions of the medium. But these differences become quite significant in some cases. This becomes especially apparent in analyzing properties of quantum field excitations ('photons in a medium') and in the quantum interpretation of the main quantities of quantum electrodynamics, including the energy, momentum, and angular momentum.

4. Quantization of a macroscopic electromagnetic field

Next, we replace the canonical variables $Q_{\mathbf{k}\sigma}(t)$ and $P_{\mathbf{k}\sigma}(t)$ with operators $\hat{Q}_{\mathbf{k}\sigma}$ and $\hat{P}_{\mathbf{k}\sigma}$ and subject them to the

Heisenberg commutation relations

$$[\hat{Q}_s, \hat{P}_s] = \mathrm{i}\hbar, \qquad s = (\mathbf{k}, \sigma), \tag{43}$$

where we introduce a single mode index s. All other pairs of operators commute. After that, the Hamiltonian function turns into the Hamiltonian operator $\hat{H}$; the vector potential and complex amplitudes b also become operators. We use the Schrödinger representation, in which operators $\hat{P}$ and $\hat{Q}$ do not depend on time.

The creation and annihilation operators of photons are introduced in the usual way,

$$\begin{aligned} \hat{c}_s^{\dagger} &= \sqrt{\frac{2\omega_s}{\hbar}}\,\hat{b}_s^{\dagger} = \sqrt{\frac{\omega_s}{2\hbar}}\left(\hat{Q}_s - \frac{\mathrm{i}\hat{P}_s}{\omega_s}\right), \\ \hat{c}_s &= \sqrt{\frac{2\omega_s}{\hbar}}\,\hat{b}_s = \sqrt{\frac{\omega_s}{2\hbar}}\left(\hat{Q}_s + \frac{\mathrm{i}\hat{P}_s}{\omega_s}\right), \end{aligned} \tag{44}$$

and satisfy the commutation relations

$$[\hat{c}_{s'}, \hat{c}_s^{\dagger}] = \delta_{ss'}\,. \tag{45}$$

The field energy operator is

$$\hat{H} = \sum_s \hbar\omega_s\left(\hat{c}_s^{\dagger}\hat{c}_s + \frac{1}{2}\right). \tag{46}$$

In a similar way, using (36) and subsequent formulas, we find the field momentum operator:

$$\hat{\mathbf{G}}^{\mathrm{A}} = \sum_s \frac{\mathbf{u}_s}{c^2}\,\hbar\omega_s\hat{c}_s^{\dagger}\hat{c}_s\,. \tag{47}$$

Both operators are expressed in terms of operators of the number of quanta in a mode s:

$$\hat{n}_s = \hat{c}_s^{\dagger}\hat{c}_s\,. \tag{48}$$

By specifying the Fock state of the field (i.e., the number of quanta $n_s = 0, 1, \ldots$ in each mode), we can find eigenvalues of the energy $\mathcal{E}$ and momentum $\mathbf{G}^{\mathrm{A}}$ of the field in this state:

$$\mathcal{E} = \sum_s \hbar\omega_s\left(n_s + \frac{1}{2}\right), \qquad \mathbf{G}^{\mathrm{A}} = \sum_s \frac{\mathbf{u}_s}{c^2}\,\hbar\omega_s n_s\,. \tag{49}$$

It follows that the energy $\mathcal{E}_{\omega}$ and momentum $\mathbf{p}_{\omega}^{\mathrm{A}}$ of an individual 'photon in a medium' are expressed as

$$\mathcal{E}_{\omega} = \hbar\omega\,, \qquad \mathbf{p}_{\omega}^{\mathrm{A}} = \frac{\mathbf{u}}{c^2}\,\hbar\omega\,. \tag{50}$$

The energy of a quantum is expressed in terms of the frequency in exactly the same way as in a vacuum. The momentum of a quantum, which is derived here from the classical-to-quantum correspondence principle, significantly depends on the medium properties via the group velocity and raises certain questions, which we discuss below after formula (56) and in the subsequent sections.

The vector potential and the field strength also become self-adjoint operators. We write them in the Heisenberg representation, i.e., with the time dependence:

$$\hat{\mathbf{A}}(\mathbf{r}, t) = \sum_s \left[\hat{c}_s\mathbf{A}_s(\mathbf{r}, t) + \hat{c}_s^{\dagger}\mathbf{A}_s^*(\mathbf{r}, t)\right], \tag{51}$$

$$\hat{\mathbf{E}}(\mathbf{r}, t) = \mathrm{i}\sum_s \frac{\omega_s}{c}\left[\hat{c}_s\mathbf{A}_s(\mathbf{r}, t) - \hat{c}_s^{\dagger}\mathbf{A}_s^*(\mathbf{r}, t)\right], \tag{52}$$

$$\hat{\mathbf{B}}(\mathbf{r}, t) = \mathrm{i}\sum_s \mathbf{k} \times \left[\hat{c}_s\mathbf{A}_s(\mathbf{r}, t) - \hat{c}_s^{\dagger}\mathbf{A}_s^*(\mathbf{r}, t)\right]. \tag{53}$$

With all substitutions, the wave functions change normalization, which now includes quantities characterizing the medium:

$$\mathbf{A}_s(\mathbf{r}, t) = \mathbf{e}_s\sqrt{\frac{2\pi\hbar c|u_s|}{\omega_s V}\left(\frac{\mu}{\varepsilon}\right)^{1/2}}\,\exp\left(\mathrm{i}\mathbf{kr} - \mathrm{i}\omega_s t\right). \tag{54}$$

However, the main difference from the vacuum case is in the photon momentum definition. In a vacuum, the photon momentum is

$$\mathbf{p}^{\mathrm{vac}} = \hbar\mathbf{k} = \frac{\hbar\omega}{c}\,\mathbf{n}\,, \tag{55}$$

where $\mathbf{n}$ is a unit vector. This expression follows both from the wave equation for $\mathbf{A}$ and from the classical expression for the momentum density after its quantization.

In a medium, the wave equation leads to formulas (27), which imply the momentum

$$\mathbf{q} = \hbar\mathbf{k} = \frac{\hbar\omega\sqrt{\varepsilon\mu}}{c}\,\mathbf{n}\,. \tag{56}$$

This expression is distinctly different from expression (50) for the photon momentum defined according to the quantum-to-classical correspondence principle for Abraham's electromagnetic field momentum (31) [9], leading to a symmetric energy–momentum tensor for the electromagnetic field (see [2, 11, 16, 31]). Hence, in this case, we are dealing with a violation of the correspondence principle, which is valid in all other cases and leads to the quantum mechanical means of physical quantities ('observables' in Dirac's terminology [32]) being related by the same relations as the corresponding classical quantities (the Ehrenfest theorem [33]).

To clarify this contradiction, we consider the definition of the electromagnetic field momentum proposed by Minkowski [7, 8]. In [17], the representation for the density of Minkowski's momentum of a wavepacket was obtained:

$$\mathbf{g}^{\mathrm{M}} = \frac{\mathbf{n}}{4\pi c}\,E_{\mathbf{k}\sigma}^2\varepsilon(\omega)\sqrt{\varepsilon\mu}\left(1 + \frac{\omega}{2\varepsilon}\frac{\partial\varepsilon}{\partial\omega} + \frac{\omega}{2\mu}\frac{\partial\mu}{\partial\omega}\right), \tag{57}$$

which is written here in terms of the non-time-averaged electromagnetic field strength. Using group velocity (32), we arrive at

$$\mathbf{g}_{\mathbf{k}\sigma}^{\mathrm{M}} = \frac{\mathbf{n}\varepsilon(\omega_k)}{4\pi u}\,E_{\mathbf{k}\sigma}^2\,, \tag{58}$$

where

$$\mathbf{u} = \mathbf{n}\,\frac{c}{\sqrt{\varepsilon\mu}}\left(1 + \frac{\omega}{2\varepsilon}\frac{\partial\varepsilon}{\partial\omega} + \frac{\omega}{2\mu}\frac{\partial\mu}{\partial\omega}\right)^{-1} \tag{59}$$

is the group velocity. We consider the ordinary isotropic dielectric medium in which the vector $\mathbf{u}$ is directed along the wave vector and $u = \mathbf{un} > 0$. Comparing (58) with Abraham's momentum density (33),

$$\mathbf{g}_{\mathbf{k}\sigma}^{\mathrm{A}} = \frac{\mathbf{n}}{4\pi c}\sqrt{\frac{\varepsilon(\omega_k)}{\mu(\omega_k)}}\,E_{\mathbf{k}\sigma}^2\,, \tag{60}$$

also written in terms of the electric field, we find

$$\mathbf{g}^{\mathrm{M}}_{\mathbf{k}\sigma} = \mathbf{g}^{\mathrm{A}}_{\mathbf{k}\sigma}\, \frac{c\sqrt{\varepsilon\mu}}{u}\,. \tag{61}$$

This formula allows us to relate the quantized field excitations in a medium for two classical representations of the momentum density:

$$\mathbf{p}^{\mathrm{M}}_{\omega} = \frac{c}{u}\sqrt{\varepsilon\mu}\,\mathbf{p}^{\mathrm{A}}_{\omega} = \frac{\hbar\omega\sqrt{\varepsilon\mu}}{c}\,\mathbf{n}\,. \tag{62}$$

It is Minkowski's momentum of an individual excitation that enters the exponents in quantum operators of the electromagnetic field. This result was first obtained and used by Ginzburg in his pioneering paper [6] on the quantum theory of Vavilov–Cherenkov radiation. Therefore, representation (62) for the photon momentum in a medium can be justly referred to as the Minkowski–Ginzburg representation.

Here, we encounter a major inconsistency: the quantum-to-classical correspondence principle is violated if Abraham's momentum is used, while it holds for Minkowski's momentum. This fundamental principle is used to construct quantum operators of all quantities that have classical counterparts. Its violation suggests the incorrectness of Abraham's theory for this problem.

To confirm this conclusion, we consider other quantum phenomena, but first discuss the correctness of the use of the classical energy–momentum tensor constructed for wavepackets as a basis for the quantum description of the field.

5. Quantum interpretation of classical wavepackets

In quantizing the electromagnetic field, we used classical expressions for the energy density (30) (Brillouin's formula) and the momentum density (31) (Abraham's formula) and (57) (Minkowski's formula). All of them are related to a narrow wavepacket, i.e., to a collection of harmonics with close frequencies and wave vectors and with well-defined phases. The packet shape significantly depends on phase shifts between harmonics, which can be set arbitrarily. The quantization resulted in field operators (51)–(53) with quite definite phases [see (54)]. For this reason, the legitimate questions arise as to whether the classical packets are consistent with the quantum description and how we can construct a pre-defined wavepacket (with specified phase shifts between harmonics) using quantum operators (51)–(53).

This problem cannot be solved only by using the Fock quantum field states defined by the occupation numbers n_s in each mode. Exactly such an approach is used in quantum electrodynamics [29] in the absence of matter. This approach does not apply in our case due to the 'number of quanta–phase' uncertainty relation: if the number of quanta in some mode is given, the phase of this wave is undefined. This purely quantum property of photons results, in particular, in the vanishing of the mean field strengths in a state with a fixed n_s:

$$\langle n_s|\hat{\mathbf{E}}|n_s\rangle = 0\,, \qquad \langle n_s|\hat{\mathbf{B}}|n_s\rangle = 0\,. \tag{63}$$

Therefore, a quantum analog of classical wavepackets cannot be constructed from the Fock states.

For this, we should use coherent (or Glauber) states (see [30, 34–36]; information for beginners can also be found in [37]). These states, denoted by the symbol $|z_s\rangle$ for a mode s, are defined as eigenstates of the non-self-adjoint photon annihilation operator $\hat{c}_s$:

$$\hat{c}_s|z_s\rangle = z_s|z_s\rangle\,. \tag{64}$$

The eigenvalue z_s of a non-Hermitian operator $\hat{c}_s$ can be any complex number, i.e., the spectrum of eigenvalues is continuous. The eigenfunction $|z\rangle$ can be represented as an infinite series in Fock states $|n\rangle$, and the Fock eigenfunction $|n\rangle$, in turn, can be represented as an integral over $\mathrm{d}^2 z$, i.e., over the argument and phase of the complex number z. The number of photons in a mode $|z\rangle$ is undefined, but the quantum mechanical mean of the photon number is simply expressed through z:

$$\overline{n} = \langle z|\hat{n}|z\rangle = \langle z|\hat{c}^{\dagger}\hat{c}|z\rangle = z\langle z|\hat{c}^{*}|z\rangle = |z|^2\,. \tag{65}$$

We can expand in vectors of coherent states, although they are nonorthogonal for $z' \neq z$:

$$\langle z|z'\rangle = \exp\left(z^{*}z' - \frac{|z|^2}{2} - \frac{|z'|^2}{2}\right),$$
$$\left|\langle z|z'\rangle\right|^2 = \exp\left(-|z-z'|^2\right) \neq 0\,. \tag{66}$$

We use the 'Ehrenfest theorem' [33] relating the observed variables to quantum mechanical means and, as the observable field $\mathbf{E}(\mathbf{r},t)$ of a wavepacket, consider the mean of the electric field operator $\hat{\mathbf{E}}(\mathbf{r},t)$ over some set of coherent states $|Z\rangle = \prod_s |z_s\rangle$:

$$\mathbf{E}(\mathbf{r},t) = \left\langle Z\middle|\hat{\mathbf{E}}(\mathbf{r},t)\middle|Z\right\rangle = \mathrm{i}\sum_s \frac{\omega_s}{c}\left[z_s\mathbf{A}_s(\mathbf{r},t) - z_s^{*}\mathbf{A}_s^{*}(\mathbf{r},t)\right]. \tag{67}$$

This real expression is made of harmonics that acquire known phases due to the complex numbers z_s. By varying them, we can construct any wavepacket. A similar representation can be written for the field vector $\mathbf{B}$.

As a result, we have realized two complementary descriptions of the electromagnetic field in a medium — the classical one using wavepackets based on relations (26)–(31) and the quantum one using coherent states — and traced the direct and inverse transitions between these treatments.

6. Angular momentum of a field in Minkowski's representation

The next significant effect depending on the momentum is the spin (the proper angular momentum) of a photon in a medium. We calculate its value using Minkowski's and Abraham's expressions for the field momentum.

We construct the field momentum using Minkowski's nonsymmetric energy–momentum tensor

$$T^{\mathrm{M}}_{ki} = \begin{pmatrix} w & -cg^{\mathrm{M}}_{\beta} \\ -\dfrac{\gamma_{\alpha}}{c} & -\sigma^{\mathrm{M}}_{\alpha\beta} \end{pmatrix}. \tag{68}$$

The quantities w (energy density) and $\boldsymbol{\gamma}$ (energy density flux vector) are identical in Abraham's and Minkowski's representations. The quantities $\mathbf{g}^{\mathrm{M}}$ and $\sigma^{\mathrm{M}}_{\alpha\beta}$ differ from those defined according to Abraham:

$$\mathbf{g}^{\mathrm{M}} = \frac{\sqrt{\varepsilon\mu}}{cu}\,\boldsymbol{\gamma}\,, \qquad \sigma^{\mathrm{M}}_{\alpha\beta} = -\frac{\sqrt{\varepsilon\mu}}{ck}\,\gamma_\alpha k_\beta = -\frac{\sqrt{\varepsilon\mu}}{ck}\,\gamma_\beta k_\alpha\,. \quad (69)$$

We construct a third-rank 4-tensor representing the angular momentum densities,

$$I^{\mathrm{M}}_{kij} = T^{\mathrm{M}}_{ki} x_j - T^{\mathrm{M}}_{kj} x_i\,, \quad (70)$$

which is antisymmetric in the last two indices, $I^{\mathrm{M}}_{kij} = -I^{\mathrm{M}}_{kji}$. Taking the divergence, we obtain

$$\partial^k I^{\mathrm{M}}_{kij} = (\partial^k T^{\mathrm{M}}_{ki}) x_j + T^{\mathrm{M}}_{ji} - (\partial^k T^{\mathrm{M}}_{kj}) x_i - T^{\mathrm{M}}_{ij}\,. \quad (71)$$

Using Eqns (68) and (69), we calculate the divergence

$$\partial^k T^{\mathrm{M}}_{ki} = \begin{cases} \dfrac{1}{c}\left(\dfrac{\partial w}{\partial t} + \nabla\boldsymbol{\gamma}\right) = 0\,, & i = 0\,, \\ -\dfrac{\partial g^{\mathrm{M}}_i}{\partial t} + \dfrac{\partial \sigma^{\mathrm{M}}_{\alpha i}}{\partial x_\alpha} = 0\,, & i = 1, 2, 3\,. \end{cases} \quad (72)$$

Thus, the original divergence is expressed as the difference between energy–momentum tensors:

$$\partial^k I^{\mathrm{M}}_{kij} = T^{\mathrm{M}}_{ji} - T^{\mathrm{M}}_{ij}\,. \quad (73)$$

Turning to nonsymmetric 4-tensor (68) and taking the symmetry of the three-dimensional tensor (i.e., the stress tensor) into account, we obtain

$$\partial^k I^{\mathrm{M}}_{kij} = 0\,, \quad i, j = 1, 2, 3\,, \quad (74)$$

$$\partial^k I^{\mathrm{M}}_{ki0} = -\partial^k I^{\mathrm{M}}_{k0i} = \frac{1}{c}\,\gamma_i - c g^{\mathrm{M}}_i \neq 0\,, \quad i = 1, 2, 3\,. \quad (75)$$

The vanishing of the four-dimensional divergence in (74) means that the integral over three-dimensional space is time-independent:

$$Q_{\alpha\beta} = \int I^{\mathrm{M}}_{0\beta\alpha}\,\mathrm{d}^3 x = \mathrm{const}\,, \qquad \alpha, \beta = 1, 2, 3\,. \quad (76)$$

This is clear after integrating Eqn (74) over three-dimensional space and using the three-dimensional Gauss–Ostrogradsky theorem. The antisymmetic three-dimensional tensor $Q_{\alpha\beta}$ is equivalent (dual) to the pseudovector $\mathbf{J}^{\mathrm{M}}$ of the field angular momentum defined according to Minkowski:

$$\mathbf{J}^{\mathrm{M}} = \int \mathbf{r} \times \mathbf{g}^{\mathrm{M}}\,\mathrm{d}^3 x = \mathrm{const}\,, \quad (77)$$

which is a conserved quantity. Abraham's three-dimensional angular momentum is also an integral of motion:

$$\mathbf{J}^{\mathrm{A}} = \int \mathbf{r} \times \mathbf{g}^{\mathrm{A}}\,\mathrm{d}^3 x = \mathrm{const}\,, \qquad \mathbf{g}^{\mathrm{A}} = \frac{1}{4\pi c}\,\mathbf{E} \times \mathbf{H}\,. \quad (78)$$

But for Abraham's symmetric tensor T^{A}_{ik}, the mixed space–time components $Q_{0\alpha}$ unrelated to the three-dimensional angular momentum are also conserved. In the Minkowski case, these components are not conserved.

7. Photon spin in a medium

The proper angular momentum — spin — is a feature of each elementary particle. It is well known that a photon, one of the most well-studied and widespread fundamental bosons, also has this important feature. But the spin of a photon cannot be defined in a way similar to that of an electron, i.e., as the angular momentum in the particle rest-frame. There is no such frame for a photon in a vacuum. Therefore, most authors [25, 28–31] consider the total angular momentum $\mathbf{J}$ of the electromagnetic field according to its classical definition and then identify two components: the orbital angular momentum $\mathbf{L}$ and the proper angular momentum $\mathbf{S}$,

$$\mathbf{J} = \mathbf{L} + \mathbf{S}\,. \quad (79)$$

The criterion for such a separation is the dependence of the orbital angular momentum on the coordinate origin, as well as the appearance of the operator $\mathbf{r} \times \nabla$ characteristic of the orbital angular momentum in quantum mechanics. The proper angular momentum does not have any dependence on the reference frame and is determined by the properties of the field itself. We also use this possibility here to define the spin angular momentum of a photon in a medium.

We write the total angular momentum of a wavepacket of a classical electromagnetic field relative to the coordinate origin in terms of the momentum density, which is defined according to expression (31) or (78), i.e., according to Abraham:

$$\mathbf{J}^{\mathrm{A}} = \int \mathbf{r} \times \mathrm{d}\mathbf{G}^{\mathrm{A}} = \sum_{s, s'} \frac{1}{4\pi c\mu(\omega_s)} \int \mathbf{r} \times [\mathbf{E}_{s'} \times \mathbf{B}_s]\,\mathrm{d}^3 r\,. \quad (80)$$

The integral extends to the entire periodicity domain V; at the region boundary, the field is zero. By expressing the field $\mathbf{B}_s$ in terms of the vector potential, we integrate by parts to represent the total angular momentum as the sum of two terms:

$$\mathbf{J}^{\mathrm{A}} = \sum_{s, s'} \frac{1}{4\pi c\mu(\omega_s)} \int E_{s'\alpha} [\mathbf{r} \times \nabla] A_{s\alpha}\,\mathrm{d}^3 r + \sum_{s, s'} \frac{1}{4\pi c\mu(\omega_s)} \int \mathbf{E}_{s'} \times \mathbf{A}_s\,\mathrm{d}^3 r\,. \quad (81)$$

In accordance with the criterion given above, the first term in the right-hand side is interpreted as the orbital angular momentum relative to the chosen coordinate origin, and the second term as the proper angular momentum (spin) of the field in the volume V. The second term is more interesting for our purpose:

$$\mathbf{S}^{\mathrm{A}} = \sum_{s, s'} \frac{1}{4\pi c\mu(\omega_s)} \int \mathbf{E}_{s'} \times \mathbf{A}_s\,\mathrm{d}^3 r\,. \quad (82)$$

We calculate possible values of this quantity in the quantum field description. For this, we symmetrize expression (82) and consider it as a self-adjoint quantum operator by expressing it in terms of field operators (52)–(54):

$$\hat{\mathbf{S}}^{\mathrm{A}} = \sum_{s, s'} \frac{1}{8\pi c\mu(\omega_s)} \int \{\hat{\mathbf{E}}_{s'} \times \hat{\mathbf{A}}_s - \hat{\mathbf{A}}_s \times \hat{\mathbf{E}}_{s'}\}\,\mathrm{d}^3 r\,. \quad (83)$$

When integrating vector products over coordinates, we use the orthogonality of wave functions (54):

$$\int_V \mathbf{A}^*_{s'}(\mathbf{r},t)\times\mathbf{A}_s(\mathbf{r},t)\,\mathrm{d}^3r = \delta_{\mathbf{k}\mathbf{k}'}[\mathbf{e}^*_{\mathbf{k}\sigma'}\times\mathbf{e}_{\mathbf{k}\sigma}]\,\frac{2\pi\hbar c u_k}{\omega_k}\left(\frac{\mu(\omega_k)}{\varepsilon(\omega_k)}\right)^{1/2}. \tag{84}$$

This enables us to write (83) in the form

$$\hat{\mathbf{S}}^{\mathrm{A}} = -\mathrm{i}\hbar\sum_{\mathbf{k}\sigma,\sigma'}\frac{u_k}{c\sqrt{\varepsilon\mu}}[\mathbf{e}^*_{\mathbf{k}\sigma'}\times\mathbf{e}_{\mathbf{k}\sigma}]\left(\hat{c}^\dagger_{\mathbf{k}\sigma'}\hat{c}_{\mathbf{k}\sigma}+\frac{1}{2}\,\delta_{\sigma\sigma'}\right). \tag{85}$$

If the photon polarization vectors are real, $\mathbf{e}_{\mathbf{k}1}=\mathbf{e}^*_{\mathbf{k}1}$ and $\mathbf{e}_{\mathbf{k}2}=\mathbf{e}^*_{\mathbf{k}2}$ (linear polarization), the vector product in (85) is nonzero only for $\sigma'\neq\sigma$. But in this case, the spin vector vanishes when averaging over states with a fixed number of photons, i.e., with fixed $n_{\mathbf{k},1}$ and $n_{\mathbf{k},2}$. The vector product is nonzero for the circular polarization of photons if $\sigma'=\sigma$: $\mathbf{e}_{\mathbf{k},\pm1}=(\mathbf{e}_{\mathbf{k}1}\pm\mathbf{e}_{\mathbf{k}2})/\sqrt{2}$. Because the polarization vectors are orthogonal to $\mathbf{k}$, we obtain

$$\mathbf{e}^*_{\mathbf{k},\pm1}\times\mathbf{e}_{\mathbf{k},\pm1}=\pm\frac{\mathrm{i}\mathbf{k}}{k}\,. \tag{86}$$

After averaging operator (85) over the state with a fixed number of photons, we obtain

$$\mathbf{S}^{\mathrm{A}}=\sum_{\mathbf{k}}\frac{\hbar\mathbf{k}}{k}\,\frac{u_k}{c\sqrt{\varepsilon\mu}}(n_{\mathbf{k},+1}-n_{\mathbf{k},-1})\,. \tag{87}$$

According to this result, one photon with a given circular polarization contributes $\hbar u_k/c\sqrt{\varepsilon\mu}$ to the spin. This result is unsatisfactory because, according to the general properties of the angular momentum in quantum mechanics and its relation to spatial isotropy, its eigenvalues can take only integer or half-integer values $0, 1/2, 1, \ldots$ in units of $\hbar$. In the formula, however, the factor $u_k/c\sqrt{\varepsilon\mu}$ can take continuous values, depending on the medium properties.

This contradiction provides grounds to assert that Abraham's definition of the momentum density, Eqn (31), which we have used to calculate the spin angular momentum of the field, does not enable the correct description of the quantum properties of photons in a medium. However, a self-consistent description of spin can be obtained if Minkowski's momentum density (61) or (77), and, correspondingly, the photon momentum defined according to Ginzburg (62), are used in Eqns (80)–(87). The undetermined extra factor then disappears, and the desired integer value remains:

$$\mathbf{S}^{\mathrm{M}}=\sum_{\mathbf{k}}\frac{\hbar\mathbf{k}}{k}(n_{\mathbf{k},+1}-n_{\mathbf{k},-1})\,. \tag{88}$$

Thus, we arrive at the conclusion that the photon momentum defined according to Minkowski and Ginzburg provides a correct description of the spin of a photon in a transparent medium. Abraham's momentum, in spite of the full symmetry of the energy–momentum tensor, gives rise to contradictions in the quantum description.

8. Quantum theory of Vavilov–Cherenkov radiation

Vavilov–Cherenkov radiation [38, 39] was the first experimentally discovered radiation effect from a fast charged particle uniformly moving in a medium or near its boundary. The theoretical explanation of this effect and necessary conditions for its appearance were provided by Tamm and Frank [40, 41] based on classical electrodynamics.

The classical theory can explain this effect with quite a high accuracy because a low-energy quantum is emitted (for example, in the optical range) by a particle moving with a subluminal velocity. Therefore, in most textbooks on electrodynamics, the radiation intensity is calculated classically (see, e.g., [2, 42, 43]). However, the quantum explanation of this effect was necessary from the fundamental standpoint and for completeness. Such an interpretation was proposed by Ginzburg [6]. The quantum treatment is very useful for understanding the properties of photons in transparent media, which are significantly different from the well-known properties of a photon in a vacuum. Contradictions related to the use of Abraham's photon momentum in a medium have been discussed in Section 4.

In a vacuum, the emission of a photon by a free particle is prohibited by the energy and momentum conservation. But in a transparent medium, the radiation from a free particle becomes possible due to the change in the dispersion equation relating the energy and momentum of a photon in the medium. To calculate the probability of the process, we use the well-known formula of the perturbation theory for the transition probability of a quantum system per unit time from an initial state $|i\rangle$ to a final state $|f\rangle$:

$$\mathrm{d}w^{\mathrm{rad}}=\frac{2\pi}{\hbar}\left|\langle f|\hat{V}|i\rangle\right|^2\delta\big(\mathcal{E}(\mathbf{p})-\mathcal{E}(\mathbf{p}')-\hbar\omega\big)\,\mathrm{d}\nu\,. \tag{89}$$

Here, $\mathrm{d}\nu$ is the number of quantum states of the emitting particle and of the quantum in the continuum, and

$$\hat{V}=\frac{1}{c}\,\hat{\mathbf{j}}^{\,k}(\mathbf{r})\hat{A}_k(\mathbf{r}) \tag{90}$$

is the operator coupling the relativistic electron to a quantized electromagnetic field, derived from classical Lagrangian (18).

We choose the Coulomb gauge, in which the scalar potential $\varphi=0$ and $\nabla\mathbf{A}(\mathbf{r})=0$. Then, instead of the four-dimensional current $\hat{j}^k$ $(k=0,1,2,3)$, we can use the Dirac three-dimensional current:

$$\hat{\mathbf{j}}(\mathbf{r})=ec\psi^+_{\mathbf{p}'\mu'}(\mathbf{r})\hat{\boldsymbol{\alpha}}\psi_{\mathbf{p}\mu}(\mathbf{r})\,, \tag{91}$$

where $\hat{\boldsymbol{\alpha}}$ is the vector of Dirac matrices and $\psi(\mathbf{r})$ are bispinors describing the initial $(\mathbf{p}\mu)$ and final $(\mathbf{p}'\mu')$ states of the emitting electron. Explicitly, the bispinors are given by

$$\begin{gathered}\psi_{\mathbf{p}\mu}(\mathbf{r})=\frac{u_{\mathbf{p}\mu}}{\sqrt{V}}\exp\left(\frac{\mathrm{i}}{\hbar}\,\mathbf{pr}\right),\\ u_{\mathbf{p}\mu}=N\begin{pmatrix} w_\mu\\ \dfrac{c(\mathbf{p}\hat{\boldsymbol{\sigma}})w_\mu}{\mathcal{E}+mc^2}\end{pmatrix},\qquad N=\left(\frac{\mathcal{E}+mc^2}{2\mathcal{E}}\right)^{1/2},\end{gathered} \tag{92}$$

where w_μ is the two-component spinor describing the spin state in the electron rest frame and normalized to unity. As with photons, the bispinor field is subject to periodic boundary conditions, which yields the normalization factor $V^{-1/2}$ in (92) containing the periodicity volume. We assume the electron moving in the medium to be free, as was assumed in the pioneering paper by Tamm and Frank [40].

The matrix element $\langle f|\hat{V}|i\rangle$ in (89) includes integration over the volume and the normalization of three exponentials containing momenta $\mathbf{p}$ and $\mathbf{p}'$ of the emitting particle and the momentum $\mathbf{q}$, Eqn (56), of a photon in the medium. The periodicity condition for the bispinors and the electromagnetic field leads to the momentum conservation law:

$$\int_V \exp\left[\frac{\mathrm{i}}{\hbar}(\mathbf{p}-\mathbf{p}'-\mathbf{q})\mathbf{r}\right]\mathrm{d}^3r = \begin{cases} V, & \mathbf{p}'=\mathbf{p}-\mathbf{q}, \\ 0, & \mathbf{p}'\neq\mathbf{p}-\mathbf{q}. \end{cases} \quad (93)$$

Energy and momentum conservation laws (89) and (93) allow us to find the angle θ of the radiated quantum relative to the initial particle velocity $\mathbf{v}$:

$$\cos\theta = \frac{1}{\beta n(\omega)}\left[1+\frac{\pi\Lambda}{n(\omega)\lambda}\left(n^2(\omega)-1\right)\sqrt{1-\beta^2}\right],$$

$$\Lambda = \frac{\hbar}{m_e c}, \quad n(\omega)=\sqrt{\varepsilon(\omega)\mu(\omega)}, \quad \beta=\frac{v}{c}.$$

The quantum correction proportional to Λ/λ is of the order of 10^{-6} for the electron, and we ignore such corrections in what follows.

Using formulas (51) and (54) for the quantized field, as well as quantities (92) and (93), we can write the required matrix element in the form

$$\langle f|\hat{V}|i\rangle = e\sqrt{\frac{2\pi\hbar c|u_s|}{\omega V}}\left(\frac{\mu}{\varepsilon}\right)^{1/2}\left(u^+_{\mathbf{p}-\mathbf{q}\,\mu'}(\hat{\boldsymbol{\alpha}}\mathbf{e}^*_s)u_{\mathbf{p}\mu}\right). \quad (94)$$

In this and subsequent expressions, $\mathbf{q}$ can be treated as the change in the momentum of the emitting electron, $\mathbf{q}=\mathbf{p}-\mathbf{p}'$, and as the momentum of the emitted photon.

The number of quantum states $\mathrm{d}\nu$ in probability (89), after using conservation law (92) and eliminating the momentum $\mathbf{p}'$, relates only to the photon, and with its dispersion law (55) can be written in terms of the frequency in the form

$$\mathrm{d}\nu = \frac{V\omega^2\,\mathrm{d}\omega\,\mathrm{d}\Omega_q}{(2\pi)^3 v_{\mathrm{ph}}^2|u_q|}, \quad (95)$$

where $v_{\mathrm{ph}} = c/\sqrt{\varepsilon\mu}$ is the phase velocity and $u_q = \mathrm{d}\hbar\omega/\mathrm{d}q$ is the group velocity. When calculating the emitted energy, probability (89) should be averaged over the initial spin states μ of the electron and summed over its final states μ' as well as over polarizations s of the emitted transverse photon. These operations can be carried out using projection operators and are well known [25–29].

Omitting the technical details of the calculation, we present the final result:

$$\frac{1}{2}\sum_{\mu,\mu',s}\left|\left(u^+_{\mathbf{p}-\mathbf{q}\,\mu'}(\hat{\boldsymbol{\alpha}}\mathbf{e}^*_s)u_{\mathbf{p}\mu}\right)\right|^2 = \beta^2\left(1-\frac{1}{\beta^2 n^2(\omega)}\right), \quad \beta=\frac{v}{c}. \quad (96)$$

The energy $\mathrm{d}I(\omega)$ of particle emission per unit frequency interval $\mathrm{d}\omega$ can be obtained by multiplying the energy of one quantum $\hbar\omega$ by the emission probability $\mathrm{d}w^{\mathrm{rad}}$ integrated over possible photon angles $\mathrm{d}\Omega_q$ using the δ-function. Its argument is represented in the form

$$\mathcal{E}(\mathbf{p})-\mathcal{E}(\mathbf{p}-\mathbf{q})-\hbar\omega = \mathbf{vq}-\hbar\omega = vq\cos\theta-\hbar\omega \quad (97)$$

and, after integration, yields the factor

$$\int\delta(vq\cos\theta-\hbar\omega)\,\mathrm{d}\Omega_q = \frac{2\pi}{\hbar\omega\beta n(\omega)}. \quad (98)$$

Collecting all necessary factors in (89) and subsequent equations, we obtain

$$\mathrm{d}I(\omega) = \frac{e^2 v\mu(\omega)}{c^2}\left(1-\frac{1}{\beta^2 n^2(\omega)}\right)\omega\,\mathrm{d}\omega. \quad (99)$$

The total radiation per unit time is found by integrating (99) over the frequency range where the energy and momentum are simultaneously conserved, i.e., $|\cos\theta| = 1/\beta n(\omega) < 1$:

$$I = \frac{e^2\beta}{c}\int_{\beta n>1}\left(1-\frac{1}{\beta^2 n^2(\omega)}\right)\mu(\omega)\omega\,\mathrm{d}\omega. \quad (100)$$

For $\mu(\omega)=1$, the obtained result coincides with that of Tamm and Frank [23], in which the authors assumed $\mu=1$ and $n(\omega)=\sqrt{\varepsilon(\omega)}$.

The presented calculation using Dirac's theory was carried out for a charged particle with spin 1/2 (fermion). In this case, the radiation is produced, generally speaking, not only by the charge of the moving particle but also by its spin magnetic moment. However, the contribution of the magnetic moment to the radiation is very small: according to Ginzburg's estimate [6], the relative contribution of this effect is less than 10^{-5}. Therefore, expression (100) can in fact be applicable to any charged particle regardless of its spin moment.

The well-known quantum formula (89), which also includes the energy conservation law and which has been used to derive the correct Vavilov–Cherenkov radiation intensity, leaves no doubt that the full energy $\hbar\omega = \mathcal{E}(p)-\mathcal{E}(p')$ and momentum of the Cherenkov photon are taken from the moving particle but not from the medium. The quantum state of the medium does not change.

Processes in which other excitations of matter besides the Cherenkov quantum participate have been considered by various authors (see [44–46]). However, all such processes are described using higher orders of the perturbation theory and have a much lower intensity than the Vavilov–Cherenkov radiation.

The presence of matter not only gives rise to macroscopic radiation mechanisms that are absent in a vacuum (Vavilov–Cherenkov radiation and transitional radiation [47, 48]) but also affects other radiation processes, for example, the spontaneous radiation of excited atoms in a medium. It is easy to verify that the probability of the spontaneous electric dipole emission of an atom in a medium compared to the probability of this emission in a vacuum acquires the factor $\sqrt{\varepsilon(\omega)\mu^3(\omega)}$. This results from the change in the number of quantum states of a photon in the continuum (95) and the change in the normalization constant.

9. Photon mass in a medium

It is well known that the mass of a photon in a vacuum is zero: $m_{\mathrm{ph}}=0$. In special relativity, the mass of an elementary particle can be correctly defined only as a relativistically invariant quantity (the widespread but not quite appropriate term for this quantity is the 'rest mass'). There is no correct definition of a 'velocity-dependent mass'. This important property of the mass notion was relatively recently — in the 21st century — explained in several papers and notes by Okun (see [49, 50] and especially [51]).

The mass of a photon in a medium can be most straightforwardly found from the Klein–Gordon–Fock equation for a free vector particle with mass m (see, e.g., [52]):

$$\left[\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2c^2}{\hbar^2}\right]\psi_k = 0\,, \qquad k = 0, 1, 2, 3\,. \tag{101}$$

As is well known, this equation implements the relation between the energy and momentum of a particle on the quantum level, irrespective of the internal degrees of freedom of the particle. Therefore, it is applicable to both bosons and fermions.

For photons in a medium, the vector potential satisfying Eqn (24) acts as an analog of the wave function:

$$\Delta\mathbf{A}_\omega - \frac{\varepsilon\mu}{c^2}\frac{\partial^2\mathbf{A}_\omega}{\partial t^2} = 0\,. \tag{102}$$

A comparison of the last two equations shows that the mass term is absent in the equation for photons and that the mass of a photon in a medium, as in a vacuum, is zero: $m_{\rm ph} = 0$. Because both equations have covariant forms, this result is valid in any inertial reference frame, i.e., the zero photon mass is invariant.

This result, quite obvious from general considerations, is presented here because papers [53, 54] appeared recently, treating the quantum theory of Vavilov–Cherenkov radiation incorrectly. There, the 'mass of a photon in a medium' $m_{\rm ph}$ defined by the author similarly to the mass of an ordinary relativistic particle is considered:

$$m_{\rm ph}^2c^2 = \frac{\mathcal{E}_{\rm ph}^2}{c^2} - p^2\,, \tag{103}$$

with Abraham's momentum p. This leads to the mass in the form

$$m_{\rm ph}^2c^2 = \left(\frac{\hbar\omega}{c}\right)^2\left(1 - \frac{1}{\varepsilon\mu}\right). \tag{104}$$

This mass is not invariant and can take imaginary values for $\varepsilon\mu < 1$. In addition, the author believes that the Cherenkov photon is emitted by the medium and not by the fast particle and asserts that "during the emission of a photon with energy $\mathcal{E}_{\rm ph}$ the energy of the medium should decrease by the amount $m_{\rm ph}c^2$, where $m_{\rm ph}$ is the mass of the photon in the medium with $\sqrt{\varepsilon\mu} \neq 1$" (see [53], p. 20). Both statements about the final and noninvariant photon mass and about the emission of the photon by the medium are incorrect.

By criticizing the studies by Tamm and Frank [40, 41], as well as by Ginzburg [11, 55], the author of [53] restricts himself to considering only kinematic relations — the energy and three-momentum conservation laws involving the 'photon mass'. The pretentious section "A novel quantum theory of Vavilov–Cherenkov radiation" does not contain calculations of probabilities or intensity of the radiation produced by a fast particle. The initial assumptions of this criticism are erroneous and the criticism is wrong.

10. Conclusions

The analysis of the macroscopic energy–momentum tensor presented in this paper continues the history of more than a century (see recent papers [56–60]) of discussions of the fundamental physical quantities involved in this notion. Our consideration is applicable not to the most general case but to a sufficiently high-frequency electromagnetic field in the form of quasimonochromatic transverse waves (wavepackets) in the absence of external charges, currents, and energy dissipation in a chosen space region. The second feature of our approach is the use of quantum theory, which, as the author believes, allows a deeper insight into physical phenomena than does a classical approach.

We quote the opinion of the outstanding physicist, V L Ginzburg, on the issues considered here [15]; he frequently returned to this field in his research: "Everything asserted allows us to consider the Abraham tensor 'correct', but in our opinion it is possible to claim the Minkowski tensor 'incorrect' only in a somewhat formal approach to the problem. In fact, in most situations, the results obtained by using the Abraham and Minkowski tensors are fully identical. This gives us the opportunity in the corresponding cases to use the Minkowski tensor and makes this quite expedient as long as some simplifications can be achieved." And later, when discussing Abraham's force (2): "... the account for the action of this force in the classical approach is very simple (see above), but quantum mechanically it would turn out to be rather cumbersome. In any case, as far as we know, such a quantum treatment has not yet been given."

In this paper, we have taken the Abraham force into account at the quantum level because it is included (see [17]) in Abraham's stress tensor $\sigma^{\rm A}_{\alpha\beta}$, and we have found quantum mechanical eigenvalues for all components of the energy–momentum tensor. However, this has been done not in the general case, which could encompass static, quasistatic, and rapidly varying fields, but for a transverse high-frequency field that only requires a quantum treatment. Static and quasistatic fields are fully described by classical methods [61]. No contradictions arise here. However, the transverse electromagnetic field is significantly different from the field generated by immobile or slowly moving charged particles. Therefore, there are no a priori grounds to believe that formulas describing the energy and momentum density of the field preserve the form in all unbounded frequency and wavenumber ranges.

As noted in the preceding sections, quantum relations appropriate for high-frequency transverse fields in a medium make the use of characteristics of 'photons in the medium' defined by Minkowski's tensor and by the related Ginzburg formula (62) for the photon momentum not only efficient but also necessary. The use of Abraham's momentum density does not agree with experimental data.

We note that in the paper by Polevoi and Rytov [62], the energy–momentum tensor of the electromagnetic field is written in terms of the group velocity exactly in the nonsymmetric Minkowski form (see Eqns (53) in [62]). As the quantum approach described above shows, just this form turns out to be correct for the high-frequency field. Therefore, our criticism in paper [17] of the expression $g_\alpha = wk_\alpha/\omega$ for Polevoi's and Rytov's momentum density is untenable and does not correspond to the results obtained here, which the author ought to state here with regret.

Thus, the above calculations lead to the following conclusions.

(1) The correct expression for the momentum density of transverse electromagnetic waves in a transparent medium is given by Minkowski's formula (2), and the correct expression for the individual photon momentum is given by Minkowski–Ginzburg's formula (62) first used by Ginzburg [6] in 1940.

(2) This conclusion is justified by the classical-to-quantum correspondence for the momentum density of transverse

waves in a medium, by the agreement with experimental values of the photon spin in a medium, and by the correct result for the probability of photon emission by a fast charged particle (the Vavilov–Cherenkov effect). The use of Abraham's expression for these purposes leads to contradictions and is inconsistent with experimental data.

(3) The quantum description of the transverse field in a medium includes its classical description by macroscopic electrodynamics equations as a particular case; therefore, the result derived from quantum theory holds in classical macroscopic electrodynamics for a high-frequency transverse electromagnetic field. This conclusion is inapplicable to static and quasistatic fields.

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