METHODOLOGICAL NOTES

PACS numbers: 03.65.Ca, 41.20.Jb, **42.79.–e**

Antilaser: resonance absorption mode or coherent perfect absorption?

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DOI: https://doi.org/10.3367/UFNe.2016.11.037963

Abstract. The main idea of the recent studies of 'coherent perfect absorption' is shown to coincide with that published in 1962 by A P Khapalyuk. Generalizing this approach to the case of layered systems with alternating absorbing and amplifying media requires a more thorough analysis of possible instabilities.

Keywords: resonance absorption mode, coherent perfect absorption, lasing threshold, absolute instability, convective instability

In 1962, Belarusian physicist Aleksandr Petrovich Khapalyuk (1925–2010) predicted the so-called 'resonance absorption mode' [1], in which all incident electromagnetic radiation is absorbed in an object, such that the object surroundings contain no radiation at all, whether reflected or scattered from it. This is to some extent opposite to the operation of the laser, which emits radiation into but is subject to no coherent radiation from the external space. The analysis in Ref. [1] was carried out for a layer of a uniform absorbing medium, with each of its opposite sides subjected to a wave of coherent monochromatic radiation. Khapalyuk's further work [2] revealed that complete resonance absorption is also possible if there is no sharp interface between the media involved, i.e., in an absorbing medium with smoothly varying complex-valued refractive index.

To understand the realization conditions for Khapalyuk's mode, we simplify the setup in Ref. [1] to a layer of an absorbing nonmagnetic medium (magnetic permittivity $\mu = 1$) in a vacuum (Fig. 1). The layer has a thickness $2L$ and a complex dielectric constant $\varepsilon = \varepsilon' + i\varepsilon''$ (with ε' and ε'' being the real and imaginary parts), and its left and right sides are subjected to plane monochromatic waves of a frequency ω incident normally (along the z axis).

The problem is analyzed in a standard way. For normal propagation, the complex electric field strength satisfies the scalar Helmholtz equation

$$\frac{d^2E}{dz^2} + \frac{\omega^2}{c^2}\,\varepsilon(z)E = 0\,, \tag{1}$$

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Received 9 May 2016, revised 4 September 2016
Uspekhi Fizicheskikh Nauk **187** (8) 879 – 882 (2017)
DOI: https://doi.org/10.3367/UFNr.2016.11.037963
Translated by E G Strel'chenko; edited by A M Semikhatov

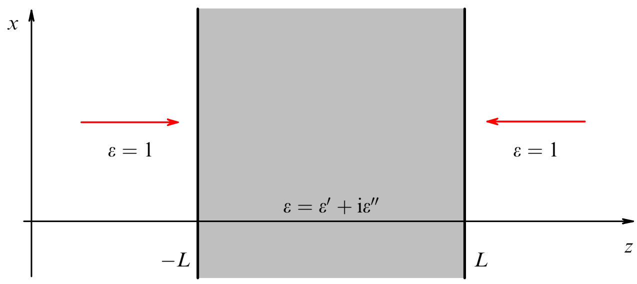

Figure 1. Schematic of total resonance absorption. Radiation waves from a vacuum are incident on the left and right sides of the layer $-L < z < L$ and are totally absorbed in it (reflected waves or waves transmitted through the absorbing layer are absent).

where we have omitted the factor $\exp(-i\omega t)$, with t the time. In the situation given,

$$\varepsilon(z) = \begin{cases} 1\,, & z < -L\,, \\ \varepsilon' + i\varepsilon''\,, & -L < z < L\,, \\ 1\,, & z > L\,, \end{cases} \tag{2}$$

yielding the solution of Eqn (1) in the form

$$\begin{aligned} &E = A\exp\left[ik_0(z+L)\right], \quad z < -L\,, \\ &E = B\exp(ikz) + C\exp(-ikz)\,, \quad -L < z < L\,, \\ &E = D\exp\left[-ik_0(z-L)\right], \quad z > L\,. \end{aligned} \tag{3}$$

Here, A and D are the amplitudes of the respective waves incident from free space on the layer boundaries $z = -L$ and $z = L$, B and C are the amplitudes of opposite waves inside the layer, $k_0 = \omega/c$ and $k = k_0 n$ are the wave numbers in the free space and in the layer, c is the speed of light in a vacuum, and $n = (\varepsilon' + i\varepsilon'')^{1/2}$ is the complex refractive index of the layer medium. The choice of the square root branch does not matter much because changing the sign of n is equivalent to redefining $B \leftrightarrow C$; in what follows, we primarily focus on the weak absorption case ($\varepsilon'' > 0$) and the amplification case ($\varepsilon'' < 0$) where $\varepsilon' > 1$, $|\varepsilon''| \ll \varepsilon'$, and $n \approx \varepsilon'^{1/2}[1 + i\varepsilon''/(2\varepsilon')]$.

From the continuity of E and dE/dz at the layer boundaries, we conclude that

$$\begin{aligned} &A = B\exp(-ikL) + C\exp(ikL)\,, \\ &A = n\left[B\exp(-ikL) - C\exp(ikL)\right], \\ &B\exp(ikL) + C\exp(-ikL) = D\,, \\ &n\left[B\exp(ikL) - C\exp(-ikL)\right] = -D\,. \end{aligned} \tag{4}$$

The necessary condition for a nontrivial solution of this homogeneous system of linear algebraic equations for the amplitudes A, B, C, and D is the vanishing of the system determinant. From this requirement, we obtain

$$\exp(4\mathrm{i}kL) = \left(\frac{n-1}{n+1}\right)^2, \tag{5}$$

or

$$\exp\left(2\mathrm{i}\,\frac{\omega}{c}\,n'L\right)\exp\left(-2\,\frac{\omega}{c}\,n''L\right) = \pm\frac{n-1}{n+1}, \tag{6}$$

giving

$$2\,\frac{\omega}{c}\,n'L = \pi s + \arg\frac{n-1}{n+1}, \qquad 2\,\frac{\omega}{c}\,n''L = \ln\left|\frac{n+1}{n-1}\right|, \tag{7}$$

where s is a nonnegative integer. We note that the fraction $(n-1)/(n+1)$ is the Fresnel amplitude reflection coefficient at the layer boundary r, allowing Eqn (5) to be rewritten in a form suitable for comparison with the laser version of the problem (see below), namely, $\exp(4\mathrm{i}kL) = r^2$.

Relations (5)–(7) can be interpreted as the condition for the interferential suppression of both the waves reflected directly from the layer boundaries and those transmitted through the layer. It follows from Eqn (7) that the total resonance absorption mode is only possible for $n'' > 0$ (an absorbing medium) and for a certain relation between the indices of refraction and reflection in the medium,

$$n'\ln\left|\frac{n+1}{n-1}\right| = n''\left(\pi s + \arg\frac{n-1}{n+1}\right). \tag{8}$$

Figure 2 illustrates this relation for various values of s. A deviation from Eqn (8) gives rise to waves that move outward from the object. For a fixed complex refractive index n satisfying relation (8), Eqn (7) determines the product $(\omega/c)L$, and hence different values of the radiation frequency correspond to different layer thicknesses. Using Eqn (4), we find that the amplitudes of the waves incident from free space on the layer should be equal up to a sign, $D = \pm A$. Thus, for the mode under study to be realized, the incident wave intensities should be equal, their phases should be matched, and the incident frequency should be determined by the layer thickness. Equation (8) and other relations change their form in the case of oblique incidence.

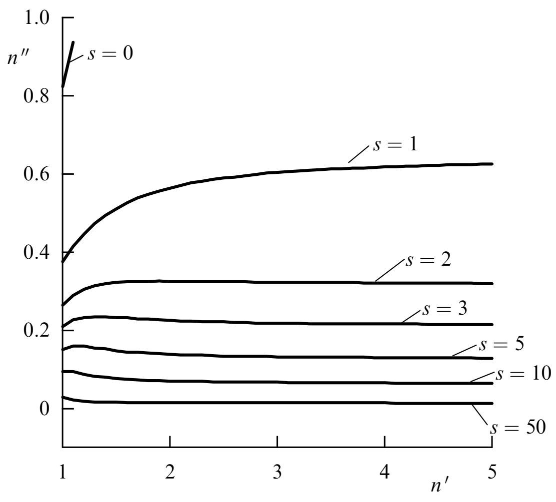

Figure 2. Relation between the real and imaginary parts of the refractive index in the regime of the total absorption of radiation.

In [2], the effect was also predicted for three-dimensional objects and possible applications for inside-resonance spectroscopy, wave heating, plasma diagnostics, and the precision optical or radiophysical diagnostics of matter were suggested. Also, importantly, a correspondence between the resonant absorption mode and the laser generation threshold in the presence of amplification in the layer was established in [2]. Formally, this correspondence follows from the invariance of Maxwell's equations for monochromatic waves under the replacement of amplification with absorption ($\varepsilon \to \varepsilon^*$) and a simultaneous replacement of waves incident on the layer with waves moving away from it. Indeed, we recall that the 'laser' solution of Eqn (1) has form (3) for the replacement $k_0 \to -k_0$. Accordingly, the generation threshold occurs at $r^2\exp(4\mathrm{i}kL) = 1$, and hence, instead of Eqns (5)–(8), we obtain ($n'' < 0$)

$$\exp(4\mathrm{i}kL) = \left(\frac{n+1}{n-1}\right)^2, \qquad \exp\left(2\mathrm{i}\,\frac{\omega}{c}\,n'L\right)\exp\left(-2\,\frac{\omega}{c}\,n''L\right) = \pm\frac{n+1}{n-1}, \tag{9}$$

$$2\,\frac{\omega}{c}\,n'L = \pi s + \arg\frac{n+1}{n-1}, \qquad -2\,\frac{\omega}{c}\,n''L = \ln\left|\frac{n+1}{n-1}\right|, \tag{10}$$

$$n'\ln\left|\frac{n+1}{n-1}\right| = -n''\left(\pi s + \arg\frac{n+1}{n-1}\right). \tag{11}$$

Thereby, the threshold generation conditions could suggest the same conclusions as for the resonance absorption mode, namely that for a fixed value of n', the threshold is reached for a discrete set of $n'' < 0$ such that the labeling numbers of the longitudinal modes can be put into correspondence with integer values of s. For low gains, Eqn (11) is satisfied only for large values of s, which, by Eqn (10), correspond to large laser lengths $2L$. If the relation between the refractive index and the gain deviates from that in Eqn (11), the radiation frequency turns out to be complex-valued, its imaginary part changing sign and vanishing when the threshold condition (the absolute instability of the no-generation mode, $E = 0$) is satisfied. However, the case of an amplifying medium greatly (in theory, infinitely) extended transversely requires a more thorough analysis. Considering the extensions that have been developed, sometimes inconsistently, to systems of layers (including amplifying ones), it is necessary to pause over this point.

We first recall that the concept of a generation threshold in laser physics refers to the fact that below the threshold, the no-generation mode is stable (with both absolute and convective instabilities considered possible). We note that in models with transversely infinite layers of an amplifying medium, the waves traveling in transverse directions (orthogonal to the z axis) are especially 'dangerous'. It is easily seen that in the 'laser version' of the problem considered above, when the layer refractive index (or more precisely, its real part) exceeds that of the environment ($\varepsilon' > 1$), the generation threshold corresponds to zero gain in the layer, $\varepsilon''_{\mathrm{thr}} = 0$. Indeed, for oblique wave propagation with the field strength varying with the transverse coordinate x as $\exp(\pm\mathrm{i}\kappa x)$, Helmholtz equation (1) is replaced with

$$\frac{\mathrm{d}^2E}{\mathrm{d}z^2} + \left[\frac{\omega^2}{c^2}\,\varepsilon'(z) + \mathrm{i}\,\frac{\omega^2}{c^2}\,\varepsilon''(z)\right]E = \kappa^2E. \tag{12}$$

(the case of s-polarization). In what follows, the term containing the imaginary part of the dielectric constant $\varepsilon''(z)$ is treated as a small perturbation (small gain and/or small absorption, with their values depending in an arbitrary way on the longitudinal coordinate z). Then, in the zeroth order (which we indicate by the subscript 0), Eqn (12) is identical to the Schrödinger equation for a particle in a one-dimensional potential well or to the equation for planar dielectric waveguide modes (with κ_0^2 playing the role of an eigenvalue). For $\varepsilon' > 1$, there is always at least one solution with a real eigenvalue κ_0^2 and an eigenfunction $E_0(z)$. In quantum mechanics, this corresponds to the presence of a discrete-spectrum wave function for an arbitrarily shallow, symmetric, rectangular quantum well [3]. In this case, the first-order perturbations theory correction to the eigenvalue turns out to be imaginary [4]:

$$\delta\kappa^2 = \frac{\mathrm{i}(\omega^2/c^2)\int \varepsilon''(z)E_0^2(z)\,\mathrm{d}z}{\int E_0^2(z)\,\mathrm{d}z}\,. \tag{13}$$

Therefore, from the laser model corresponding to Eqn (3) with $\varepsilon' > 1$ and $\varepsilon''(z) < 0$, we obtain for $\kappa' = \mathrm{Re}\,\kappa > 0$ that $\kappa'' = \mathrm{Im}\,\kappa < 0$. Then, for $x \to \infty$, the perturbation increases infinitely, giving rise to a convective instability. The frequency ω is here real and arbitrary (within the laser amplification line).

Convective instability can be understood physically as an infinite enhancement of noise (within the adopted model), including spontaneous radiation that unavoidably accompanies laser amplification (amplified spontaneous radiation). This phenomenon causes a decrease in gain and thus makes laser generation unachievable. More realistic model systems should have finite transverse dimensions and permit wave reflection from their boundaries, thereby allowing a convective-to-global instability transition [5]. The condition for exceeding the threshold for this global instability has the form $(\omega/c)|n''|D > G$, where D is a characteristic transverse size of the system and G is weakly (logarithmically) dependent on the characteristic reflection index value on the distant lateral boundaries of the system. For a typical laser setup, $G \sim 1$, although this value can be increased by taking measures to reduce reflection from the lateral surfaces, in particular, by applying the immersion method (i.e., bringing these surfaces into contact with a medium with a similar refractive index). If the laser setup is angularly selective (for example, if it consists of a number of single-mode waveguides), the gain in the transverse directions is limited, and the obliquely propagating waves experience a greater loss than those propagating along the axis. A one-dimensional setup model with an amplifying medium is justified if the mode structure is taken into account correctly (we note that we should no longer use plane waves but should employ waveguide modes instead in order to describe radiation in this case) [6]. The convective instability condition does not reduce to requiring that singularities be present in elements of the characteristic matrix or the transmission matrix [7] in its standard form, nor to requiring that the imaginary part of the radiation frequency change its sign [8], because its frequency remains real, and it is the imaginary part of the transverse wave number which changes sign.

Such a simple and, as we have seen, insufficient model as Eqn (3) could be considered legitimate only during the infancy period of laser physics. Still, surprising though it may seem, it can even now be found in the literature, albeit ornamented with high-brow mathematics. We note, for example, that the modes considered in Ref. [9] are unstable under the conditions assumed there, because the convective instability threshold already corresponds, as we have just seen, to zero gain in the layer medium.

After the pioneering publications by Khapalyuk, a long break in the study of the resonance absorption mode followed until the 2010 rediscovery of the effect in Ref. [10], now under a different name — coherent perfect absorption, CPA — and without any reference to previous work. We note that the acronym CPA for this effect was an unsuccessful proposal because in the laser literature CPA traditionally stands for chirped pulse amplification. Paper [10] was 'selected for a viewpoint in physics' by the editors of the journal and was followed by a series of prestigious publications [11–16]. Indeed, even a recent review [8] by Russian authors makes no reference to the pioneering studies [1, 2] when discussing the subject.

At a new stage, the idea of resonance absorption is being intensely investigated (in parallel with laser generation) for layered media with some of the layers exhibiting amplification. Of special note are (1) a version of so-called CPA lasers [11, 17], with which it is possible to simultaneously achieve total resonance absorption and reach the laser generation threshold, and (2) the idea of using layers with laser amplification to compensate losses in photon crystals and metallodielectric metamaterials. Without pausing to discuss research in the spirit of PT symmetry (optical systems with the property $\varepsilon(-\mathbf{r}) = \varepsilon^*(\mathbf{r})$; see review [8]), we note that the presence of metal leads to unwelcome loss for optical radiation. This can in principle be overcome by alternating metal layers and dielectric laser-amplifying layers as proposed in Refs [18, 19]. The PT symmetry property is not mandatory for such systems, but it is necessary to ensure that the significant decrease in loss for the radiation propagating at a small angle to the system axis not be accompanied by parasitic generation or enhanced spontaneous radiation unrelated to the incident signals. Such studies are numerous and are partly cited in literature reviews [20, 21]. The point to note here is that the possibility of the development of convective instability is no less important for layered systems containing layers whose amplification region is unbounded in the transverse direction. As an example, we take a system of two amplifying layers with respective dielectric constants $\varepsilon_1 = 1.1 - 0.3\mathrm{i}\beta$ and $\varepsilon_2 = 1.1 - 0.3\mathrm{i}\alpha$, which was considered in Ref. [8] for $0 \leqslant \alpha \leqslant 1$ and $\beta = 1$. From Eqn (13), here again the convective instability threshold corresponds to a medium of zero gain ($\alpha = \beta = 0$), and hence for $\beta > 0$ and $\alpha \geqslant 0$, the system is necessarily above the convective instability threshold, and the modes described in Ref. [8] are unobservable in this case. A correct analysis of layered systems with amplification requires numerical simulations to be performed for specific system parameters of the type presented in Refs [4, 22]. Thus, Fig. 3 presents the gain threshold values for the convective (transverse) and absolute (longitudinal) instability for a system of p periods of alternative passive (absorbing) and active (amplifying) layers. It is seen that for $p \leqslant 10$, the convective instability arises for a lower gain coefficient than the absolute instability does, whereas for $p > 10$ the situation reverses.

The effects of enhanced spontaneous radiation can be ignored and the use of one-dimensional geometry with purely longitudinal radiation propagation can be justified if the restrictions indicated above apply. We also note that our

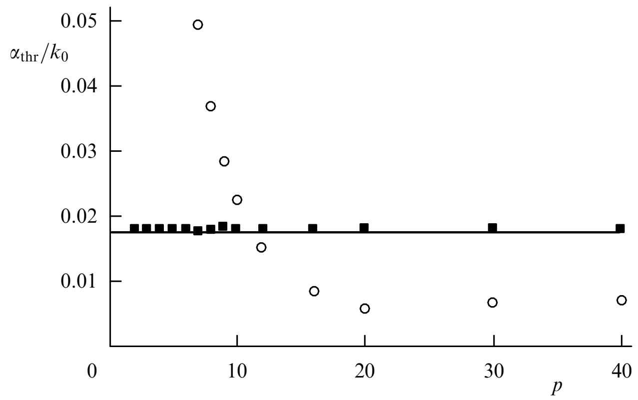

Figure 3. Threshold values of the amplitude gain coefficient $\alpha_{thr} = -k_0 n''$ in dielectric layers for the convective (s-polarization, black squares) and absolute (circles) instability as a function of the number of periods p in a metal–dielectric system under the conditions in [4]. The horizontal straight line indicates the p-dependent value of the gain coefficient corresponding to the compensation condition for a fixed value.

discussion has not touched on nonlinear optical phenomena unrelated to the generation threshold concept itself, nor on physical phenomena such as the marked influence of a metallic surface on the gain coefficient in a nearby region (the Purcell effect) [23].

Thus, the basic ideas behind recent work on 'coherent perfect absorption' are identical to those that started to be published in 1962, which can probably be explained by generational change and insufficient continuity in science. The discussion of the laser generation threshold undertaken in this paper serves as a warning against an excessive formalization of this concept and highlights the necessity of considering the specific features of laser amplifying systems — two points whose importance was already noted at the dawn of laser physics research.

The author thanks S V Fedorov for permission to use Fig. 3. This study was supported in part by the Government of the Russian Federation (grant 074-U01) and the program "Fundamental Problems of Nonlinear Dynamics in Mathematics and Physics" of the Presidium of the Russian Academy of Sciences.

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