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Unconventional magnetism of the Kondo lattice

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Contents

Abstract. A review is given of the experimental situation and theoretical predictions concerning unconventional magnetic ordering in dense Kondo 4f- and 5f-systems, including recently investigated on cerium-, ytterbium-, and actinide-based ternary systems. This unconventional ordering is characterized by a small magnetic moment and exhibits features typical of both localized-spin and itinerant-electron magnets. Particular attention is paid to ferromagnetic systems and the competition among different types of magnetic ordering. The perturbation theory method, renormalization group approach, and auxiliary pseudofermion representation are considered as tools for describing the formation of the Kondo lattice ground state. The problem of non-Fermi-liquid behavior is also discussed.

Keywords: magnetism, Kondo lattices, heavy-fermion systems

1. Introduction

Systems with heavy fermions (giant values of electron heat capacity) and other 4f- and 5f-compounds with unconventional properties remain the most interesting subjects of studies in the many-electron physics of solids: they are a very nontrivial example of strongly correlated systems, in which exotic states of matter are realized. From the theoretical viewpoint, they are usually considered to be Kondo lattices or dense (concentrated) Kondo systems, i.e., periodic lattices of f-spins, in which the interaction with the conduction electrons leads to screening and suppression of localized magnetic moments and to anomalies of electronic properties [1] (infrared catastrophe, first discovered by Kondo [2]). For the theoretical description of these systems, the entire arsenal of contemporary methods developed in the quantum field theory is applied.

On the other hand, experimental studies show that many such compounds are characterized by magnetic ordering, frequently with a small magnetic moment, and/or by developed spin fluctuations. For systems with 'moderately' heavy fermions (coefficient in the linear term of heat capacity $\gamma \approx 100$ mJ mol^{-1} K^{-2}), the co-existence of incomplete Kondo screening with anomalous magnetic ordering (with a sharply suppressed, but nonzero moment) is sufficiently typical. Antiferromagnetic ordering is especially frequently encountered, although ferromagnetic ordering is also rather frequent. The related references and the discussion of the early experimental data concerning concrete compounds and the corresponding theoretical concepts can be found in monograph [3] and paper [4].

The class of 'Kondo' magnets is characterized by the following features [4, 5].

(1) A logarithmic temperature dependence of the resistivity at temperatures that exceed the Kondo temperature T_K, $T > T_K$, which is inherent in Kondo systems. This dependence can be obtained in the third-order perturbation theory and in Kondo's original approach as well [2].

(2) A low value of the magnetic entropy at the point of ordering in comparison with the value $R \ln (2J + 1)$ (R is the molar gas constant, and J is the nominal value of the magnetic moment), which corresponds to usual magnets with localized moments. This phenomenon is connected with the suppression of the magnetic heat capacity as a result of the Kondo effect (screening of moments): only a small part of the change in the entropy is connected with the long-range magnetic order.

(3) Ordered magnetic moment μ_s, which is small in comparison with the high-temperature moment μ_{eff} found from the Curie constant. The latter has, as a rule, a normal value close to the corresponding value for a rare-earth ion [for example, $\mu_{eff} \approx 2.5\mu_B$ for the Ce^{3+} ion (μ_B is the Bohr magneton)]. This behavior resembles that of weak itinerant (band) magnets.

(4) Paramagnetic Curie point θ, which has, as a rule, a negative (even in the case of ferromagnets) value noticeably

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exceeding in magnitude the temperature of magnetic ordering. Such a behavior can be caused by a large single-site Kondo contribution to the paramagnetic susceptibility χ, which dominates over the contribution from intersite exchange interactions. In particular, the interpolation formula obtained from the exact solution to the Kondo model, which is valid for $0.5T_K < T < 16T_K$, takes on the form $\chi(T) = 0.17/(T + 1.5T_K)$, where T_K is the Kondo temperature according to Wilson [6]. On the other hand, a certain effect can be due to the frustration of exchange interactions.

As to 'classical' systems with heavy fermions (with large $\gamma > 400$ mJ mol^{-1} K^{-2} [7]), the situation here is more complex. There are clear data indicating the existence of antiferromagnetism in UCd_{11} and U_2Zn_{17} with the same μ_s by order of magnitude [7]. For the compounds UPt_3 and URu_2Si_2, the saturation moments are extremely small: $\mu_s \approx (2-3) \times 10^{-2}\mu_B$. Some signs of antiferromagnetic ordering with very small μ_s have also been observed in $CeAl_3$, UBe_{13}, $CeCu_2Si_2$, and $CeCu_6$ (although part of the data for these systems is open to question (see, e.g., review [8])).

A typical feature of heavy-fermion magnetic materials is a high sensitivity of μ_s to external parameters, such as the pressure and the level of doping with small amounts of impurities. For example, UBe_{13} becomes antiferromagnetic with a noticeable magnitude of μ_s under a pressure $P > 23$ kbar; $CeAl_3$, on the contrary, becomes paramagnetic under a pressure higher than $P = 3$ kbar. The magnetic moment in UPt_3 increases to values on the order of $1\mu_B$ upon substituting palladium for 5% Pt or thorium for 5% U. Some systems with heavy fermions undergo metamagnetic transitions in weak magnetic fields, with a sharp increase in the magnetic moment. From this point of view, the title of the review by P Coleman [9] is characteristic: "Heavy fermions: electrons at the edge of magnetism."

In the 1990s, one more class of f-electron systems was discovered: a non-Fermi-liquid (NFL) behavior was revealed [10–12] for a number of compounds and alloys based on cerium and uranium, which manifests itself in the unconventional temperature dependences of the electron heat capacity $C(T)$ of the $T \ln T$ or $T^{1-\lambda}$ type with small exponents λ, in the anomalous power dependences of the magnetic susceptibility [such as T^{ζ} (with $\zeta < 1$)] and of the resistivity [such as T^{μ} (with $\mu < 2$], etc. Frequently, NFL behavior appears at the boundary of magnetic ordering (quantum phase transition) [8], although other diverse mechanisms are also discussed.

In this survey, we consider contemporary theoretical concepts and experimental data concerning the unconventional type of magnetic ordering in dense Kondo f-electron systems and its manifestation in electronic properties, including the non-Fermi-liquid behavior.

2. Experimental results

There are numerous examples of compounds in which the Kondo anomalies in the thermodynamic and kinetic properties coexist with a magnetic ordering, and the values of the magnetic-moment saturation μ_s are on the order of μ_B. In particular, these compounds involve such ferromagnets as CePdSb, $CeSi_x$, Sm_3Sb_4, Ce_4Bi_3, $NpAl_2$, and such antiferromagnets as $CeAl_2$, TmS, CeB_6, $UAgCu_4$ (for a detailed discussion, see book [3]); systems with more exotic magnetic properties are also encountered. In Sections 2.1–2.3, we give some of the latest experimental results on the magnetism of Kondo systems.

2.1 Cerium systems

In the 1980s, after the active studies of classical systems with heavy fermions and of Kondo lattices, the time for examining ternary systems came. One of the most vivid examples among them is the Kondo ferromagnet $CeRh_3B_2$ with the Curie temperature $T_C = 115$ K, the paramagnetic temperature $\theta = -370$ K [13], and a comparatively small heat capacity coefficient $\gamma = 16$ mJ mol^{-1} K^{-2}.

New antiferromagnetic systems with a low Néel temperature T_N and reduced moment of the ground state are constantly revealed, for example, $Ce_8Pd_{24}Ga$ ($T_N = 3.6$ K, $\mu_s = 0.36\mu_B$ per Ce atom [14]) and layered systems, such as $Ce_3(Pd, Pt)In_{11}$ [15].

The number of Kondo ferromagnets is also growing, e.g., CePt [16], $CeRu_2Ge_2$ [17], $CeAgSb_2$ [18], CeRuPO [19], $CeRu_2M_2X$ (M = Al, Ga; X = B, C) [20, 21], $CeIr_2B_2$ [22], hydrogenated CeNiSn [23], $CeFe_4Sb_{12}$ [24], and $Ce_4Sb_{1.5}Ge_{1.5}$ ($T_C = 13$ K, $\theta = -9$ K) [25].

In the $Ce_{1-x}La_xPdSb$ system, where there is a high-temperature logarithmic contribution to the resistance, the ferromagnetic Kondo state changes gradually into the NFL state in the range of $x > 0.7$, with the latter, apparently, having a single-site nature [26].

A good example of the competition between the Kondo effect and antiferro- and ferromagnetism is represented by the system $(Ce_{1-x}Nd_x)_3Al$ [27]. Here, for $x > 0.2$, the antiferromagnetic ordering is changed to ferromagnetic; moreover, at low temperatures, the Kondo effect and the ferromagnetism coexist for $0.2 < x < 0.3$.

The authors of Refs [28–30] investigated a number of compounds of the system $CeRu_2M_2X$ (M = Al, Ga and X = B, C), which demonstrate ferromagnetism with T_C from 12.8 K ($CeRu_2Al_2B$) to 17.2 K ($CeRu_2Ga_2C$). The electron heat capacity of these systems is small (γ is approximately 30 mJ mol^{-1} K^{-2}), so that the Kondo effect appears in them only weakly. According to Ref. [31], in the $CeRu_2Al_2B$ system for $T_C < T < T_N = 14.2$ K there is an incommensurate magnetic order. It also appears in a number of other Kondo cerium systems, for example, in $CeAu_2Ge_2$ [32] and $Ce_2M_mIn_{3n+2m}$ [33].

In the $CeCd_{0.7}Sb_2$ system [34], highly anisotropic ferromagnetism is observed with $T_C = 3$ K and with a negative paramagnetic Curie temperature $\theta = -24$ K; at $T_N = 0.8$ K, a transition into the antiferromagnetic state occurs. An analogous situation also takes place in the $CeZn_{0.66}Sb_2$ system [35] ($T_C = 3.6$ K, $T_N = 0.8$ K, $\theta = -11$ K).

Cerium-based ternary intermetallic compounds of the $CeTX_2$ type (where T is a transition metal, and X = Si, Ge, Sn) are of interest in view of the unconventional properties of their ground state; in particular, $CePtSi_2$ represents a heavy-fermion system in which the Sommerfeld coefficient reaches a value of $\gamma = 1.7$ J mol^{-1} K^{-2} at a temperature of 1.25 K [36], while $CeRhSi_2$ [37] and $CeNiSi_2$ [38] are systems with strong valence fluctuations. A description of such intermetallic compounds must be based on the allowance for a number of factors: the environment of cerium, the hybridization of the 4f-level with the conduction band, and the values of the exchange interaction constants.

The $CeRuSi_2$ system has been investigated in detail in Refs [39–42]. The coexistence of the Kondo effect and ferromagnetism with small magnetic moments on the order

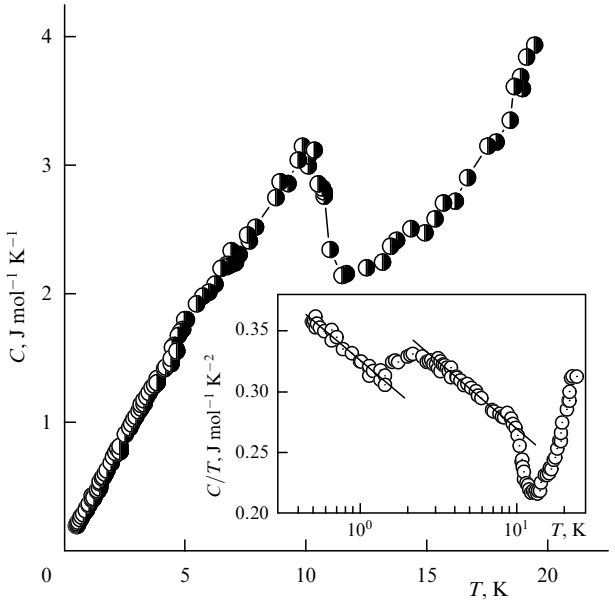

Figure 1. Behavior of the heat capacity of $CeRuSi_2$. In the inset, the variation of the heat capacity is shown on a logarithmic scale.

of $0.1\mu_B$, which appears at temperatures below 11.2 K, is confirmed by both the study of kinetic properties [41] and by muon measurements [40]. The field dependences of the magnetization in strong magnetic fields, up to a very high value of 150 T, showed the absence of saturation of the magnetization, which resembles the behavior of weak itinerant magnets.

Figure 1 plots the temperature dependence of the heat capacity of $CeRuSi_2$ [42]. Against the background of a rather high electron heat capacity, the magnetic heat capacity $C(T)$ has a Λ-shaped peak in the temperature region close to 11 K (the corresponding change in the entropy is $\Delta\mathcal{S} = 2.7$ J K^{-1} mol^{-1} $= 0.5R\ln 2$). The value of the Sommerfeld coefficient obtained by the standard extrapolation of the C/T dependence (at temperatures above 20 K) is $\gamma \approx 140$ mJ mol^{-1} K^{-2}, which corresponds to moderately heavy fermions. At low temperatures, γ grows, and there are signs indicative of a logarithmic NFL behavior of the heat capacity, which is particularly interesting for a stoichiometric compound (the anomaly near $T = 2$ K may be connected with the influence of the crystal field).

In Ref. [42], a large number of other Ce–Ru-based compounds were also investigated: $CeRuX_2$, $CeRu_2X_2$, $Ce_2Ru_3X_5$, and $CeRuX_3$ (X = Si, Ge). In the $CeRu_2Si_2$ system, no magnetic ordering was observed, but there is a Kondo maximum of the resistance at a temperature of 50 K (in $CeRuSi_2$, such a maximum appears at a much higher temperature of 150 K). This difference is probably connected with the lower Kondo temperature (the volume of a unit cell in $CeRu_2Si_2$ is substantially greater). It should be noted that the metamagnetism, which was previously discussed for $CeRu_2Si_2$, is now considered as a continuous crossover [43, 44]; moreover, the itinerant character of the 4f-electrons is retained in very strong magnetic fields, up to ~ 50 T [44], which resembles situation for $CeRuSi_2$.

2.2 Ytterbium systems

Ferromagnetic ordering in combination with manifestations of the Kondo effect is especially frequently encountered in the ytterbium-based ternary systems. Sometimes, it is connected with the formation of a noncollinear magnetic structure; for example, a small canted ferromagnetic moment arises in YbNiSn [45].

In a number of anomalous ytterbium systems, a linear-in-temperature resistance is observed, which indicates an NFL behavior. It is interesting that a similar anomalous behavior of the resistance is also observed in the $YbMn_2Sb_2$ system with nonmagnetic ytterbium, where the conventional Kondo effect must be absent. These anomalies can be connected with scattering on the pseudospin degrees of freedom [46].

In Ref. [47], new ytterbium-based ternary compounds have been obtained and investigated: $YbPd_2Ge$, $YbPd_2Si$, YbPdGe, YbPdSi, YbPtGe, and also $YbPdGe_2$ and the previously known compound $YbPd_2Si_2$. Based on the analysis of galvanomagnetic properties and magnetic susceptibility, it was revealed that the germanium compounds YbPdGe, YbPtGe, and $YbPd_2Ge$ at low temperatures (about 16, 10, and 12 K, respectively) demonstrate ferromagnetic ordering.

In later work, results concerning magnetic and thermodynamic properties were supplemented. In Ref. [48], for YbPtGe, values of $T_C = 5.4$ K and $\gamma = 209$ mJ mol^{-1} K^{-2} were found for $T < T_C$. The magnetic entropy at T_C was found to be $0.52R\ln 2$. For YbPdGe, the values of $T_C = 11.4$ K and $\gamma = 150$ mJ mol^{-1} K^{-2} were obtained in Ref. [49]; the situation in this system resembles that observed in $CeRuSi_2$.

In Ref. [50], different manifestations of the Kondo effect in YbPdGe and YbPtGe were discussed, including the logarithmic contribution to the resistance, and a comparison was carried out of their kinetic properties with those of the corresponding cerium systems CePdGe and CePtGe (formally, the trivalent ions of cerium and ytterbium are similar, since they correspond to one electron or one hole in the 4f-shell). As was shown in Ref. [51], the compounds CeTX (T = Pd, Pt, and X = Ga, Ge, Sn) come under the heading of Kondo systems that are antiferromagnetic at low temperatures, in contrast to YbTGe. In review [52], the difference between these cases is discussed in detail based on the example of $YbRh_2Si_2$, which is a typical antiferromagnetic compound of ytterbium with heavy fermions. In spite of the similarity with $CeRh_2Si_2$ (close values of the Kondo temperature T_K and of the amplitudes of the maximum of the resistance, and the linearity of the temperature dependence of the resistance for $T > T_N$) there are noticeable differences, which can be connected with the greater localization (smaller degree of hybridization with the conduction electrons), and also with the stronger spin–orbit interaction in the case of the 4f-orbitals in ytterbium.

According to Ref. [53], the YbPdSi system constitutes a heavy-fermion ferromagnet with $T_C = 8$ K and γ of about 300 mJ mol^{-1} K^{-2}. The study of the compounds YbTGe (T = Rh, Cu, Ag) [54] showed that YbRhGe is an antiferromagnet with $T_N = 7$ K, and YbCuGe is a ferromagnet with a moment of $0.7\mu_B$ and $T_C = 8$ K; in YbAgGe, a very large value of γ was found, equal to 570 mJ mol^{-1} K^{-2}. Subsequently, a low-temperature noncollinear magnetic ordering was revealed in YbAgGe [55]. According to Ref. [56], YbAgGe is a frustrated heavy-fermion antiferromagnet with a complex magnetic phase diagram, which exhibits an NFL behavior. In the antiferromagnetic system $Yb(Rh_{1-x}Co_x)_2Si_2$ at $x = 0.27$ and a temperature $T_C = 1.3$ K, ferromagnetism with a magnetic moment of $0.1\mu_B$ was revealed [57].

The systems $Yb_2(Pd_{1-x}Ni_x)_2Sn$ [58] and $Yb_2Pd_2In_{1-x}Sn_x$ [59] demonstrate frustrated magnetism: an unconventional magnetic ordering appears in them upon alloying. In the latter system, the ordering occurs in a narrow region of x concentrations near $x = 0.6$ against the background of the NFL behavior.

In the quasi-one-dimensional system $YbNi_4P_2$, exotic ferromagnetism was revealed with $T_C = 0.17$ K and an extremely small saturation moment $\mu_s = 0.05\mu_B$ [60]. In this case, the resistance is linear for $T > T_C$, while the coefficient γ diverges more strongly than logarithmically.

2.3 Actinide systems

Metals of the uranium group manifest a strong tendency toward Pauli paramagnetism as a result of their specific electronic structure. However, their magnetism can be substantially strengthened in compounds with transition metals because of an increase both in the interatomic distances (which, according to Hill's criterion, favors the formation of localized moments) and in the density of states near the Fermi level [61]. An important difference between anomalous uranium compounds and cerium compounds is a stronger delocalization of 5f electrons, which is also reflected in the spectrum of spin fluctuations, which become stronger [62].

The coexistence of Kondo anomalies and long-range magnetic order in actinide compounds is not a rare phenomenon. In a number of actinide systems, ferromagnetic ordering coexists with the Kondo logarithmic contributions to the resistance (see also the discussion in Ref. [63]), for example, in compounds such as UTe ($T_C = 102$ K [64]), UAsSe ($T_C = 109$ K [65]), $UCu_{0.9}Sb_2$ ($T_C = 113$ K [66]), $UCo_{0.5}Sb_2$ ($T_C = 64.5$ K [67]), $UNiSi_2$ ($T_C = 95$ K [68]), and $NpNiSi_2$ ($T_C = 51.5$ K [69]). In the compound Np_2PtGa_3 ($T_C = 26$ K), a high value of $\gamma = 180$ mJ mol^{-1} K^{-2} was observed [70].

The system UIr_2Zn_{20} [71] can rightfully be called a ferromagnetic system with heavy fermions. Here, γ reaches a value of 450 mJ mol^{-1} K^{-2} and remains large at temperatures lower than $T_C = 2.1$ K, and the saturation moment is $0.4\mu_B$.

In the compound UAu_2Si_2 at T less than $T_C = 19$ K, an uncompensated antiferromagnetism is observed with a spontaneous magnetization along the crystallographic c-axis; moreover, $\gamma = 180$ mJ mol^{-1} K^{-2} at low temperatures [72].

A number of actinide systems demonstrate a non-Fermi-liquid behavior. In the $Th_{1-x}U_xCu_2Si_2$ system [73, 74], the NFL behavior (C/T is proportional to $\ln T$, and the resistance is linear in temperature) appears in the region of compositions where the ferromagnetism is suppressed.

The compound URhGe is ferromagnetic. In the $URh_{1-x}Ru_xGe$ alloy [75], the critical concentration corresponding to the suppression of the magnetic order is $x_{cr} = 0.38$; the Curie temperature approaches zero according to a linear law with increasing x, and the ferromagnetic moment μ_s decreases continuously ($\mu_s = 0.4\mu_B$ at $x = 0$). At the point x_{cr}, the heat capacity behaves as $T \ln T$, and the exponent in the power dependence of the resistance on temperature reaches a minimum: $\mu = 1.2$. The total magnetic entropy obtained by the integration of C/T is $0.48R \ln 2$ at $x = 0$ and diminishes to $0.33R \ln 2$ at x_{cr}.

At the same time, the compound NpRhGe [76] is an antiferromagnet with $T_N = 21$ K, an ordered moment $\mu_{Np} = 1.14\mu_B$, and a large value of $\gamma = 195$ mJ mol^{-1} K^{-2}.

In the UCoGe system, the ferromagnetism is also changed by an NFL state upon substituting ruthenium for cobalt [77].

One further example of NFL behavior in a ferromagnetic phase is represented by a heavy-fermion system $URu_{2-x}Re_xSi_2$ [78], in which a small moment of the ground state ($\mu_s = 0.44\mu_B$) and $-\ln T$ (or $T^{-0.1}$) dependences were revealed for C/T and T^μ with $\mu = 1.2$ for the resistance in a wide range of temperatures below 20 K at $x = 0.6$. At the same time, no anomalies in the heat capacity or in the resistance were observed at the point of magnetic transition in this system.

The preparation of pure UGe_2 crystals and, recently, also of URhGe, led to the discovery of ferromagnetic superconductors. It is assumed that the factor of key importance for the coexistence of ferromagnetism and superconductivity is triplet pairing [52, 75, 79].

3. Models, perturbation theory, and renormalization group for Kondo lattices

It was long assumed that the competition between the Ruderman–Kittel–Kasuya–Yosida (RKKY) intersite exchange interaction and the Kondo effect must lead to the formation of either the conventional magnetic ordering with large atomic magnetic moments (as in pure rare-earth metals) or of the nonmagnetic Kondo state with suppressed magnetic moments [1]. This assumption was based on the Doniach criterion [80] in which the energy scales of the phases indicated are compared. However, the experimental data given in Section 2 made it necessary to reexamine this point of view, which has led to the development of more complex theoretical approaches.

Let us examine the Hamiltonian of an s–f exchange model:

$$H = H_s + H_f + H_{int} = \sum_{\mathbf{k}m\sigma} t_{\mathbf{k}} c^\dagger_{\mathbf{k}m\sigma} c_{\mathbf{k}m\sigma} - \sum_{\mathbf{q}} J_{\mathbf{q}} \mathbf{S}_{-\mathbf{q}} \mathbf{S}_{\mathbf{q}}$$

$$- I \sum_{im\sigma\sigma'} (\mathbf{S}_i \boldsymbol{\sigma}_{\sigma\sigma'}) c^\dagger_{im\sigma} c_{im\sigma'} , \qquad (1)$$

where $c_{\mathbf{k}m\sigma}$ and $c^\dagger_{\mathbf{k}m\sigma}$ are the operators of the conduction electrons; $t_{\mathbf{k}}$ is the bare band energy; $\mathbf{S}_i$ are the operators of localized spins; $\boldsymbol{\sigma}$ is the Pauli matrices vector, and I is the parameter of the s–f exchange interaction. Frequently (for example, in rare-earth metals), the f–f interaction between the localized spins $J_{\mathbf{q}}$ represents the indirect RKKY exchange through the conduction electrons, which is due to the same s–f interaction (although in a number of compounds an important role can be played by the superexchange interaction). However, when formulating the perturbation theory it is convenient to include the f–f interaction in the zero Hamiltonian (in the Western literature, this model was called the Kondo–Heisenberg model).

In contrast to the intraatomic Coulomb (Hubbard) interaction, the s–f exchange interaction is not, as a rule, strong; however, it leads to substantial effects in the electronic spectrum. From a microscopic point of view, it can have different natures. In a number of rare-earth systems (for example, in magnetic semiconductors), this is the intraatomic Hund exchange possessing ferromagnetic nature. In rare-earth compounds, the s–f exchange is frequently not genuine but rather is effective — caused by the hybridization between the conduction s bands and atomic levels of f electrons; in this case, the exchange is antiferromagnetic ($I < 0$). This condition is necessary for the appearance of

the Kondo effect: in this case, the effective (renormalized) exchange interaction becomes infinite in the regime of strong coupling, so that magnetic scattering leads to the complete screening of the magnetic moments [6, 81].

In Hamiltonian (1), the orbital degeneracy was taken into account for the generality: the subscript $m = 1, \ldots, M$ was introduced, where M is the number of scattering channels for conduction electrons, which can be an additional formal large parameter when constructing the perturbation theory. As was first shown by Nozières and Blandin [82] when analyzing the splitting of the levels in a crystal field in the case of a single Kondo impurity, the scaling behavior substantially depends on the relationship between the value of the localized spin and the orbital degeneracy of electrons. In the case of $S > M/2$, we encounter a situation of undercompensation (incomplete screening) when in the regime of strong coupling the substitution $S \to S - M/2$ comes about. At $S = M/2$, the conventional Kondo effect (complete screening, formation of a heavy Fermi liquid) takes place; the case of $S < M/2$ (overcompensation) is most complex: a nontrivial fixed point and an NFL behavior appear here already in the single-impurity regime.

In the more general $\mathrm{SU}(N) \otimes \mathrm{SU}(M)$ model $\sigma = 1, \ldots, N$, and the Hamiltonian is expressed as [83]

$$H = \sum_{\mathbf{k}m\sigma} t_{\mathbf{k}} c^{\dagger}_{\mathbf{k}m\sigma} c_{\mathbf{k}m\sigma} - I \sum_{im\sigma\sigma'} |i\sigma'\rangle\langle i\sigma| c^{\dagger}_{im\sigma} c_{im\sigma'} + H_{\mathrm{f}} . \qquad (2)$$

At $N = 2$, we return to the s–f model with a spin $S = 1/2$, and the case of $M = N$ corresponds to the Coqblin–Schrieffer model in which complete compensation also takes place. A more realistic model, which includes angular momenta, is discussed in Ref. [5]; a generalization to the case of an arbitrary spin S is also possible (see, e.g., papers [12, 83]).

Let us discuss various means of forming a magnetic state. The simplest mechanism involves the appearance of the regime of an undercompensation of magnetic moments ($S > M/2$ at $N = 2$). Apparently, this mechanism is realized in some uranium ferromagnetic compounds, where the Kondo effect is exhibited noticeably only in the paramagnetic phase [63].

A more conventional situation deals with complete compensation: $S = M/2$, which appears naturally in real systems with a complex electronic structure and degenerate bands. In such systems, the relative roles of the Kondo effect and of the intersite RKKY interaction are assigned by two energy scales: the Kondo temperature $T_{\mathrm{K}} \sim D \exp(1/2I\rho)$ (where $\rho = \rho(E_{\mathrm{F}})$ is the bare density of states at the Fermi level, and D is the width of the conduction band) which determines the crossover between the regime of free moments and the region of strong coupling, and the value of $T_{\mathrm{RKKY}} \sim I^2\rho$. The latter is on the order of the temperature T_{m} of magnetic ordering in the absence of the Kondo effect. The ratio $T_{\mathrm{K}}/T_{\mathrm{m}}$ can change depending on the external parameters and the composition of the system upon alloying.

In the nonmagnetic case one has $T_{\mathrm{RKKY}} \sim \overline{\omega}$, where $\overline{\omega}$ is the characteristic frequency of spin fluctuations. For the majority of the compounds discussed, the inequality $T_{\mathrm{K}} > T_{\mathrm{RKKY}}$ is satisfied. However, there are anomalous magnets (containing cerium and uranium), with $T_{\mathrm{K}} \ll T_{\mathrm{N}}$, for example, $CeAl_2Ga_2$ and $UAgCu_4$. An analogous case occurs in the ferromagnetic systems $CeRu_2T_2M$ [28, 29]. This situation is close to that observed in elementary rare-earth magnetic metals, where the Kondo effect, which is almost completely suppressed by the magnetic ordering, makes only small corrections to the magnetic moment.

As the first step toward describing the formation of the state of a Kondo magnet, it is useful to examine the perturbation theory corrections to magnetic characteristics [5, 84]. To this end, the effects of impurity–impurity exchange interactions (absent in the single-impurity Kondo problem) should be considered, which lead to the appearance of spin dynamics. The calculation of the magnetic susceptibility gives the following result:

$$\chi = \frac{S(S+1)}{3T}(1 - 4I^2 L) ,$$
$$L = \frac{MN/2}{S(S+1)} \sum_{\mathbf{pq}} \int K_{\mathbf{p}-\mathbf{q}}(\omega) \frac{n_{\mathbf{p}}(1 - n_{\mathbf{q}})}{(t_{\mathbf{q}} - t_{\mathbf{p}} - \omega)^2} \, \mathrm{d}\omega , \qquad (3)$$

where $K_{\mathbf{q}}(\omega)$ is the spin spectral density.

From a simple logarithmic evaluation of the integral in formula (3), we obtain

$$\chi = \frac{S(S+1)}{3T}\left(1 - MNI^2\rho^2 \ln \frac{D^2}{T^2 + \overline{\omega}^2}\right) , \qquad (4)$$

where the expression in the parentheses describes the suppression of the effective moment.

The Kondo corrections to the magnetic moment in the ferro- and antiferromagnetic states are obtained by the substitution into the standard spin-wave result

$$\delta\overline{S} = -\sum_{\mathbf{q}} N_{\mathbf{q}}, \quad N_{\mathbf{q}} = \langle b^{\dagger}_{\mathbf{q}} b_{\mathbf{q}} \rangle \qquad (5)$$

of the correction to the occupation numbers $N_{\mathbf{q}}$ of magnons at the zero temperature, which is caused by the damping of spin waves as a result of their scattering from the conduction electrons. In the case of ferromagnets (assuming for simplicity that $N = 2$, $M = 1$), we have

$$\delta N_{\mathbf{q}} = 2I^2 S \sum_{\mathbf{k}} \frac{n_{\mathbf{k}\downarrow}(1 - n_{\mathbf{k}-\mathbf{q}\uparrow})}{(t_{\mathbf{k}\downarrow} - t_{\mathbf{k}-\mathbf{q}\uparrow} - \omega_{\mathbf{q}})^2} , \qquad (6)$$

where $n_{\mathbf{k}\sigma}$ are the occupation numbers of electrons (Fermi functions). The calculations for both the ferromagnet and antiferromagnet give [5, 85]

$$\frac{\delta\overline{S}}{S} = -MNI^2\rho^2 \ln \frac{D}{\overline{\omega}} . \qquad (7)$$

These corrections to the moment in the ground state appear in any conducting magnets, including pure f-metals, in which case, however, the corrections are small (on the order of 10^{-2}).

Corrections to the electronic characteristics can be calculated in a similar manner; in the electronic self-energy $\Sigma_{\mathbf{k}}(E)$, the Kondo contributions of the third order (determining the renormalization of the s–f exchange parameter) are also cut off at $\overline{\omega}$. It should be noted that in the presence of spin dynamics the Kondo type divergences in $\Sigma_{\mathbf{k}}(E)$ arise already in terms of the second order, which make a contribution to the increase in the effective mass [84]. Formally, these divergences are connected with the Fermi function, and the corresponding contributions are similar to the electron–phonon or spin-fluctuation (paramagnon) renormalization.

To obtain a self-consistent picture for a magnet with noticeable Kondo renormalizations, it is necessary to calculate corrections to the characteristic frequencies $\overline{\omega}$ of spin fluctuations. In the paramagnetic phase, the estimation of the second-order correction to the dynamic susceptibility yields [86]

$$\omega_{\mathbf{q}}^2 = \frac{4}{3}\, S(S+1) \sum_{\mathbf{p}} (J_{\mathbf{q}-\mathbf{p}} - J_{\mathbf{p}})^2 \left[1 - 4I^2 L(1-\alpha_{\mathbf{q}})\right]. \quad (8)$$

Here, the quantity L is determined in Eqn (3):

$$\alpha_{\mathbf{q}} = \sum_{\mathbf{R}} J_{\mathbf{R}}^2 \left(\frac{\sin k_{\mathrm{F}} R}{k_{\mathrm{F}} R}\right)^2 \left[1 - \cos(\mathbf{qR})\right] \times \left\{\sum_{\mathbf{R}} J_{\mathbf{R}}^2 \left[1 - \cos(\mathbf{qR})\right]\right\}^{-1}, \quad (9)$$

where k_{F} is the Fermi vector, and the summation is performed over the lattice vectors $\mathbf{R}$. Since $0 < \alpha_{\mathbf{q}} < 1$, the Kondo effect leads to a decrease in $\overline{\omega}(T)$ upon a decrease in temperature. In the nearest-neighbor approximation (with a lattice parameter d) for $J(\mathbf{R})$, the value of α is independent of $\mathbf{q}$, so that $\alpha = \sin^2(k_{\mathrm{F}} d)/(k_{\mathrm{F}} d)^2$. The calculation of the corrections to the frequency of spin waves in the ferromagnetic and antiferromagnetic phases with allowance for magnon–magnon interactions and electron–magnon scattering leads to the following relationship [5, 85]:

$$\frac{\delta\omega_{\mathbf{q}}}{\omega_{\mathbf{q}}} = -2MNI^2\rho^2 a \ln\frac{D}{\overline{\omega}}, \quad (10)$$

where the factor a depends on the type of magnetic ordering.

The above perturbation theory results make it possible to qualitatively describe the state of the Kondo lattice as a magnet with a small magnetic moment [86]. We will lower the temperature starting from the paramagnetic state. In this case, the magnetic moment is 'compensated', but the degree of compensation is determined as $(T^2 + \overline{\omega}^2)^{1/2}$ rather than as T, in contrast to the single-impurity case. At the same time, $\overline{\omega}$ decreases according to expression (8). This process cannot be described analytically in terms of the perturbation theory. However, if we intend to obtain a united universal energy scale on the order of T_{K}, we should assume that $\overline{\omega} \sim T_{\mathrm{K}}$ for $T < T_{\mathrm{K}}$. This is confirmed by a large amount of experimental data concerning the quasielastic neutron scattering in Kondo systems, which show that the typical width of the central peak, $\Gamma \sim \overline{\omega}$, at low temperatures has the same order of magnitude as the Fermi degeneracy temperature determined from the thermodynamic and kinetic properties, i.e., an order of T_{K}. Consequently, the process of the compensation of the magnetic moment is completed somewhere at the boundary between the region of strong coupling and leads to a state with a finite (although probably small) saturation moment μ_{s}.

A more consistent consideration of the problem concerning the magnetism of Kondo lattices can be performed within the framework of the renormalization group approach. In the simplest form of the Anderson 'poor-man's scaling' [87], the renormalization group equations for the effective s–f parameter and $\overline{\omega}$ are written on the basis of examining the momentum integrals in the many-electron Kondo corrections to the electron self-energy and frequency of spin fluctuations [5, 88, 89].

In order to construct the scaling procedure, it is necessary to separate contributions from an energy layer with $C < E < C + \delta C$ near the Fermi level $E_{\mathrm{F}} = 0$. For example, in the case of a ferromagnet, we find from the effective splitting in the electronic spectrum, determined from the second-order self-energy

$$2I_{\mathrm{eff}} S = 2IS - \left[\Sigma_{\mathbf{k}\uparrow}^{\mathrm{FM}}(E_{\mathrm{F}}) - \Sigma_{\mathbf{k}\downarrow}^{\mathrm{FM}}(E_{\mathrm{F}})\right]_{k=k_{\mathrm{F}}} \quad (11)$$

that

$$\delta I_{\mathrm{eff}} = I^2 \sum_{C < t_{\mathbf{k}+\mathbf{q}} < C+\delta C} \left(\frac{1}{t_{\mathbf{k}+\mathbf{q}} + \omega_{\mathbf{q}}} + \frac{1}{t_{\mathbf{k}+\mathbf{q}} - \omega_{\mathbf{q}}}\right) = \frac{\rho I^2}{\overline{\omega}}\, \delta C \ln\left|\frac{C - \overline{\omega}}{C + \overline{\omega}}\right|, \quad (12)$$

where $\overline{\omega} = 4\mathcal{D}k_{\mathrm{F}}^2$, and $\mathcal{D}$ is the spin-wave stiffness. By introducing dimensionless coupling constants $g = -NI\rho$ and $g_{\mathrm{eff}}(C) = -NI_{\mathrm{eff}}(C)\rho$ and making a change $g \to g_{\mathrm{eff}}(C)$ in the corrections of the perturbation theory, we obtain a set of equations of the renormalization group in the one-loop approximation:

$$\frac{\partial g_{\mathrm{eff}}(C)}{\partial C} = -\Lambda, \quad \frac{\partial \ln \overline{\omega}_{\mathrm{eff}}(C)}{\partial C} = \frac{aM\Lambda}{N}, \quad \frac{\partial \ln \bar{S}_{\mathrm{eff}}(C)}{\partial C} = \frac{M\Lambda}{N}, \quad (13)$$

where

$$\Lambda = \Lambda\left(C, \overline{\omega}_{\mathrm{eff}}(C)\right) = \frac{g_{\mathrm{eff}}^2(C)}{C}\, \eta\left(\frac{\overline{\omega}_{\mathrm{eff}}(C)}{C}\right). \quad (14)$$

The scaling function for the paramagnetic (PM), ferromagnetic (FM), and antiferromagnetic (AFM) phases has the following forms:

$$\eta(x) = \begin{cases} x^{-1} \arctan x, & \mathrm{PM}, \\ \dfrac{1}{2x} \ln\left|\dfrac{1+x}{1-x}\right|, & \mathrm{FM}, \\ -x^{-2} \ln|1 - x^2|, & \mathrm{AFM}. \end{cases} \quad (15)$$

As follows from the results of an investigation of Eqns (13)–(15) [5, 88], three regimes are possible, depending on the relationship between the single-impurity Kondo temperature and the bare frequency of spin fluctuations.

(1) The strong coupling regime, in which g_{eff} diverges at a certain C. This regime is roughly defined by the condition $\overline{\omega} < T_{\mathrm{K}} = D\exp(-1/g)$. Here, $I_{\mathrm{eff}}(C \to 0) = -\infty$, so that all conduction electrons are bound into singlet states and the spin dynamics are suppressed.

(2) The 'Kondo' magnet regime with a noticeable, but incomplete, compensation of magnetic moments, which is realized in the interval of $T_{\mathrm{K}} < \overline{\omega} < AT_{\mathrm{K}}$ (A is a numerical coefficient on the order of unity), which corresponds to the small interval of $\delta g \sim g^2$. In this interval, the renormalized values of the magnetic moment and of the frequency of spin fluctuations ($\bar{S}_{\mathrm{eff}}(0)$ and $\overline{\omega}_{\mathrm{eff}}(0)$, respectively) increase from zero to almost bare values.

(3) The regime of 'conventional' magnets with small logarithmic corrections to the ground-state moment [see formula (7)], which appears for $\overline{\omega} > AT_{\mathrm{K}}$. In this case, there is no essential dependence on the sign of I.

In the case of regime 2, there is a high sensitivity of the magnetic state to external factors, so that the magnetic moment changes significantly upon small variations in the bare interaction parameter.

The transition to the magnetic state upon a change in the bare coupling constant g is described as a quantum phase transition with its critical indices [5]. In the small vicinity of this transition, an NFL behavior is observed, where g_{eff} increases linearly with an increase in $\xi = \ln|D/C|$ (see discussion in Section 4). The critical value g_c depends substantially on the type of magnetic ordering and on the magnon spectrum structure (for example, on the presence of a gap in it), the dimensionality of space, etc. Thus, the Doniach criterion $g_c \simeq 0.4$ [79] obtained for the simple one-dimensional model can hardly be applied to real systems. These problems are discussed in Ref. [90] within the framework of first-principles calculations.

It is natural that when examining quantum magnetic phase transitions it is necessary to more precisely take into account magnetic fluctuations, including momentum dependences. Recently, for the analysis of the phase diagram of the two-dimensional antiferromagnetic Kondo lattice, an ϵ-expansion was performed within the renormalization group method using nonlinear sigma-model [91]. A change in the topology of the Fermi surface can also play an important role [92].

4. Magnetism and non-Fermi-liquid behavior. Multichannel Kondo model

A number of ideas have been suggested to explain the NFL behavior of f-systems. Since many NFL compounds are disordered alloys, mechanisms connected with the disordering in Kondo lattices have been proposed [93], and the Griffiths–McCoy singularities model was also utilized [94]. As was mentioned in the Introduction, the NFL behavior is most frequently observed on the border of magnetic instability during quantum phase transitions (QPTs), in the immediate vicinity of which (controlled by the composition or by an external pressure) the magnetic order is suppressed at all temperatures up to zero. Therefore, mechanisms connected with spin fluctuations [95] and approaches which examine the behavior near a QPT have been proposed in the 'pure' limit [96], or taking into account the disordering [97].

As was shown by perturbation theory calculations in the s–f model of antiferromagnets [98], in the two-dimensional case ($d = 2$) or in the situation of a frustrated magnon spectrum for $d = 3$, the interband contributions to the electron heat capacity give a $T\ln(T/T^*)$ contribution instead of linear dependence, and instead of the quadratic dependence of the resistance at $d = 2$ or at $d = 3$ in the situation of 'nesting' for the electronic spectrum we have $R(T) \sim T\ln(T/T^*)$. Naturally, these dependences are valid in a limited temperature interval for $T > T^*$, where $T^* \sim T_N\bar{\Delta}/E_F$ is determined by the magnitude of the spin splitting $\bar{\Delta} = 2|I|S$ (in the case of ferromagnets, T^* is on the order of $T_C I^2$ because of the quadratic spectrum of spin waves). Similar dependences appear in the weak itinerant models at the border of the magnetic phase transition [95, 99, 100]. On the other hand, as we have seen above, in some substances the NFL behavior is retained at temperatures that substantially exceed the ordering temperature or in a magnetically ordered phase, rather far from the critical point of the QPT.

The NFL behavior can be described in the multichannel Kondo model [83, 101–103], where it arises in the regime of an overscreening of magnetic moments. Here, the state of complete compensation is unstable: a whole complex of conduction electrons is connected with the localized spin (even in the case of $S = 1/2$), and the composite obtained continues interacting with the electron sea. As a result, an infrared fixed point appears.

In this situation, it is necessary to generalize the renormalization group consideration represented in Section 3 by including a higher-order scaling. In the single-impurity case, the scaling behavior is determined by the beta-function $\beta(g) = \partial g_{\text{eff}}(C)/\partial \ln|C|$, whose expansion gives [103]

$$\beta(g) = -g^2 + \frac{M}{N}\, g^3 + \dots . \tag{16}$$

For $M > N$ (for simplicity, we discuss the case of $S = 1/2$), the fixed point $g^* = N/M$ [zero of $\beta(g)$] lies in the region of weak coupling, so that the use of the perturbation theory and renormalization group approach is justified. On the contrary, in the case of compensation ($M = 1$) discussed in Section 3, this approach is not applicable: the fixed point with a large g^* has no physical sense.

The solution to the scaling equation yields

$$\frac{g^* - g_{\text{eff}}(C)}{g^* - g} = g^* \left|\frac{C}{T_K}\right|^{\Delta} \exp\left(-\frac{g^*}{g_{\text{eff}}(C)}\right), \tag{17}$$

where $\Delta = N/M$, and the Kondo temperature is given by

$$T_K = D g^{M/N} \exp\left(-\frac{1}{g}\right). \tag{18}$$

In this approach, the divergence of $g_{\text{eff}}(\xi)$ is absent, and a power-law critical behavior (17) is observed in a wide range, including the region of $|C| > T_K$ [103]. The critical indices are determined by the slope of $\Delta = \beta'(g)$. Calculations taking into account the higher orders with respect to $1/M$ lead to the result

$$\Delta = \frac{N}{M}\left(1 - \frac{N}{M}\right) \simeq \frac{N}{M+N}, \tag{19}$$

which is in agreement with the accurate results obtained from the Bethe ansatz and conformal field theory (see Ref. [83]). For the corresponding value of g^* at $N = 2$, the expansion gives $g^* = (2/M)(1 - 2\ln 2/M)$ [103], which differs slightly from Δ.

The important case of $M = N = 2$ is more complex from a theoretical viewpoint, and it cannot be examined with the aid of simple analytical methods. Here, the susceptibility and the heat capacity coefficient diverge logarithmically with decreasing temperature, as given again by the Bethe ansatz and conformal field theory (see review [83]).

Let us now consider the case of a Kondo lattice [85]. Similarly to how this was done in Section 3, we find

$$\frac{\partial g_{\text{eff}}(C)}{\partial C} = -\left(1 - \frac{N}{M}\, g_{\text{eff}}(C)\right)\Lambda . \tag{20}$$

In this case, the renormalization of the frequency $\overline{\omega}$ in the leading order in M is described by the same equation as in the case of equation (13). Then, we obtain (at $\gamma = M/N$) the

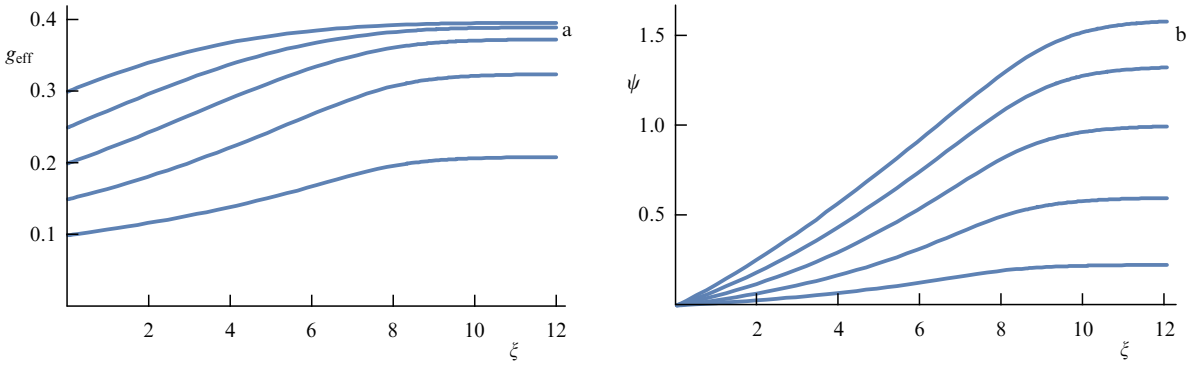

Figure 2. Scaling trajectories for the case of a paramagnet: (a) $g_{\text{eff}}(\xi)$, and (b) $\psi(\xi)$ [85]. Values of the parameters are as follows: $\gamma = 5/2$, $\lambda = 7$, $a = 1/2$, $g = 0.10$, 0.15, 0.20, 0.25, and 0.30 (from bottom to top).

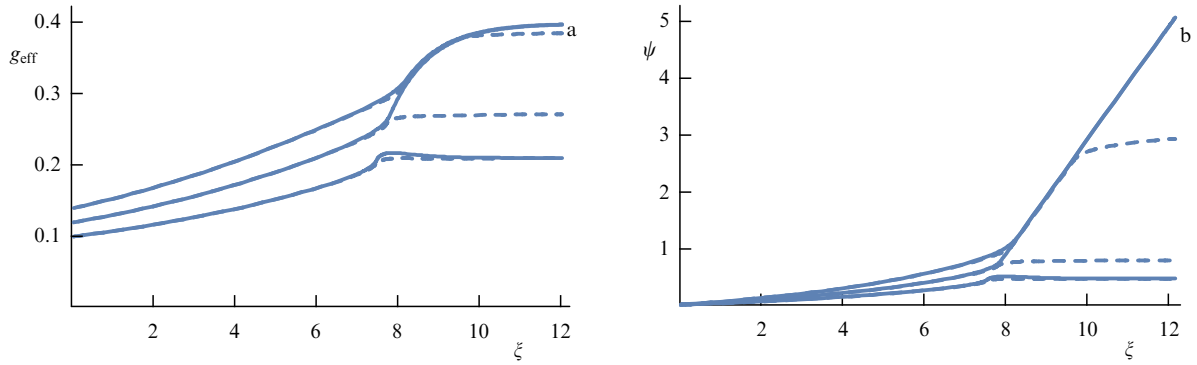

Figure 3. Scaling trajectories for the case of a three-dimensional antiferromagnet: (a) $g_{\text{eff}}(\xi)$, and (b) $\psi(\xi)$ [85] at $\gamma = 5/2$, $\lambda = 7$, the damping parameter $\delta = 2 \times 10^{-4}$, $a = 1$, $g = 0.10$, 0.12, and 0.14 (from bottom to top). The dashed curves correspond to the allowance for the noncoherent contribution to the spin spectral density.

expression

$$\frac{\overline{S}_{\text{eff}}(C)}{S} = \left(\frac{\overline{\omega}_{\text{eff}}(C)}{\overline{\omega}}\right)^{1/a} = \frac{1-\gamma g_{\text{eff}}(C)}{1-\gamma g} = \frac{g^* - g_{\text{eff}}(C)}{g^* - g}\,. \quad (21)$$

Thus, we have a situation of a soft boson mode when approaching the fixed point $g^* = 1/\gamma$.

By introducing the function

$$\psi(\xi) = \ln\frac{\overline{\omega}}{\overline{\omega}_{\text{eff}}(\xi)} = a\ln\frac{\overline{S}_{\text{eff}}(C)}{S}\,, \quad (22)$$

where a was introduced in relation (10), we can write out the scaling equation in the form

$$\frac{\partial\psi}{\partial\xi} = \frac{a}{\gamma}\left[1-(1-\gamma g)\exp\left(-\frac{\psi}{a}\right)\right]^2 \Psi(\lambda+\psi-\xi)\,, \quad (23)$$

where

$$\Psi(\xi) = \eta\big(\exp(-\xi)\big)\,, \qquad \xi = \ln\left|\frac{D}{C}\right|\,, \qquad \lambda = \ln\frac{D}{\overline{\omega}} \gg 1\,.$$

These equations are written down in terms of γ rather than through M and N separately. In order to establish correspondence with the single-impurity case (19), it can be assumed that $\gamma = M/N + 1 = 1/\Delta$. This gives, at least for $M > 2$, correct critical indices for the magnetic susceptibility, the heat capacity, and the resistance.

In the paramagnetic phase, at large $\xi = \ln|D/C|$, it is possible to assume, for an estimation, that $g_{\text{eff}}(\xi) \simeq g^* = 1/\gamma$. We then arrive at a power-law behavior:

$$\overline{\omega}_{\text{eff}}(C) \simeq \overline{\omega}\left(\frac{|C|}{T_{\text{K}}}\right)^{\beta}, \quad \beta = \frac{a}{\gamma} = a\Delta\,, \quad \overline{S}_{\text{eff}}(C) \simeq \left(\frac{|C|}{T_{\text{K}}}\right)^{\Delta}. \quad (24)$$

Such a behavior is observed in a limited interval of the ξ values determined by the spin dynamics — up to $\xi = \xi_1 \simeq (\lambda - \beta/g)/(1-\beta)$. As $\xi \to \infty$, the magnitude of $\psi(\xi)$ remains finite, while $g_{\text{eff}}(\xi)$ tends to a value that is somewhat less than the single-impurity g^* value (Fig. 2).

The behavior for $\xi < \xi_1$ is analogous in the case of magnetic ordering, but for $\xi > \xi_1$ an important role belongs to the Van Hove singularities (VHSs) in the scaling function: $\Psi(\lambda + \psi - \xi)$ can begin increasing rather than decreasing when approaching ξ_1, the argument of the function Ψ in equation (23) becomes almost constant (fixed), and we have (see Fig. 3)

$$\psi(\xi) \simeq \xi - \lambda\,, \qquad \overline{\omega}_{\text{eff}}(C) \simeq |C|\,. \quad (25)$$

Here, a sharp crossover takes place with the change in g: regime (25) cannot be achieved for $g < g_{\text{c}}$. The value of g_{c} is determined by the value of the magnon damping parameter δ at which the singularities of the function (15) are cut off. The behavior specified by formulas (25) has a critical nature: in the case of the single-channel Kondo model, it appears only at a critical value $g = g_{\text{c}}$, which corresponds to the quantum magnetic phase transition.

Generally, one-loop scaling considered in Section 3 gives an NFL behavior at finite M only in a very narrow variation

interval of the bare coupling constant g, since as g grows we rapidly move into the regime of strong coupling, where $g_{\text{eff}}(\xi > \lambda) \to \infty$. However, there are mechanisms which lead to an expansion of the region of the NFL behavior in the case of $M = 1$ as well (when the mechanism of the appearance of the fixed point, described in this section, is not realized). In particular, this occurs if we take into account the spin excitation damping [104].

As was shown in Ref. [104], in the case of a special form of the spectral function of spin excitations, which can be realized in multisublattice magnets, the argument of the function Ψ can also be fixed on the scaling trajectory. This leads to a linear growth of $g_{\text{eff}}(\xi)$, so that an NFL behavior shows up in a wide variation interval of the bare coupling constant, $g_{c1} < g < g_{c2}$.

In the presence of VHSs in the electronic spectrum, the region of NFL behavior is also expanded [105]. Such singularities can significantly influence the structure of the perturbation theory. In this case, the scaling behavior will be determined not only by the proximity of the Fermi level ($C \to 0$), but also by the distance to the singularity; in certain situations, a weakening of the dependence of the coupling constant on its bare value is possible. The pinning of the Fermi level at the VHS is an interesting possibility [106].

There is also a possibility that a fixed point similar to that observed in the case of the multichannel Kondo model will appear in the case of the single-channel model with a singularity in the density of states or nesting [107] (notice that giant VHSs are formed upon the intersection of the weaker singularities [108], i.e., they are connected with the degeneracy of the bands).

Now, let us discuss the behavior of physical properties for the case of $N = 2$. The temperature dependences of the magnetic moment and of the susceptibility in the paramagnetic case are determined directly from the above results with the aid of the $|C| \to T$ substitution:

$$S_{\text{eff}}(T) \propto \left(\frac{T}{T_K}\right)^{\Delta}, \quad \chi(T) \propto \frac{S_{\text{eff}}^2(T)}{T} \propto \frac{(T/T_K)^{2\Delta}}{T}. \quad (26)$$

A similar dependence is also obtained for the electron heat capacity [103]. These dependences, in contrast to those in the single-impurity case, are valid in a limited temperature interval.

The temperature correction to the magnetoresistance assumes the form [103]

$$\delta R_m(T) \propto g_{\text{eff}}(T) - g^* \propto -\left(\frac{T}{T_K}\right)^{\Delta}. \quad (27)$$

The $T^{1/2}$ dependence (which corresponds to $M = 2$) is indeed observed in a number of f-systems [83]. On the other hand, as was discussed above, the approximation in question does not take into account the logarithmic divergences that appear in the susceptibility and heat capacity at $M = 2$ ($\Delta = 1/2$).

In the spin-wave region, we have $\chi \propto \overline{S}/\overline{\omega}$ for an antiferromagnet. Using substitutions $\overline{\omega} \to \overline{\omega}_{\text{eff}}(C)$ and $\overline{S} \to \overline{S}_{\text{eff}}(C)$ with $|C| \sim T$ in the spirit of the scaling consideration, we find

$$\chi(T) \propto T^{-\zeta}, \quad \zeta = \begin{cases} \dfrac{\Delta(a-1)}{a} \\ \dfrac{a-1}{a} \end{cases}, \quad (28)$$

for regimes (24) and (25), respectively. The nonuniversal exponent ζ, which is determined by the details of the magnetic structure, can be both positive and negative.

For the electron heat capacity, in the antiferromagnetic case we have

$$\frac{C_{\text{el}}(T)}{T} \propto \frac{g_{\text{eff}}^2(T)\overline{S}_{\text{eff}}(T)}{\overline{\omega}_{\text{eff}}(T)} \propto \chi(T), \quad (29)$$

which is in agreement with the experimental data for NFL systems [83, 101].

Thus, the multichannel model of a Kondo lattice in a natural way describes the formation of a magnetic state with a low value of the magnetic moment. Moreover, it gives an example of an essential renormalization of the coupling constant in accordance with formula (21). This can be of interest for the general theory of metallic magnetism (in particular, for weak itinerant ferro- and antiferromagnets): the magnetic state is determined by the process of renormalization rather than by the Stoner criterion with the bare parameters.

An important problem is the stability of the fixed point: the removal of the degeneracy of the electronic subbands with different m in Hamiltonian (1) must lead to a change in the scaling behavior, so that the anomalous temperature dependences will remain only in a limited interval. The application of the two-channel model to rare-earth and actinide systems and the corresponding difficulties of the interpretation are discussed in Ref. [83]. In the case of uranium systems, the realization of this model is possible owing to the symmetry of the subbands relative to the time reversal.

5. Problem of the ground state. Mean-field approximation in the pseudofermion representation

The methods of the perturbation theory and scaling examined in Sections 3 and 4 do not make it possible to describe the ground state of Kondo lattices in the strong coupling regime (however, study [109] should be noted, in which this description was obtained within the single-impurity model by the diagrammatic method). Therefore, the problem of the strongly correlated ground state of dense Kondo systems is very complex.

Of fundamental importance is the fact that the f electrons in systems with heavy fermions behave unconventionally: they exhibit delocalization, which is confirmed by the observation of large electron effective masses in the experiments on the de Haas–van Alphen effect. This behavior is very nontrivial in the s–f exchange model (which distinguishes the latter from the Anderson model with the f states near the Fermi level, where there is a bare s–f hybridization and the state of intermediate valence appears). Such a delocalization is similar to the appearance of a Fermi branch of excitation in the theory of the resonant valence bonds for high-temperature superconductors, so that the Fermi surface can be spinonic and not only in the two-dimensional but also in the three-dimensional cases [110]. As a result of the collectivization of the f states, the Luttinger theorem on the constancy of the volume under the Fermi surface is violated and the so-called large Fermi surface appears (this change in the statistics is in part analogous to the formation and destruction of Hubbard subbands [111]).

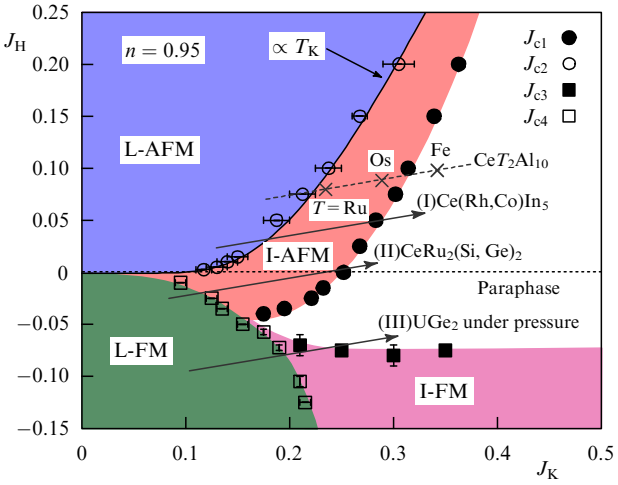

Figure 4. Phase diagram of a Kondo–Heisenberg lattice in the $J_K - J_H$ plane (where $J_K = -2I$) at $T = 0.001$ [112]. The abbreviations L and I mean 'localized' and 'itinerant', respectively. The values along the axes are given in the units of the bandwidth.

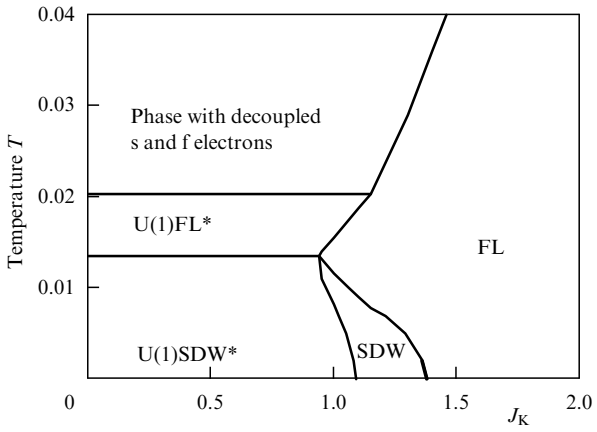

Figure 5. Magnetic phase diagram in the mean-field approximation (borrowed from Ref. [110]). $J_K = -2I$. The values along the axes are given in units of the transfer integral.

In the ground state, a quantum phase transition to the metallic state with the conventional small Fermi surface can occur upon a change in the interaction parameter. Such a jump of the Fermi surface is indeed observed in $YbRh_2Si_2$ and $CeRhIn_5$ [92]. However, the question remains of how does the magnetism form in this case: whether the magnetic ordering appears upon the destruction of the Kondo state or the magnetic transition occurs in the borders of the Kondo phase via the formation of a spin density wave (SDW). In the latter case, the formation of small moments occurs in a natural way, at least in the case of antiferromagnetism. At the same time, the magnetic ordering can by itself substantially influence the band structure and the Fermi surface by changing the Brillouin zone.

The transition from the small to the large Fermi surface can be considered as a transition from localized to itinerant magnetism. The corresponding consideration based on Monte Carlo calculations in combination with the dynamical mean-field theory (DMFT) method is carried out in Ref. [112] in connection with phase transitions in $CeRh_{1-x}Co_xIn_5$, $CeRu_2(Si_xGe_{1-x})_2$, and UGe_2, CeT_2Al_{10} (T = Fe, Ru, Os) systems. To this end, both ferromagnetic and antiferromagnetic Heisenberg interactions have been considered (Fig. 4).

The competition of magnetic interactions (frustrations) can lead to a variety of phase transitions, including those between the magnetic and paramagnetic states with large and small Fermi surfaces (see discussion in Refs [92, 113, 114]). For example, the Kondo screening in $Ce_3Pd_{20}Si_6$ and $YbRh_2Si_2$ apparently disappears upon the magnetic transition, and, upon alloying the latter compound with cobalt or iridium, two separate phase transitions appear [115]. According to neutron diffraction studies for $CeCu_2Si_2$, the magnetic ordering is connected with the nesting effect of the large Fermi surface of heavy quasiparticles [116]. The exotic magnetic state in CePdAl caused by frustrations is discussed in Ref. [117]. The role of frustrations in the physics of Kondo lattices was discussed earlier in Refs [118, 119].

A new type of Fermi-liquid state called FL* was proposed in Ref. [110]. In the FL* state, the localized moments do not take part in the formation of the Fermi surface, but they are adiabatically connected with the spin liquid described by the gauge theory and possessing relevant exotic excitations in the deconfinement phase. In the case of spatial dimensionalities $d \geqslant 2$, a spin liquid of the Z_2 type is stable; however, of greater interest is the spin liquid U(1) that exists for $d \geqslant 3$. In this phase, the electron heat capacity coefficient C/T diverges logarithmically. Against the background of such a state, a metallic magnetic state SDW* appears, which can be characterized by a small moment. In this case, magnetic instability is developed as the wave of spin density for the spinonic Fermi surface. With an increase in the coupling constant, a transition occurs to the conventional (but heavy) Fermi liquid (FL) (the phase diagram is illustrated in Fig. 5).

In Ref. [110], a field that stabilized the unsaturated state with a small moment was artificially introduced. A certain modification of the model and of the physical picture in Ref. [114] made it possible to avoid the introduction of such a field and a state in which the Kondo effect coexists with antiferromagnetism was obtained, although with an unrealistically high s–f exchange parameter.

Let us now turn to the discussion of the mathematical formalism. In the ground state of anomalous f-systems, the Kondo compensation (formation of a singlet) should be considered in the zero approximation. To describe the ground state of the Kondo lattice in the regime of strong coupling, a special mean-field approximation was developed, within which Coleman and Andrei [120] considered the formation of the state of a spin liquid in the two-dimensional case. This approximation was then applied to the cases of ferro- and antiferromagnetic ordering [86, 110, 121, 122]. Within such an approach, a pseudofermionic representation is used for the operators of localized spins $S = 1/2$:

$$\mathbf{S}_i = \frac{1}{2}\sum_{\sigma\sigma'} f_{i\sigma}^{\dagger}\boldsymbol{\sigma}_{\sigma\sigma'} f_{i\sigma'}\,, \tag{30}$$

with an auxiliary condition

$$f_{i\uparrow}^{\dagger} f_{i\uparrow} + f_{i\downarrow}^{\dagger} f_{i\downarrow} = 1\,. \tag{31}$$

Earlier, a similar idea on the introduction of pseudofermions was suggested in Ref. [123]. Variants related to the use of auxiliary bosons [124] prove to be substantially less successful, since they are justified only in the limit of large degeneracy N (at finite N and, especially, at $N = 2$, they give an exponent

in the expression for the Kondo temperature that is different from that for the one-impurity case). An analogous problem arises in approaches of the type used in the Gutzwiller method [125].

Using the saddle-point approximation for the path integral which describes the spin–fermion interaction system [120], the Hamiltonian of the s–f exchange interaction can be reduced to an effective hybridization model:

$$-I\sum_{\sigma\sigma'} c_{i\sigma}^{\dagger}c_{i\sigma'}\left(\boldsymbol{\sigma}_{\sigma\sigma'}\mathbf{S}_i-\frac{1}{2}\,\delta_{\sigma\sigma'}\right) \to \mathbf{f}_i^{\dagger}V_i\,\mathbf{c}_i+\mathbf{c}_i^{\dagger}V_i^{\dagger}\mathbf{f}_i-\frac{1}{2I}\,\mathrm{Sp}\,(V_iV_i^{\dagger})\,, \tag{32}$$

where the following designations were introduced: $\mathbf{f}_i^{\dagger}=(f_{i\uparrow}^{\dagger},f_{i\downarrow}^{\dagger})$, $\mathbf{c}_i^{\dagger}=(c_{i\uparrow}^{\dagger},c_{i\downarrow}^{\dagger})$, and V is the effective hybridization matrix, which is determined from the condition of the free-energy minimum.

Thus, the f-pseudofermions in this situation become itinerant by themselves. In the mean-field approximation, such a picture encounters some conceptual difficulties. In particular, a problem arises concerning the elimination of nonphysical states in the Hilbert space: condition (31) is taken into account only on the average by the introduction of a chemical potential of pseudofermions (in the approach under consideration, the numbers of pseudofermions and electrons are preserved separately, so that $n_{\mathrm{f}}=1$). The true wave function should be obtained from the mean-field function upon the application of an additional Gutzwiller projection operator, which eliminates the unoccupied and doubly occupied f-states at a site [120]. Similar problems in the spin-liquid picture have been discussed in detail in Refs [126, 127], where fluctuation effects leading to interactions with boson gauge fields were considered. It should be noted that the Kondo effect in the s–f exchange model can lead to the stabilization of exotic states like spin-liquid, in comparison with the situation in the simple Heisenberg model [120].

In the ferromagnetic state, we have

$$H-\mu\hat{n}=\sum_{\mathbf{k}\sigma}\left[(t_{\mathbf{k}}-\mu)c_{\mathbf{k}\sigma}^{\dagger}c_{\mathbf{k}\sigma}+W_{\sigma}f_{\mathbf{k}\sigma}^{\dagger}f_{\mathbf{k}\sigma} +V_{\sigma}(c_{\mathbf{k}\sigma}^{\dagger}f_{\mathbf{k}\sigma}+f_{\mathbf{k}\sigma}^{\dagger}c_{\mathbf{k}\sigma})\right]-\sum_{\sigma}\frac{V_{\sigma}^2}{I}\,, \tag{33}$$

where V_{σ} is the effective hybridization, and

$$W_{\sigma}=W-\sigma J_0\bar{S}\,,$$

where W is the position of the f-level counted from the electron chemical potential μ, the quantity $-W$, which is on the order of the Kondo temperature T_{K}, is the chemical potential of the pseudofermions, and $J_0=J(\mathbf{q}=0)$.

The corresponding density of states has a hybridization form and includes sharp peaks of the pseudofermionic states:

$$N_{\sigma}(E)=\left[1+\frac{V_{\sigma}^2}{(E-W_{\sigma})^2}\right]\rho\left(E-\frac{V_{\sigma}^2}{E-W_{\sigma}}\right). \tag{34}$$

In this case, the capacity of the band is twice as large as that of the bare band. Furthermore, the function $N(E)$ reproduces and even strengthens the singularities in the bare density of states.

Let us first discuss the half-metallic ferromagnetic (HMF) solution, for which the chemical potential lies in the energy gap at $\sigma=\uparrow$ (Fig. 6), so that

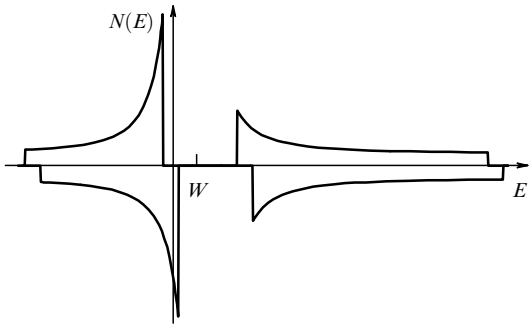

Figure 6. Partial densities of states for the spin up (upper half of the figure) and spin down (lower half of the figure) in the case of a rectangular band [122]. The ordinate axis is drawn through the value of the chemical potential for the half-metallic ferromagnetic state. Notice that the polarizations of the conduction electrons and pseudofermions are opposite.

$$W_{\downarrow}>\frac{V_{\downarrow}^2}{D-\mu}\,,\qquad -\frac{V_{\uparrow}^2}{\mu}<W_{\uparrow}<\frac{V_{\uparrow}^2}{D-\mu}\,. \tag{35}$$

Since the capacity of the hybridization subband is equal to unity, we have

$$n_{\uparrow}^{\mathrm{f}}\simeq 1-\frac{n}{2}\,,\qquad n_{\downarrow}^{\mathrm{f}}\simeq\frac{n}{2}\,,\qquad \bar{S}\simeq\frac{1-n}{2}$$

(where n is the concentration of conduction electrons), whereas the magnetization of conduction electrons is small, $n_{\uparrow}\simeq n_{\downarrow}\simeq n/2$. In this state, each conduction electron compensates for one localized spin because of the negative sign of the s–f-exchange parameter I, and the magnetic ordering is caused by the exchange interaction of the uncompensated moments. This picture resembles the situation taking place in narrow-band ferromagnets in the Hubbard model or the s–d exchange model with a strong intraatomic interaction (double exchange). However, the bare interaction is small in our case, but the effective interaction in the regime of strong coupling is large.

For a rectangular band ($\rho(E)=1/D, 0<E<D$), the condition for the existence of the HMF solution takes on the form

$$j=\frac{J_0}{W_0}<\frac{2}{n(1-n)} \tag{36}$$

(W_0 corresponds to the paramagnetic state). Thus, the HMF solution exists at the arbitrarily small J_0, as in the case of a Heisenberg magnet. In the mean-field approximation, the total energy $\mathcal{E}$ of the magnetic state is always lower than the energy $\mathcal{E}_{\mathrm{nonmag}}$ of the nonmagnetic Kondo state, and $\mathcal{E}$ decreases with increasing moment in the following way:

$$\mathcal{E}-\mathcal{E}_{\mathrm{nonmag}}=-J_0\bar{S}^2\,. \tag{37}$$

The energy of the ferromagnetic Kondo state should also be compared with the energy of the conventional (Heisenberg) ferromagnetic state, in which the Kondo effect is suppressed ($W=0$, $\bar{S}=1/2$). It becomes energetically advantageous at sufficiently large values of j:

$$j>j_{\mathrm{c}}=\frac{1}{1/4-\bar{S}^2}\,. \tag{38}$$

At the critical point, a magnetic first-order phase transition must occur, which is also confirmed by the general phenomenological consideration in Ref. [128].

Notice that the picture of half-metallic magnetism is tightly connected with the hybridization nature of the spectrum, as in the case of intermetallic d-systems [129].

Thus, the HMF state is sufficiently stable. In paper [130], this state was rediscovered in the Kondo model within the framework of the DMFT and was called the spin-selective Kondo insulator. In Refs [131, 132], such a state was obtained within the framework of the Hartree–Fock approximation where, together with the Kondo anomalous averages, decouplings were considered that describe the magnetic RKKY interaction (polarization of conduction electrons). It should be noted that in the last approach the Kondo state appears only at large $|I|$, on the order of the half-width of a band, so that the low energy scale T_K is already absent. To construct a quantitative theory, it is probably necessary to take into account the renormalization of the bare parameters, as was noted in Section 3.

Now, let us discuss the unsaturated ferromagnetic solution with a magnetization $\bar{S} < (1-n)/2$, where $W_\sigma > V_\sigma^2/(D-\mu)$ for both σ and the Fermi level lies in the lower hybridization subband (lower than the energy gap), just as in the nonmagnetic case.

Taking into account the renormalizations of the hybridization in our model, it is possible to obtain a self-consistent equation for the magnetization:

$$\frac{J_0\bar{S}}{W} = L(\bar{S},n)\,,$$
$$L(\bar{S},n) = \tanh\left(\frac{1}{2\rho_n}\int_{\mu(n+1-2\bar{S})}^{\mu(n+1+2\bar{S})} \mathrm{d}E\ \frac{\rho(E)-\rho}{E-\mu}\right), \tag{39}$$

and an expression for the renormalized Kondo temperature (energy of the f-level):

$$W = W_0 P(\bar{S},n)\,,$$
$$P(\bar{S},n) = \frac{1}{2}\sum_\sigma \exp\left(\frac{1}{\rho}\int_{\mu(n+1)}^{\mu(n+1+2\sigma\bar{S})} \frac{\rho(E)-\rho}{E-\mu}\ \mathrm{d}E\right). \tag{40}$$

These equations can be represented as

$$j\bar{S} = P(\bar{S},n)L(\bar{S},n)\,, \qquad j = \frac{J_0}{W_0}\,. \tag{41}$$

The solutions with $\bar{S} \neq 0$ can exist if the left-hand and right-hand sides of equation (39) are on the order of unity, i.e., $J_0 \sim W_0$. However, the realistic conditions for this case prove to be quite tough (this situation is similar to that in the case of the Hubbard-I approximation, where there is a strong dependence of the criterion for magnetism on the bare density of states [111]). In particular, equation (39) does not have nontrivial solutions at $\rho(E) = \text{const}$: the magnetization appears only as a result of the energy dependence of ρ.

The necessary condition for the existence of a solution with small $\bar{S}$ takes on the following form:

$$k_n = \left.\frac{\mathrm{d}L(\bar{S},n)}{\mathrm{d}\bar{S}}\right|_{\bar{S}=0} = \frac{1}{\rho_{n+1}\rho_n}\,\frac{\rho_{n+1}-\rho_n}{\mu_{n+1}-\mu_n} > 0\,, \tag{42}$$

where $\rho_n = \rho(\mu_n)$. Thus, in contrast to the criterion for half-metallic ferromagnetism, which is determined by the global behavior of $\rho(E)$, the criterion for ferromagnetism with small $\bar{S}$ is determined as $\rho(E \simeq \mu_{n+1})$ (which corresponds to a 'large' Fermi surface). The situation for the appearance of solutions with small $\bar{S}$ is more favorable at large n and in the presence of high narrow peaks in $\rho(E)$.

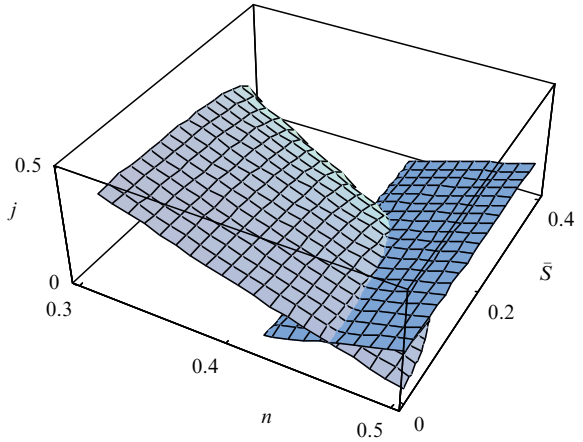

Figure 7. Diagram of the surface of existence of the unsaturated ferromagnetic solution for the bare semielliptical band according to equation (41) (left-hand sheet) [122]. The saturated (half-metallic) solution exists above the right-hand sheet.

For simple lattices with a symmetrical bare density of states, the unsaturated solution can exist only in the region where $n < 1/2$ (when $\rho_{n+1} < \rho_n$). In particular, for the semielliptical band as $J_0 \to 0$, the ferromagnetism disappears for $n > 0.5$, and the saturated ferromagnetic solution comes about for $n < 0.42$ (Fig. 7).

The condition for the occurrence of ferromagnetism with $\bar{S} \to 0$ is as follows: $k = j$, so that the ferromagnetic solution originates with $j = k$, and with decreasing j the moment grows. This unconventional behavior is connected with a peculiar character of the unsaturated solution. In this state, there are two competing tendencies. The growth of the exchange splitting $2J_0\bar{S}$ with increasing j 'pushes' the chemical potential in the energy gap, but the sharp pseudofermionic peak does not let itself to be crossed and remains higher than the Fermi level. Thus, the magnitude of $\bar{S}$ must decrease with increasing j. Notice that in the case of the RKKY exchange (when $J \sim I^2\rho$, and T_K is exponentially small), the growth of j corresponds to a decrease in $|I|$ and, therefore, the situation is not so paradoxical as it seems at first glance.

As was shown by numerical calculations [122], the unsaturated solution exists for simple square and cubic lattices only inside the domain of existence of the saturated solution and is energetically disadvantageous in comparison with it. For the appearance of magnetism with a small moment, a situation is favorable in which a narrow peak of the density of states resides at μ_{n+1} or a little higher, so that the ordering shifts it from the Fermi level, which turns out to be in a weak local minimum for states with the spin up (Fig. 8).

The region of the unsaturated solution for the bare density of states with a Lorentzian peak is shown in Fig. 9. It can be seen that magnetism with a small magnetic moment appears in a rather wide region near the values of n where $\mu_{n+1} = E_1$, and the HMF state does not exist in this region of concentrations, at least at small j.

Thus, Kondo ferromagnetism, in comparison with the Stoner picture for conventional systems of itinerant electrons,

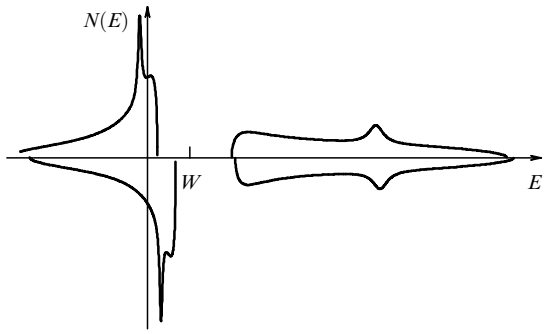

Figure 8. Partial densities of states for a semielliptical band with a Lorentzian peak.

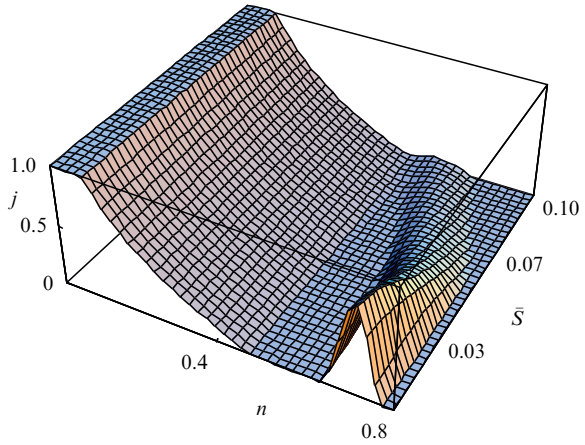

Figure 9. Diagram of the surface of existence of the unsaturated ferromagnetic solution for the semielliptical band with a Lorentzian peak ($\rho(E) = \rho_0(E) + h\Gamma/[(E - E_1)^2 + \Gamma^2]$) at $\Gamma = 0.01D$, $E_1 = 0.8D$, and $h = 0.01$ [122].

has a number of special features. In particular, the dependence of the ferromagnetism criterion on the bare density of states is much more complex. The reason is that the key role for this criterion is played by the spin dependence of the effective hybridization.

It should be noted that the above approach gives a universal description in terms of the relation $j \sim J_0/T_K$ and the functions depending only on the bare density of states rather than on s–f exchange parameter I.

The hybridization form of the electronic spectrum (existence of peaks in the density of states) in the Kondo lattices is confirmed by numerous experimental studies: by direct optical data [133], by the observation of large effective electron masses in the measurements of the de Haas–van Alphen effect, etc. Sometimes, it is difficult to clearly distinguish the regimes of the intermediate valence (IV) and the Kondo effect in the formation of magnetism, since the f-states in both cases play an important role in the electronic structure near the Fermi level. The occurrence of real hybridization among the s, d, and f states in the IV systems can lead to the appearance of a peak in the bare density of states, which, in turn, will affect the pseudofermionic ferromagnetism. The ferromagnetic $CeRh_3B_2$ system may serve here as a possible example with a small moment and a high Curie temperature, which was ascribed to the strong d–f hybridization. At the same time, the analysis of data of

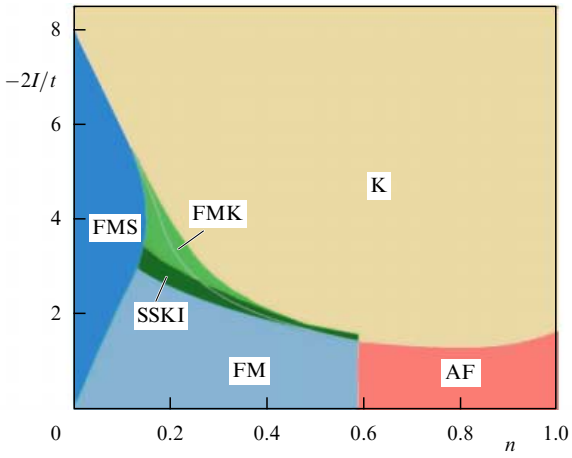

Figure 10. Magnetic phase diagram for a Kondo model on a square lattice [131]. FMS and FM are the saturated and unsaturated ferromagnetism without the Kondo compensation; SSKI and FMK are the phases of the co-existence of the Kondo effect and saturated (half-metallic) and unsaturated ferromagnetism, respectively.

neutron-diffraction studies for this system did not reveal magnetization at the rhodium and boron sites, so that the ferromagnetism in this system is connected with the ordering of the cerium local moments rather than to the itinerant magnetism in the rhodium band, as it was assumed earlier [134].

In the case of antiferromagnetic ordering, the electronic spectrum is distorted by both the Kondo type hybridization and antiferromagnetic gaps. Here, the analytical study of the equations of the mean-field theory is hindered, and they should be solved numerically. In the case of antiferromagnetism with small moments, the dependence of the hybridization on σ can be ignored (which is inadmissible for the case of ferromagnets), since the corrections due to spin polarization have the structure of $(J_{\mathbf{Q}}\bar{S})^2/(t_{\mathbf{k}+\mathbf{Q}} - t_{\mathbf{k}})$ (where $\mathbf{Q}$ is the wave vector of the magnetic structure) and are proportional to $(J_{\mathbf{Q}}\bar{S})^2/D$. Thus, the antiferromagnetism criterion takes on the conventional form:

$$J_{\mathbf{Q}}\chi_{\mathbf{Q}} > 1\,, \tag{43}$$

where $\chi_{\mathbf{Q}}$ is the susceptibility of noninteracting f-pseudofermions in the effective hybridization model. Due to the hybridization peaks, the contribution from the interband transitions proves to be large: $\chi_{\mathbf{Q}} \sim 1/T_K$. Consequently, antiferromagnetism appears at

$$J_{\mathbf{Q}} > \nu T_K\,, \tag{44}$$

where the constant $\nu \sim 1$ is determined by the band structure. It should, however, be noted that, according to numerical results for a square lattice [131], the magnetic moment is not small, the phase diagram is rather complex (Fig. 10), and the coexistence of the Kondo and antiferromagnetic states takes place at quite large $|I|$ and J. For consistent consideration, it is also important here to take into account the spiral magnetic states [135]. These problems require further study.

The question of how the magnetism is affected by fluctuations beyond the limits of the mean-field approximation also presents a nontrivial problem. The simple spin-wave

corrections, which were discussed in Ref. [86], make only formally small contributions to the magnetization of the ground state, which are on the order of $j \ln j$ (and which, furthermore, are absent in the HMF state). A more consistent consideration of fluctuations can be carried out with the employment of the approach of auxiliary bosons and a $1/N$ expansion within the framework of the Anderson or Coqblin–Schrieffer periodic models [136]. For the case of an exotic Fermi liquid, the fluctuations were studied in Ref. [110] within the framework of the pseudofermionic concept with the simultaneous use of an auxiliary boson (a Higgs field type).

The behavior of Kondo magnets at finite temperatures is usually treated in the context of a theory that is analogous to the Stoner theory with a hybridization spectrum [131, 132], which is mainly not justified: the fluctuation effects must be in this case even more important than in the ground state (as in conventional band magnets [95]). These effects are discussed in Ref. [110], where it is shown that the temperature-induced phase transition from the Kondo state to a conventional Fermi liquid is, in reality, smooth (a crossover).

6. Conclusions

The magnetism of Kondo systems is a complex phenomenon: Kondo lattices exhibit features of magnets with both itinerant electrons and with localized moments. Under certain conditions, the magnetic instability can take place at very low (even in comparison with T_K) values of intersite exchange J, which is characteristic of Heisenberg systems. On the other hand, the magnetic ordering is very sensitive to the electronic structure, just as in the case of itinerant systems. This duality manifests itself especially vividly in the approaches that use the idea of pseudofermions, which practically become real and participate in the formation of the Fermi surface.

The described mechanisms of forming the magnetic state with a suppressed saturation moment μ_s differ significantly from the conventional mechanism for band ferromagnets, which, as is assumed, in the rough approximation are described by Stoner's theory. However, since both the energy spectrum of new Fermi quasiparticles and the effective interaction among them undergo strong renormalizations, the Stoner criterion is not applicable for Kondo ferromagnets (or, in the case of antiferromagnetism, the Overhauser criterion for the formation of the SDW) with the bare parameters. Rather, it can be applied to the effective model in the strongly correlated state with a 'large' Fermi surface; however, the situation here turns out to be substantially more complex as well.

As we saw, the modern theory describes the magnetic Kondo lattices as highly correlated systems. Since there is a continuous transition between the Kondo lattices and 'standard' systems of itinerant electrons (in particular, conventional Pauli paramagnets can be considered systems with a high T_K, on the order of the Fermi energy), a question arises about the role played by many-electron effects in 'classical' weak band magnets such as $ZrZn_2$. It may be that the proximity of the ground state to the point of the Stoner instability, i.e., the smallness of μ_s, in the latter systems appears as a result of the renormalizations of the interaction parameter U and of the chemical potential, rather than because of the random character of the bare values of $N(E_F)$. Indeed, such a randomness is hardly probable, since the deviation from the Stoner boundary condition $UN(E_F) = 1$ is extremely small.

All this gives a new view on the physics of conventional weak itinerant electron magnets: they can be considered based not on the band theory, but rather on the concept of local magnetic moments that are almost compensated. At present, it is already normal to discuss Kondo lattices and band magnets together [137, 138] and to treat UPt_3, $CeSi_x$, and $CeRh_3B_2$ compounds as weak itinerant magnets (see, e.g., Ref. [139]). Thus, the theory of Kondo magnets acquires significant importance for the general theory of magnetism.

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