

METHODOLOGICAL NOTES

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Nondiagonal cross-transport phenomena in a magnetic field

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Contents

Abstract. **This brief paper supplements the review by A F Barabanov et al. (*Physics–Uspekhi* 58 446 (2015)) concerning the Hall and Righi–Leduc effects. Both effects are diagonal in the sense that the initial current perpendicular to the magnetic field and the transverse response (perpendicular to both the magnetic field and the initial current) are of the same nature (the electric current in the Hall effect and the heat current in the Righi–Leduc effect). We here take a similar perspective in discussing the nondiagonal Ettingshausen and Nernst effects, in which the transverse current is different in nature from the initial longitudinal one. A summary of transverse effects in a magnetic field is also given.**

Keywords: Hall effect, anomalous Hall effect, spin Hall effect, magnon Hall effect, Righi–Leduc effect, Ettingshausen effect, Nernst effect

1. Introduction

There was a recent discussion [1] on transport phenomena that are odd with respect to a magnetic field: the Hall effect (electric conductivity) and its thermomagnetic analog, the Righi–Leduc effect (thermal conductivity). It was demonstrated in [1] that either effect can be of both a *dynamic* character (magnetically curved trajectories, the left-hand side of the kinetic equation) and a *dissipative* character (the effect of the magnetic field on the collision term, the right-hand side of the kinetic equation). A number of mechanisms that implement either of these channels were also presented in [1].

Both these effects are *diagonal* in the sense that the transverse current that arises in a magnetic field is of the same nature as the original longitudinal current. For the Hall effect [2], this is the transverse component of the electric field

$$\mathbf{J} \sim \mathbf{B} \times \mathbf{E}\,.$$

For the Righi–Leduc effect [3, 4], this is the transverse component of the heat flow

$$\mathbf{q} \sim \mathbf{B} \times \nabla T\,,$$

where $\mathbf{B}$ is the magnetic induction vector.

It makes sense to extend this list of phenomena by including the *transverse* effects in which the transverse current that arises in a magnetic field is of a different nature than the original longitudinal current. One of these is the Ettingshausen effect (see Ref. [5] and its reprints [6–11]), in which a transverse heat flow

$$\mathbf{q} \sim \mathbf{B} \times \mathbf{E}$$

arises in a current-carrying sample placed in a magnetic field. The second is the Nernst effect, also known as the Nernst–Ettingshausen effect [12] (see also reprints [13–18]), i.e., the appearance of a transverse current

$$\mathbf{J} \sim \mathbf{B} \times \nabla T$$

in a sample with a longitudinal heat flow placed in a magnetic field. (As mentioned in Ref. [1], the copious bibliography is due to the common nineteenth century practice of publishing a paper in several journals in different countries, resulting in the fact that any reference could be cited in later bibliographies.)

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Although the calculations below are in many respects similar to those in Ref. [1], we repeat them here for the reader's convenience. We assume for simplicity that the body is isotropic and $\mathbf{B} \parallel z$. As before, we consider the case of a weak magnetic field and ignore the quantum Hall effect. Because our concern is with the transverse effects, only results linear in the magnetic field are of interest to us.

2. Nondiagonal effects

Similarly to the discussion in Ref. [1], there are two mechanisms for effects that are odd in a magnetic field. The first is the bending of the current- or energy-carrier paths in a magnetic field, and the second is the scattering of carriers by magnetic impurities (or magnetization fluctuations).

In metals, kinetic phenomena are dominated by conduction electrons. The Boltzmann equation has the form

$$\frac{\partial f}{\partial t} + (\mathbf{V}\nabla) f + \mathbf{F}\,\frac{\partial f}{\partial \mathbf{p}} + \mathrm{St}\, f = 0\,, \qquad (1)$$

where f is the distribution function, $\mathbf{V}$ is the velocity, $\mathbf{p}$ is the momentum, $\mathbf{F}$ is the external force, and St is the collision operator. For the reason that is clarified below, the sign of the collision term is redefined in [1] compared with the standard definition.

We consider the steady-state case, $\partial f/\partial t = 0$. Equation (1) for the magnetic field $\mathbf{B}$ perpendicular to the sample surface, the longitudinal electric field $\mathbf{E}$, and the temperature gradient ∇T (Fig. 1) takes the form

$$(\mathbf{V}\nabla) f + \left(e\mathbf{E} + \frac{e}{c}\,[\mathbf{V}\times\mathbf{B}]\right)\frac{\partial f}{\partial \mathbf{p}} + \mathrm{St}\, f = 0\,. \qquad (2)$$

By introducing a nonequilibrium correction to the Fermi distribution function, $f^{(1)} = f - f^{(0)}$, and keeping linear terms in the generalized forces (the electric field and the temperature gradient), we obtain

$$(\mathbf{V}\nabla) f \approx (\mathbf{V}\nabla) f^{(0)} = (\varepsilon-\mu)(\mathbf{V}\nabla)\ln T\left|\frac{\partial f^{(0)}}{\partial \varepsilon}\right|, \qquad (3)$$

where $\varepsilon-\mu$ is the energy measured from the chemical potential level; as usual, we disregard the term $\sim \nabla\mu$,

$$e\mathbf{E}\,\frac{\partial f}{\partial \mathbf{p}} \approx e\mathbf{E}\,\frac{\partial f^{(0)}}{\partial \mathbf{p}} = -e\mathbf{E}\mathbf{V}\left|\frac{\partial f^{(0)}}{\partial \varepsilon}\right|, \qquad (4)$$

and in the last two terms in Eqn (2), the only nonzero contribution comes from the nonequilibrium term $f^{(1)}$. Finally, the Boltzmann equation becomes

$$\mathbf{V}\big(-e\mathbf{E} + (\varepsilon-\mu)(\nabla\ln T)\big)\left|\frac{\partial f^{(0)}}{\partial \varepsilon}\right| + \frac{e}{c}\,[\mathbf{V}\times\mathbf{B}]\,\frac{\partial f^{(1)}}{\partial \mathbf{p}} + \mathrm{St} f^{(1)} = 0\,. \qquad (5)$$

We first consider the *Ettingshausen effect*. We find the heat flow

$$q_i = \sum_{p\sigma}(\varepsilon-\mu)V_i f^{(1)} \qquad (6)$$

produced by the electric field $\mathbf{E} \neq 0$ when $\nabla T = 0$. The Boltzmann equation is then written as

$$-e(\mathbf{V}\mathbf{E})\left|\frac{\partial f^{(0)}}{\partial \varepsilon}\right| + \frac{e}{c}[\mathbf{V}\times\mathbf{B}]\,\frac{\partial f^{(1)}}{\partial \mathbf{p}} + \mathrm{St} f^{(1)} = 0\,. \qquad (7)$$

We now simplify Eqn (7) by introducing the notation

$$f^{(1)} = eE_i\left|\frac{\partial f^{(0)}}{\partial \varepsilon}\right|\chi_i, \quad \hat{\Lambda} = \frac{e}{c}\,[\mathbf{V}\times\mathbf{B}]_l\,\frac{\partial}{\partial p_l}\,,$$

$$\mathrm{St} f^{(1)} = \hat{\Omega} f^{(1)} = eE_i\left|\frac{\partial f^{(0)}}{\partial \varepsilon}\right|\hat{\Omega}\chi_i\,, \quad \langle\varphi\rangle = \sum_{p\sigma}\left|\frac{\partial f^{(0)}}{\partial \varepsilon}\right|\varphi\,,$$

which gives

$$-eE_i\left|\frac{\partial f^{(0)}}{\partial \varepsilon}\right|V_i + eE_i\left|\frac{\partial f^{(0)}}{\partial \varepsilon}\right|\hat{\Lambda}\chi_i + eE_i\left|\frac{\partial f^{(0)}}{\partial \varepsilon}\right|\hat{\Omega}\chi_i = 0\,, \qquad (8)$$

which, after canceling $eE_i\,|\partial f^{(0)}/\partial\varepsilon|$, transforms into the equation for the standard (diagonal) Hall effect (Eqn (8) in Ref. [1]):

$$V_i = (\hat{\Lambda} + \hat{\Omega})\chi_i\,. \qquad (9)$$

We next calculate the heat flow:

$$q_i = \sum_{p\sigma}(\varepsilon-\mu)V_i f^{(1)} = \big\langle(\varepsilon-\mu)V_i\chi_k\big\rangle eE_k = \big\langle\chi_k(\varepsilon-\mu)(\hat{\Lambda}+\hat{\Omega})\chi_i\big\rangle eE_k\,. \qquad (10)$$

Here, $\chi_i = \chi_i^{(0)} + \chi_i^{(1)}$, up to the canceled factors, is the nonequilibrium part of the distribution function, where $\chi_i^{(0)}$ and $\chi_i^{(1)}$ are the zeroth and first-order contributions in the magnetic field. It can then be shown that the transverse part of the heat flow that is linear in the magnetic field has

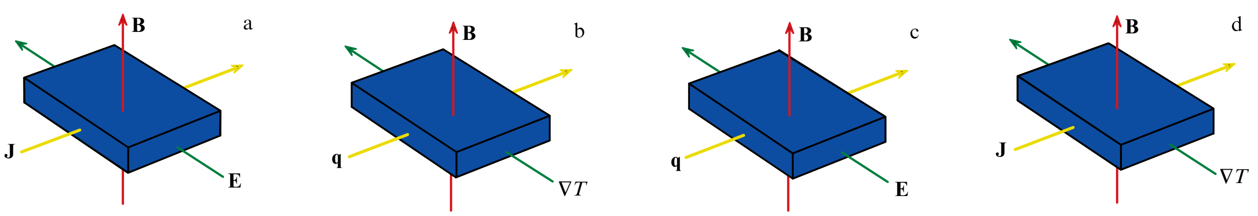

Figure 1. (Color online.) Geometry of the standard transverse effects (an external magnetic field). Diagonal: (a) Hall effect, (b) Righi–Leduc effect. Nondiagonal: (c) Ettingshausen effect, (d) Nernst (or Nernst–Ettinshausen) effect. The red, green, and yellow arrows in Figs a–c indicate the magnetic field, original flow, and transverse (Hall) flow. $\mathbf{B}$ is the magnetic field (the Weiss field, $\mathbf{B}_{\mathrm{eff}} = \gamma\mathbf{M}$, $\mathbf{B}_{\mathrm{eff}}$ is the effective magnetic field, $\mathbf{M}$ is the spontaneous magnetization), $\mathbf{E}$ is the electric field, ∇T is the temperature gradient, $\mathbf{J}$ is the transverse electric current component, and $\mathbf{q}$ is the transverse component of the heat flow. The transverse flow is given up to a sign to provide uniformity in the figure.

the form

$$q_i^{\text{lin}} = \langle \chi_k^{(0)}(\varepsilon - \mu)\hat{\Lambda}\chi_i^{(0)} \rangle e E_k \,. \tag{11}$$

For the Hall effect, the energy in Eqn (11) is simply replaced by the electron charge: in the Ettingshausen effect, the integrand in the integral for the kinetic effect odd in the field is qualitatively different only in the energy measured from the Fermi level, and hence has a sharper dependence on the ratio T/μ. For example, if we make the τ-approximation $\chi_i^{(0)} = \tau V_i$, then

$$\langle \chi_k^{(0)}(\varepsilon - \mu)\hat{\Lambda}\chi_i^{(0)} \rangle = \tau^2 \left\langle V_k(\varepsilon - \mu)\,\frac{e}{c}\,[\mathbf{V}\times\mathbf{B}]_l\,\frac{\partial}{\partial p_l}\,V_i \right\rangle \sim e_{ikz} B T^3 \,. \tag{12}$$

This result is obtained by assuming the dominant role of the 'dynamic' mechanism whereby the Lorentz force $(e/c)[\mathbf{V}\times\mathbf{B}]$ bends the trajectory of an electron. However, in a metal with strong spin–orbit coupling, a transverse flow arises even in the absence of an external magnetic field, and in the linear order in the magnetization $\mathbf{M}$, the Lorentz force term in Eqn (2) is replaced by

$$\hat{\Lambda}^M = \frac{e}{c}\,[\mathbf{V}\times\mathbf{B}_{\text{eff}}]_l\,\frac{\partial}{\partial p_l} \,. \tag{13}$$

Thus, we here have nothing other than the anomalous Ettingshausen effect. Here, $\mathbf{B}_{\text{eff}} = \gamma\mathbf{M}$ is the effective Weiss field.

A 'dissipative' mechanism related to the anisotropic scattering of an electron by a magnetization fluctuation (alternatively, the term 'asymmetric scattering' is used) also exists. In the simplest case, this is the spin–orbit contribution to the Hamiltonian, proportional to

$$\sum \mathbf{M}[\mathbf{r}\times\mathbf{p}] \,, \tag{14}$$

where the summation is over all particles.

As shown in Ref. [1], to evaluate the magnitude of the 'dissipative' mechanism, it suffices to drop the $\hat{\Lambda}$ term in Boltzmann equation (9) and replace it with the spin–orbit frequency $\hat{\Omega}^{(\text{SL})}$,

$$\left(\hat{\Omega}^{(\text{SL})}\right)_{ik} \sim e_{ikz} B \,. \tag{15}$$

This again produces an equation like (12) (details of the calculation leading from Eqn (12) to Eqn (15) can be found in Ref. [1]).

We briefly comment on the *Nernst effect*. In this case, we need to find the electric current density

$$J_i = \sum_{p\sigma} e V_i f^{(1)} \tag{16}$$

produced by the temperature gradient $\nabla T \neq 0$ for $E = 0$. Then the electric field term vanishes in Eqn (5), and the only remaining term is the gradient term

$$\mathbf{V}(\varepsilon - \mu)(\nabla \ln T) \,, \tag{17}$$

and therefore the Boltzmann equation becomes

$$\mathbf{V}(\varepsilon - \mu)(\nabla \ln T)\left|\frac{\partial f^{(0)}}{\partial \varepsilon}\right| + \frac{e}{c}\,[\mathbf{V}\times\mathbf{B}]\,\frac{\partial f^{(1)}}{\partial \mathbf{p}} + \mathrm{St}\, f^{(1)} = 0 \,. \tag{18}$$

As in the preceding case, we introduce the notation

$$f^{(1)} = (\varepsilon - \mu)(\nabla \ln T)_i \left|\frac{\partial f^{(0)}}{\partial \varepsilon}\right| \chi_i, \qquad \hat{\Lambda} = \frac{e}{c}\,[\mathbf{V}\times\mathbf{B}]_l\,\frac{\partial}{\partial p_l} \,,$$

$$\mathrm{St}\, f^{(1)} = \hat{\Omega} f^{(1)} = (\varepsilon - \mu)(\nabla \ln T)_i \left|\frac{\partial f^{(0)}}{\partial \varepsilon}\right| \hat{\Omega}\chi_i \,,$$

$$\langle \varphi \rangle = \sum_{p\sigma} \left|\frac{\partial f^{(0)}}{\partial \varepsilon}\right| \varphi \,.$$

Equation (18) can then be written as

$$(\varepsilon - \mu)(\nabla \ln T)_i \left|\frac{\partial f^{(0)}}{\partial \varepsilon}\right| V_i + (\varepsilon - \mu)(\nabla \ln T)_i \left|\frac{\partial f^{(0)}}{\partial \varepsilon}\right| \hat{\Lambda}\chi_i + (\varepsilon - \mu)(\nabla \ln T)_i \left|\frac{\partial f^{(0)}}{\partial \varepsilon}\right| \hat{\Omega}\chi_i = 0 \,, \tag{19}$$

which, after cancelations, takes the same form as for the standard (diagonal) Righi–Leduc effect

$$V_i = -(\hat{\Lambda} + \hat{\Omega})\chi_i \,. \tag{20}$$

We note a misprint — the omission of the sign — in the corresponding equation (31) in Ref. [1].

We next calculate the current density to obtain

$$J_i = \sum_{p\sigma} e V_i f^{(1)} = \langle (\varepsilon - \mu) V_i \chi_k \rangle e (\nabla \ln T)_k = -\langle \chi_k (\varepsilon - \mu)(\hat{\Lambda} + \hat{\Omega})\chi_i \rangle e (\nabla \ln T)_k \,. \tag{21}$$

As in Eqn (11), the electric field component linear in the magnetic field is

$$J_i^{\text{lin}} = \langle \chi_k^{(0)}(\varepsilon - \mu)\hat{\Lambda}\chi_i^{(0)} \rangle e (\nabla \ln T)_k \,.$$

Clearly, all the discussion above about the anomalous effect (in our case, the Nernst effect) remains valid.

A comparison of flow expressions (10) and (21) shows that the Onsager–Casimir symmetry principle is obeyed, as it should be.

3. List of transverse effects

Presented in Figs 1–3 is the geometry of the transverse kinetic effects — the Hall effect and its analogs — that are mentioned in Ref. [1] and in this paper. Figures 1, 2, and 3 present the respective geometries of the normal effects arising in an external magnetic field, of anomalous effects possible in the absence of an external magnetic field, and of spin transverse effects, in which an external field is not necessary. The table summarizes the basic mechanisms of the transverse effects and presents some references.

The transverse effects are covered highly nonuniformly in the literature. While there is a vast literature on some of them (for example, the Hall effect) — so much so that the corresponding references in the table are by no means comprehensive — others (for example, the Nernst effect) are poorly studied. Our aim is only to provide a simple and clear classification and to mention the pioneering works, some typical studies, reviews, and the most recent papers in which more detailed bibliographies can be found.

Table. Transverse kinetic effects: the Hall effect and its analogs.*

N	Name of effect	Type of effect	Dynamic mechanism	Dissipative mechanism	References **
I	Hall effect in metals	Hall $\mathbf{J} \sim \mathbf{B} \times \mathbf{E}$	Electron path bending in an external magnetic field	Anisotropic electron scattering by magnetic impurities	[2′–17′, *36′*, 55′]
II	Anomalous Hall effect in ferromagnetic metals	Hall $\mathbf{J} \sim \mathbf{B}_{\mathrm{eff}} \times \mathbf{E}$	Electron path bending in a Weiss field ($\mathbf{B}_{\mathrm{eff}}$)***	Anisotropic electron scattering by magnetic atoms	[19′–21′, 24′, 33′, 34′, *36′*, 66′–68′], [19, 20]
III	Righi–Leduc effect in metals	Righi–Leduc $\mathbf{q} \sim \mathbf{B} \times \nabla T$	Electron path bending in an external magnetic field	Anisotropic electron scattering by magnetic impurities	[46′–55′]
IV	Anomalous Righi–Leduc effect in ferromagnetic metals	Righi–Leduc $\mathbf{q} \sim \mathbf{B}_{\mathrm{eff}} \times \nabla T$	Electron path bending in a Weiss field	Anisotropic electron scattering by magnetic atoms	[21]
V	Binaker–Sentfleben effect in molecular gases****	Righi–Leduc $\mathbf{q} \sim \mathbf{B} \times \nabla T$	Precession of rotating moments in a magnetic field	Anisotropic collisions of non-spherical molecules	[57′–61′]
VI	Transverse heat flow in a lattice of rotating molecules ****	Righi–Leduc $\mathbf{q} \sim \mathbf{B} \times \nabla T$	Precession of rotating moments in a magnetic field	Anisotropic phonon scattering by nonspherical molecules	[62′]
VII	Phonon Hall effect****	Righi–Leduc $\mathbf{q} \sim \mathbf{B} \times \nabla T$	Phonon polarization change in a magnetic field	Anisotropic phonon scattering by magnetic impurities	[22′, 23′, 63′–65′, 69′–71′]
VIII	Magnon Hall effect*****	Hall $\mathbf{J} \perp \mathbf{E}$	Spinon deconfinement	Spin frustration, Berry phase, ...	[76′–86′], [23–25]
IX	Magnon Righi–Leduc effect*****	Righi–Leduc $\mathbf{q} \perp \nabla T$	Spinon deconfinement	Spin frustration, Berry phase, ...	[76′, 77′], [21, 22]
X	Ettingshausen effect	Ettingshausen $\mathbf{q} \sim \mathbf{B} \times \mathbf{E}$	Electron path bending in an external magnetic field	Anisotropic electron scattering by magnetic impurities	[55′], [5–11]
XI	Anomalous Ettingshausen effect in ferromagnetic metals	Ettingshausen $\mathbf{q} \sim \mathbf{B}_{\mathrm{eff}} \times \mathbf{E}$	Electron path bending in a Weiss field	Anisotropic electron scattering by magnetic atoms	*The authors found no research on this effect*
XII	Nernst (Nernst–Ettingshausen) effect	Nernst $\mathbf{J} \sim \mathbf{B} \times \nabla T$	Electron path bending in an external magnetic field	Anisotropic electron scattering by magnetic atoms	[55′], [12–18]
XIII	Anomalous Nernst (Nernst–Ettingshausen) effect in ferromagnetic metals	Nernst $\mathbf{J} \sim \mathbf{B}_{\mathrm{eff}} \times \nabla T$	Electron path bending in a Weiss field	Anisotropic electron scattering by magnetic atoms	[26–31]
XIV	Magnon Ettingshausen effect*****	Ettingshausen $\mathbf{q} \perp \mathbf{E}$	Spinon deconfinement	Spin frustration, Berry phase, ...	[5–11]
XV	Magnon Nernst (Nernst–Ettingshausen) effect*****	Nernst $\mathbf{J} \perp \nabla T$	Spinon deconfinement	Spin frustration, Berry phase, ...	[12–18]
XVI	Spin Hall effect*****	Spin Hall $\mathbf{J}_\sigma \perp \mathbf{E}$	Spin polarization in a magnetic field (Zeeman effect)	Anisotropic spin scattering by Coulomb centers	[37′–45′], [*32*, *33*, 35]
XVII	Spin Nernst (Nernst–Ettingshausen) effect*****	Spin Nernst $\mathbf{J}_\sigma \perp \nabla T$	Spin polarization in a magnetic field (Zeeman effect)	Anisotropic spin scattering by Coulomb centers	[34, 35]

* I–IX — diagonal effects (Fig. 1a, b; Fig. 2a, b), X–XV — nondiagonal effects (Fig. 1c, d; Fig. 2c, d), XVI–XVII — spin effects (Fig. 3a, b).

** [N′] denotes a reference [N] from review [1]. Unprimed references are taken from the bibliography in this paper. A number in italics denotes a reference to a review. The notation is the same as in the captions to Figs 1 and 2.

*** $\mathbf{B}_{\mathrm{eff}} = \gamma \mathbf{M}$, $\mathbf{M}$ is the total magnetization.

**** This effect is not a separate type of transverse effect but rather a variety — or a specific implementation mechanism — of the Righi–Leduc effect. The reason for a separate line is that the effect received detailed discussion in review [1].

***** The geometric scheme of the effect presented in the table is clearly not complete because an external magnetic field (and hence the 'third perpendicular') is absent. For a definition of the effect, see the relevant references.

4. Notes on the figures and the table

1. Formally, the spin Hall effect ($\mathbf{J}_\sigma \perp \mathbf{E}$) can equally well be called the spin Ettingshausen effect, because both in the Hall effect and in the Ettingshausen effect, a transverse flow arises in the presence of a longitudinal electric field. The literature commonly uses the former term. At the same time, the spin Nernst effect ($\mathbf{J}_\sigma \perp \nabla T$) might well be called the spin Righi–Leduc effect (transverse flow in the presence of a longitudinal temperature gradient). It is interesting that in the former case, the 'thermodynamic victory' is with the diagonal effects, while in the latter case, it is with nondiagonal ones — for the obvious reason that a spin flow is perceived as having more similarity to an electric flow (Hall and Nernst effects) than to a thermal energy flow (Ettingshausen and Righi–Leduc effects).

2. The terminology is not entirely settled in the field under review and (as is especially the case for present-day research)

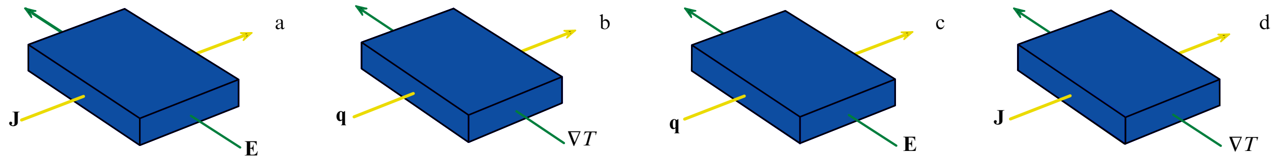

Figure 2. (Color online.) Geometry of anomalous transverse effects (that are possible without an external magnetic field). Diagonal: (a) anomalous Hall effect, (b) anomalous Righi–Leduc effect. Nondiagonal: (c) anomalous Ettingshausen effect, (d) anomalous Nernst (Nernst–Ettingshausen) effect. The geometry of the magnon transverse effects is identical to that of the anomalous transverse effects. The notation is the same as in Fig. 1.

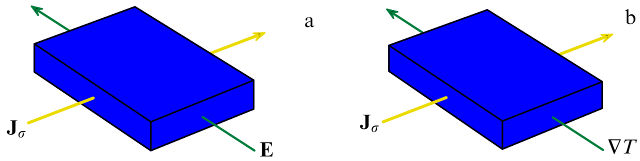

Figure 3. (Color online.) Geometry of spin transverse effects (that are possible without an external magnetic field): (a) spin Hall effect, (b) spin Nernst effect. $\mathbf{J}_\sigma$ is the spin current. The other notation is the same as in Fig. 1.

for any of the transverse effects there is an alternative, often-used name that describes the effect as a variation of the Hall effect. Thus, the Righi–Leduc effect is also referred to as the thermal Hall effect, and the spin Nernst effect as the thermal spin Hall effect.

3. An important point to be noted when analyzing the literature is the following. Unlike the standard terminology we employ here (in which *anomalous* is an analog of *normal*, but in a Weiss field), the current literature occasionally calls an effect anomalous if a deviation, however small, from a simple theory is observed. This does not necessarily mean that the effect is assigned to the chosen class, but can be due, for example, to the insufficiency of the τ-approximation.

4. For any standard transverse effect, there should be an anomalous analog — the same effect but in a Weiss field rather than in an external field. Some of these effects (anomalous Hall effects) are much studied, others less so (see the table), and still others barely or not at all (we are unaware of any work on the anomalous Ettingshausen effect).

5. It is hypothesized that for any standard transverse effect, there should exist a magnon analog (because of the lack of established terminology, the terms chiral, topological, or frustration-induced are also used). Without going into a detailed analysis of possible mechanisms (see the corresponding references in the table), it suffices to mention that all of them are based on various phase effects (the phase shift in a frustrated spin system, the Berry phase, etc.). Loosely, these effects can be referred to as transverse effects in a 'synthetic' magnetic field (although the theory of ultracold atoms uses this term in a somewhat different sense). Despite its very rapid development, the field is to a large extent of a theoretically speculative nature, with experimental evidence available only incidentally (see, e.g., Refs [78, 81] in review [1]).

6. The table does not present all the mechanisms of transverse kinetic effects; furthermore, the literature on the subject has expanded tremendously in recent years. Our goal here was to offer a general classification capable of integrating new mechanisms.

5. Conclusion

In conclusion, as in Ref. [1], two points should again be noted.

First, although geometrically similar, the kinetic phenomena in a magnetic field we have discussed are very different in their formation mechanisms, both dynamic and dissipative.

Second, although this field of physics has a history of almost 140 years, new branches continue to emerge in it, topically interesting and rapidly 'coming into fashion' (as is exemplified by the chirality induced magnon Hall effect). Indeed, as seen from the table, even some of the standard effects are still studied poorly, if at all.

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References

1. Barabanov A F et al. *Phys. Usp.* **58** 446 (2015); *Usp. Fiz. Nauk* **185** 479 (2015)
2. Hall E H *Am. J. Math.* **2** 287 (1879)
3. Righi A *Mem. Acc. Lincei* **4** 433 (1887)
4. Leduc S A *J. Physique 2* **6** 378 (1887)
5. von Ettingshausen A *Anz. Akad. Wiss. Wien* 16 (1887)
6. von Ettingshausen A *Am. J. Sci. 3* **34** 309 (1887)
7. von Ettingshausen A *Lum. Electr.* **24** 436 (1887)
8. von Ettingshausen A *Phil. Mag. 5* **24** 369 (1887)
9. von Ettingshausen A *Z. Phys. Chem.* **2** 105 (1887)
10. von Ettingshausen A *Nuovo Cimento 3* **24** 261 (1888)
11. von Ettingshausen A *Wied. Ann.* **33** 126 (1888)
12. von Ettingshausen A, Nernst W *Anz. Akad. Wiss. Wien* **23** 114 (1886)
13. von Ettingshausen A, Nernst W *Fortschr. Phys.* **2** 689 (1886)
14. von Ettingshausen A, Nernst W *Lum. Electr.* **22** 613 (1886)
15. von Ettingshausen A, Nernst W *Wied. Ann.* **29** 343 (1886)
16. von Ettingshausen A, Nernst W *Z. Phys. Chem.* **2** 103 (1888)
17. von Ettingshausen A, Nernst W *J. Phys. 2* **6** 202 (1887)
18. von Ettingshausen A, Nernst W *Nuovo Cimento 3* **22** 192 (1887)
19. Kondo J *Prog. Theor. Phys.* **27** 772 (1962)
20. Karplus R, Luttinger J M *Phys. Rev.* **95** 1154 (1954)
21. Madon B et al., arXiv:1412.3723
22. Matsumoto R, Murakami S *Phys. Rev. Lett.* **106** 197202 (2011)
23. Sumiyoshi H, Fujimoto S *Phys. Rev. B* **90** 184518 (2014)
24. Yu G et al. *Phys. Rev. B* **89** 104421 (2014)
25. Li X et al. *Nature Commun.* **5** 5174 (2014)
26. Lee W-L et al. *Phys. Rev. Lett.* **93** 226601 (2004)
27. Miyasato T et al. *Phys. Rev. Lett.* **99** 086602 (2007)
28. Pu Y et al. *Phys. Rev. Lett.* **101** 117208 (2008)
29. Hanasaki N et al. *Phys. Rev. Lett.* **100** 106601 (2008)
30. Mizuguchi M et al. *Appl. Phys. Express* **5** 093002 (2012)
31. Kelekci O et al. *J. Magn.* **18** 225 (2013)
32. Vignale G *J. Supercond. Novel Magn.* **23** 3 (2010)
33. Sinova J et al. *Rev. Mod. Phys.* **87** 1213 (2015)
34. Tauber K et al. *Phys. Rev. B* **87** 161114(R) (2013)
35. Bobkova I V, Bobkov A M *Phys. Rev. B* **93** 024513 (2016)