# Unexpected properties of interaction of high-energy protons* 

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#### Abstract

Experimental data on proton-proton interactions in high-energy collisions show that the elastic-to-inelastic scattering ratio varies in an unexpected way with the collision energy: the decrease at comparatively low energies is followed by an increase by a factor of over 1.5 (!) in the energy range from 1160 GeV at the Intersecting Storage Rings (ISR) to $7-13 \mathrm{TeV}$ at the Large Hadron Collider (LHC). Intuitive expectations are that, classically, proton break-up processes continue increasing in number compared to proton survivals. It can be assumed that this surprising effect is due to either the asymptotic freedom property or the collision time being extremely short at such high energies. The unquestionable unitarity principle is combined with the available elastic scattering data to gain new insight into the spatial shape of the interaction region of colliding protons. We discuss how this region evolves at energies currently used and make some predictions on its behavior at still higher energies under different assumptions concerning the relative roles of elastic scattering and inelastic processes. The shape can transform rather drastically if elastic processes keep increasing in proportion. There is an unexpected corollary to this unexpected property. The possible origins of the effect and its relation to strong interaction dynamics are discussed.


Keywords: proton, elastic and inelastic processes, interaction region, impact parameter, torus, black disk

[^0]
## 1. Foreword

If a cup falls to the floor, it breaks up into pieces but sometimes stays intact. The harder it hits the floor, the less is the chance it remains unbroken.

If two high-energy protons collide, many new particles (mostly pions) are produced, but sometimes the protons scatter elastically and retain their entity. It is surprising enough that at very high collision energies, the proportion of elastic processes increases as the energy increases from the ISR to the LHC one.

This unexpected and paradoxical phenomenon and its consequences at present and higher energies are discussed in this review.

## 2. Introduction

We are often accustomed to unexpected facts, and they just become either everyday reality or a trivial observation. However, sometimes they stay unexplained for a long time.

In the 1950s, the strong interactions of hadrons impressed the physics community by producing resonances in pionproton collisions. Afterwards, the resonances filled in all the tables of elementary particles and became a well-known phenomenon. This process continues today with the discovery of the famous Higgs boson or with the 'closure' of the massive two-photon resonance. The phenomenon is described in terms of the dynamical levels of the system.

However, not all discoveries can be given the desired explanation. In the early 1970s, it was unexpectedly found that the total cross section of the interaction of positively charged kaons with protons increased with an energy increase already at energies of the Protvino accelerator up to 70 GeV in

[^1]the rest (laboratory) system of one of the protons or about 12 GeV in the center-of-mass system. We recall that up to that time it had commonly been believed that hadronic cross sections must either decrease or tend to constant values with an energy increase. The first blow to this belief came with the discovery of the so-called 'Serpukhov effect'. Nowadays, it is well known that the total cross section of interaction of highenergy protons steadily increases with an increase in the energy of colliding partners. The elastic scattering cross section, as well as the cross section of inelastic processes, also increases with energy. Both the larger intensity of the interaction due to the larger number of actively participating partons (mostly gluons) and its larger spatial extension can be responsible for that behavior. Moreover, it happens that all hadronic cross sections increase with energy. Almost half a century has passed since then, but no fundamental explanation of such behavior has been proposed within quantum field theory. It has been shown that the cross sections cannot increase faster than the squared logarithm of energy. However, phenomenologically, this is usually described at present energies by a slow power-law energy dependence due to an exchange by the so-called supercritical pomeron. Its dynamical origin is yet unclear.

It is less known that experimental data hide another quite surprising and completely unexpected phenomenon: the increase in the ratio of elastic-to-inelastic (or total) cross sections with an energy increase in the interval from ISR energies [1, 2] to the highest explored accelerator energies at the LHC [3-5]. The share of elastic collisions in the total outcome of all processes used to decrease at lower energies, which coincided with our expectations. However, it reversed the tendency at the ISR (the corresponding data were analyzed by the author and the table with them was demonstrated earlier in Physics-Uspekhi [6, 7]). Their relative roles evolve drastically. The inelastic cross section is about 5 times larger than the elastic one at the ISR, while their ratio decreases to 3 at LHC energies. According to intuitive classical ideas, we would expect the opposite behavior with the probability of the breakdown of both colliding protons into more and more 'pionic pieces' increasing compared to their survival probability, when protons are scattered purely elastically. Moreover, this increasing proportion of elastic scattering approaches a critical value at the LHC energies [810], which probably indicates the transition to some fundamentally new regime of interactions. Somehow, the protons tend to keep their entity while colliding with higher and higher energies. No reliable explanation for this fact exists either. There are only some simplistic proposals.

Here, we show the consequences of such an increase at present energies in a pictorial view of the spatial interaction regions of colliding protons. We describe their possible nontrivial evolution at higher energies if this tendency persists. The adopted approach relies only on the unitarity condition and experimental data about elastic scattering of protons. No phenomenological input has been used to ensure the validity of the conclusions. The results of some phenomenological models are discussed just to provide additional support to our statements.

The general indubitable principle of the conservation of total probability known in particle physics as the unitarity condition relates elastic and inelastic processes. The sum of their ratios to the total outcome must be equal to 1 . Therefore, some knowledge about inelastic processes can also be gained using elastic scattering data, which depend on a smaller
number of variables, and can hence be analyzed more easily. Surely, on the other hand, that leads to a somewhat restricted sample of conclusions about inelastic processes deduced from the unitarity condition. Nevertheless, some knowledge about the spatial interaction region of protons at present energies and its possible evolution at higher energies can be gained.

From the heuristic standpoint, the increase in the share of elastic scattering to the critical value attained at the LHC can for the first time reveal the transition from the traditionally considered branch of the unitarity condition dominated by inelastic processes (where elastic scattering is treated as the shadow of inelastic collisions) to another branch with the dominance of elastic scattering. That would require a completely new physical interpretation of the mechanism of proton (hadron) interactions and the formulation and further studies of new dynamical equations.

The increase in the proportion of elastic scattering processes reveals itself, first of all, in the spatial evolution of the elastic and inelastic interaction regions of colliding protons from $E_{\text {IRS }}$ to $E_{\text {LHC }}$ energies. It is instructive to learn that the inelastic interaction region becomes more Black (absorptive) at the center, has steeper Edges (sharper decrease), and enLarges in size due to its periphery (the socalled BEL scenario [11]) with an energy increase in this energy interval. Even though the shape of these regions cannot be measured directly in experiment, this knowledge has been used, for example, to interpret some peculiar features of experimental data on jet production at 7 TeV . It also inspires theoretical ideas about possible experimental implications of their further evolution at higher energies. If the noticed tendency persists at higher energies, the profiles of both elastic and inelastic interactions can change drastically and show quite unexpected features, especially in the case of head-on collisions. Thus, the BEL scenario can be replaced by the absolutely new toroid-like regime with an enhanced role of elastic scattering for central collisions. It could be named the TEH (Toroidal Elastic Hollow) regime.

No explanation of this phenomenon at present energies has yet been proposed. Regarding our attempts to extrapolate it to higher energies, we hope that experimental studies of elastic scattering of polarized protons or charge asymmetries of pions produced in inelastic collisions (or other still unexploited observations) could help in properly choosing different possibilities. From the theoretical side, we can try to use the more traditional QCD approach with enhanced fluctuations of gluon fields at collisions or revolutionarily speculate on the peculiar properties of solitons and instantons using the corresponding equations in attempts to find a reasonable explanation.

We stress once again that the approximations adopted in the considered approach are completely justified, and it can therefore be claimed that all results at present energies are obtained directly from a combination of two well-grounded sources: the unitarity condition and experimental data on elastic scattering. Their extrapolation to higher-energy regions relies on the assumption that the tendency of the increase in the share of elastic scattering experimentally observed in the energy interval from $E_{\mathrm{ISR}}$ to $E_{\mathrm{LHC}}$ as well as the exponential shape of the diffraction cone will persist there.

The structure of this review is as follows. In Section 3, we start with a description of the general features of experimental results on the elastic scattering of protons. Then the effective theoretical tool of the unitarity condition is introduced in Section 4, where we discuss the accuracy of the main
approximations for the elastic scattering amplitude, which is necessary for reliable estimates in the framework of the unitarity condition. It is applied further in Section 5 to the special case of central head-on collisions of protons, which allows demonstrating typical features of unitarity constraints. Then, in Section 6, the transverse spatial shapes of the inelastic and elastic interaction regions at current energies are demonstrated and their energy evolution is discussed. Possible extrapolations of the profiles of interactions beyond modern (LHC) energies to the asymptotic form are presented in Section 7 for different assumptions about the energy behavior of the proportion of elastic scattering. Finally, some conclusions are given at the end of the paper. Some assumptions about the possible dynamical origin of the observed effect are also discussed.

## 3. Elastic scattering

Information about elastic scattering of protons comes from the measurement of the differential cross section $\mathrm{d} \sigma / \mathrm{d} t$ at some energy $s$ as a function of the transferred momentum $t$ at its experimentally accessible values. The cross section is related to the scattering amplitude $f(s, t)$ as

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{~d} t}=|f(s, t)|^{2} \equiv(\operatorname{Re} f(s, t))^{2}+(\operatorname{Im} f(s, t))^{2} . \tag{1}
\end{equation*}
$$

The variables $s$ and $-t$ are the squared total energy $2 E$ and the squared transferred momentum of the two colliding protons in the center-of-mass system: $s=4 E^{2}=4\left(p^{2}+m^{2}\right)(p$ is the proton momentum) and $-t=2 p^{2}(1-\cos \theta)$ at the scattering angle $\theta$. From this measurement, only knowledge about the modulus of the amplitude is obtained, i.e., about the sum of the squared values of its real and imaginary parts, but not about their signs. The Coulomb scattering contribution to it can be neglected everywhere except at small angles. However, it is precisely there that the Coulomb scattering of the electrically charged protons is comparable to their nuclear interaction. The interference between the nuclear and Coulomb contributions to the amplitude $f$ becomes quite large and allows finding the ratio of the real and imaginary parts of the elastic scattering amplitude $\rho(s, t)=\operatorname{Re} f(s, t) / \operatorname{Im} f(s, t)$ from the shape of the experimental differential cross section. This can be done only for the forward direction $t=0$, $\rho(s, 0)=\rho_{0}$ (to be more precise, for directions extremely close to it) but not at any other values of $t$.

The typical shape of the experimentally measured differential cross section at high energies shown in Figs 1, 2 contains some characteristic features: the above-mentioned interference region at extremely small values of $|t|$, almost invisible in Fig. 1, the exponentially decreasing (with an increase in $|t|$ ) diffraction cone with the energy-dependent slope $B(s)$ (Fig. 1), the dip (Fig. 2), and the more slowly decreasing tail at larger transferred momenta with much smaller values of the cross section than for the diffraction cone (Fig. 2).

### 3.1 Diffraction cone

The diffraction cone is shown in Fig. 1. Protons scatter mainly in processes with small transferred momenta. The differential cross section is much larger there than at higher transferred momenta. Its exponential parameterization is demonstrated by the straight line on the logarithmic scale.

There are several tiny features of this plot. In the very narrow region of extremely small transferred momenta, the


Figure 1. (Color online.) Differential cross section of elastic protonproton scattering at $\sqrt{s}=7 \mathrm{TeV}$ measured by the TOTEM collaboration (Fig. 4 in [3]). The region of the diffraction cone with the $|t|$-exponential decrease is shown.


Figure 2. (Color online.) Differential cross section of elastic proton-proton scattering at $\sqrt{s}=7 \mathrm{TeV}$ measured by the TOTEM collaboration (Fig. 5 in [3]). The region beyond the diffraction peak is shown. The predictions of five phenomenological models are demonstrated.
amplitude is the sum of the nuclear and Coulomb amplitudes. Their interference produces some increase in the differential cross section there. It has been used for estimates of the real part of the amplitude. Moreover, small deviations of the order of 1 percent from the exponential shape (invisible in Fig. 1) were noticed in extremely precise measurements at 8 TeV [4]. Finally, some steepening of the cone can be seen at the very end of the diffraction peak, approximated by another exponential (the dashed line), which differs from the leading one, albeit not very strongly, and the whole effect is noticeable only in a very small interval of transferred momenta. We note that at lower energies, the shape is slightly flattened but not steepened. The impact of all these specific features on our further calculations is easily estimated. It is shown in what follows to be very small because we use the averaged
integrated parameters. Therefore, we adopt the simple exponential parameterization of the diffraction cone, which is precise enough for the corresponding transferred momenta and has been used by experimentalists:

$$
\begin{equation*}
|f(s, t)| \approx \frac{\sigma_{\mathrm{tot}}(s)}{4 \sqrt{\pi}} \exp \left(B(s) \frac{t}{2}\right) \tag{2}
\end{equation*}
$$

where $\sigma_{\text {tot }}(s)$ is the total cross section and $B(s)$ is the energydependent slope of the diffraction cone.

### 3.2 Real part of the elastic scattering amplitude

Some theoretical information about the energy behavior of the real part of the forward scattering amplitude can be obtained from the dispersion relations that follow from the analyticity of the amplitude. They relate it to the integral of the imaginary part at a zero angle, i.e., to the total cross section according to the optical theorem [see Eqn (5) below]. Using reasonable extrapolations of the total cross section to higher energies, it was predicted long ago [12-14] that at high energies the real part is small compared to the imaginary part, and their ratio is about $0.12-0.15$, with a slow decrease at asymptotic energies. Both real and imaginary parts are positive at $t=0$ due to the positivity of the latter. These predictions were confirmed by experiment. At LHC energies, the measured ratio range is $0.12-0.145[3,5,15]$. Hence, the real part only contributes about $1-2 \%$ to differential cross section (1) at $t=0$.

As regards the behavior of the real part as a function of the transferred momentum, some general theoretical guesses in $[16,17]$ indicated that it can become zero somewhere within the diffraction cone. Therefore, its decrease inside the diffraction cone should be steeper than for the imaginary part, and therefore its integral contribution from this region to the elastic cross section must be even smaller. No definite position was ascribed in $[16,17]$ to the point where it crosses the abscissa axis. Recently, some possibilities of using the analytic properties of the elastic scattering amplitude to gain some knowledge about its real part were considered in Ref. [18].

Nevertheless, from the data presented in Figs 1 and 2, we can easily estimate the upper limit of the real part of the amplitude at the dip. Its ratio to the imaginary part at $t=0$ is calculated as the square root of the ratio of the differential cross sections at those points and is very small, $\leqslant 0.006$. This estimate supports our intention to ignore the contribution of the real part of the amplitude in further calculations, where only its integrally averaged characteristics are used.

Further hints regarding the behavior of the amplitude can only be obtained from particular models of proton interactions. Those of them that attempt to make precise fits of a wide variety of present experimental data are certainly preferred. Even then, they should not be trusted absolutely, because we have some experience that several details were wrong even at present energies and could become worse at extrapolations to new energy regions. Nevertheless, as an example, in Fig. 3 (borrowed from Ref. [19]) we show the behavior of the real and imaginary parts of the elastic scattering amplitude at an energy of 7 TeV within a large interval of the transferred momenta. Its shape is derived with the help of a particular phenomenological model [19], which happened to be very successful in fitting many experimental characteristics in a wide range of energies up to the LHC energies.


Figure 3. Real $(\operatorname{Re} f)$ and imaginary $(\operatorname{Im} f)$ parts of the proton-proton amplitude at 7 TeV according to a particular phenomenological model [19]. The contribution of the real part to $\mathrm{d} \sigma / \mathrm{d} t$ becomes noticeable only near the dip, where $\mathrm{d} \sigma / \mathrm{d} t$ is small. It can be completely disregarded inside the diffraction cone. Moreover, it becomes equal to zero inside th cone, as predicted $[16,17]$. It is quite interesting that the imaginary part also dominates in the Orear region of intermediate transferred momenta.

In particular, we can see that the real part at 7 TeV is much smaller than the imaginary part everywhere within the diffraction cone and crosses the abscissa axis in accordance with theoretical expectations [16, 17]. Its relative contribution to differential cross section (1) is given by the term $\rho^{2}(s, t)$ where $\rho(s, t)=\operatorname{Re} f(s, t) / \operatorname{Im} f(s, t)$. It can be disregarded in the model considered. The accuracy of experimental data is not yet high enough for such small contributions to be taken into account. That corresponds well with our prejudice that the diffraction cone is somehow a shadow of inelastic processes, because the elastic amplitude is substantially imaginary there. It is interesting to note that according to the model in [19], the imaginary part dominates everywhere except in the dip interval, which is very short. However, the differential cross section is already very small there compared to the diffraction cone. Thus, in our analytic estimates, we can ignore the real part of the amplitude, although we sometimes come back to it to show once again how irrelevant for our conclusions its contribution is.

The steep exponential decrease in differential cross sections in the diffraction cone implies that precisely this region contributes mostly to Eqn (7). There, the integral contribution of the real part of the amplitude $f$ must even be noticeably smaller than its overestimated value $\rho_{0} \operatorname{Im} f$. That is why it is possible to disregard it in analytic calculations in what follows.

### 3.3 Differential cross section outside the diffraction cone

In what follows, we need to estimate the contribution of the region outside the diffraction peak to some analyzed variables. Comparing Figs 2 and 1 shows that the differential cross section is much lower (by more than 4 orders of magnitude!) at the dip and at the tail than at the beginning of the diffraction cone. Moreover, it decreases approximately as $\exp (-c(s) \sqrt{|t|})$ in this region. It is usually called the Orear region after its discoverer, and can be explained (see [20]) by subsequent iterations (rescattering) in the solution of the unitarity equation in the $(s, t)$ representation. The $t$-exponential parameterization used in (2) for the diffraction cone certainly underestimates the contribution of the tail with the $-\sqrt{|t|}$ exponent at high transferred momenta. However, the
integral contribution of the excess to our variables can easily be estimated, and we show in Section 5 that it is negligibly small. The interplay of the real and imaginary parts of the amplitude $f$ can be more complicated there, as seen, for example, from Fig. 3. However, the smallness of the modulus, i.e., of $\sqrt{\mathrm{d} \sigma / \mathrm{d} t}$, implies the smallness of both of them in this region even though their ratio $\rho$ becomes infinitely large if the imaginary part vanishes.

## 4. Unitarity condition

Our main goals here are to obtain knowledge about the spatial region of interactions of high-energy protons at current energies, to draw a pictorial view of its evolution with increasing energy, and to discuss possible theoretical and experimental implications of these findings.

The most precise and reliable (albeit rather limited) information about the interrelation of elastic and inelastic processes comes from the unitarity of the $S$-matrix,

$$
\begin{equation*}
S S^{+}=1 \tag{3}
\end{equation*}
$$

or, for the scattering matrix $T(S=1+\mathrm{i} T)$,

$$
\begin{equation*}
2 \operatorname{Im} T_{a b}=\sum_{n} \int T_{a n} T_{n b}^{*} \mathrm{~d} \Phi_{n} \tag{4}
\end{equation*}
$$

where $a, b, n$ denote the number of particles. The integral is taken over the whole $n$-particle phase space $\Phi_{n}$. For the elastic scattering amplitude $a=b=2$, the unitarity condition relates the elastic scattering amplitude $f \propto T_{22}$ to the amplitudes of $n$-particle inelastic processes $T_{2 n}$, stating that the total probability of all interaction outcomes (elastic and inelastic) must be equal to $1 .{ }^{1}$

In the $s$-channel, this unquestionable condition is usually expressed in the form of the well-known integral relation (for more details, see, e.g., $[6,20,21])$. This relation is quite complicated for arbitrary values of the transferred momentum $t$. However, for forward scattering at $t=0$, it leads to the widely used optical theorem, showing the normalization of the imaginary part of the amplitude $\operatorname{Im} f(s, 0)$ by its direct connection with the total cross section $\sigma_{\mathrm{tot}}$,

$$
\begin{equation*}
\operatorname{Im} f(s, 0)=\frac{\sigma_{\mathrm{tot}}(s)}{4 \sqrt{\pi}} \tag{5}
\end{equation*}
$$

and to the general statement that the total cross section is the sum of cross sections of elastic and inelastic processes,

$$
\begin{equation*}
\sigma_{\mathrm{tot}}=\sigma_{\mathrm{el}}+\sigma_{\mathrm{inel}} \tag{6}
\end{equation*}
$$

i.e., that the total probability of all processes equals 1 .

We can use the Fourier-Bessel transform of the amplitude $f$ to reduce the integral relation to a simpler algebraic one. This transformation retranslates the momentum data to the shortest transverse distance between the trajectories of the centers of colliding protons, called the impact parameter $b$, and is written as

$$
\begin{equation*}
\mathrm{i} \Gamma(s, b)=\frac{1}{2 \sqrt{\pi}} \int_{0}^{\infty} \mathrm{d}|t| f(s, t) J_{0}(b \sqrt{|t|}) \tag{7}
\end{equation*}
$$

where $J_{0}$ is the Bessel function. The unitarity condition in the $b$-representation then takes the form (for reviews, see, e.g.,

[^2]$\operatorname{Refs}[6,78])$
\[

$$
\begin{equation*}
G(s, b)=2 \operatorname{Re} \Gamma(s, b)-|\Gamma(s, b)|^{2} \tag{8}
\end{equation*}
$$

\]

This relation establishes the connection between the distributions of the intensity of all processes in the transverse configuration space:

$$
\begin{equation*}
\frac{\mathrm{d}^{2} \sigma_{\text {inel }}}{\mathrm{d} b^{2}}=\frac{\mathrm{d}^{2} \sigma_{\text {tot }}}{\mathrm{d} b^{2}}-\frac{\mathrm{d}^{2} \sigma_{\mathrm{el}}}{\mathrm{~d} b^{2}} \tag{9}
\end{equation*}
$$

The left-hand sides in Eqns (8) and (9) describe the transverse impact-parameter profile of inelastic collisions of protons, satisfy the inequalities $0 \leqslant G(s, b) \leqslant 1$, and determine how absorptive the interaction region is at a given impact parameter (with $G=1$ for the full absorption and $G=0$ for the complete dominance of elastic scattering). The profile of elastic processes is determined by the subtrahend in Eqns (8) and (9). Thus, we obtain a spatial view of the whole process if the elastic scattering amplitude $f$ is integrated in Eqn (7).

We note from the very beginning that these profiles cannot be measured directly in experiments because the impact parameters are not measurable quantities. Nevertheless, their energy behavior has an important heuristic value, because it can reveal the evolution of the process dynamics. We describe in what follows how knowledge of the spatial extension of the inelastic interaction region has been used to describe the processes of jet production at the LHC energy of 7 TeV . One can use various models of the interactions and confront different assumptions. Also, one can try to relate the impact parameters, for example, to the multiplicities of inelastic collisions, as is done for interactions of relativistic nuclei. However, we do not speculate on this here.

If $G(s, b)$ is integrated over the impact parameter, it yields the cross section of inelastic processes. The terms in the righthand sides of Eqns (8) and (9) would respectively produce the total cross section and the elastic cross section, in accordance with Eqn (6).

It follows from the above relations that, strictly speaking, we should know both real and imaginary parts of the elastic scattering amplitude to deduce results about the impactparameter profiles of inelastic and elastic processes from the unitarity condition. However, its modulus can only be found from experimental data, as follows from Eqn (1), and some very limited knowledge about its real part for forward scattering. Nevertheless, the accuracy of any assumption can easily be estimated in calculations in accordance with Eqns (7) and (8).

In particular, we use the fact that the modulus of the amplitude decreases approximately exponentially in the diffraction cone [see Eqn (2)] and becomes much smaller at the tail than at the top of the diffraction peak. The slight decline from a simple exponential inside the cone of the order of $1 \%$ noticed recently by the TOTEM Collaboration [22] for small transferred momenta and a somewhat steepened behavior at the very end of the diffraction cone near the dip seen in Fig. 2 do not influence its integral contribution to (7) within the accuracy of the determination of the slopes. In what follows, we use the exponential parameterization of the imaginary part of the amplitude $f$ to proceed with analytic calculations and argue that it is very precise:

$$
\begin{equation*}
\operatorname{Im} f(s, t)=\frac{\sigma_{\mathrm{tot}}(s)}{4 \sqrt{\pi}} \exp \left(B(s) \frac{t}{2}\right) \tag{10}
\end{equation*}
$$

Formally, this approximation is not valid for differential cross sections at large transferred momenta. However, for our purposes, only the integral contribution of $f$ at large $|t|$ to Eqn (7) is important. It is negligibly small there compared to the peak of the diffraction cone. Approximation (10) is justified, as we show below. In fact, that was clear earlier when it was demonstrated in [23] that such an approximation and direct integration of experimental data lead to practically indistinguishable results. The accuracy of calculations is very high. Thus, we can claim that the results obtained analytically rely only on the unitarity condition and experimentally measured exponential decrease of the differential cross section in the diffraction cone.

## 5. Central collisions

Before using detailed formulas for the spatial extension of the interaction region as a function of the impact parameter $b$, we study the simpler case of the energy dependence of the intensity of interaction for central (head-on) collisions of impinging protons at $b=0$. We introduce the variable $\zeta$ as

$$
\begin{equation*}
\zeta(s)=\operatorname{Re} \Gamma(s, 0) . \tag{11}
\end{equation*}
$$

For the dominant contribution of the diffraction cone [Eqn (10)], we conclude that $\zeta$ is directly related to the share of elastic processes:

$$
\begin{equation*}
\zeta(s) \approx \frac{4 \sigma_{\mathrm{el}}}{\sigma_{\mathrm{tot}}} \tag{12}
\end{equation*}
$$

We can also write

$$
\begin{equation*}
|\Gamma|^{2}=\zeta^{2}+\frac{1}{4 \pi}\left(\int_{0}^{\infty} \mathrm{d}|t| \operatorname{Re} f\right)^{2} \tag{13}
\end{equation*}
$$

The last term here can be disregarded, unlike the first one. This is easily seen from

$$
\begin{align*}
\int_{0}^{\infty} \mathrm{d}|t| \operatorname{Re} f & \leqslant \int_{0}^{\infty} \mathrm{d}|t||\operatorname{Re} f| \\
& =\int_{0}^{\infty} \mathrm{d}|t| \sqrt{\frac{\rho^{2}(s, t) \mathrm{d} \sigma / \mathrm{d} t}{1+\rho^{2}(s, t)}} \tag{14}
\end{align*}
$$

The factor $\rho^{2}(s, t) /\left(1+\rho^{2}(s, t)\right)$ is very small in the diffraction cone. It can become approximately 1 at large values of $\rho^{2}(s, t)$ (say, at the dip), but the cross section is already small there (compare Figs 2 and 3). Unitarity condition (8) can then be written as

$$
\begin{equation*}
G(s, b=0)=\zeta(2-\zeta) . \tag{15}
\end{equation*}
$$

Thus, according to unitarity condition (15), the darkness of the inelastic interaction region for central collisions (absorption) is defined by the only experimentally measured parameter $\zeta(s)$ depending on energy. It has its maximum $G(s, 0)=1$ at $\zeta=1$. Any decline in $\zeta$ from $1(\zeta=1 \pm \epsilon)$ results in a parabolic decrease in the absorption $\left(G(s, 0)=1-\epsilon^{2}\right)$, i.e., in an even much smaller decline from 1 for small $\epsilon$. The elastic profile, equal to $\zeta^{2}$ in central collisions, also reaches the value 1 for $\zeta=1$.

The unitarity condition imposes the limit $\zeta \leqslant 2$ on the increase in the share of elastic scattering. This is required by the positivity of the inelastic profile. In this case, there are no inelastic processes for central collisions [ $G(s, 0)=0$ according
to Eqn (15)]. This limit corresponds to the widely discussed 'black disk' picture, which must lead to the relation

$$
\begin{equation*}
\sigma_{\mathrm{el}}=\sigma_{\mathrm{inel}}=\frac{\sigma_{\mathrm{tot}}}{2} . \tag{16}
\end{equation*}
$$

The value of the profile of central $(b=0)$ elastic collisions $\zeta^{2}$ completely saturates the total profile $2 \zeta$ for $\zeta=2$. Below, we discuss the implications of these findings for physics.

We can describe $\zeta$ with quite high precision by the formulas

$$
\begin{equation*}
\zeta(s) \approx \frac{\sigma_{\mathrm{tot}}(s)}{4 \pi B(s)} \approx(4 \pi)^{-0.5} \int_{0}^{\infty} \mathrm{d}|t| \sqrt{\frac{\mathrm{d} \sigma / \mathrm{d} t}{1+\rho^{2}(s, t)}} \tag{17}
\end{equation*}
$$

We especially note that all formulas contain only experimentally measurable quantities $\sigma_{\text {tot }}(s), \sigma_{\text {el }}(s)$, and $B(s)$. The interpretation of $\zeta(s)$ as a share of elastic processes is the most convenient for our further discussion because, in particular, it is proportional to the experimentally measurable dimensionless ratio of the elastic cross section $\sigma_{\text {el }}$ to the total cross section $\sigma_{\text {tot }}$ in Eqn (12).

From the first formula, we reach the conclusion that the increase in $\zeta(s)$ with increasing energy demonstrates that the height of the diffraction cone (the numerator) increases faster than its width shrinks (the denominator).

From the second relation in (17), we can reach very definite conclusions about the role of different regions of the differential cross section for the variable $\zeta$ and hence for the unitarity condition. In practice, the squared root of the differential cross section should just be integrated over the corresponding interval of transferred momenta. It is clearly seen that its value is mainly determined by the transferred momenta at which the differential cross section is large and the real part of the amplitude is small compared to the imaginary part. This is valid in the diffraction cone. The simplest estimates with a constant value $\rho_{0}(s) \approx 0.02$ in place of $\rho(s, t)$ in Eqn (17) show that this contribution is at the level of $1 \%$. It is greatly reduced if its values from Fig. 3 are used, because the values of $\operatorname{Re} f$ are smaller there and, moreover, their contribution is exponentially weighted within the diffraction cone in (17). Moreover, small deviations from the simple exponential shape both inside and at the end of the diffraction cone can be ignored because their contribution becomes very small after integration in Eqn (17). In fact, it can definitely be stated that the exponential parameterization of the imaginary part of amplitude (10) can be used to describe experimental data in our formulas. The conclusions of the phenomenological model in [19] just support our estimates, as was shown in [24].

As regards the tail of the differential cross section, the convenient approximation of $\mathrm{d} \sigma / \mathrm{d} t$ by a pure exponential [in Eqn (10)] is most easily verified by directly taking the published distribution and carrying out integration using the measured data. Numerically, we find that when the region above the dip is included, the data yield the values of $\zeta$ that are less than $3.9 \%$ higher than those obtained with the exponential approximation.

This results in a less than $2 \times 10^{-3}$ correction to the calculation of $G(7 \mathrm{TeV}, 0)$ in Eqn (15). These two approximations ( $\rho_{0}^{2} \approx 0.02$ and the exponential form) allow us to greatly simplify the discussion of the profile function and in any case do not contradict known data and experimental uncertainties. The discussion of the accuracy of estimates can be found in [25].

More detailed estimates of different contributions according to the phenomenological model in [19] are given in Ref. [24]. The imaginary part of the amplitude becomes negative after the dip in this model. The contribution to the definition of $\zeta$ is also negative. Its numerical value decreases, but again only within several percent.

The experimentally measured proportion of elastic processes $\sigma_{\mathrm{el}}(s) / \sigma_{\mathrm{tot}}(s)=0.25 \zeta$ demonstrates the nontrivial dependence on energy shown in the Table. The values of the absorption at central collisions $G(s, 0)$ and the ratios of inelastic to elastic cross sections $\sigma_{\text {inel }} / \sigma_{\text {el }}$ are also shown. All values are derived directly from experimental data at the corresponding energies $s$. The change in the tendency of the behavior of elastic processes with an energy increase looks especially surprising. One would naively expect that their proportion would decrease, being replaced by inelastic processes with higher multiplicities at higher energies. That happens only at low energies up to $E_{\text {IRS }}$, where the parameter $\zeta$ decreases from about 1 to about $2 / 3$. At higher energies, protons reveal unexpected stability. The share of elastic scattering increases with energy. The parameter $\zeta$ reaches the critical value 1 for 7 TeV data at the LHC, where the elastic cross section is about one quarter of the total cross section.

This looks even more impressive in terms of the ratio of the inelastic cross section to the elastic one:

$$
\begin{equation*}
\frac{\sigma_{\text {inel }}}{\sigma_{\mathrm{el}}}=\frac{4}{\zeta}-1 . \tag{18}
\end{equation*}
$$

The ratio decreases from 5 at the ISR to 3 at the LHC, as shown in the Table.

It is intriguing whether this increase in the proportion of elastic scattering will really show up in experiments at higher energies or will be saturated asymptotically with $\zeta$ tending to 1 from below. The asymptotic saturation would lead to a conservative stable situation on the same branch of the unitarity condition, while a further increase above 1 would require a transition to another branch of the unitarity equation and a new physics interpretation.

To explain the last statement, we rewrite Eqn (15) as

$$
\begin{equation*}
\zeta(s)=1 \pm \sqrt{1-G(s, 0)} . \tag{19}
\end{equation*}
$$

The critical value $\zeta=1$ reveals itself in the use of different signs in front of the square root term (different branches of the unitarity condition) for $\zeta<1$ and $\zeta>1$. Elastic scattering is typically treated as a shadow of inelastic processes. This statement is valid when the branch with the negative sign in Eqn (19) is considered, because it leads to the proportionality of elastic and inelastic contributions $(\zeta \approx G(s, 0) / 2)$ for $G(s, 0) \ll 1$. That is typical for electrodynamical forces in particle interactions (e.g., for processes like ee $\rightarrow$ ee $\gamma$ ) and for optics (photon interactions), where the inelastic cross sections

Table. Energy behavior of $4 \sigma_{\mathrm{el}} / \sigma_{\mathrm{tot}} \approx \zeta, G(s, 0)$, and $\sigma_{\text {inel }} / \sigma_{\mathrm{el}}$.

| $\sqrt{s}, \mathrm{GeV}$ | 4.11 | 4.74 | 7.62 | 13.8 | 62.5 | 546 | 1800 | 7000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $4 \sigma_{\text {el }} / \sigma_{\text {tot }}$ | 0.98 | 0.92 | 0.75 | 0.69 | 0.67 | 0.83 | 0.93 | $1.00 \pm 1.04$ |
| $G(s, 0)$ | 1.00 | 0.993 | 0.94 | 0.904 | 0.89 | 0.97 | 0.995 | 1.00 |
|  |  |  |  |  | ISR | Sp $\bar{p} S^{*}$ | FNAL** | LHC |
| $\sigma_{\text {inel }} / \sigma_{\text {el }}$ |  |  |  |  | 5 |  |  | 3 |

[^3]are small and their values are governed by the fine structure constant $\alpha$.

The large value of the inelastic cross sections in hadronic collisions with a subsequent increase in the elastic proportion as the role of inelastic production diminishes destroys the analogy. That is why the observation of this effect comes as a surprise. For strong interactions, the shares of inelastic and elastic processes are comparable in magnitude (see the Table). The approach of $\zeta$ to 1 at 7 TeV corresponds to complete absorption in central collisions. This value is considered to be critical, because from (19) the significant conclusion is reached that the excess of $\zeta$ over 1 implies that the unitary branch with the plus sign in front of inelastic processes is at work. This branch was first considered in [26] with application to highenergy particle scattering. That changes the interpretation of the role of elastic processes as a simple 'shadow' of inelastic ones.

Present experimental data at the LHC cannot distinguish definitively between the two possibilities: the asymptotic saturation and the increase in the elastic share. A slight tendency of $\zeta$ to increase and become larger than 1 can be noticed by comparing the TOTEM data at 7 TeV [3], where it can be estimated ${ }^{2}$ in the limits 1.00 and 1.04 , and at $8 \mathrm{TeV}[4]$, where, according to the data of the same collaboration, it is approximately 1.05 , although within the accuracy of experimental data of about $\pm 0.024$. Data from the ATLAS collaboration at 8 TeV do not reveal any increase in the proportion of elastic scattering, albeit with approximately the same accuracy. More precise data at these energies and at 13 TeV are needed.

A further increase in the share of elastic scattering with energy is favored by extensive fits of available experimental information for a wide energy range and their extrapolations to ever higher energies done in the phenomenological models in Refs [19, 27], as well as by some theoretical speculations (e.g., see Ref. [28]). The asymptotic values of $4 \sigma_{\mathrm{el}} / \sigma_{\text {tot }}$ are about 1.5 in Refs [19, 27] and 1.8 in [28]; if the exponential shape of the diffraction cone survives, they would correspond to an incomplete but rather noticeable decrease in the absorption at the center of the interaction region. The corresponding values of the attenuation at the center, $G(\infty, 0)$, are 0.75 and 0.36 . This is discussed in more detail in the next section.

## 6. Shape of the inelastic interaction region at present energies

The detailed shape of the inelastic interaction region at arbitrary values of the impact parameters can be obtained with the help of relations (7) and (8) if the behavior of the amplitude $f(s, t)$ is known. Its modulus and the $\rho_{0}$ values are obtained from experiment. The most prominent feature of experimental results at present energies in the range from $E_{\mathrm{ISR}}$ to $E_{\mathrm{LHC}}$ is the rapid exponential decrease in $\mathrm{d} \sigma / \mathrm{d} t$ with increasing the transferred momentum $|t|$, especially in the near-forward diffraction cone. It is just this region of transferred momenta that contributes most to Eqns (7) and (17). Inserting the exponential shape of the cone in there, we can write

$$
\begin{align*}
\mathrm{i} \Gamma(s, b) & \approx \frac{\sigma_{\mathrm{tot}}(s)}{8 \pi} \int_{0}^{\infty} \mathrm{d}|t| \exp \left(-B(s) \frac{|t|}{2}\right) \\
& \times(\mathrm{i}+\rho(s, t)) J_{0}(b \sqrt{|t|}) . \tag{20}
\end{align*}
$$

[^4]We stress that the diffraction cone dominates the contribution to $\operatorname{Re} \Gamma$ in Eqns (12) and (20) so strongly that the tail of the differential cross section at larger $|t|$ can be completely ignored at the level of several percent even for central collisions, as was estimated in the preceding section. Furthermore, it is suppressed additionally by the Bessel function $J_{0}$ at larger impact parameters. Therefore, the accuracy of the approximation increases. It was estimated using fits of the experimental differential cross section outside the diffraction cone by the simplest analytic expressions. Moreover, this was shown in [23,29] by computing how well the versions with direct fits of experimental data and with their exponential approximation coincide if used in the unitarity condition. Therefore, expression (10) can be treated as following directly from experiment and being very precise. Thus, we calculate

$$
\begin{equation*}
\operatorname{Re} \Gamma(s, b)=\zeta \exp \left(-\frac{b^{2}}{2 B}\right) . \tag{21}
\end{equation*}
$$

Correspondingly, the shape of the inelastic profile for small $\rho_{0}$ is given by

$$
\begin{equation*}
G(s, b)=\zeta \exp \left(-\frac{b^{2}}{2 B}\right)\left[2-\zeta \exp \left(-\frac{b^{2}}{2 B}\right)\right] \tag{22}
\end{equation*}
$$

It scales as a function of $b / \sqrt{2 B}$. Its energy dependence is determined by the two measured quantities, the diffraction cone slope $B(s)$ and its ratio to the total cross section, i.e., by the variable $\zeta(s)$. It has a maximum at

$$
\begin{equation*}
b_{\mathrm{m}}^{2}=2 B \ln \zeta . \tag{23}
\end{equation*}
$$

It is located in the unphysical region of impact parameters $b_{\mathrm{m}}^{2}<0$ for $\zeta<1$, i.e., at all energies below the LHC ones. Therefore, the absorption is incomplete, $G(s, b)<1$, at any physical impact parameter $b \geqslant 0$. Its largest value is reached at the very center, $b=0$. The inelastic interaction region has the shape of a disk, with absorption strongly diminishing to its edges. The disk is semi-transparent at ISR energies. This is demonstrated by the corresponding line ( $\zeta=0.7$ ) in Fig. 4 [9], shown below.

At $\zeta=1$, which is only reached at the LHC energy of 7 TeV , the maximum is located exactly at the center $b=0$, and the maximum absorption occurs just for central collisions, i.e., $G(s, 0)=1$. The disk center becomes black. The strongly absorptive core of the inelastic interaction region grows in size compared to ISR energies (see [23]) because of an increase in the slope $B(s)$. The enlarged size of the inelastic interaction region can be clearly seen from the Taylor expansion of Eqn (23) at small impact parameters:

$$
\begin{equation*}
G(s, b)=\zeta\left[2-\zeta-\frac{b^{2}}{B}(1-\zeta)-\frac{b^{4}}{4 B^{2}}(2 \zeta-1)\right] . \tag{24}
\end{equation*}
$$

The negative term proportional to $b^{2}$ vanishes at $\zeta=1$, and $G(b)$ develops a wide strongly absorbing plateau that extends to the comparatively large values of the impact parameter $b$ (up to about 0.5 fm ). The plateau is very flat, because the last negative term in Eqn (25), which diminishes the absorption, starts to play a role at 7 TeV (where $B \approx 20 \mathrm{GeV}^{-2}$ ) only for larger values of $b$. Therefore, the absorption decrease becomes steeper at the periphery. The earlier proposed BEL scenario is therefore realized at present energies in such a way. The two lines in Fig. 4 demonstrate the evolution of the shape
of the inelastic interaction region from the $\operatorname{ISR}(\zeta=0.7)$ to LHC $(\zeta=1.0)$ energies. The enhancement of the blackness for central collisions at the LHC compared to the ISR can be ascribed to the increased role of soft gluons in the proton structure function. It is claimed in several papers [31-33] that already at LHC energies the hollowness of the plateau can be seen at $b=0$. Actually, the accuracy of experiments is still insufficient for definitive conclusions. Only at higher energies (or if a higher accuracy at the LHC could be achieved) can it be clearly observed as displayed in Fig. 4. We discuss these predictions in Section 7.

Before discussing the predictions at higher energies, we note that the cross sections of inelastic processes are determined not only by the strength of the interaction inside the interaction region but also by a purely geometrical factor. Even though the proton interaction region is very dark at central collisions ( $G(s, b) \approx 1$ inside the plateau), the cross sections of processes with small impact parameters $b \leqslant r$ are very small, because the corresponding areas proportional to $r^{2}$ are small for integrals over $b \leqslant 0.5 \mathrm{fm}$. Integrating the total and elastic terms in Eqn (22) up to impact parameters $b \leqslant r$, we can estimate their roles for different radii $r$ as

$$
\begin{align*}
& \sigma_{\mathrm{el}}(s, b \leqslant r)=\sigma_{\mathrm{el}}(s)\left[1-\exp \left(-\frac{r^{2}}{B(s)}\right)\right],  \tag{25}\\
& \sigma_{\mathrm{tot}}(s, b \leqslant r)=\sigma_{\mathrm{tot}}(s)\left[1-\exp \left(-\frac{r^{2}}{2 B(s)}\right)\right] . \tag{26}
\end{align*}
$$

We find that the contribution of processes at small impact parameters $b^{2} \ll 2 B$ diminishes quadratically as $r \rightarrow 0$. In particular, inelastic processes contribute at $r \rightarrow 0$ as

$$
\begin{equation*}
\sigma_{\text {inel }}(s, b \leqslant r) \rightarrow \pi r^{2} G(s, 0)+O\left(r^{4}\right), \quad r^{2} \ll B . \tag{27}
\end{equation*}
$$

The maximum intensity of central inelastic collisions, equal to 1 , is attained at $\zeta=1$. The high intensity must result in high multiplicities of inelastic events. The integral contribution of


Figure 4. Energy evolution of the shape of the inelastic interaction region for different values of the survival probability. The values $\zeta=0.7$ and 1.0 correspond to the ISR and LHC energies and agree well with the results of detailed fitting to the elastic scattering data [23, 30, 31]. A further increase in $\zeta$ leads to a toroid-like shape with a dip at $b=0$ (TEH regime) shown at values $\zeta=1.5,1.8,2.0$. The values $4 \sigma_{\mathrm{el}} / \sigma_{\text {tot }}$ in [19, 27] and 1.8 in [28] are proposed as corresponding to asymptotic regimes. We recall that Eqn (12) is only valid if the exponential shape of the diffraction cone persists at higher energies. The value $\zeta=2$ corresponds to the 'black disk' regime $\left(\sigma_{\mathrm{el}}=\sigma_{\mathrm{in}}=0.5 \sigma_{\mathrm{tot}}\right)$. For more discussion of the black disk and the geometrical scaling, see Refs [34-36].
the near-central region of collisions is small. The cross sections of very high-multiplicity events are also small. The estimates show that they are quite comparable to one another.

This property has been used in Ref. [29] to explain the jet excess observed for very high-multiplicity events at 7 TeV compared to predictions of the well-known Monte Carlo models PYTHIA and HERWIG. This excess was interpreted as an indication of the active role of the high-density gluonic component of the internal structure of protons at that energy. Thus, it was concluded that such a component should be more properly accounted for in new versions of the Monte Carlo models. This demonstrates how knowledge about the spatial view of the inelastic interaction region helps in reaching some conclusions about possible omissions in the models used nowadays to describe experimental data on jet production at the LHC energies.

The spatial region of elastic scattering as derived from the subtrahend in Eqn (22) is strongly peaked at low impact parameters and rapidly decreases at larger values of $b$ in accordance with the Gaussian exponent law. The contribution to the elastic cross section is nevertheless suppressed at small $b$ and comes mainly from impact parameters $b^{2} \approx 2 B$. The mean value of the squared impact parameter for elastic scattering can be estimated as

$$
\begin{equation*}
\left\langle b_{\mathrm{el}}^{2}\right\rangle=\frac{\sigma_{\mathrm{el}}(s)}{\pi \zeta^{2}(s)} . \tag{28}
\end{equation*}
$$

Inelastic processes are much more peripheral. The ratio of the corresponding values of squared impact parameters is

$$
\begin{equation*}
\frac{\left\langle b_{\text {inel }}^{2}\right\rangle}{\left\langle b_{\text {el }}^{2}\right\rangle}=\zeta \frac{8-\zeta}{4-\zeta} . \tag{29}
\end{equation*}
$$

This ratio exceeds 2 already at LHC energies and would become equal to 6 for $\zeta=2$ if that were possible. The peripherality of inelastic processes compared to elastic ones increases with an increase in the proportion of elastic collisions. Elastic collisions are more effective in the most central interactions.

## 7. Some predictions at higher energies

What can we expect at higher energies?
The profiles shown in Fig. 4 are valid so long as we can assume that the differential cross section of elastic scattering decreases exponentially with $|t|$ within the diffraction cone. They can change if this traditional behavior is no longer valid at higher energies. Even though we commonly believe that this feature will be preserved because of finite proton sizes, it cannot be ruled out that drastic changes can be found if the internal structure of protons starts playing role, à la Rutherford, at higher energies.

Any new results can only be deduced from the extrapolation of experimental data at present energies to new regimes, even though our previous experience teaches us how uncertain and even erroneous such extrapolations can be. Nevertheless, we try to use some assumptions relying on the fact that we have used only the most reliable methods of acquiring the necessary information such as the unitarity condition and quite precise experimental data on elastic scattering.

First, we can assume that $\zeta$ will increase but approach 1 asymptotically without exceeding it. In principle, such an assumption can be valid because the present accuracy of
experimental data at 7 and 8 TeV is not high enough and allows that behavior. That would imply that the precise value of $\zeta$ at these energies is still slightly lower than 1 within the present experimental errors. This is the only possibility to keep the present status of the shape of the interaction region (BEL) where the inelastic profile stays quite stable with a slow approach to complete blackness at central collisions and a steady increase in its range with asymptotic saturation (see Fig. 7 in [19]). That is a kind of 'black tube' if we assume rather long longitudinal distances, as is commonly done for the picture with soft wee partons. This situation seems most appealing to our theoretical intuition.

Just such a situation is favored by the phenomenological model presented in Ref. [19], where numerous experimental data at present energies have been successfully described. However, it leads to a very important prediction of a nonexponential shape of the diffraction cone at higher energies. Only in this case is the further increase in the share of elastic scattering compatible with $\zeta$ saturation, which leads to violation of Eqn (12). Do not the data about some substructure of the diffraction cone at 8 TeV signify such a situation?

Surely, it cannot be ruled out that the share of elastic scattering will suddenly decrease again. Then we would come to the picture that we dealt with, say, at ISR energies, and nothing interesting would happen. This possibility, however, looks quite improbable. In both cases, we are dealing with the same branch of the unitarity condition.

Another more interesting and intriguing possibility is a further increase in the share of elastic processes with an assumed stable exponential diffraction cone at increasing energies. We then have to consider the values $\zeta>1$. The transition to another branch of the unitarity condition occurs. The BEL scenario described above changes drastically. The maximum absorption occurs at nonzero impact parameters, and shifts to positive values of impact parameters (23) for $\zeta>1$. Then the inelastic interaction region inevitably acquires a toroid-like shape (TEH) with a dip exactly at the center, $b=0$. Most probably, if the accuracy of experimental data becomes sufficiently high, we will observe at 13 TeV both an increase in the share of elastic processes at approximately the same rate as happened in the ISR-to-LHC range, where it changed from 0.67 to 1.0 (with intermediate values 0.8 at Sp $\bar{p} S$ at 546 GeV and 0.9 at Tevatron at 1.8 TeV if the protonantiproton data are included) and a stable exponential diffraction cone. Then the blackness at very central collisions $G(s, b=0)$ diminishes with an increase in $\zeta$. The center becomes more transparent. The dip at the center of the interaction region with a minimum at $b=0$ should appear instead of a flat plateau. The 'black plateau' described at 7 TeV transforms into a toroid-like structure with somewhat lower darkness at the center and the maximum blackness equal to 1 at more peripheral impact parameter $b_{\mathrm{m}}$ (see [7, 8 , 37]). As follows from the above formulas, this dependence is very slow near $\zeta=1$, and hence the darkness at the center would only become smaller, e.g., by $6 \%$ if $\zeta$ increases to 1.2 . Therefore, we can hardly expect immediate drastic changes with an increase in LHC energies to 13 TeV . Nevertheless, the forthcoming TOTEM + CMS results on elastic scattering at 13 TeV can be very conclusive about the general trend if the precise values of the diffraction cone slope $B$ and the total cross section $\sigma_{\text {tot }}$ (or, equivalently, of the proportion of elastic processes) become available and the corresponding value of $\zeta$ turns out to be above 1 .

The central dip becomes even deeper at larger $\zeta$. The limit value $\zeta=2$ leads to a complete dominance of elastic scattering at the center $b=0$ with $\zeta^{2}=4$. It coincides with the total profile $2 \zeta=4$ there. No inelastic absorption can be observed at the center $G(s, 0)=0$. The maximum absorption is shifted to $b_{\mathrm{m}}=\sqrt{2 B \ln 2}$. Such a situation can only be reached if the positive branch of the unitarity condition is applicable.

All these features are demonstrated in Fig. 4 borrowed from [9]. In addition to the present-energy results at $\zeta=0.7$ and 1.0 and the limit plot of the attenuation at $\zeta=2$, intermediate values 1.5 and 1.8 are shown. These values illustrate the regimes with a further increase and asymptotic saturation of the share of elastic scattering.

Some phenomenological models [19, 27] favor the situation of an increasing proportion of elastic scattering at higher energies. It is related to the increase in $\zeta$ if the shape of the diffraction cone stays exponential. These models lead to good fits of a large set of experimental data at present energies and provide some extrapolations to higher energies. Realistic estimates of their predictions at 13 TeV and 100 TeV [38] show that an extremely high accuracy of elastic scattering experiments will be necessary to observe some effects. It is predicted that $\zeta$ will be only $3-4 \%$ higher at 13 TeV than at 7 TeV . In accordance with the above formulas, the darkness decrease at the center of the inelastic interaction region is quadratically small compared to the change in $\zeta$ itself and becomes noticeable at the third digit only. That requires a very high precision of forthcoming TOTEM + CMS results at 13 TeV . At the newly planned 100 TeV collider, the value of $\zeta$ can increase by $13-20 \%$ from 1 , which would imply a $3-4 \%$ lower value of $G(b=0)$. The maximum blackness 1 will be reached at the impact parameters about 0.5 fm . The formation of a toroid-like structure proceeds very slowly with energy. No model predicts a fast increase in $\zeta$ to values close to 2 . The asymptotic values of $4 \sigma_{\mathrm{el}} / \sigma_{\text {tot }}$ preferred by both models are about 1.5. The corresponding asymptotic profiles of inelastic processes are shown in Fig. 4. A somewhat different asymptotic value equal to 1.8 is favored in theoretical paper [28]. Its prediction of a deeper dip is also demonstrated in Fig. 4. The whole impact-parameter structure is reminiscent of a toroid (tube) with absorbing black edges, which looks as if it is more and more transparent for the elastic component at the very center. The inelastic cross section will be only about 1.5 times larger than the elastic cross section at the asymptote. It is most fascinating in the presented scenario that the density of central inelastic interactions tends to 0 as $\zeta \rightarrow 2$, which would lead to the 'black disk' limit with equal elastic and inelastic cross sections. However, nobody predicts such a high increase in the share of elastic scattering even at asymptotically high energies.

Concerning the inelastic processes, there are no predictions of any drastic evolution of the interaction region with increasing energy beyond the LHC range. The (almost) black plateau with a small dip at the central part near $b=0$ will become somewhat enlarged in size. Therefore, the jet cross sections due to central collisions will also slightly increase at the beginning. Step by step, the inelastic profile will become even more peripheral and the role of peripheral collisions will increase.

As was discussed, central collisions are responsible for the rare events with the highest multiplicities. The decrease in their intensity at ever higher energies would result in a lower
tail of the multiplicity distributions and in their steepening. In particular, a diminished role of jet production from central collisions is also predicted with a further increase in $\zeta$. Once again, these effects will develop very slowly, unfortunately.

## 8. Conclusions

The intriguing purely experimental phenomenon of an increase in the share of elastic processes in the total outcome observed in proton interactions at energies from the ISR to LHC ones is currently attracting much attention. It has not been explained yet. One of the possibilities can be related to the fact that the larger number of high-energy constituents (quarks and gluons) are exchanged with high momenta. Due to the QCD asymptotic freedom property, the role of such processes would decrease and hence the relative role of elastic scattering is to increase. We note that the mutual influence of a smaller number of these processes and larger transferred momenta must lead to some increase in the transverse momenta of created particles, as observed in experiment. Another possibility is connected to the fluctuations of the partonic picture of colliding protons. The time of flight of protons through one another becomes shorter with increasing energy. The pointlike partons have almost no chance to interact during such a short time. ${ }^{3}$ Therefore, the role of elastic processes can increase.

Despite the lack of an explanation for the observed effect, the increase in the proportion of elastic processes has been used in this review to examine its consequences. In particular, important information about the spatial regions of proton interactions has been obtained. That the share of inelastic scattering approaches $1 / 4$ at the LHC energies (or, equivalently, $\zeta$ approaches 1) can become a critical sign of the changing character of processes of hadron interactions if the above tendency to increase persists. If the exponential shape of the diffraction cone persists, the concave central part of the inelastic interaction region will be formed. The inelastic interaction region would then look like a toroid (tube) hollowed out inside and strongly absorbing in its main body at the edges. The role of elastic scattering in central collision is increasing. This is surprising and somewhat contradictory to our everyday experience and theoretical prejudices. Intuitively, we would expect the steady increase in the proportion of inelastic processes with increasing energy as was the case for energies below the ISR energy. Instead, we are faced with the problem that from the formal theoretical standpoint, the new tendency now requires considering another branch of the unitarity condition, which requires a physical interpretation.

It is hard to believe that protons become more penetrable at higher energies after being so dark in central collisions with $G(s, 0)=1$ at 7 TeV , unless some special coherence within the internal region develops. Moreover, it seems somewhat mystifying that the coherence is more significant just for central collisions but not at other impact parameters where inelastic collisions become dominant.

Several very speculative ideas come to mind and have been proposed, but not a single one looks satisfactory. We try to describe some of them independently of how fantastic they look.

For example, the role of the string junction in three-quark hadrons can become crucial. This effect would not be

[^5]observed, say, in pion-proton interactions. However, we have no chance to obtain any experimental information about these processes. Moreover, the success of the quark-diquark models adds some sceptical attitude to this approach. The relative strengths of the longitudinal and transverse components of gluon (string) fields can probably help to explain the new physics of the TEH scenario of the 'hollowed interactions' of protons.

In classical terms, the transparency in central collisions could reveal itself in collisions of the two tori with the radii so strongly different that one of them penetrates through the hole in the other at $b=0$. In a more general situation, they can be some stratified objects in which the empty spaces of one of them coincide in the collision with the dense regions in the other. They overlap in peripheral collisions and thus lead to inelastic processes. Such fluctuations of the size and the structure of high-energy protons seem very improbable.

One could also imagine that 'black' protons start scattering backward [9] like billiard balls in head-on collisions. Snell's law allows such a situation for equal reflective indices of colliding bodies. However, the forward and backward scattering cannot be distinguished for two equivalent colliding objects. It can only be checked if forward and backward scattered protons can somehow be identified in experiment. They should then have different labels. The proton spin can be used as such a label. In principle, experiments with oppositely polarized protons can resolve the problem. Unfortunately, no polarized protons are available now even at the LHC. Thus, it is improbable that the TEH structure will be observed directly. Moreover, backward scattering would require that all partons have coherently large transferred momenta. The asymptotic freedom of QCD means that the probability of such processes must be extremely low.

Besides the case of two billiard balls colliding head-on, one could consider the hypothesis that centrally colliding protons at $\zeta=2$ are like solitons, which "pass through one another without losing their identity. Here we have a nonlinear physical process in which interacting localized pulses do not scatter irreversibly" [39]. Again, in the case of two identical colliding objects, it is impossible to decide whether they scatter forward or backward. In the case of solitons, it is known that the nonlinearity and dispersive properties (the chromopermittivity [40]) of a medium compete to produce this effect. The dynamics of the whole process must then be understood. To describe it, the Korteweg-de Vries and sineGordon equations, the nonlinear Schrödinger equation [41], the Skyrme model [42], and instantons [43] are used. It is not at all clear yet how the QCD nonlinearity and the properties of the quark-gluon medium could be responsible at the quantum field level for these new features of proton interactions. Again, the asymptotic freedom of QCD seems to forbid such processes.

Coherence of the parton structures inside the interaction region of colliding hadrons can probably lead to the observed effects. It can reveal itself in 'squeezing' (or complete absorption) of intermediate created inelastic channels. That would lead to the antishadowing effect with an increasing role of the elastic channel, which is similar to the self-focusing of laser beams. At the model level of reggeon interactions, these possibilities were considered in Refs [44, 45] with the discussion of different variants of the absorbing disk.

Another more exotic hypothesis [46] that could be used to treat the hollowed internal TEH region is the formation of cooler disoriented chiral condensate inside it ('baked-Alaska'
DCC). A signature of this squeezed coherent state would be some disparity between the production of charged and neutral pions [47], probably noticed in some high-energy cosmic ray experiments. However the cross sections for central collisions seem to be extremely small, as discussed above. The failure to find such events at the Fermilab is probably connected with the low available energies. This leaves some hope for higher energies in view of the discussions above. Total internal reflection of coherent states from the dark edges of the toroid can be blamed for enlarged elastic scattering (like the transmission of laser beams in optical fibers).

The transition to the deconfined state of quarks and gluons in central collisions could also be claimed to be responsible for new effects (see Ref. [48]). The optical analogy with the scattering of light on a metallic surface as induced by the presence of free electrons is used. Again, it is hard to explain why that happens for central collisions, while peripheral ones with impact parameters near $b_{\mathrm{m}}$ are strongly inelastic.

To conclude, the problem of the increasing proportion of the elastic scattering of high-energy protons, which requires its own solution, can be further studied only with the advent of experimental facilities of higher-energy accelerators. Cosmic ray studies do not look very promising because of the relatively low accuracy of measurements. However, the detailed analyses of extensive air showers and observed 'anomalies' can probably say something about 'escaping' high-energy protons. Only very precise experimental results can lead to definite conclusions, because the theoretically predicted energy dependence of the darkness of the interaction region discussed above is very mild. However, the heuristic value of the foreseen results should not be underestimated. If the tendency towards the increased proportion of elastic scattering processes persists, it will pose the problem of a new view of mechanisms of high-energy proton (hadron) interactions. New ways of explaining the transition to the quite uncommon regime of proton interactions with peculiar shapes of the interaction region would then have to be invented.

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[^2]:    ${ }^{1}$ The nonlinear contribution from the elastic amplitude appears in the right-hand side for $n=2$.

[^3]:    * Superconducting proton-antiproton Synchrotron.
    ** Fermi National Accelerator Laboratory.

[^4]:    ${ }^{2}$ The experimental values of the ratios of the elastic to the total cross section and $\rho_{0}$ have been used.

[^5]:    ${ }^{3}$ The classical analogy of this effect to a bullet passing through a glass was pointed out to me by B L Altshuler.

