

LETTERS TO THE EDITORS

PACS number: 03.65.Ta

The uncertainty principle and measurement accuracy

A V Belinsky, V B Lapshin

DOI: https://doi.org/10.3367/UFNe.2017.02.038069

Abstract. **A gedanken experiment is proposed in which the momentum and position of a photon can be measured simultaneously in such a way that the product of their respective errors is less than the right-hand side of the appropriate Heisenberg uncertainty relation.**

Keywords: quantum measurements, measurement accuracy, Heisenberg uncertainty principle

Heisenberg's uncertainty principle [1] in its general form — the Robertson–Schrödinger relation [2–5] — is a fundamental theorem in quantum theory and any doubts in its validity can shatter the whole quantum theory, because the operator description of physical observables automatically entails the uncertainty relation due to the noncommutativity of operators. However, this principle is rather often interpreted as the impossibility simultaneously measuring canonically conjugate physical quantities. This was the original formulation by Heisenberg [1]. He believed that the measurement, for example, of a quantum particle coordinate has a reverse effect on measuring its momentum. As a result of this distortion, the exact measurement of momentum is impossible. Is this correct? The authors of experiment [6], based on earlier work [7, 8], managed to destroy this persistent prejudice. Such an outstanding result with measurement accuracy exceeding the Heisenberg limit was achieved in a series of complicated experiments with 'weak' measurements, during which the system's quantum state was teleported and thus the information about the state before the measurement was preserved to a certain extent.

At the same time, we believe that this fundamental result can be proved using a simple thought experiment, which is quite realizable, and the same result can be achieved much more easily by a single coordinate measurement at a given momentum of a specially prepared photon.

Let us consider Fig. 1. Single atom *1* residing in the ground state is excited by a resonant laser pulse which transfers the atomic state into the excited state with a unity probability (π-pulse). After some time (lifetime T of an atomic excited state), a photon will be spontaneously emitted into a sphere with the solid angle 4π sr. If this photon is absorbed by the identical nonexcited atom *2*, the photon momentum will be known: $\mathbf{p} = \hbar\mathbf{k}$. The momentum is determined by the transition energy or the wavelength λ, as well as by the coordinates of two atoms, which define the wave vector $\mathbf{k}$ direction. After exciting this atom, the photon will be re-emitted into 4π sr. The photon count on the detector D will mean that the momentum and coordinate of the first photon are established simultaneously; this coordinate is determined by the atom *2* coordinates at the moment in time when it is absorbed. Let us now elucidate with what an accuracy these measurements were done.

Photon coordinate measurement error is defined by the size of atom *2*, trap design (for example, an incorporated or substituted impurity atom in the crystal lattice site), and the atom transition cross section or, just the same, the cross section of its resonant interaction with a linear size on the order of the wavelength λ. If we are concerned with optical radiation, the latter error is clearly dominant. In fact, although the atom is small, it can 'see' the radiation around itself at distances of approximately λ (see, for example, book [9, p. 25]).

The photon momentum uncertainty is linked, in turn, to the natural width $1/T$ of the spectral line of the spontaneous transition, Doppler broadening, and Heisenberg's quantum uncertainty caused by the accuracy in fixing the atom *1* coordinates. Among these causes, the natural line width $1/T$ is clearly dominant, because the Doppler broadening can be suppressed by atomic cooling, and the positioning of atom *1* within the boundaries of a certain spatial region is not critical: you see that the error in forming a specific $\mathbf{k}$ vector direction is due not to the linear dimensions of the atom *1* location region, but to its angular size. This means that by increasing the distance between the atoms one can unlimitedly decrease the angular size of the first atom with respect to the second one, and the $\mathbf{k}$ vector direction will be determined quite precisely. At the same time, this will significantly decrease the prob-

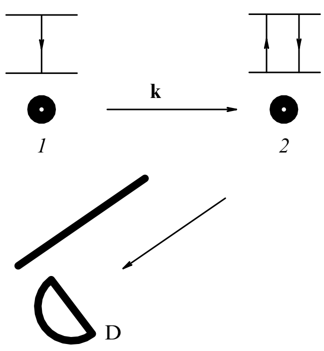

Figure 1. Thought experiment setup. Excited atom *1* emits a photon, which is re-emitted by atom *2*. A nontransparent screen is placed between atom *1* and detector D in order to prevent the direct interaction between a photon emitted by this atom and detector D.

A V Belinsky, V B Lapshin Faculty of Physics,
Lomonosov Moscow State University,
Leninskie gory 1, str. 2, 119991 Moscow, Russian Federation
E-mail: belinsky@inbox.ru, lapshin-vb1@mail.ru

Received 14 July 2016
Uspekhi Fizicheskikh Nauk **187** (3) 349 – 350 (2017)
DOI: https://doi.org/10.3367/UFNr.2017.02.038069
Translated by A L Chekhov; edited by A Radzig

ability of realizing the whole re-emission cascade, but for a gedanken experiment such a situation is acceptable: we are interested in the principal possibility of the process considered.

Now, let us estimate the errors in our measurements and calculate their product

$$\Delta x \Delta p_x = 2\pi\lambda \, \frac{\hbar}{cT} \approx 10^{-6}\,\hbar \tag{1}$$

for a visible spectral region. It is clear that the obtained value is much smaller than $\hbar$.

What have we achieved? The photon coordinate measurement accuracy is not limited for a known momentum by the Heisenberg uncertainty principle. However, because the concrete values of the coordinate and momentum did not exist (*a priori*) before the measurement (see, for example, Refs [10–18]), the experimentally obtained values need to lie in the known Heisenberg uncertainty limits, while the concrete values appear at the moment of the measurement accompanied by the photon demolition.

At the same time, one can justly object that, during the photon coordinate measurement, its momentum acquires an additional uncertainty. But in fact this happens simultaneously with the photon demolition, so it does not care anymore.

Does our example call into question Heisenberg's uncertainty principle in its general form, i.e., the Robertson–Schrödinger relation [2–5]? Not at all. These uncertainties objectively exist and cancelling them would mean destroying all the quantum theory apparatus. However, one can measure the quantities more precisely, because the measurement is just a projection of the initial state on the measured quantity. In order to increase the informational value of these measurements, one can perform a series of experiments which would define quantum uncertainties of the measurable quantities. The achieved improvement in measurement accuracy will be helpful for the solution of such a problem. But first of all, from a fundamental point of view, the key possibility of taking such measurements with an accuracy exceeding the Heisenberg uncertainty principle is the most important.

In conclusion, let us note that one can use a crystal lattice site for an atom trap and incorporate a single impurity atom with given radiative transitions different from the spectra of surrounding atoms, so the latter would be transparent for the wavelengths of these transitions. The spatial position of a single impurity atom in such a structure will be fixed up to a characteristic distance between the crystal lattice sites.

This study was supported by RFBR grant 17-07-00693 a.

References

1. Heisenberg W *Z. Phys.* **43** 172 (1927); Translated into English: in *Quantum Theory and Measurement* (Eds J A Wheeler, W H Zurek) (Princeton, NJ: Princeton Univ. Press, 1983) p. 62
2. Kennard E H *Z. Phys.* **44** 326 (1927)
3. Weyl H *Gruppentheorie und Quantenmechanik* (Leipzig: S. Hirzel, 1928); Translated into English: *The Theory of Groups and Quantum Mechanics* (London: Methuen, 1931)
4. Robertson H P *Phys. Rev.* **34** 163 (1929)
5. Schrödinger E *Sitzungsber. Preuß. Akad. Wiss. Phys. Math. Kl.* **14** 296 (1930)
6. Rozema L A et al. *Phys. Rev. Lett.* **109** 100404 (2012)
7. Ozawa M *Phys. Rev. A* **67** 042105 (2003)
8. Lund A P, Wiseman H M *New J. Phys.* **12** 093011 (2010)
9. Klyshko D *Physical Foundations of Quantum Electronics* (Eds M Chekhova, S Kulik) (Singapore: World Scientific, 2011); Translated from Russian: *Fizicheskie Osnovy Kvantovoi Elektroniki* (Moscow: Nauka, 1986)
10. Belinskii A V, Klyshko D N *Phys. Usp.* **36** 653 (1993); *Usp. Fiz. Nauk* **163** (8) 1 (1993)
11. Klyshko D N *Phys. Usp.* **37** 1097 (1994); *Usp. Fiz. Nauk* **164** 1187 (1994)
12. Klyshko D N, Masalov A V *Phys. Usp.* **38** 1203 (1995); *Usp. Fiz. Nauk* **165** 1249 (1995)
13. Belinsky A V, Klyshko D N *Laser Phys.* **6** 1082 (1996)
14. Belinskii A V *Phys. Usp.* **40** 305 (1997); *Usp. Fiz. Nauk* **167** 323 (1997)
15. Belinskii A V *Phys. Usp.* **46** 877 (2003); *Usp. Fiz. Nauk* **173** 905 (2003)
16. Belinsky A V *Kvantovye Izmereniya* (Quantum Measurements) (Moscow: BINOM. Laboratoriya Znanii, 2008)
17. Belinsky A V, Chirkin A S *Phys. Usp.* **56** 1126 (2013); *Usp. Fiz. Nauk* **183** 1231 (2013)
18. Belinsky A V, Shulman M Kh *Phys. Usp.* **57** 1022 (2014); *Usp. Fiz. Nauk* **184** 1135 (2014)