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Is it possible to measure the electromagnetic radiation of an instantly started charge?

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Contents

Abstract. In considering the propagation of high-energy multiply charged ions through a condensed medium, the paper looks at how the emerging electromagnetic radiation is influenced by the processes in which the ions capture (or lose) electrons. The arising changes in radiation characteristics are due to the additional contribution to the output from the emission of the electrons captured or lost by multiply charged ions in a medium. This contribution is similar to that from instantly started or stopped charges.

Keywords: multiply charged ions, Vavilov – Cherenkov radiation, transition radiation

1. Introduction

The problem of electromagnetic radiation emitted from a system under an instantaneous change of its state has been discussed in a scientific community for quite a long time (see, for example, one of the early reviews regarding this question [1]). These issues are interesting mainly due to their fundamental nature related to the basics of classical electrodynamics. This is most clearly demonstrated in the rapidly developing recent concept of 'half-dressed' charges [2, 3], those which temporarily lose their equilibrium electromagnetic field. This concept has already helped to elegantly explain the Landau–Pomeranchuk–Migdal effect [4, 5] and the Ternovskii–Shul'ga–Fomin effect [6, 7], which was recently confirmed in an experiment [8]. Physical systems in which such effects may occur can be, for example, ones with an instantaneously changing electric dipole moment, instantaneously accelerated or decelerated charge, or rapidly changing (fluctuating) charge. When speaking about alternating charge, we naturally assume that part of the system charge stops its motion and stays in the medium, while the total charge of the entire system remains constant. An example of the last processes may be the formation and ensuing disappearance of excess electrons in a nuclear-electromagnetic cascade [9] or the loss and capture of electrons by accelerated multiply charged ions in a medium [10, 11]. Changes in the parameters of such systems are always followed by the emission of electromagnetic waves.

The concept of an instantaneous change in the parameters characterizing the radiating source has a range of applicability, which can be defined in the following way [1]. Let us consider some small source at rest emitting radiation, when one of its parameters is changed (dipole moment, charge, velocity, etc.). We will assume that this parameter is changing during time T from a given initial instant to some final moment. Let us consider one monochromatic wave with frequency ω from the spectrum of radiation. If the radiation source is resting and the inequality $\omega T < 1$ is fulfilled, then the radiation at frequency ω is defined only by the initial and final values of the parameter and do not depend on time T which it took this parameter to change. Thus, time T can be excluded from the expressions that define the radiation, and we can consider the change to be instant. For example, a well-known expression for a spectral-angular density of radiation emitted by an instantly stopped or started charge has the following form (see, for example, book [12]):

$$\frac{\mathrm{d}^2 W}{\mathrm{d}\omega\,\mathrm{d}\Omega} = \frac{e^2 \sin^2\vartheta}{4\pi^2 c}\,\frac{\beta^2}{(1-\beta\cos\vartheta)^2}\,, \tag{1}$$

where ϑ is the angle between the particle velocity vector and the photon emission direction, and $\beta = v/c$ (c is the speed of light). The qualitative form of the radiation angular distribution (1) for an instantly started or stopped charge in a vacuum is illustrated in Fig. 1.

It would seem that the easiest way to experimentally expose the effect of such electromagnetic radiation is to

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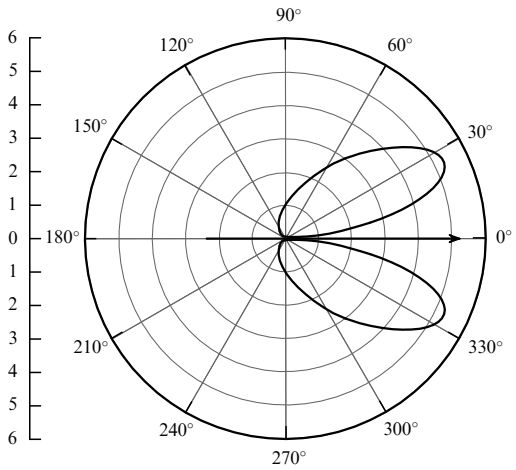

Figure 1. Angular distribution of radiation emitted by an instantly started or stopped charge in a vacuum at $\beta = 0.9$. The arrow indicates the direction of charge motion.

watch over the process of radioactive α- or β-decay of nuclei. However, on the one hand, a significant obstacle is posed in this case by α-decay isotropy. On the other hand, β-decay anisotropy would need strong magnetic fields and low temperatures in order to align the spins of decaying nuclei (for example, the experimental verification of the spatial parity conservation law in weak interactions was performed with β-active ^{60}Co source placed in the magnetic field [13]. In order for thermal motion not to destroy the nucleus polarization, the sample was cooled to low temperatures of order 0.01 K).

The most suitable object for the observation of the effects described above could be processes when electrons are captured (or lost) by multiply charged accelerated ions in a medium, giving rise to an additional contribution to the electromagnetic radiation. Indeed, the charge exchange processes are 'fast' and the characteristic time of charge exchange between the accelerated multiply charged ion and the medium is $\tau_c \sim (\sigma v n_e)^{-1}$, where σ is the charge exchange cross section, v is the ion velocity, and n_e is the target electron concentration. If we consider the electromagnetic radiation formed along the path $l_{coh} \approx v/(\omega - \mathbf{kv})$ greatly exceeding the characteristic spatial scale of the charge exchange, $l_{eq} \sim v\tau_c$, where ω, $\mathbf{k}$ are the frequency and the wave vector of the radiated electromagnetic wave, then the electron capture and loss processes can be assumed to be 'prompt'. Estimates show that for a broad range of ion velocities and radiation wavelengths the condition $l_{coh} \gg l_{eq}$ is not violated.[1] This means that it is absolutely safe to consider the electron loss or capture process by relativistic ions entering the medium as prompt.

[1] It is obvious that the hypothesis of the instantaneous (prompt) change in system parameters does not allow a correct description of the high-energy part of the spectrum. Let us consider the process of electron loss by relativistic ions in condensed matter. The upper energy limitation of emitted quanta, for which the 'prompt' hypothesis becomes inapplicable, can be estimated from the following considerations. For strictly forward radiation, the radiation formation length is defined in the relativistic limit by the expression $l_{coh} \approx 2c\gamma^2/\omega$, where $\gamma = 1/(1-\beta^2)^{-1/2}$ is the Lorentz factor of the relativistic ion. On the other hand, electron loss cross section can be estimated for a fast ion according to Ref. [15] as $\sigma \approx \pi a_0^2 (Z_2^{2/3}/Z_1)(v_0/v)$, where a_0, v_0 are the Bohr radius and velocity, and Z_1, Z_2 are charge numbers of the ion and target, respectively. By substituting the characteristic values of variables in the given relations, we can obtain an estimate for the maximum energy of quanta: $\hbar\omega < 10^2\gamma^2$ eV. For example, if $\gamma \approx 10$, the 'prompt' approximation is valid for the energies of emitted photons up to 10 keV.

2. Multiply charged ion radiation close to the Cherenkov threshold

If during the flight through the medium the ion multiply loses or captures electrons, then one can characterize this process by charge fluctuation. Correlational effects in Vavilov–Cherenkov radiation, connected with such charge fluctuations of multiply charged accelerated ions with moderate energies in a medium, will be considered in Section 3. In the case of high-energy ions and thin targets, it is worth talking about single processes of an electron loss or capture by the ion during the flight through the target, rather than about charge fluctuations. One should expect that the single charge exchange processes in thin targets will also lead to some peculiarities and will make a contribution to the radiation in the velocity region below the threshold.

Let us represent the ion charge as a function of time $Z(t)\,e$, where $Z(t)$ is a random variable running through a series of discrete values from zero to the ion charge number. Then, charge and current densities of a charged particle with a time-dependent charge moving along the trajectory $\mathbf{r}(t)$ with the velocity $\mathbf{v}(t)$ can be written out as [9, 10]

$$\mathbf{j}(\mathbf{r},t) = eZ(t)\,\mathbf{v}(t)\,\delta\big(\mathbf{r}-\mathbf{r}(t)\big)\,, \tag{2}$$

$$\rho(\mathbf{r},t) = eZ(t)\,\delta\big(\mathbf{r}-\mathbf{r}(t)\big) - e\int_{-\infty}^{t} \mathrm{d}\tau\, Z'(\tau)\,\delta\big(\mathbf{r}-\mathbf{r}(\tau)\big)\,. \tag{3}$$

It is readily seen that these expressions for $\mathbf{j}(\mathbf{r},t)$ and $\rho(\mathbf{r},t)$ satisfy both the continuity equation and the charge conservation law — at every point in space and any time moment the total charge is constant and equal to zero. Assuming that the magnetic permeability of the medium $\mu = 1$, we write down the Maxwell equations for the electromagnetic field potentials created in the medium by currents and charges (2), (3):

$$\Delta\mathbf{A} - \frac{\varepsilon}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi}{c}\,\mathbf{j}(\mathbf{r},t)\,,\quad \Delta\varphi - \frac{\varepsilon}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -\frac{4\pi}{\varepsilon}\,\rho(\mathbf{r},t)\,. \tag{4}$$

We seek solutions to equations (4) by expressing all quantities as Fourier integrals:

$$\mathbf{A}(\mathbf{r},t) = \int \mathbf{A}(\mathbf{k},\omega)\,\exp\left(\mathrm{i}\mathbf{kr} - \mathrm{i}\omega t\right)\mathrm{d}\mathbf{k}\,\mathrm{d}\omega\,, \tag{5}$$

$$\mathbf{A}(\mathbf{k},\omega) = \frac{4\pi}{c}\,\frac{\mathbf{j}(\mathbf{k},\omega)}{k^2 - \omega^2\varepsilon(\omega)/c^2}\,,$$

$$\varphi(\mathbf{k},\omega) = \frac{4\pi}{\varepsilon(\omega)}\,\frac{\rho(\mathbf{k},\omega)}{k^2 - \omega^2\varepsilon(\omega)/c^2}\,, \tag{6}$$

with $\varepsilon(\omega) = \varepsilon'(\omega) + \mathrm{i}\varepsilon''(\omega)$, where $\varepsilon'(\omega)$ and $\varepsilon''(\omega)$ are real and imaginary parts of the medium dielectric constant, respectively. Then, the Fourier components of the electric field strength are expressed as

$$\mathbf{E}(\mathbf{k},\omega) = \frac{\mathrm{i}\omega}{c}\,\mathbf{A}(\mathbf{k},\omega) - \mathrm{i}\mathbf{k}\varphi(\mathbf{k},\omega)\,. \tag{7}$$

The transverse component of the electric field strength (7) with respect to wave vector $\mathbf{n} = \mathbf{k}/k$ can be expressed in the absence of spatial dispersion as

$$\mathbf{E}_\perp(\mathbf{r}, t) = \int \mathbf{E}_\perp(\mathbf{k}, \omega) \exp(\mathrm{i}\mathbf{k}\mathbf{r} - \mathrm{i}\omega t)\, \mathrm{d}\mathbf{k}\, \mathrm{d}\omega$$
$$= \frac{4\pi \mathrm{i}}{c^2} \int \omega \frac{\mathbf{n} \times \big(\mathbf{j}(\mathbf{k}, \omega) \times \mathbf{n}\big)}{k^2 - \omega^2 \varepsilon(\omega)/c^2} \exp(\mathrm{i}\mathbf{k}\mathbf{r} - \mathrm{i}\omega t)\, \mathrm{d}\mathbf{k}\, \mathrm{d}\omega\,. \quad (8)$$

Then, the energy lost by the charged particle due to the emission of electromagnetic waves during the full time of its flight through the medium will take on the form

$$W = -\int \mathbf{E}_\perp(\mathbf{r}, t)\, \mathbf{j}(\mathbf{r}, t)\, \mathrm{d}\mathbf{r}\, \mathrm{d}t\,. \quad (9)$$

An expression similar to Eqn (9), but with the longitudinal component of the electric field strength with respect to the wave vector $\mathbf{n} = \mathbf{k}/k$, will describe the particle energy losses due to excitation of longitudinal plasma oscillations in the medium. An analysis of such effects is beyond the scope of issues explored in this article and will not be performed further.

In expression (8) we will take into account the relationship which is valid for small values of the imaginary part of the dielectric constant: [2]

$$\lim_{\varepsilon'' \to 0} \frac{\mathrm{i}}{k^2 - \omega^2 \varepsilon(\omega)/c^2} = -\pi\delta\left(k^2 - \frac{\omega^2 \varepsilon'(\omega)}{c^2}\right). \quad (10)$$

Then after integrating over the $\mathbf{k}$-vector modulus, we will finally obtain for the radiation emitted in the frequency range ω, $\omega + \mathrm{d}\omega$ into the solid angle $\mathrm{d}\Omega$ in the transparent region approximation (i.e., as $\varepsilon''(\omega) \to 0$):

$$\frac{\mathrm{d}^2 W}{\mathrm{d}\omega\, \mathrm{d}\Omega} = \frac{\omega k}{(2\pi c)^2} \left|\mathbf{n} \times \mathbf{j}(\mathbf{k}, \omega)\right|^2, \quad k = \frac{\omega\sqrt{\varepsilon'(\omega)}}{c}\,; \quad (11)$$

$$\mathbf{j}(\mathbf{k}, \omega) = e \int_{-\infty}^{\infty} \mathrm{d}t\, Z(t)\, \mathbf{v}(t)\, \exp\big(\mathrm{i}\omega t - \mathrm{i}\mathbf{k}\mathbf{r}(t)\big)\,. \quad (12)$$

For definiteness, let us consider the emission pattern arising as an electron is captured by an ion in medium. The initial ion charge number is denoted by Z_1. Then, as the electron is captured at time instant $t = 0$, the dependence $Z(t)$ can be expressed as $Z(t) = Z_1 - \vartheta(t)$, where the unit stepwise function is defined as $\vartheta(t) = 0$ for $t < 0$, and $\vartheta(t) = 1$ for $t \geqslant 0$, and the current density can be written out as

$$\mathbf{j}(\mathbf{k}, \omega) = eZ_1 \int_{-\infty}^{\infty} \mathrm{d}t\, \mathbf{v}(t) \exp\big(\mathrm{i}\omega t - \mathrm{i}\mathbf{k}\mathbf{r}(t)\big)$$
$$- e \int_0^{\infty} \mathrm{d}t\, \mathbf{v}(t) \exp\big(\mathrm{i}\omega t - \mathrm{i}\mathbf{k}\mathbf{r}(t)\big)\,. \quad (13)$$

If we ignore the braking and multiple scattering of ion in the medium, assuming the ion motion to be uniform and rectilinear with velocity v, then from the last formula we obtain

$$\mathbf{j}(\mathbf{k}, \omega) = 2\pi e Z_1 \mathbf{v}\, \delta(\omega - \mathbf{k}\mathbf{v}) - \pi e \mathbf{v} \delta_+(\omega - \mathbf{k}\mathbf{v})\,. \quad (14)$$

Using the well-known relationship

$$\delta_\pm(\omega - \mathbf{k}\mathbf{v}) = \delta(\omega - \mathbf{k}\mathbf{v}) \pm \frac{\mathrm{i}}{\pi}\, \mathrm{P}\left(\frac{1}{\omega - \mathbf{k}\mathbf{v}}\right), \quad (15)$$

where the symbol P indicates that the integral of the corresponding quantity is assigned its principal value, we transform Eqn (14) into

$$\mathbf{j}(\mathbf{k}, \omega) = 2\pi e\left(Z_1 - \frac{1}{2}\right) \mathbf{v}\delta(\omega - \mathbf{k}\mathbf{v}) - \mathrm{i}e\mathbf{v}\mathrm{P}\left(\frac{1}{\omega - \mathbf{k}\mathbf{v}}\right). \quad (16)$$

If the threshold condition for the appearance of Vavilov–Cherenkov radiation is not fulfilled — that is, when the ion speed is small and does not satisfy the condition $v \geqslant c_\mathrm{p}$, where $c_\mathrm{p} = c/\sqrt{\varepsilon'(\omega)}$ is the phase velocity of light in the medium, then only the second term in current density (16) will contribute to the radiation. After simple transformations, we obtain — according to general expression (11) — the spectral-angular density of radiation:

$$\frac{\mathrm{d}^2 W}{\mathrm{d}\omega\, \mathrm{d}\Omega} = \frac{e^2 \sqrt{\varepsilon'(\omega)} \sin^2\theta}{4\pi^2 c} \frac{\beta^2}{\big(1 - \beta\sqrt{\varepsilon'(\omega)}\cos\theta\big)^2}\,. \quad (17)$$

As expected, expression (17) coincides with the spectral-angular radiation density of the charge, which at time moment $t = 0$ instantly starts moving with velocity v [see equation (1) for $\varepsilon' = 1$]. Radiation described by distribution (17) is directed forward in a strongly diffused cone close to the Cherenkov angle. This radiation should be observed for ion velocities below the threshold.

Similar angular distribution patterns were observed in experiments [14] studying the salient features of visible electromagnetic radiation ($\lambda = 500$ nm) emitted by multiply charged gold ions in a transparent medium close to the Cherenkov threshold. The energy of fully ionized gold ions was varied in the experiment conducted by Ruzicka et al. [14] from 640 to 990 MeV per nucleon. The threshold for the appearance of Vavilov–Cherenkov radiation in an SiO_2 radiator (refractive index $n = 1.17$, thickness $L = 7.7$ mm) was 863 MeV per nucleon. Notably, the authors of Ref. [14] detected forward-directed radiation in a strongly diffused cone, which appeared for ion velocities below the threshold at energies of 641 MeV per nucleon. The corresponding angular distribution is illustrated by the photograph in Fig. 2a (the central bright spot is a trace from the ion beam, and the bright luminous ring is the Vavilov–Cherenkov ion radiation at an approximately 27° angle emitted into a focusing lens with the refractive index $n = 1.52$). Similar peculiarities appear in the angular distribution of radiation at velocities below the threshold, if one takes into account the processes of an electron capture by an ion in medium. Indeed, if we use the Bohr–Lindhard expression for the electron capture cross section [15], it is easy to see that the probability of electron capture by the fully ionized gold ion during its passage through the radiator is close to unity under experimental conditions in work [14]. Therefore, one should expect the electron capture process by the ion to influence the spectral-angular characteristics of radiation in the way described

[2] Indeed, if we assume that the imaginary part of the dielectric constant is infinitely small and introduce new variables $k^2 - \omega^2\varepsilon'(\omega)/c^2 = x$, $\mathrm{i}\omega^2\varepsilon''(\omega)/c^2 = \mathrm{i}/\alpha$ $(\alpha \to \infty)$, then we arrive at the relation $\mathrm{i}/(k^2 - \omega^2\varepsilon(\omega)/c^2) = -\alpha/(x^2\alpha^2 + 1) + \mathrm{i}\alpha^2 x/(x^2\alpha^2 + 1)$. The first term approximates a delta function in the limit of $\alpha \to \infty$: $\lim_{\alpha\to\infty} \alpha/(x^2\alpha^2 + 1) = \pi\delta(x)$, and the second term equals zero at $k = \omega\sqrt{\varepsilon'(\omega)}/c$.

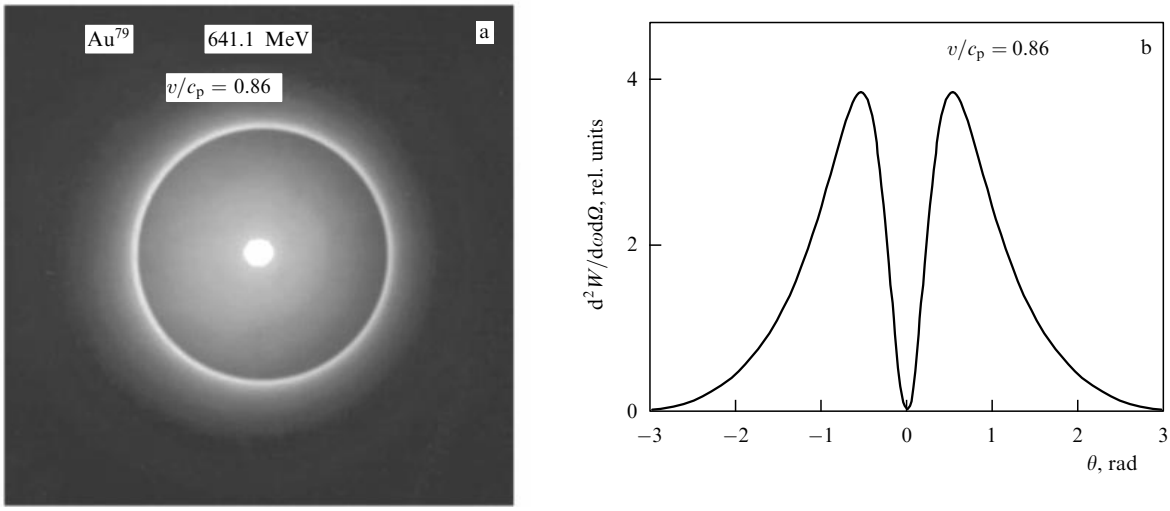

Figure 2. (a) Angular distribution of radiation emitted by gold ions with an energy below the Cherenkov radiation threshold in an SiO_2 aerogel, measured in Ref. [14]. (b) Emission angular distribution calculated using expression (17). The radiation intensity reaches the maximum value for the 'forward' angle of approximately 29°.

above. Since formula (17) is valid in the transparency region, it can be applied to analyze the spectral-angular distribution in the optical region. The angular distribution of emission calculated using expression (17) is given in Fig. 2b.

By integrating (17) over the angles, we obtain the spectral density of emission:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega} = \frac{2e^2}{\pi c\sqrt{\varepsilon'(\omega)}}\left(\frac{1}{2\beta\sqrt{\varepsilon'(\omega)}}\ln\frac{1+\beta\sqrt{\varepsilon'(\omega)}}{1-\beta\sqrt{\varepsilon'(\omega)}} - 1\right). \quad (18)$$

It is remarkable that the spectral-angular density of emission (17) and the spectral density (18) do not depend on the initial charge of the ion, its mass, or target thickness (in the framework of our approximations). This could provide a way to verify the observation of the described effect of subthreshold radiation emitted by high-energy multiply charged ions in the medium. Let us note that the considered classical problem is similar to the well-known 'Tamm problem', which is related to the study of the field created by the Cherenkov charge that starts moving at some instant of time and stops at another moment of time [16]. In this case, the existence of a uniformly moving charge is insignificant in the subthreshold regime, which results in well-known results (17) and (18) for an instantly starting (or stopping) charge.

Let us now consider the case when the condition for the Vavilov–Cherenkov radiation threshold is fulfilled and $v \geqslant c_{\mathrm{p}}$. Taking into account the expression for current (14) and performing renormalization of the full radiation energy over the whole time of flight to the energy emitted per unit of time (or flight path), we obtain

$$\frac{1}{L}\frac{\mathrm{d}^2W}{\mathrm{d}\omega\,\mathrm{d}\Omega} = \frac{\omega^2\beta e^2\sqrt{\varepsilon'(\omega)}\sin^2\theta}{2\pi c^2}\left(Z_1 - \frac{1}{2}\right)^2\delta(\omega - \mathbf{kv})\,. \quad (19)$$

After integrating expression (19) over the angle, we obtain the well-known expression for the Vavilov–Cherenkov radiation spectral density per unit flight path [17], but with a modified value of the particle charge. As the electron is captured by an ion during its flight, the ion charge is naturally replaced by its 'effective' value, equal to the mean value $[Z_1 + (Z_1 - 1)]/2 = Z_1 - 1/2$:

$$\frac{1}{L}\frac{\mathrm{d}W}{\mathrm{d}\omega} = \frac{\omega}{c^2}\left(Z_1 - \frac{1}{2}\right)^2 e^2\left(1 - \frac{c_{\mathrm{p}}^2}{v^2}\right)\vartheta\left(1 - \frac{c_{\mathrm{p}}^2}{v^2}\right). \quad (20)$$

It is clear that the effect of electron capture under the conditions of the Vavilov–Cherenkov threshold, described by expressions (19) and (20), is noticeable only for light ions with small Z_1. For example, the intensity of radiation emitted by helium ions ($Z_1 = 2$) due to single electron capture during a flight through the target decreases by a factor of almost two.

3. Influence of charge fluctuations of multiply charged accelerated ions

As an accelerated ion enters the target, its charge state quickly changes due to the electron exchange between the ion and the medium. This fast enough leads to the stabilization of the ion charge at some equilibrium value depending on the ion velocity. If we express the ion charge as a function of time $Z(t)\,e$, where $Z(t)$ is a random variable running through a series of discrete values from zero to the ion charge number, the equilibrium charge equals the average over the charge state equilibrium distribution: $Z_{\mathrm{eq}}e = \langle Z(t)\rangle\, e$. The equilibrium distribution itself can be found from the corresponding equations which describe the stochastic charge exchange process with given transition probabilities.

Since the charge exchange processes are 'fast', the characteristic time of charge exchange between the accelerated ion and a medium can be comparable with the periods of longitudinal and transverse oscillations excited by a particle in the medium. Averaging the medium-absorbed energy over the charge state equilibrium distribution in the linear response approximation will lead to correlational effects in ion braking ability and generated electromagnetic (bremsstrahlung, transition, and Cherenkov) radiation. Particularly, the influence of such effects on the polarization losses of ion energy in a medium were considered in papers [18, 19].

The influence of multiple charge exchange on Vavilov–Cherenkov radiation effect can be qualitatively described based on the Huygens principle usually invoked for its

interpretation. Specifically, the field component with frequency ω of a particle traveling through the medium can be expressed as a superposition of fields emitted by oscillators with the same frequency placed along the particle trajectory. Let us assume that at some trajectory section the particle charge abruptly changes. This will lead to a change in the energy of its interaction with the medium and in field amplitudes of oscillators. As a result, the interference of oscillator fields at coherence length from trajectory regions with different charge states will not fully cancel out the resulting field outside the Cherenkov cone. If the charge is changing randomly, it will lead to the blurring of the radiation wave front and transformation of the spectral-angular density.

It should be noted that similar effects are caused by another stochastic process — multiple scattering [20] — although its mechanism is slightly different. Namely, the reason lies in the disturbance of oscillator wave coherence brought about by the phase change in particle scattering on individual atoms of the medium. The influence of further described charge fluctuation effects in Cherenkov radiation will prevail over multiple scattering effects if the root-mean-square (rms) angle of multiple scattering of ions along the whole effective ion path in the medium (target thickness or the photon absorption length) is less than the rms angular spread of the correlational contribution to the radiation.

According to the above considerations, the expression for the spectral-angular radiation density (1) should be averaged over the equilibrium distribution of the particle charge states. As a result, for a uniformly and rectilinearly moving particle with velocity $\mathbf{v}$, we obtain

$$\frac{\mathrm{d}^2 W}{\mathrm{d}\omega\,\mathrm{d}\Omega} = \frac{\omega^2\beta^2 e^2\sqrt{\varepsilon'(\omega)}\sin^2\theta}{4\pi^2 c}\,\Delta(\omega-\mathbf{kv})\,, \tag{21}$$

$$\Delta(\omega-\mathbf{kv}) = \int \mathrm{d}t\,\mathrm{d}t'\big\langle Z(t)\,Z(t')\big\rangle \exp\big[\mathrm{i}(\omega-\mathbf{kv})(t-t')\big]\,.$$

A significant difference between equation (21) and result (11), (12) is the presence of an autocorrelation function $\big\langle Z(t)\,Z(t')\big\rangle$ in the integrals over time. If the correlational effects are disregarded, the autocorrelation function $\big\langle Z(t)\,Z(t')\big\rangle$ should be replaced with a constant value Z_{eq}^2 equal to the square of the equilibrium charge of the particle. In this case, the electromagnetic radiation appears at frequency ω for which the particle velocity is larger than the wave phase velocity in the medium, $c_{\mathrm{p}} = c/\sqrt{\varepsilon'(\omega)}$, and is directed under the characteristic angle θ_0 defined from the well-known relationship $\cos\theta_0 - c_{\mathrm{p}}/v = 0$. After integrating (21) over the angles, the spectral density of radiation per unit path length is described by the Frank–Tamm formula:

$$\frac{\mathrm{d}^2 W^{\mathrm{TF}}}{\mathrm{d}\omega\,\mathrm{d}l} = \frac{\omega}{c^2}\,Z_{\mathrm{eq}}^2 e^2\left(1-\frac{c_{\mathrm{p}}^2}{v^2}\right)\vartheta\left(1-\frac{c_{\mathrm{p}}^2}{v^2}\right). \tag{22}$$

The presence of the autocorrelation function $\big\langle Z(t)\,Z(t')\big\rangle$ in Eqn (21) leads to the spreading of the threshold condition and to the modification of the radiation spectral density (22). It is convenient to express the autocorrelation function $\langle Z(t)\,Z(t')\rangle$ in the following way:

$$\big\langle Z(t)\,Z(t')\big\rangle = Z_{\mathrm{eq}}^2 + \big\langle \xi(t)\,\xi(t')\big\rangle\,, \tag{23}$$

where $\xi(t) = Z(t) - Z_{\mathrm{eq}}$. For a stationary stochastic process like the ion charge exchange in the medium, the last term in Eqn (23) is an even function of the $t - t'$ argument. In order to qualitatively analyze the effect, we will use the following approximation (see, for example, book [21]):

$$\big\langle \xi(t)\,\xi(t')\big\rangle = \Lambda^2 \exp\big(-\Gamma|t-t'|\big)\,, \tag{24}$$

where Λ^2 is the dispersion of the charge random quantity $Z(t)$, $\Lambda^2/Z_{\mathrm{eq}}^2$ is the standard deviation of the charge from its equilibrium value, and $1/\Gamma$ is the characteristic charge exchange time, which can be estimated from the relation $\Gamma \approx n(\sigma_{\mathrm{c}} + \sigma_{\mathrm{l}})\,v$, where σ_{c} and σ_{i} are the electron capture and loss cross sections, respectively, and n is the atom concentration in the medium.

The spectral-angular density of radiation per unit path length (21), with (23) and (24) taken into account, can be written out as

$$\frac{\mathrm{d}^3 W}{\mathrm{d}\omega\,\mathrm{d}\Omega\,\mathrm{d}l} = \frac{\mathrm{d}^3 W^{\mathrm{TF}}}{\mathrm{d}\omega\,\mathrm{d}\Omega\,\mathrm{d}l} + \frac{\Lambda^2 e^2\omega\sin^2\theta}{2\pi^2 c^2}\,\frac{xy}{x^2y^2+(y-\cos\theta)^2}\,, \tag{25}$$

where we designated $x = \Gamma/\omega$ and $y = c_{\mathrm{p}}/v$. Performing the integration of equation (25) over the angles, we obtain the following expression for the spectral density of radiation:

$$\frac{\mathrm{d}^2 W}{\mathrm{d}\omega\,\mathrm{d}l} = \frac{\mathrm{d}^2 W^{\mathrm{TF}}}{\mathrm{d}\omega\,\mathrm{d}l} + \frac{\mathrm{d}^2 W^{\mathrm{cor}}}{\mathrm{d}\omega\,\mathrm{d}l}\,, \tag{26}$$

$$\begin{aligned}\frac{\mathrm{d}^2 W^{\mathrm{cor}}}{\mathrm{d}\omega\,\mathrm{d}l} &= \frac{\Lambda^2 e^2\omega}{\pi c^2}\\ &\times\left[(x^2y^2-y^2+1)\left(\arctan\frac{1-y}{xy}+\arctan\frac{1+y}{xy}\right)\right]\\ &-\frac{\Lambda^2 e^2\omega}{\pi c^2}\,xy\left(2+y\ln\left|\frac{x^2y^2+(1-y)^2}{x^2y^2+(1+y)^2}\right|\right).\end{aligned} \tag{27}$$

The second terms on the right-hand sides of formulas (25) and (26) describe the contribution from charge exchange correlational effects to the spectral-angular and spectral densities of Vavilov–Cherenkov radiation, respectively. As one can see, these terms unlike expression (22) make a nonzero contribution to the radiation yield, even when the threshold condition is not fulfilled and $y \geqslant 1$. The appearance of additional radiation is caused by the ion charge fluctuations during the process of electron capture or loss by the ion in the medium.

If the threshold condition is satisfied, $y < 1$, but the characteristic charge exchange time $1/\Gamma$ is much longer than the electromagnetic wave period (or the ion mean free path $1/(n\sigma)$ is much larger than the coherence length, which is the same thing) and $x \ll 1$, then it can be easily obtained from Eqn (26) that

$$\frac{\mathrm{d}^2 W}{\mathrm{d}\omega\,\mathrm{d}l} \approx \frac{\mathrm{d}^2 W^{\mathrm{TF}}}{\mathrm{d}\omega\,\mathrm{d}l}\left(1+\frac{\Lambda^2}{Z_{\mathrm{eq}}^2}\right). \tag{28}$$

This means that the influence of charge exchange correlational effects is reduced in this case to the replacement of the equilibrium charge Z_{eq} in Frank–Tamm formula (22) with some effective charge Z_{eff}, being, according to Eqn (28), $Z_{\mathrm{eff}} = Z_{\mathrm{eq}}(1+\Lambda^2/Z_{\mathrm{eq}}^2)^{1/2}$. On the other hand, if the threshold condition is not fulfilled, $y \geqslant 1$, and the characteristic charge exchange time $1/\Gamma$ is still much longer than the electromagnetic wave period, the correlational contribution to the spectral density (26) tends to zero and formula (22) is valid.

Finally, if the characteristic charge exchange time $1/\Gamma$ is much shorter than the electromagnetic wave period, so that $x \gg 1$, then, for any values of y, the correlational contribution to the spectral density (26) also tends to zero. When the Vavilov–Cherenkov radiation threshold condition is not fulfilled and $y \geqslant 1$, the radiation spectral density (26) is only defined by the correlational contribution (27), which at a given frequency ω has a maximum for the value of parameter $\Gamma \approx \omega$ [22].

4. Transition radiation of multiply charged ions

Processes of electron capture and loss by an ion crossing the interface between two media lead to a situation where the fields in each medium are generated by different ion currents. The 'prompt' condition for the electron capture or loss process will help us to find the corresponding fields from the continuity conditions for normal and tangential field components with respect to the interface.

Let the ion velocity be directed normal to the interface, and the z-axis be parallel to the particle velocity. We will denote the ion charge before entering the medium as $Z_1 e$, and the charge attained after entering the medium as $Z_2 e$. Assuming the magnetic permeability of a medium to be $\mu = 1$, we write out the Maxwell equations for the potentials in the left-hand medium (vacuum):

$$\Delta \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = -\frac{4\pi}{c} Z_1 e \mathbf{v} \delta(\mathbf{r} - \mathbf{v}t)\,,$$
$$\Delta \varphi - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = -4\pi Z_1 e \delta(\mathbf{r} - \mathbf{v}t)\,. \tag{29}$$

When writing out the Maxwell equations in the second medium, we will assume that, first, the fields are created by the current of the charge Z_2 moving with the same speed as in the first medium and, second, the charge $Z_1 - Z_2$ acquired by the ion due to the electron loss (or the 'hole' charge formed in the medium due to electron capture) creates some charge density in the medium, which quickly stops close to the interface. Then, for the medium with dielectric constant ε, located on the right of the interface, the relevant equations take on the form

$$\Delta \mathbf{A} - \frac{\varepsilon}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = -\frac{4\pi}{c} Z_2 e \mathbf{v} \delta(\mathbf{r} - \mathbf{v}t)\,, \tag{30}$$

$$\Delta \varphi - \frac{\varepsilon}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = -\frac{4\pi}{\varepsilon} Z_2 e \delta(\mathbf{r} - \mathbf{v}t) - \frac{4\pi}{\varepsilon}(Z_1 - Z_2)\, e \delta(\mathbf{r})\,. \tag{31}$$

Note that according to previous discussions a term is added to the right-hand side of equation (31), which contains the charge difference and ensures charge conservation. We will search the solutions of equations (29)–(31) by representing all the quantities in terms of Fourier integrals. Then, the Fourier components of electric field strengths in the first (on the left-hand side — vacuum) and in the second (on the right-hand side) media will have the following form:

$$\mathbf{E}_1(\mathbf{k}, \omega) = \frac{\mathrm{i} Z_1 e}{2\pi^2}\left(\frac{\omega \mathbf{v}}{c^2} - \mathbf{k}\right)\frac{\delta(\omega - \mathbf{k}\mathbf{v})}{\mathbf{k}^2 - \omega^2/c^2} + \mathbf{E}_1'(\mathbf{k}, \omega)\,\delta\left(\mathbf{k}^2 - \frac{\omega^2}{c^2}\right), \tag{32}$$

$$\mathbf{E}_2(\mathbf{k}, \omega) = \frac{\mathrm{i} Z_2 e}{2\pi^2}\left(\frac{\omega \mathbf{v}}{c^2} - \frac{\mathbf{k}}{\varepsilon}\right)\frac{\delta(\omega - \mathbf{k}\mathbf{v})}{\mathbf{k}^2 - \varepsilon\omega^2/c^2} - \frac{\mathrm{i}(Z_1 - Z_2)\, e}{2\pi^2 \varepsilon}\frac{\mathbf{k}}{\mathbf{k}^2}\,\delta(\omega) + \mathbf{E}_2'(\mathbf{k}, \omega)\,\delta\left(\mathbf{k}^2 - \frac{\varepsilon\omega^2}{c^2}\right), \tag{33}$$

where $\mathbf{E}_1'(\mathbf{k}, \omega)$ and $\mathbf{E}_2'(\mathbf{k}, \omega)$ are the Fourier components of the free field amplitudes in the first medium and in the second medium, respectively, which we will obtain from the continuity condition for the tangential and normal field components at the interface. Here, we should also take into account that, first, $\operatorname{div} \mathbf{E}_1'(\mathbf{r}, t) = \operatorname{div} \mathbf{E}_2'(\mathbf{r}, t) = 0$ and, second, the wave in the first medium propagates in the opposite direction with respect to the ion speed, while in the second medium it moves in the same direction. We will denote the projection of the vector $\mathbf{k}$ on the interface as $\mathbf{q}$. In the considered case of normal incidence of an ion on the interface, the tangential components of the radiation fields satisfy the equality $\mathbf{q}\mathbf{E}_{1,2t}' = q E_{1,2t}'$, and the generated radiation will be polarized in the plane parallel to the vector $\mathbf{k}$ and the z-axis. Moreover, we can also take advantage of scalar relationships, which bind the normal and tangential Fourier components of the fields:

$$E_{1t}' = \frac{\chi_1 E_{1n}'}{q}\,, \quad E_{2t}' = -\frac{\chi_2 E_{2n}'}{q}\,,$$

where $\chi_1 = \sqrt{\omega^2/c^2 - q^2}$, and $\chi_2 = \sqrt{\varepsilon\omega^2/c^2 - q^2}$.

Omitting the intermediate calculations, we provide the solution to the system of equations for free-field normal components in the first and second media (see details in Ref. [11]):

$$E_{1n}' = \frac{\mathrm{i} e}{\pi^2}\frac{q^2}{v}\frac{\chi_1}{\varepsilon\chi_1 + \chi_2} \times \left[\frac{Z_1(\varepsilon - \chi_2 v/\omega)}{q^2 + \omega^2/v^2 - \omega^2/c^2} + \frac{Z_2(-1 + \chi_2 v/\omega)}{q^2 + \omega^2/v^2 - \varepsilon\omega^2/c^2}\right], \tag{34}$$

$$E_{2n}' = \frac{\mathrm{i} e}{\pi^2}\frac{q^2}{v}\frac{\chi_2}{\varepsilon\chi_1 + \chi_2} \times \left[\frac{Z_1(1 + \chi_1 v/\omega)}{q^2 + \omega^2/v^2 - \omega^2/c^2} - \frac{Z_2(1/\varepsilon + \chi_1 v/\omega)}{q^2 + \omega^2/v^2 - \varepsilon\omega^2/c^2}\right]. \tag{35}$$

Angular and frequency distributions of the transition radiation on the left ('backward' radiation) and on the right ('forward' radiation) to the interface will be obtained from the relations [11]

$$\frac{\mathrm{d}^2 W_1}{\mathrm{d}\omega\,\mathrm{d}\Omega'} = \frac{\pi^2 c}{\sin^2\vartheta'}\,|E_{1n}'|^2\,, \quad \frac{\mathrm{d}^2 W_2}{\mathrm{d}\omega\,\mathrm{d}\Omega} = \frac{\pi^2 c\sqrt{\varepsilon}}{\sin^2\vartheta}\,|E_{2n}'|^2\,, \tag{36}$$

where ϑ is the angle between vectors $\mathbf{v}$ and $\mathbf{k}$, and ϑ' is the angle between vectors $-\mathbf{v}$ and $\mathbf{k}$.

Relations (34)–(36) provide us with the angular and frequency distributions of the transition radiation on the left ('backward' radiation) and on the right ('forward' radiation) to the interface:

$$\frac{\mathrm{d}^2 W_1}{\mathrm{d}\omega\,\mathrm{d}\Omega'} = \frac{e^2 \sin^2\vartheta' \cos^2\vartheta'}{\pi^2 c}\left|\frac{\beta}{\varepsilon\cos\vartheta' + \sqrt{\varepsilon - \sin^2\vartheta'}}\,F_1(\vartheta')\right|^2, \tag{37}$$

$$F_1(\vartheta') = \frac{Z_1\left(\varepsilon - \beta\sqrt{\varepsilon - \sin^2\vartheta'}\right)}{1 - \beta^2\cos^2\vartheta'} - \frac{Z_2}{1 + \beta\sqrt{\varepsilon - \sin^2\vartheta'}}\,, \tag{38}$$

$$\frac{\mathrm{d}^2 W_2}{\mathrm{d}\omega\,\mathrm{d}\Omega} = \frac{e^2 \varepsilon^{5/2} \sin^2\vartheta \cos^2\vartheta}{\pi^2 c} \times \left| \frac{\beta}{\cos\vartheta + \varepsilon^{1/2}\sqrt{1-\varepsilon\sin^2\vartheta}}\, F_2(\vartheta) \right|^2, \tag{39}$$

$$F_2(\vartheta) = \frac{Z_1}{1-\beta\sqrt{1-\varepsilon\sin^2\vartheta}} - \frac{Z_2(1+\beta\varepsilon\sqrt{1-\varepsilon\sin^2\vartheta})}{\varepsilon(1-\beta^2\varepsilon\cos^2\vartheta)}\,. \tag{40}$$

If we set $Z_1 = Z_2$, then expressions (37), (39) transform into the well-known relations obtained by Ginzburg and Frank [12]. For $Z_1 = 1$, $Z_2 = 0$, and $\varepsilon \to \infty$, Eqn (37) transforms into a well-known expression for spectral-angular density of the 'backward' transition radiation of a charged particle entering an ideal conductor from a vacuum:

$$\frac{\mathrm{d}^2 W_1}{\mathrm{d}\omega\,\mathrm{d}\Omega'} = \frac{e^2 \sin^2\vartheta'}{\pi^2 c}\,\frac{\beta^2}{(1-\beta^2\cos^2\vartheta')^2}\,. \tag{41}$$

Assuming $\varepsilon \to 1$ and taking allowance for $\vartheta' = \pi - \vartheta$ in Eqn (37) we obtain from formula (37) at $Z_1 = 1$, $Z_2 = 0$ and from formula (39) at $Z_1 = 0$, $Z_2 = 1$ a well-known expression for spectral-angular density of radiation emitted by an instantly stopped or started charge:

$$\frac{\mathrm{d}^2 W_{1,2}}{\mathrm{d}\omega\,\mathrm{d}\Omega} = \frac{e^2 \sin^2\vartheta}{4\pi^2 c}\,\frac{\beta^2}{(1-\beta\cos\vartheta)^2}\,. \tag{42}$$

As we should have expected, when the Vavilov–Cherenkov radiation appearance condition is fulfilled in the transparent medium, namely, when $\mathrm{Im}\,(\varepsilon) = 0$ and $\cos\vartheta = 1/\beta\sqrt{\varepsilon}$, the spectral-angular density of the 'forward' radiation (39) becomes infinite. This is connected with the fact that Vavilov–Cherenkov radiation in the transparent medium is coherently integrated over the whole infinite trajectory. In order to obtain a correct expression for the transparent medium, one needs to calculate the radiation yield per unit charged particle path length. However, in real systems there is always arbitrarily small absorption which leads to the finite trajectory length on which the Vavilov–Cherenkov radiation amplitudes sum up coherently. We should also note that the spectral-angular density of the 'forward' radiation in the second medium (39) in the cases of both electron capture and loss for $\varepsilon > 1$ contains waves that experience total internal reflection from the interface. As expected, this is valid for radiation angles which satisfy the condition $\sin\theta > 1/\sqrt{\varepsilon}$ — that is, when $(1-\varepsilon\sin^2\vartheta)^{1/2}$ in expressions (39) and (40) becomes imaginary. A similar influence of the 'total external reflection' effect on the 'backward' radiation for $\varepsilon < 1$ is described by expressions (37) and (38), when $(\varepsilon - \sin^2\vartheta)^{1/2}$ becomes imaginary.

Particularly, in the case of 'backward' radiation of a neutral particle entering the medium and losing an electron in it, the spectral-angular radiation density is described by

$$\frac{\mathrm{d}^2 W_1}{\mathrm{d}\omega\,\mathrm{d}\Omega'} = \frac{e^2 \sin^2\vartheta' \cos^2\vartheta'}{\pi^2 c} \times \left| \frac{\beta}{\left(\varepsilon\cos\vartheta' + \sqrt{\varepsilon - \sin^2\vartheta'}\right)\left(1+\beta\sqrt{\varepsilon-\sin^2\vartheta'}\right)} \right|^2. \tag{43}$$

However, if a singly charged ion captures one electron after entering the medium, then, instead of formula (43), we will have

$$\frac{\mathrm{d}^2 W_1}{\mathrm{d}\omega\,\mathrm{d}\Omega'} = \frac{e^2 \sin^2\vartheta' \cos^2\vartheta'}{\pi^2 c} \times \left| \frac{\beta\left(\varepsilon - \beta\sqrt{\varepsilon - \sin^2\vartheta'}\right)}{\left(\varepsilon\cos\vartheta' + \sqrt{\varepsilon-\sin^2\vartheta'}\right)(1-\beta^2\cos^2\vartheta')} \right|^2. \tag{44}$$

5. What to measure and how?

The most convenient object to conduct measurements on is the multiply charged ions accelerated to relativistic velocities and having one or two residual electrons on their shells. Such ions will lose their electrons soon after entering the medium. Backward electromagnetic radiation will have characteristic features which are absent for radiation process without electron loss. Some calculated results are given in Fig. 3 (spectral-angular density of radiation in dimensionless units $e^2/\pi^2 c$). The narrow maxima of the backward radiation in the X-ray range for angles close to $\pi/2$ (along the interface) are related to the effect of total external reflection.[3]

It is known that the full number of emitted quanta of the transition radiation diverges logarithmically at small frequencies [12]. On the other hand, not all emitted waves propagate in the medium. In metals, for example, these are frequencies smaller than the plasma frequency ω_p, which for most of the media is not higher than a value corresponding to an energy of 15–20 eV. Then, the number of emitted quanta in the energy range from $\hbar\omega_1$ to $\hbar\omega_2$ can be estimated by assuming that the upper boundary of the energy interval $\hbar\omega_2$ does not exceed the limitation put on the 'prompt' hypothesis of the charge variation given in the Introduction, and the lower boundary is larger than the plasma frequency. Thus, the number of backward-emitted quanta in such an energy range can be estimated from formulas (37), (38):

$$\Delta N = \int_{\omega_1}^{\omega_2} \mathrm{d}\omega\,\mathrm{d}\Omega\,\frac{1}{\hbar\omega}\,\frac{\mathrm{d}^2 W_1}{\mathrm{d}\omega\,\mathrm{d}\Omega}\,. \tag{45}$$

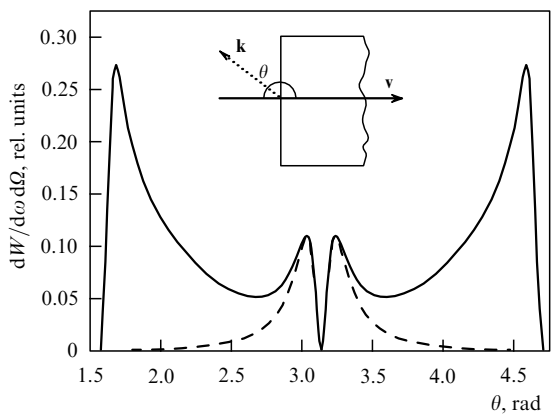

Figure 3. Angular distribution of the backward radiation in the spectral region of 0.5 keV for argon relativistic ions ($\beta = 0.995$), as they enter a gold film in the cases of fully ionized ions ($Z_1 = 18$, $Z_2 = 18$, dashed curve) and singly ionized ions ($Z_1 = 17$, $Z_2 = 18$, solid curve).

[3] In computations, we invoked the data on the frequency dependence of gold complex dielectric constant obtained at the Center for X-ray Optics of Lawrence Berkeley National Laboratory (http://henke.lbl.gov/optical_constants/index).

By using the given data, it is easy to verify that in the photon energy range from, for example, 1 to 2 keV, according to formula (45), approximately 0.001 photons are emitted per relativistic argon ion ($\beta = 0.995$, $\gamma = 10$). For an ion beam current on the order of 10 mA, several photons will be emitted per second, which is enough for the reliable detection of an effect.

6. Conclusion

Summing up, we can conclude that the charge exchange processes of accelerated multiply charged ions in a medium lead to some characteristic features in the accompanying electromagnetic radiation. Similar effects can be observed in charge exchange processes in so-called tandem accelerators, as noted in paper [23]. Physically, this effect is connected to the appearance of an additional contribution to the radiation yield from the electrons, which are captured or lost by a multiply charge ion in the medium. As follows from the performed analysis, this contribution is similar to that of instantly started or stopped charges. Therefore, we can give an affirmative answer to the question posed in the title of the present paper.

Detailed experimental investigations of the considered phenomena will give an opportunity to study the characteristics of electromagnetic radiation emitted by instantly starting or stopping charges.

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