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Review of a book on the anniversary of the theory of the Berezinskii – Kosterlitz – Thouless transition — a book which proved to be a precursor of the 2016 Nobel Prize in physics

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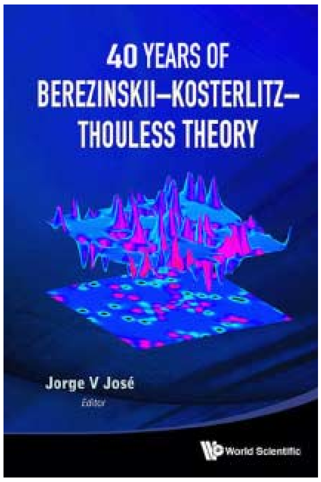
40 years of Berezinskii – Kosterlitz – Thouless theory (Ed. J V José) (Singapore: World Scientific Publ., 2013) 351 pp. ISBN: 978-981-4417-62-4 [1]

On 4 October, in Stockholm, three people were awarded the Nobel Prize for physics for 2016 "for theoretical discoveries of topological phase transitions and topological phases of matter", according to the Nobel citation. The trio, all of them British-born long-time US-based researchers, was David Thouless (University of Washington, Seattle), Duncan Haldane (Princeton University), and John Michael Kosterlitz (Brown University, Providence). Duncan Haldane, although undoubtedly an outstanding theoretical physicist known well for his work on topological states in one-dimensional integer-spin magnetic chains, seems, however, to be somewhat incidental in this company. Half of the prize went to D Thouless, whereas J M Kosterlitz and D Haldane each received a quarter, which seems justifiable and consistent with the respective contributions of each. What is not justifiable, though — and this is obvious for any one even remotely familiar with the history of this discovery — is the absence of the name of Vadim L'vovich Berezinskii in the Nobel Committee press release. Unfortunately, the Nobel Prize may only be awarded to living people; otherwise, he would also have to be among the recipients (see, for example, the comment in [2] by American Institute of Physics reviewer Yuen Yiu on the 2016 Nobel Prize).

While the term "Kosterlitz–Thouless transition" appears quite frequently in the English literature (as, unfortunately, exemplified by the Nobel Committee press release), scientists in Russia (as well as their colleagues elsewhere — judging, for example, even from the very name of the book [1] to be reviewed below) speak more often of the "Berezinskii–Kosterlitz–Thouless (BKT) transition".

Vadim Berezinskii was a talented Soviet physicist [3] remarkably skilled in mathematics. While the problems he dealt with were many, the areas where he was outstanding were the theory of phase transitions in two-dimensional systems and the theory of localization in one-dimensional conductors. Berezinskii died in 1980 at the age of only 45, and



Introduction

More than 40 years ago, Vladimir Berezinskii[1] (1971) and the team of Michael Kosterlitz and David Thouless[2] (1973) (BKT) published the results of their independent investigations into the puzzling question of whether or not long-range order exists in two-dimensional systems with continuous symmetry. This work would have a seminal impact upon condensed matter physics, and other areas in physics, an impact we explore in the present volume. Heuristic arguments and rigorous mathematical theorems had long led to the general belief that there could not be a stable thermodynamic ordered phase at low temperatures. However, experimental and theoretical evidence had indicated that there was something else going on with this problem. BKT introduced the idea of a *topological phase transition*, where pairs of bound vortex excitations unbind at a critical temperature "T_{BKT}". The nature and characteristics of the BKT transition are different in several respects from the more common second order phase transitions. While there is no long-range order with a finite order parameter for all temperatures, as shown by the earlier theorems, we now know that there is, however, a continuous line of critical points below T_{BKT}. At low temperatures, the two-point correlation function decays algebraically, with a temperature dependent exponent up to T_{BKT}. Above T_{BKT}, the correlation function decays exponentially in distance but has a correlation length that diverges exponentially with temperature, rather than as a power law. The seminal ideas of BKT have had an impact in many areas of theoretical and experimental physics, some examples of which are presented in this volume. Given that there are thousands of papers where BKT work has been applied, in this 40th Anniversary volume, we provide only a small sampling of the ways in which BKT ideas have been used. It is interesting to note that the number of citations to the KT and BKT work is comparable to the number of citations to the seminal 1958 paper by Phil Anderson on localization theory.[3]

Soon after I started my PhD thesis in 1974, under the advice of Leo Kadanoff, I attended my first summer school conference on *The Helium Liquids* held at St. Andrews University in Scotland. In that meeting Ann Eggington presented a review of the experimental and theoretical situation

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Cover and first page of the preface of the book under review [1]. A copy of the book was sent to *Uspekhi Fizicheskikh Nauk* (*UFN*) (*Physics-Uspekhi*) journal by the publishing house World Scientific for consideration for publication of the book's review in *UFN*'s 'Bibliography' section.*

it was only in his last three years that he worked at the L D Landau Institute of Theoretical Physics. It was Berezinskii who first formulated in [4, 5] the essential features of the theory which won the Nobel Prize this year. Discussing priority issues is an extremely complex and thankless job, but it should be recognized that Berezinskii's work did not gain as much recognition as the results of Kosterlitz and Thouless [6, 7], even though the seminal paper [7] makes correct, appropriately commented reference to both of Berezinskii's papers [4, 5] on this subject. Indeed, Kosterlitz writes the following in his 2016 review [8], tellingly titled "Kosterlitz–Thouless physics: a review of key issues": "David and I congratulated ourselves on finding important new physics but our euphoria soon dissipated. We were informed that Berezinskii [4]† had discussed the vortex driven transition in a superfluid film a year earlier than our paper [6, 7]†. Since neither of us knew any Russian we were blissfully unaware of this work while we were developing the basic physics of the vortex driven transition. For some

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* A *UFN* Editorial Board decision has existed since 1999 that the reviews of foreign books on physics and related sciences are published only in those exceptional cases in which the content of the book is closely related to the development of Russian national science (see [10]). It was decided that the 2016 Nobel Prize in physics can be considered such an exceptional case, thus warranting a *UFN* review of this book. (*Editor's note.*)

† The respective reference numbers of the corresponding papers by Berezinskii and Kosterlitz & Thouless are given in accordance with their reference numbers in the bibliography of the present work. (*Editor's note.*)

unknown reason, our work seems to have had much greater impact than that of Berezinskii.'' In this connection, it should be pointed out that *Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki* (*ZhETF*) which published Berezinskii's two seminal papers [4, 5] was at the time translated into English (under the title *Soviet Physics – JETP*) on a regular basis by the American Institute of Physics (see [9]) and that, specifically, a translation of Berezinskii's paper appeared in March 1971.

Of much importance in this context is, in my view, the appearance in 2013 (i.e., three years before the 2016 Nobel Prize) of a book titled *40 years of Berezinskii–Kosterlitz-Thouless Theory*. This publication, edited by Jorge V José, an acknowledged expert in the field of phase transitions, not only reflects the obvious international recognition of Vadim Berezinskii's [1] role in the development of this new area but also highlights the fact that the field has developed to the point that the Nobel Prize for its foundation and development no longer comes as a surprise.

The selection of J José as editor of this volume seems an appropriate one considering his immediate involvement in the development of the modern physics of phase transitions. There are moments in the history of science when gradual, relaxed development, the accumulation and evaluation of experimental evidence, and the appearance of a variety of local theories give way to a sudden burst, when within a decade or two, if not the worldview as a whole, then at least our understanding of a certain major field of research undergoes a fundamental change.

Of course, there is no comparison between the profound revolutionary effect relativity and quantum mechanics had on the worldview of physics in the early 20th century and what occurred in the theory of phase transitions in the 1960s–1970s, however, in that time the fundamental change in our understanding of the physics of phase transitions has taken place. The development of the fluctuation theory of phase transitions — which relied on the scaling hypothesis (proposed by A Z Patashinskii and V L Pokrovskii and somewhat later also by Leo Kadanov) and which Kenneth Wilson used to develop the renormalization group method and (together with Michael Fisher) the famous ε-expansion — led to K Wilson being awarded the first phase transition Nobel Prize in 1982.

In describing the properties of phase transitions, these and other authors noted the role of the dimensionality of the space where the transition takes place. As Peierls and Landau, and later Bogolyubov, Mermin, and Wagner had already shown, in two-dimensional systems with continuous order parameter symmetry (Heisenberg magnet, plane rotator, or $X-Y$, model, superfluid, and superconducting systems, and two-dimensional crystal lattices), thermal fluctuations destroy long-range order (i.e., the nonzero value of the order parameter which is the same throughout the system). Because most physicists of the time attributed this transition to the appearance of long-range order, it was concluded that the phase transition in such systems is possible only at zero temperature. At the same time, experimental studies on superfluidity in thin films of liquid helium-4 have been conducted and numerical and computer simulation results obtained (in particular, on the crystallization of two-dimensional systems) that were inconsistent with this conclusion.

It was Berezinskii [4, 5] and Kosterlitz and Thouless [6, 7] who cleared things up. Berezinskii was the first to show that, despite the lack of long-range order in the system, a thin liquid helium film does exhibit superfluidity at low temperatures. Two-dimensional crystals, despite the absence of long-range translation order, have a finite shear modulus and therefore are solid. Two-dimensional magnets are resistant to a nonuniform rotation of spins. Berezinskii recognized that these phenomena are common in nature and coined the now popular term *transverse rigidity* for them. He showed that in transverse-rigidity systems two-point order-parameter correlation functions exhibit a slow power-law decay with distance, with an exponent dependent on the interaction parameters and temperature. Recall that in the presence of long-range order a similar correlation function tends to a nonzero limit as the distance between the points tends to infinity, and in a disordered high-temperature phase the correlations decay exponentially fast. The new phase, sometimes called the Berezinskii phase, differs fundamentally from what can be observed in three dimensions. Given the slowly decaying correlations, this phase is commonly referred to as a phase with quasi-long-range order. Somewhat later, similar results were obtained by Kosterlitz and Thouless [6, 7], who also corrected an inaccuracy made by Berezinskii: he argued, erroneously, that quasi-long-range order can exist in a two-dimensional Heisenberg magnet, i.e., in a system with three-component magnetic moments. Noting that correlations in the low- and high-temperature phases have different decay laws which do not change continuously into one another, it clearly follows that a phase transition between them should exist. Accordingly, the question of what the mechanism of this transition is arose. Berezinskii was the first to discover that topological defects — vortices in a superfluid helium film, dislocations in a two-dimensional crystal, vortex configurations in a two-dimensional magnet with two-component magnetic moments ($X-Y$-model) — play an important role in the transition, and proposed a qualitative mechanism for the transition. At low temperatures, defects couple into pairs, which do not destroy the quasi-long-range order. As the temperature rises, however, the coupled pairs dissociate, giving rise to free defects, which transform the quasi-long-range order to a disordered phase with fast exponentially decaying correlations. A method for calculating the transition temperature was developed in the subsequent work of Kosterlitz and Thouless [6, 7, 11].

One cannot but be amazed at the talent of V L Berezinskii, who took up the problem which most physicist of the time did not recognize at all and who opened up a new field of research, which is still alive and developing.

Kosterlitz and Thouless [6, 7] give an elegant discussion of mechanisms that destroy quasi-long-range order in two-dimensional systems with a continuous symmetry group. Based on the qualitative analysis that was once used by Thouless to describe a phase transition in a one-dimensional Ising model with $1/r^2$ interaction, Kosterlitz and Thouless calculated the formation free energy of an isolated topological defect. The reason why this proved possible is that the energy and entropy of an isolated vortex are proportional to the logarithm of the system's size. The condition of zero free energy yields the transition temperature.

[1] There are regrettably two points to make here that cannot but be somewhat painful for the prospective reader of the book under review. The book appeared in 2013 to mark the 40th anniversary of Kosterlitz's and Thouless's most known paper, whereas Berezinskii's first paper was published in *ZhETF* in 1970. Second, in the first line of the book, *Vadim* Berezinskii is misnamed *Vladimir*, and references to his papers are not always accurate and/or full even in this anniversary publication, which clearly demonstrates the lack of editorial control.

This simple picture is not totally correct physically, however, because the coupled pairs of oppositely 'charged' vortices do not destroy quasi-long-range order and have a finite energy. Such pairs even exist at low temperatures. The Hamiltonian for the vortex subsystem is equivalent to that for a two-component two-dimensional Coulomb gas (i.e., a gas of particles that interact via a long-range logarithmic potential). The mechanism of the Berezinskii–Kosterlitz–Thouless transition is the dissociation of the dilute gas of vortex pairs (account should also be taken of the screening of the Coulomb potential due to thermally excited pairs). The dissociation occurs at a temperature at which the dielectric constant of the system diverges, and the coupling constant is renormalized at the transition point T_0 to the universal limiting value, which subsequently jumps down to zero. The correlation length diverges exponentially as the transition temperature is approached from above, with the heat capacity peaking only slightly at a temperature above the transition point. The analogy with a two-dimensional gas and the renormalization group equations for describing the transition were introduced by Kosterlitz and Thouless in their papers [6, 7, 11]. It should be noted that the transition temperature obtained by the renormalization group method is equal to the temperature calculated above from simple energy considerations, with the coupling constant replaced by its renormalized value.

The Berezinskii–Kosterlitz–Thouless Theory has found increasing application in the study of a variety of two-dimensional systems, ranging from superfluid and superconducting films, thin magnetic and liquid crystal films, to systems of Josephson junctions and two-dimensional systems of ultracold atoms in optomagnetic traps. It is the impact the seminal work of Berezinskii, Kosterlitz, and Thouless has had on the current state of condensed matter physics which is the subject of the book under review.

While the fluctuation theory of phase transitions caused an immediate avalanche of publications (as is well remembered by anyone who happened to browse the mid-1970s issues of *Physical Review*), it should be noted that the Berezinskii–Kosterlitz–Thouless Theory had a less fortunate destiny. The idea of how exactly things went can be obtained from the first chapter of the book under review, in which Kosterlitz and Thouless describe the history of and the prospects for their proposed theory. This is especially interesting because research papers do not usually reveal the motivation of their authors, nor do they trace the (sometimes far-from-straight) pathways of the authors' thought. In the mid-1970s, attempts at disproving the BKT theory were made, and, according to this chapter, Kosterlitz's and Thouless's papers remained virtually uncited for the first five years. It should be noted that, whereas the preliminary paper [6] was indeed cited very little in the first years after publication according to the Web of Science Core Collection (WoS CC) data, article [7] (published in June 1973) was noted immediately by specialists. In particular, this paper was mentioned by V L Ginzburg at the scientific field session of the Department of General Physics and Astronomy already in October 1973 (i.e., four months after its publication) and was already cited [12] in the abstract of his talk published in *UFN*, its reference number being shared with Berezinskii's two seminal papers [4, 5].** The breakthrough came after 1977, due, in my view, to the work of Nelson and Kosterlitz [14] on the superfluid transition in a helium film and of Nelson and Halperin [15] and Young [16] on the theory of two-dimensional melting. At present, the papers by Kosterlitz and Thouless are among the most cited in the periodical literature (as of December 2016, paper [7] alone has been cited more than 6,000 times according to WoS CC). The paper by Berezinskii has, unfortunately, received less citation, although the WoS CC database result has already approached 2,000 only for his papers [4, 5] (which represent part 1 [4] and part 2 [5] of one paper, as the very titles of the papers suggest).

Note also that a revised version of Chapter 1 of the book [1] under review formed the basis for the already-cited paper of Kosterlitz in the journal *Reports on Progress in Physics* [8]. In my view, the publication of this review article in early 2016 is not accidental and can be understood in the context of the then-underway struggle for the Nobel Prize.

The papers published in the book under review were to a certain extent chosen based on the editor's research preferences. The second chapter, written by Jorge José himself, considers the application of dual transformations, gauge symmetry, and the renormgroup approach to understanding the phase structure, correlation functions, and excitations in the two-dimensional $X-Y$-model. This allowed the perturbative derivation of the renormalization equations for this model — equations that had been derived in Kosterlitz's and Thouless's original work by considering a two-dimensional Coulomb gas — and thereby made it possible to substantiate the basic assumptions of BKT theory. The next chapter, by G Ortiz, E Cobanera, and Z Nussinov, reviews current work and considers inconsistencies that arise when the dual transformations are applied to the $X-Y$- and p-clock models.

In their original paper [7], Kosterlitz and Thouless stated that superconducting films do not exhibit the topological phase transition, which proved to be incorrect because of their assumption that the vortex-vortex interaction behaves asymptotically as $1/r$. In Chapter 4 of the book being reviewed, A Goldman points out that there is actually an effective screening length below which the intervortex interaction in superconducting films and Josephson junction systems has a logarithmic form, making the BKT theory an adequate approximation for treating these systems. Goldman presents an elementary theory of the BKT transition as applied to superconducting films and considers evidence for the applicability of the theory to superconductors and Josephson junction systems. The development of this approach is traced in Chapter 5, written by C Benfatto, C Casterllani, and T Giamarchi, who present a critical review of the theoretical and experimental applications of the BKT transition to superconductors. Remembering that the analogy with the $X-Y$-model, although used in most studies, is not always adequate, they take as their starting point the transition picture based on the sine-Gordon model. This model allows including variations in the defect core energy, which affects significantly the transition scenario. In particular, for certain values of the nucleus energy, the transition in a superconducting film can become first order.

S Teiltel in Chapter 6 reviews the theoretical and experimental status of the completely frustrated $2D-X-Y$-model for a system of Josephson junctions in a magnetic field. The brief review by R Fazio and G Shon (Chapter 7) concentrates on the properties of Josephson junctions in the quantum regime. In particular, it is shown that, under certain

** It is interesting that, according to the WoS CC database, V L Ginsburg's reference in [12] to [7] is 11th in the world literature, and the 12th reference is given in *UFN*, in S M Stishov's review [13] in a 1974 issue. (*Editor's note*.)

conditions, charges and vortices turn out to be dual, and the system can exhibit a quantum phase transition between a conductor and a Mott insulator. Finally, Chapter 8 by V Vinokur and T Baturina, both of Russia, discusses the competition between quantum and classical fluctuations in a two-dimensional Josephson junction system and shows that a dual transformation can link the state that the BKT transition produces in a vortex-antivortex plasma with the super-insulator that forms in an insulating state of the junction system due to the charge BKT transition.

Noteworthy among recent intriguing applications of the BKT theory is the study of quasi-two-dimensional systems of ultracold atoms in optomagnetic traps. In Chapter 9, Z Hadzibabic and J Dalibard examine the experimental realization of such systems and present a theoretical interpretation of their behavior. They discuss the equation of state of this system as well as the 'competition' between the superfluid state resulting from the BKT transition and the usual Bose-Einstein condensation which can occur in a two-dimensional Bose gas in a harmonic potential.

Finally, in Chapter 10, H Fertig and G Murphy consider the further development of the ideas of Berezinskii, Kosterlitz, and Thouless as applied to a two-layer system which exhibits the quantum Hall effect with filling factor $\nu = 1$ and can be mapped into a two-dimensional superfluid system with charged vortices. This problem is of great interest both theoretically and experimentally, especially at low temperatures, where both quantum fluctuations and disorder are important.

To my mind, the book reviewed presents a sufficiently broad and detailed but still, in some important respects, limited view of the current development of the seminal ideas of Berezinskii, Kosterlitz, and Thouless. In particular, virtually no discussion is given of such an important topic as the theory of two-dimensional melting, a theory which came almost immediately after the BKT theory. In the first chapter, Kosterlitz and Thouless give a rather brief discussion of this question, but address themselves almost exclusively to the famous early efforts of Nelson and Halperin [15] and Young [16], who showed that, unlike in three dimensions (when melting always occurs via a first order transition), in two dimensions melting via two continuous BKT-type transitions is possible.

The first of these involves the dissociation of dislocation pairs (which are topological defects in this case). However, the liquid above the dissociation point of the dislocation pairs turns out to be non-isotropic in this case. The resulting new phase was called hexatic, in analogy with liquid crystals. The hexatic phase contains free dislocations and therefore its shear modulus is zero, i.e., this phase is a liquid with some amount of order. Note that a dislocation can be represented as a coupled pair of disclinations. The subsequent Berezinskii–Kosterlitz–Thouless transition transforms the hexatic phase into a usual isotropic liquid via the dissociation of disclination pairs. The presented theory, referred to as the Berezinskii–Kosterlitz–Thouless–Halperin–Nelson–Young (BKTHNY) theory, is attractive and unique, so that the suggestion even arises, in a sense, that all two-dimensional crystals should melt within this scenario. Two points of doubt arise, however, about this theory: it fails to calculate the core energy of topological defects and the effective interaction energy between disclinations in the hexatic phase.

However, for small dislocation core energies, a first-order transition is also possible [17, 18].

Vadim L'vovich Berezinskii (15.07.1935–23.06.1980) and his dissertation [21] published as a separate book (2007, Fizmatlit, prefaces by A V Polyakov and V L Pokrovskii).

This finding was followed by a flurry of studies, both experimental and computer simulation. The systems experimented on are diverse and include but are not limited to: two-dimensional colloids, electrons on a liquid helium surface, inert gas atoms on substrates (specifically, xenon on graphite), two-dimensional granulated systems, vortex systems in HTSCs and in thin superconducting films in a magnetic field, dust plasmas, and films of liquids (for instance, water). The conclusion that suggests itself currently is that the melting scenario of a two-dimensional system depends critically on exactly how the particles interact among themselves. In particular, it appears that the BKTHNY theory is valid for systems with long-range interaction, whereas for systems with short-range potentials, melting can occur through two transitions with an intermediate hexatic phase — in such a way, however, that the crystal to hexatic transition occurs according to the Berezinskii-Kosterlitz-Thouless theory, whereas the hexatic phase changes to a liquid as a result of a first-order transition [19, 20]. While much is already known about the melting mechanisms of two-dimensional systems, there is still much to be understood.

Given the interest generated by the awarding of the Nobel Prize for topological phase transitions, it is impossible not to recall book [21] published 10 years ago, which was in fact the doctoral dissertation of V L Berezinskii. Although many years have elapsed since the dissertation was written, and many of its results — first, those concerning topological phase transitions discussed here and second, those on the dynamical conductivity of a one-dimensional crystal — have become scientific classics and have been described and rediscovered many times in other studies, reading the original work of such an undoubtedly outstanding theoretical physicist as Vadim Berezinskii can enrich the reader with new ideas, as well as promote and develop the culture of how papers on theoretical physics should be written. It should be noted at once that, while clear and understandable in presentation, this book (as well as Berezinskii's original papers on topological phase transitions) is not casual or light reading but, instead, requires considerable intellectual effort to understand, providing, in my view, another answer to the question asked by Kosterlitz (see above) of why Berezinskii's papers generated less discussion than his and Thouless's work. Today, as I reread these papers, my feeling (perhaps mistaken but nevertheless firm) is that papers [6, 7] give, in a sense, a popular presentation of Berezinskii's ideas. But this may be an overly personal opinion. It should be noted that Chapter 5 of the book covers Berezinskii's results on a two-dimensional Heisenberg magnet. As already noted above, Berezinskii believed erroneously that in two dimensions as well this system should exhibit quasi-long-range order with power-law decaying correlations. As is well known, this conclusion was refuted in Ref. [7]. A nice comment on this is given in the second (V L Pokrovskii's) preface in Ref. [21].

The prefaces, the first by A M Polyakov, the second by V L Pokrovskii, seem to deserve special mention. V L Pokrovskii in his preface presents a brief but in-depth review examining the basic ideas of Berezinskii on the topic of topological phase transitions and traces their development in some subsequent work. The preface by A M Polyakov is in fact a very brief but vivid essay not so much about physics as about the atmosphere that existed at the time in the Institute of Theoretical Physics, then perhaps one of the leading world centers for research in this field. I did not know Berezinskii personally, but I have heard many stories about him, some of them funny, but all touched with sadness, as is often the case when speaking posthumously about a good person. Polyakov's essay is exactly the text showing how people felt about him personally and what their attitude was to his research style. The following paragraph caught my eye when reading this preface: "One of his (Berezinskii's — *V N R*) counter-arguments had to do with the Heisenberg antiferromagnet. He argued that the antiferromagnet contained relativistic gapless goldstones, which was inconsistent with my results. I had no objections to this and, on the other hand, I was firmly confident in my calculations (which were performed with three different methods). And then ultimately I simply forgot about Vadim's objection — which I should not do. Many years later, Haldane realized that in the case of half-integer spins, topological effects should be included, and no gap occurs — whereas for integer spins, my results are valid." This was written nearly 10 years ago. And today Haldane has received the Nobel Prize exactly for predicting the existence of a gap in an integer spin chain. I'd rather not comment on this situation... .

Finally, we note that the 2016 Nobel Prize awarded for work on phase transitions led to a general revival of interest in the field. Popular science papers [22, 23] and papers on science news sites (including those abroad; see, for example, Ref. [2]) have been published and scientific conferences conducted that emphasize the pioneering role of V L Berezinskii in developing the BKT theory. Special mention should perhaps be made of the 21 December 2016 Scientific session of the Physical Sciences Division of RAS titled "Old and new in the physics of phase transitions", where one of the four talks held was fully dedicated to the BKT theory. Reviews on the basis of these talks are in the *UFN* publication plan for 2017. It goes without saying that in all these publications (unlike the Nobel press release), the theory of low-dimensional systems will be referred to by its full name to honor all of its founders — i.e., as the Berezinskii–Kosterlitz–Thouless (BKT) theory, which is exactly how it was referred to in the book under review [1] published three years ago — and, as predicted prophetically in Ref. [3] in 1981, the name V L Berezinskii "...will remain forever in the physical literature... ."

In conclusion, I would like to note that the book [1] we have presented not only provides insight into the development of the ideas advanced already more than 45 years ago in the pioneering work of Berezinskii, Kosterlitz, and Thouless, but also shows that these ideas still serve as a powerful stimulus for the study of low-dimensional systems. The pity is that the remarkable scientist Vadim L'vovich Berezinskii did not survive until our time. He would only have been 81 this year.

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