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A drop jumps to weightlessness: a lecture demo

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Abstract. The paper discusses the lecture demonstration of the phenomenon in which a drop lying on a solid unwettable substrate jumps when making the transition to weightlessness. An elementary theory of the phenomenon is given. A jump speed estimate is obtained for small and large drops. The natural vibrational frequency of a flying drop is determined. A full-scale model of Einstein's elevator is described. Experimental and theoretical results are found to agree satisfactorily.

Keywords: lecture demonstrations, liquid drop, transition to weightlessness, natural oscillations, Einstein's elevator

1. Introduction

The authors of paper [1] studied the phenomenon of the capillary 'ball game' under the condition of weightlessness. In the experiment, a camera and a plexiglass jar with a 20% solution of hydrochloric acid were placed in a container with a mass of 100 kg, which was then dropped from a height of 20 m. A large drop of mercury with a mass of 20 g was placed on the bottom of the jar. As the container was hanging motionless in the gravity field, the mercury drop on the horizontal surface had a round flat shape with a radius of 1.2 cm and thickness of 0.35 cm. As the weightlessness took place, the drop squeezed together, jumped, and started traveling away from the bottom of the jar with a constant speed. After reaching the liquid surface, it reflected and moved towards the bottom of the jar, from which it also reflected. This phenomenon was popularly described in [2]. In order to educationally investigate the drop jump under the system transition to weightlessness, the authors of [3] suggested a laboratory setup and elementary theory.

Modern technologies of digital photography and multimedia projection give an opportunity to create lecture demonstrations of fast mechanical phenomena, which are associated with free fall in Earth's gravity field. A simple demonstration of a real model of Einstein's elevator was described in [4]. In this article, we present an educational experiment and theory for a jump of a water drop lying on a nonwettable (super-hydrophobic) surface in a gravity field and undergoing a transition to weightlessness. We also consider a similar experiment, where a metal ball is placed on a solid elastic substrate. Discussed phenomena are analyzed by means of the didactic potential for studies of general and experimental physics.

2. Experimental setup

In demonstration experiments, one needs to create such conditions so that a squeezed drop on a nonwettable substrate instantaneously becomes weightless. Therefore, a motionlessly hanging mounting with a drop should start falling at some moment. Free fall is a quite fast process, so in order to observe its different stages, one needs to take a series of instantaneous photographs of the falling mounting with the drop traveling away from it.

A functional diagram of the experimental setup built according to the described idea is shown in Fig. 1. Fluorescent lamp *1* is covered with a scattering screen *2* fabricated from frosted plexiglass. Electromagnet *3* is placed in front of the screen at a height of about 1 m from the demonstration table. The magnet core attracts steel anchor *4*, which is fixed to metal clamp *5*. The lower ends of the clamp are connected to the mounting made of brass or bronze (*6*). The working surface of the mounting has a spherical depression 0.2–0.5 mm deep, so that the lying drop does not roll down. The mounting is provided with pointed metal leg *7*, which allows flight stabilization and effective braking. A jar with sand (*8*) is placed under the mounting. A digital camera (*9*) is placed at a distance of about 1.5 m from the mounting.

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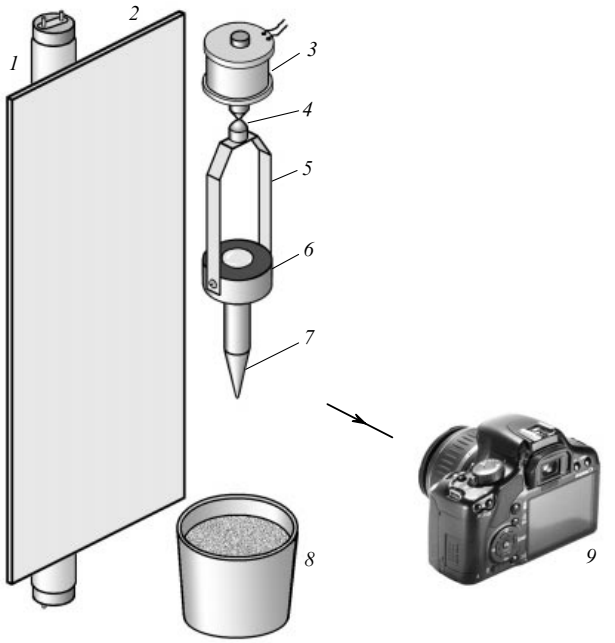

Figure 1. Experimental setup: *1* — fluorescent lamp, *2* — scattering screen, *3* — electromagnet, *4* — steel anchor, *5* — metal clamp, *6* — brass or bronze mounting, *7* — pointed metal leg, *8* — jar with sand, *9* — digital camera.

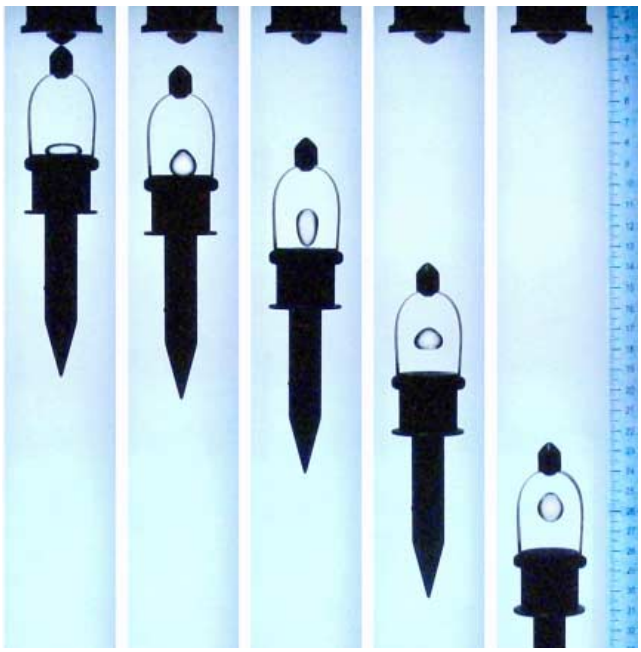

Figure 2. Demonstration experiment results. Shots are taken after equal time periods of 50 ms.

In order to photograph falling objects, a Casio camera is available for use in the time-lapse regime at 60 frames per second and 1/1000 or 1/2000 s exposure. During the mounting free fall, the camera takes a series of 18 photographs with a time spacing of 1/60 s. In order to obtain quantitative results, a transparent scale with millimeter divisions (not shown in Fig. 1) needs to be placed parallel to the free fall trajectory.

3. Phenomenon of a drop jump under a sudden transition to weightlessness

Before the experiment, the concave surface of the mounting should be covered with soot above a candle flame. In order to obtain a durable layer, after the first deposition, the mounting should be cooled and the soot deposited for a second time and, if needed, this process repeated for the third time. The surface of the drop lying on the mounting will be absolutely pure if the soot is preliminarily carefully washed with water. In demonstration experiments, it is convenient to use mountings with detachable substrates, which should be covered with soot in advance.

After fixing the mounting to the electromagnet, a drop of the size needed is placed on its surface. This can be controlled using a medical syringe with volume calibration. Then, the lamp is turned on, the sharpness of the camera image is adjusted, and the shutter is released at the same time as the electromagnet is switched off. The mounting falls, hits the sand with the pointed leg, brake, and stops in the vertical position. Often, the surface covered with soot remains intact and the experiment can be repeated with a new drop.

The series of free-fall photographs are uploaded to the computer and, using a multimedia projector, are visualized on a large screen. If the demonstration setup is built in advance, it will only take several minutes for a lecturer to explain the experimental conditions and perform the measurements with satisfactory results.

Figure 2 shows a series of photographs of a drop jump in weightlessness obtained using the method described above. On the right-hand side of the last photograph, an image of the scale can be seen. It is clear that already 0.1 s after the fall starts, the drop breaks away from the hydrophobic surface of the mounting and then, oscillating, travels away from it with a constant speed.

4. Mechanical model of a liquid drop

As a mechanical model for a liquid drop with mass m, we use an oscillator, which consists of two weights with masses $m/2$, attached at diametrically opposite points to a ring spring with rigidity k.

Under conditions of weightlessness, the model is a round ring with radius R_0. The lower weight of the drop ring model is first placed on the substrate with mass $M \gg m$. After the system is released, the upper weight will start to move downwards due to the gravitational force $\mathbf{f} = m\mathbf{g}/2$ (Fig. 3a). The model experiences damping oscillations and reaches a new equilibrium condition (Fig. 3b). The vertical size of the model $2R_0$ reduces to H and the center of mass C shifts lower at the distance

$$h = R_0 - \frac{H}{2}\,. \tag{1}$$

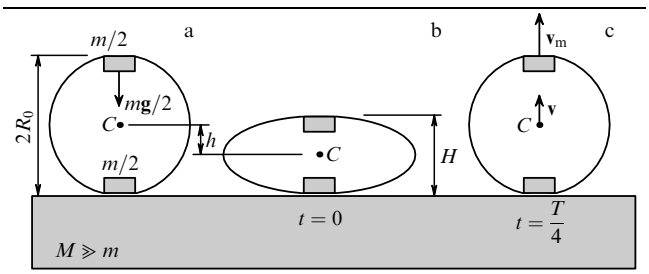

Figure 3. Mechanical model of a liquid drop.

The potential energy of the system decreases by the value mgh and the deformed spring acquires energy $W = kh^2/2$. For an elastic deformation, the spring constant is $k = mg/h$. Consequently, the model strain energy

$$W = \frac{kh^2}{2} = \frac{mgh}{2}\,. \tag{2}$$

This means that one half of the system potential energy variation is transferred to the elastic strain energy, while the other half is transferred to oscillations and is eventually dissipated as heating of the spring, weights, and the environment. If the system rests on the substrate in the gravity field, the upper weight compresses the spring (Fig. 3b), so that the potential strain energy equals W (2). As the model experiences a transition to weightlessness at time moment $t = 0$, the spring starts to straighten up. Spring deformation becomes zero after a quarter of the period T of model natural oscillations. Therefore, at the moment $t = T/4$ the upper weight will have the maximal speed v_{m} and the lower weight will lift off from the substrate (Fig. 3c). The drop model will then oscillate and travel away from the substrate, so that its center of mass will travel with constant speed $\mathbf{v}$. Since the weights oscillate symmetrically with respect to the center of mass, the oscillation period is proportional to the square root of the model mass:

$$T = \pi\sqrt{\frac{m}{k}}\,. \tag{3}$$

At the time moment $t = T/4$, the energy and momentum conservation laws in the substrate reference frame have the following forms:

$$W = \left(\frac{m}{2}\right)\frac{v_{\mathrm{m}}^2}{2}\,, \qquad \frac{m}{2}\,v_{\mathrm{m}} = mv\,. \tag{4}$$

Consequently, the model speed

$$v = \sqrt{\frac{W}{m}}\,. \tag{5}$$

This means that only half of the deformed spring potential energy is transformed into the kinetic energy of the system translational motion $mv^2/2$, while the other part of the energy is transferred to the oscillations.

Let us now consider a liquid drop. A drop in the gravity field on a nonhydrophobic surface is deformed and its surface area S is larger than a surface area S_0 of a spherical drop with the same volume. So, the increase in the surface energy of a drop due to its deformation is expressed as

$$W = \sigma(S - S_0) = \sigma\Delta S\,, \tag{6}$$

where σ is the surface tension coefficient. This energy corresponds to the spring strain energy in the mechanical model described above. As the system experiences a transition to weightlessness, the energy W (6) is equally shared between translational and vibrational motion of the drop that jumps. Taking into account (5), the speed with which a drop moves away from the substrate after the system becomes weightless

$$v = \sqrt{\frac{\sigma\Delta S}{m}} = \sqrt{\frac{\sigma\Delta S}{\rho V}}\,, \tag{7}$$

where ρ is the liquid density and V the drop volume.

If the liquid drop is small, its deformation in the gravity field is negligible, and the surface energy increase $W = \sigma\Delta S$ equals the decrease in the potential energy of a spherical drop as it is placed on a substrate, $W = mgh/2$ (2). By substituting here the expression for h from (1) and using (5), we obtain

$$v = \frac{1}{2}\sqrt{g(2R_0 - H)}\,. \tag{8}$$

Therefore, the speed with which relatively small drops move away from the falling substrate increases proportionally to the square root of the radius of a nondeformed drop.

The surface of a larger drop on a nonwettable substrate can be approximated by a cylinder. A liquid cylinder with height H and radius R has the volume $V = \pi R^2 H$, which is equal to the volume of a spherical drop $V = (4/3)\,\pi R_0^3$. Consequently,

$$R = \sqrt{\frac{4R_0^3}{3H}}\,, \qquad \Delta S = 2\pi R(R + H) - 4\pi R_0^2\,. \tag{9}$$

Using these expressions and equation (7), the speed of the drop jumping from the substrate can be estimated. Such an approach for an estimate of drop kinetic energy is presented in [1].

5. Speed of a drop that jumps

Positions of the mounting and drop that fall can be measured using images of a millimeter scale, which are present in every photograph. This allows plotting the drop motion in the reference system fixed to the falling mounting and calculating from this graph the speed of the drop with respect to the mounting.

Figure 4 shows three such plots, obtained for drops with different volumes. It is directly seen that in the mounting reference frame the drops move with a constant speed. In order to compare experimental results with the conclusions of the elementary theory described in Section 4, we need to know how the height H of a deformed drop on a hydrophobic surface depends on the radius R_0 of a drop with a spherical shape.

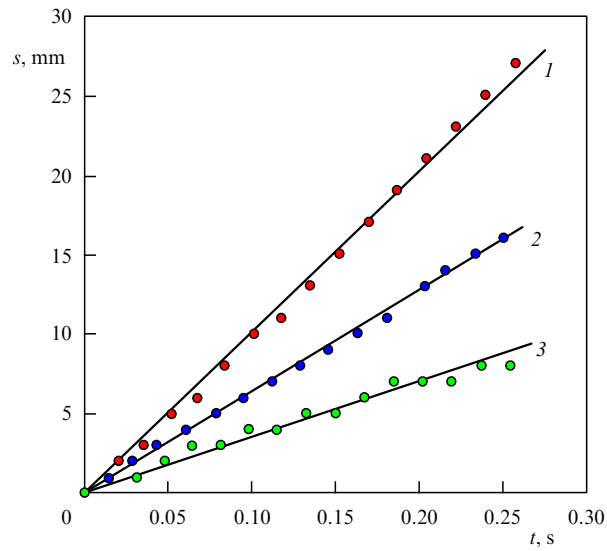

Figure 4. Coordinates of three drops in the substrate reference frame plotted versus the time: *1* — $R_0 = 5.8$ mm, $v = 102$ mm s^{-1}; *2* — $R_0 = 2.5$ mm, $v = 64$ mm s^{-1}; *3* — $R_0 = 1.6$ mm, $v = 35$ mm s^{-1}.

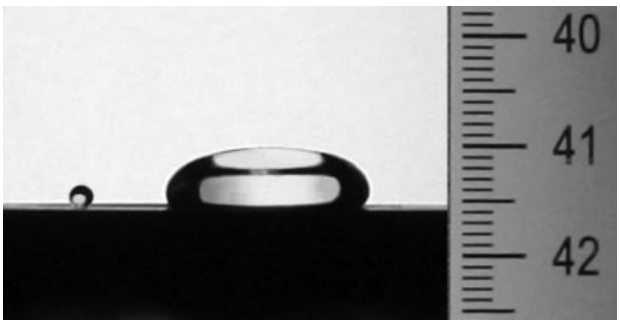

Figure 5. Water drop with 1.2 ml volume, lying on a nonwettable surface. For comparison, an almost spherical small drop is placed nearby.

This dependence can be measured experimentally by placing drops on a nearly flat hydrophobic substrate. Such a substrate was fabricated from a flat surface of a chemical glass bottom with a diameter of 80 mm, which was processed with an abrasive in such way that it acquired a rough and almost spherical shape. The largest depth of depressions on the processed surface was found to be 0.2 mm. The frosted surface of the glass was repeatedly uniformly covered with soot over a flame in order to form a durable layer with no microcracks. The substrate fabricated using this method was fixed horizontally and a specific volume of distilled water was placed on its surface with a syringe. The resting drop was then photographed together with a millimeter scale with a digital camera (Fig. 5). The drop was then filled with another portion of water and a photograph was taken again. The photographs were then uploaded to a computer and enlarged, and the heights of the drops were measured. The absolute error of the measurements was 0.1 mm.

Figure 6 shows the experimentally measured (curve *1*) height H of a drop lying on a nonwettable surface versus its radius R_0 in the weightless state. Curve *2* corresponds to results of numerical modeling of drop shape in the same conditions [5]. Curve *2* has a maximum at $R_0 \approx 8.9$ mm. The amplitude of this maximum is not large and it was not observed in our experiments.

Figure 7 shows a theoretical dependence (*1*) of the speed v of a small spherical drop moving with respect to a substrate on its radius R_0 (8), and for every R_0 we use experimental results of the height H shown in Fig. 6. Curve *2* for the speed v of a relatively large drop is plotted using expressions (9) and (7). Experimental result *3* is plotted on the same figure using the direct measurement of the drop speed with respect to the substrate. We see that experimental and theoretical results are in good agreement. Note that the experimentally observed minimal value of the water drop radius, for which the drop breaks away from the soot layer, was 1.1 mm.

6. Natural oscillation frequency of a drop

The mechanical model described in Section 4 shows that the period of jumped drop oscillations has to be proportional to the square root of its mass or volume (3). Frequencies of natural oscillations of a spherical liquid drop were found by Rayleigh [6]:

$$\omega_n = \sqrt{(n-1)\,n(n+2)}\,\sqrt{\frac{\sigma}{\rho R_0^3}}, \qquad n = 1, 2, 3, \ldots, \quad (10)$$

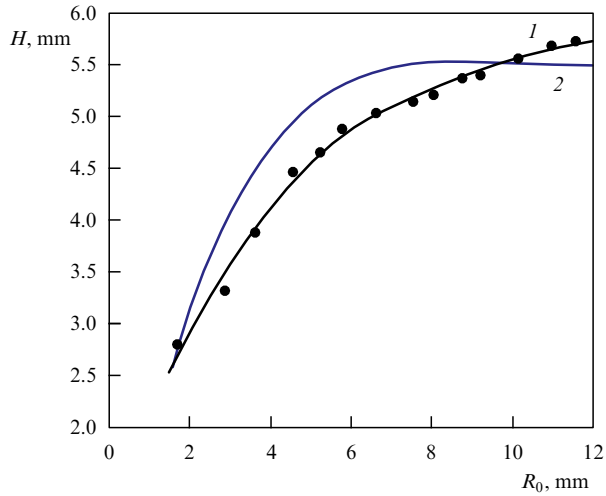

Figure 6. Height of a lying drop versus the radius of a spherical drop with the same volume. *1* — experimental results, *2* — numerical modeling results.

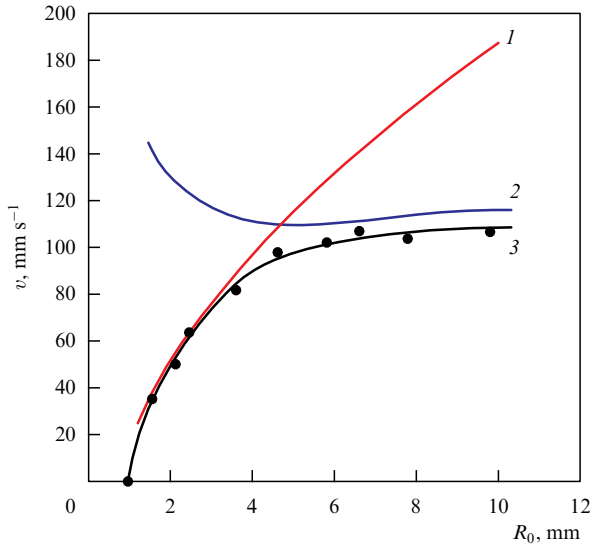

Figure 7. Drop speed with respect to the substrate versus the radius of a spherical drop with the same radius: *1* — data plotted using expression (8), *2* — data plotted according to equations (7) and (9), *3* — experimental data.

where n is the mode number of an oscillating drop. Consequently, for an ellipsoid mode ($n = 2$), we have

$$\omega = \sqrt{\frac{8\sigma}{\rho R_0^3}}. \quad (11)$$

The derivation of this expression is, for example, given in [7]. Oscillations of drops falling in air were investigated in [8]. If instead of radius R_0 of a spherical drop one uses its volume V, the period of the fundamental mode of drop oscillations will be expressed as

$$T = \sqrt{\frac{3\pi\rho V}{8\sigma}}. \quad (12)$$

A series of photographs obtained in demonstration experiments considered in this paper can be used to estimate the oscillation period of a falling drop and to compare this value with the results of theoretical calculations.

Figure 8. Mounting with two substrates for the comparison of different drops.

In such experiments, it is convenient to simultaneously observe and compare the motion and oscillations of two drops with different volumes. For this purpose, one needs a mounting with two identical removable substrates (Fig. 8). The metal clamp for such a mounting should have two pairs of holes, so its height above the mounting can be changed.

Figure 9 shows four photographs of drops jumping above the substrate. The volumes of the drops were 0.2 ml (on the left-hand side) and 0.8 ml, and the photographs are chosen from 18 consecutive shots taken experimentally. The time t indicated in the photographs corresponds to the time that passes after the fall starts. In Figs 9a, b, a small drop is in the same oscillation phase, so the period of its fundamental mode period is $T_1 \approx 0.05$ s. A large drop is in the same phase in Figs 6c, d, so its fundamental mode period is $T_2 \approx 0.12$ s. At the same time, the shapes of a small drop are similar in these images too, so the estimate for its period is $T_1 \approx 0.06$ s. This correlates with theory, because the drop volumes have a four-fold difference. The oscillation periods of small and large drops calculated using expression (12) are, correspondingly, $T_1' = 0.057$ s and $T_2' = 0.114$ s, which is in good agreement with experimental results.

Summing up, the demonstration experiment justifies expression (12), according to which the period T is proportional to the square root of the drop volume V.

7. Einstein's elevator model

In order to explain the nature of the equivalence principle, Albert Einstein suggested a thought experiment in an elevator which is freely falling in Earth's gravity field. An analysis of the phenomena in the elevator shows that the gravity field is present for an external observer, but is zero for the observer

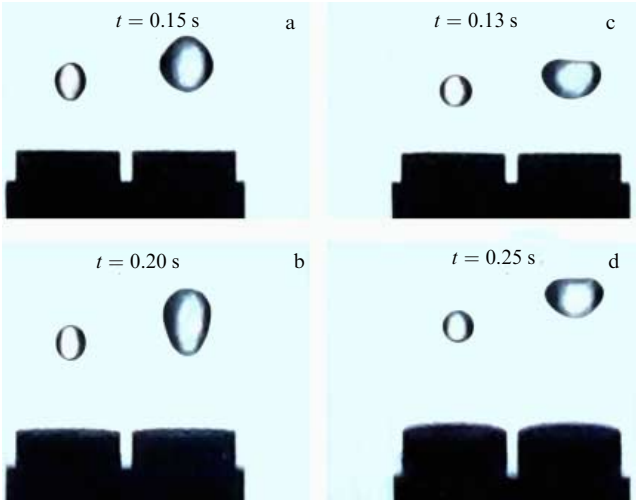

Figure 9. Photographs with the motion of drops with a four-fold difference in volumes and approximately two-fold difference in oscillation frequencies: (a, b) small drop is in the same oscillation phases, (c, d) large drop and small drop are in the same oscillation phases.

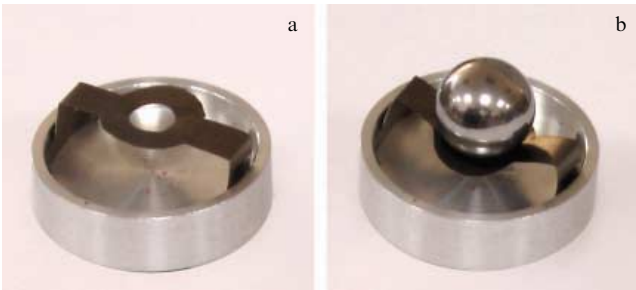

Figure 10. Substrate for the mounting used in the experimental investigation of steel ball motion in Einstein's elevator model: (a) spring is not deformed, (b) ball lies on the spring and deforms it.

inside the elevator [9]. The mounting considered above with a jumping drop is, in fact, a model of Einstein's elevator. Corresponding educational experiments are described in detail in paper [4]. Here, we suggest an experiment in which one can compare the motion of solid balls, one of which rests on the floor of an elevator model, while the other one is vertically thrown up from the floor.

For this purpose, the right-hand substrate of the mounting (Fig. 10a) is equipped with a thin metal spring with a hole for a ball (Fig. 10b). In the experiment, we use two identical steel balls 9.5 mm in diameter. The ball on the left-hand side is placed directly on a duralumin substrate, while the right-hand one is placed on the spring. During the free fall, the left-hand ball barely jumps, while the right-hand one moves away from the mounting with a relatively high speed.

Figure 11 shows a series of five photographs with the results of the described experiment, taken every 0.05 s. In order to decrease the influence of the substrate deformation, it was covered with a layer of vinyl chloride tape 0.13 mm thick. The speed of the right-hand ball defined from the photographs is $v = 15$ cm s^{-1}. The speed of the left-hand ball does not exceed $v = 0.5$ cm s^{-1}. For the experiment, we used steel balls with the mass $m = 3.5$ g. The right-hand ball bent the spring down to $\Delta h = 3$ mm. When the elevator model motionlessly hangs on the magnet, the gravity force acting on this ball $\mathbf{f} = m\mathbf{g}$ is balanced with the elastic deformation force

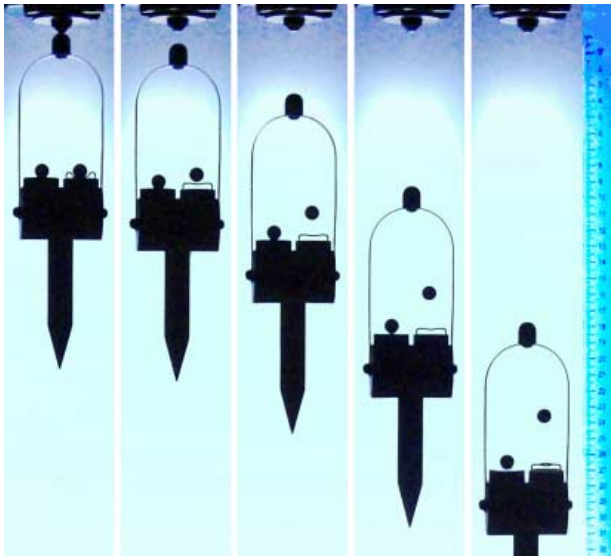

Figure 11. Results of an experiment where the water drop is replaced with a steel ball. Motion relative to the mounting occurs due to the elastic strain energy of the spring.

of the spring with the same amplitude but opposite direction. When the system becomes weightless, this force shifts the ball a distance Δh and performs work $A = f\Delta h/2$. As a result, the ball acquires kinetic energy

$$W = \frac{mv^2}{2} = \frac{f\Delta h}{2} = \frac{mg\Delta h}{2}\,.$$

This gives the speed of the ball:

$$v = \sqrt{g\Delta h}\,. \tag{13}$$

By substituting the value Δh into (13), we obtain the speed of the ball relative to the elevator model $v = 17$ cm s^{-1}, which coincides with the experimentally measured value $v = 15$ cm s^{-1} within the error limits.

8. Conclusions

The experimental setup considered in the present paper is simple and allows conducting series of important demonstration experiments during lectures or seminars. The results of these experiments justify the facts, which are usually analyzed speculatively:

(1) The surface energy of a spherical drop is minimal;

(2) During the transition to weightlessness, the surface energy of a deformed drop lying on a nonwettable substrate is transformed into mechanical energy, which consists of kinetic energy of translational motion and of the drop's natural oscillation energy;

(3) The speed with which small drops travel away from the freely falling substrate is proportional to the square root of the nondeformed drop radius;

(4) The speed of large drops does not depend on their volume and is defined by the ratio of the surface tension coefficient to the liquid density;

(5) The period of the fundamental mode of drop natural oscillations is proportional to the square root of its volume;

(6) A solid ball lying on an elastic substrate in a gravity field acts similarly to a liquid drop and jumps above the substrate as the system becomes weightless.

(7) The jump during the transition to weightlessness allows demonstrating the motion of a body thrown upwards in the real model of Einstein's elevator.

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References

1. Kirko I M, Dobychin E I, Popov V I *Sov. Phys. Dokl.* **15** 442 (1970); *Dokl. Akad. Nauk SSSR* **192** 301 (1970)
2. Geguzin Ya E *Kaplya* (A Drop) (Moscow: Nauka, 1973)
3. Mayer V V, Saranin V A *Ucheb. Fiz.* (1) 40 (1997)
4. Mayer V V, Varaksina E I *Eur. J. Phys.* **36** 055020 (2015)
5. Saranin V A, in *Aktual'nye Problemy Mekhaniki Sploshnykh Sred, Perm', 1–2 Noyabrya 2005 g. Materialy Nauchnoi Konf.* (Actual Problems of Continuum Mechanics, Perm, November 1–2, 2005. Materials of Scientific Conf.) (Ekaterinburg: UrO RAN, 2005) p. 120
6. Strutt J W (Baron Rayleigh) *The Theory of Sound* (New York: Dover Publ., 1945); Translated into Russian: *Teoriya Zvuka* Vol. 2 (Moscow: Gostekhizdat, 1955)
7. Landau L D, Lifshitz E M *Fluid Mechanics* (Oxford: Pergamon Press, 1987); Translated from Russian: *Gidrodinamika* (Moscow: Nauka, 1986)
8. Sterlyadkin V V *Izv. Akad. Nauk SSSR. Fiz. Atmos. Okeana* **18** (1) 98 (1982)
9. Einstein A, Infeld L *The Evolution of Physics: the Growth of Ideas from Early Concepts to Relativity and Quanta* (New York: Simon and Schuster, 1938); Translated into Russian: Einstein A *Sobranie Nauchnykh Trudov* Vol. 4 (Eds I E Tamm et al.) (Moscow: Nauka, 1967) p. 357