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LETTERS TO THE EDITORS

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On the phase shift in a uniformly rotating Michelson interferometer (comment on "Quadratic Sagnac effect — the influence of the gravitational potential of the Coriolis force on the phase difference between the arms of a rotating Michelson interferometer (an explanation of D C Miller's experimental results, 1921 – 1926)" by G B Malykin and V I Pozdnyakova [*Phys. Usp.* **58** 398 (2015); *Usp. Fiz. Nauk* **185** 431 (2015)])

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Abstract. It is argued that the 'quadratic Sagnac effect' recently put forward by G B Malykin and V I Pozdnyakova is the consequence of an incorrect estimation of second-order relativistic corrections and not a real physical phenomenon. The correct expression for the phase shift induced by rotations in a Michelson interferometer is presented.

Keywords: Michelson interferometer, Coriolis force, gravitational potential, Earth orbital revolution

In an article recently published in this journal [1], G B Malykin and V I Pozdnyakova considered the effect of rotations on an equal-arm Michelson interferometer deployed on a plane tangent to Earth's surface. Their computation yields a rotationally induced phase shift

$$\Delta\Phi = 2\pi \frac{L\Omega^2 R^2}{\lambda c^2} \left[\cos^2\varphi \cos(2\alpha) + \sin^2\varphi\right], \qquad (1)$$

with L the proper length of the interferometer's arms, Ω the angular speed of rotation, R the rotation radius, λ the radiation wavelength, c the speed of light, φ the interferometer's latitude, and α the azimuth (the horizontal angle measured clockwise from a north baseline) of its first arm.[1] The effect is second order in V/c, the ratio of the interferometer's speed of rotation $V = \Omega R$ to the speed of light, and formally resembles the historical expectation for the Michelson–Morley experiment [2, 3] with the ether wind speed substituted by the speed of rotation. The authors propose the name 'quadratic Sagnac effect' for their finding and use it to explain the once long disputed nonnull result of Miller's repetition of the Michelson–Morley experiment [4].

Contrary to what is stated in paper [1], the phase shift induced in a uniformly rotating, arbitrarily oriented, equal-arm Michelson interferometer, had already been calculated in full generality in work [5] (for some reason belatedly misquoted as reference [53] in paper [1]). In particular, for a right-angled interferometer deployed on a plane tangent to Earth's surface, equation (15) of work [5] gives

$$\Delta\Phi = -\sqrt{2}\pi \frac{L^2\Omega^2 R}{\lambda c^2} \sin(2\varphi) \sin(\alpha + 45^\circ), \qquad (2)$$

with the same notation used above. Besides a different angular dependence, formula (2) substantially differs in magnitude from formula (1) by a factor L/R.

Both results are claimed to be derived in the framework of the Special Theory of Relativity. Reference [5] offers an adaptation of the classical derivation of the (null) Michelson–Morley phase shift [2, 6] to the case of uniform rotations. The computation is performed from the point of view of an inertial observer and then referred to the co-rotating one. In reference [1], the derivation is carried out directly in the noninertial co-rotating frame of reference, and the result is attributed to the time dilation induced by the scalar gravitational potential of the Coriolis force. In principle, the two results should be identical. Their difference needs a clarification.

To this end, let me observe that in the limit of very large radii of rotation R, with the speed of rotation $V = \Omega R$ kept constant, a uniform rotation approaches an inertial motion, and the phase shift in an equal-arm Michelson interferometer should cancel correspondingly. This is the case for formula (2), which approaches zero as $1/R$. It is not the case for formula (1), which remains constant. Consequently, the result obtained by Malykin and Pozdnyakova seems to me to contradict the Special Theory of Relativity.

[1] In paper [1] this phase shift is presented in equation (9) in terms of the angle $\psi = 90^\circ - \alpha$, and in units of the interference fringe width 2π rad.

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In order to identify the possible issues in the derivation of formula (1), it will suffice to reconsider the special case discussed at the beginning of Section 3 in paper [1], when the interferometer's plane is orthogonal to the rotation plane with one arm perpendicular and the other one parallel to it ($\varphi = 0°$, $\alpha = 90°$). In the co-rotating frame of reference, Malykin and Pozdnyakova respectively evaluate the propagation proper time for the light in the orthogonal (subscript $\perp$) and parallel (subscript $||$) interferometer arms, forward (superscript $+$) and backward (superscript $-$) as[2]

$$\tau_{||}^{\pm} \simeq \frac{L}{c}\sqrt{1-\frac{\Omega^2 R^2}{2c^2}}\,, \quad \tau_{\perp}^{\pm} \simeq \frac{L}{c}\sqrt{1 \pm \frac{2\Omega R}{c} - \frac{\Omega^2 R^2}{2c^2}}\,. \quad (3)$$

In the authors' view: (1) the factors L/c are the forward and backward propagation coordinate times $t_{||}^{\pm}$, $t_{\perp}^{\pm}$ in the parallel and perpendicular arms, simply evaluated as the time that light needs to propagate a length L at the speed c; (2) the square root factors are the relativistic time dilation factors $(1 + 2U/c^2)^{1/2}$, with U being the nonrelativistic *scalar* potential of Coriolis and centrifugal inertial forces. After expanding in V/c and invoking other approximations, Malykin and Pozdnyakova obtain formula (1) as $2\pi c/\lambda$ times the difference in propagation proper times $\Delta\tau \simeq (\tau_{||}^{+} + \tau_{||}^{-}) - (\tau_{\perp}^{+} + \tau_{\perp}^{-})$ for light in the two interferometer arms.

At this point, some clarifications seem to be necessary.

(1) In a uniformly rotating frame of reference, null geodesics connecting different spacetime points in opposite directions are, in general, not coincident. As pointed out in paper [7] in the context of Sagnac interferometry, this implies that the paths followed by light in propagating forward and backward in the rotating interferometer arms are not the same. In particular, they do not have the same length or the length L of the interferometer arms. In addition, in a uniformly rotating frame of reference, the speed of light is not constant [8].[3] In principle, it is quite possible that these two effects compensate to simply give back L/c as the propagation coordinate time forward and backward, regardless of the orientation. However, this has to be proved. To this end, one can choose to solve the geodesic equations in co-rotating coordinates, obtaining the null geodesics connecting beam-splitter and mirror together with their parametrization, or one can look at the problem from the inertial viewpoint and then move to the co-rotating frame of reference. This second alternative was chosen in paper [5]. The specialization to the present choice for the interferometer's orientation of the detailed computation described there yields the forward and backward propagation coordinate times in the parallel and perpendicular arms as

$$t_{||}^{\pm} \simeq \frac{L}{c}\left(1+\frac{\Omega^2 R^2}{2c^2}\right), \quad t_{\perp}^{\pm} \simeq \frac{L}{c}\left(1 \pm \frac{\Omega R}{c} + \frac{\Omega^2 R^2}{2c^2}\right), \quad (4)$$

up to terms of order three in V/c. These are different from L/c, indicating that the first factors in formula (3) are mistaken.

(2) Coriolis forces are velocity-dependent interactions completely analogous to Lorentz forces in their mathematical structure [10]. As such, they cannot be described in terms of a *scalar* potential and can only be derived by a vector potential. Even if in other circumstances the authors have claimed that "the notion of scalar potential may be introduced for such forces, with certain constraints and reservations" [11], in no case do Coriolis forces induce time dilation. The simplest way of seeing this is probably in terms of the tensorial formalism of General Relativity. In standard notation, the Minkowski line element in a rotating coordinate system (t, R, ϕ, z) reads [8]

$$\mathrm{d}s^2 = \left(1-\frac{\Omega^2 R^2}{c^2}\right)c^2\,\mathrm{d}t^2 - \mathrm{d}R^2 - R^2\,\mathrm{d}\phi^2 - \mathrm{d}z^2 - 2\Omega R^2\,\mathrm{d}t\,\mathrm{d}\phi\,. \quad (5)$$

Coriolis interactions only enter the equation of motion through the off-diagonal metric element g_{02}, while the proper time interval τ between two events taking place at the same point is given in terms of g_{00} and the coordinate–time interval t as

$$\tau = \frac{1}{c}\sqrt{g_{00}}\,t = \sqrt{1-\frac{\Omega^2 R^2}{c^2}}\,t \simeq t\left(1-\frac{\Omega^2 R^2}{2c^2}\right) \quad (6)$$

up to terms of order four in V/c (see § 84 and § 88 of monograph [8]). Consequently, the time dilation factors in formula (1) are also mistaken.

In accordance with formula (2) and up to terms of order four in V/c, expressions (4) and (6) yield a null difference $\Delta\tau$ in the propagation proper times in the two arms of the interferometers with the present choice of orientation. Correspondingly, the phase shift also vanishes.

In conclusion, it seems to me that the 'quadratic Sagnac effect' put forward by Malykin and Pozdnyakova is more a consequence of a wrong estimation of second-order relativistic corrections than a real physical effect. As explained above, this is straightforwardly indicated by the fact that expression (1) does not vanish in the limit of very large R with V kept constant, as required by the Special Theory of Relativity. As far as Miller's experimental results are concerned, we can remain satisfied with the largely accepted analysis of Shankland and collaborators [4]. On the other hand, new and independent derivations of the phase shift induced by rotations in a Michelson interferometer would be of great interest.

Notes added in proof

A consistent derivation from the co-rotating point of view of the phase shift induced by a uniform rotation in a Michelson interferometer has now been published in paper [12]. This new computation is certainly not affected by the possible issues connected with the derivation in an inertial frame of reference hypothesized in the objections to this comment published hereafter [13].

In addition, I would like to remark that the main goal of this paper is not to argue the priority of my previous work [5] over the work of G B Malykin and V I Pozdnyakova [1], but rather to point out that their result cannot be correct simply because a rotationally induced noninertial effect cannot depend solely on the speed of rotation $V = \Omega R$, thus leading to identical results for all observers moving with that speed regardless of their distance from the center of rotation.

[2] In equation (1) of paper [1], L/c is written as t and the notation $t_{||}^{\pm}$, $t_{\perp}^{\pm}$ is used in place of $\tau_{||}^{\pm}$, $\tau_{\perp}^{\pm}$. For the sake of clarity, we prefer to use t, with possible sub- and superscripts, for coordinate times, and τ, with possible sub- and superscripts, for the proper times.

[3] The reader can see Ref. [9] for an illustrative visualization of these effects.

Contrary to what is stated in the objections to this comment [13], the phase shift induced by a uniform rotation in a Sagnac interferometer correctly approaches zero in the limit of large radii of rotation R, when the speed of rotation $V = \Omega R$ is kept constant.

References

1. Malykin G B, Pozdnyakova V I *Phys. Usp.* **58** 398 (2015); *Usp. Fiz. Nauk* **185** 431 (2015)
2. Michelson A A, Morley E W *Am. J. Sci. 3* **34** 333 (1887)
3. Shankland R S *Am. J. Phys.* **32** 16 (1964)
4. Shankland R S et al. *Rev. Mod. Phys.* **27** 167 (1955)
5. Maraner P *Ann. Physics* **350** 95 (2014)
6. Møller C *The Theory of Relativity* (Oxford: Clarendon Press, 1972)
7. Maraner P, Zendri J-P *Gen. Relat. Grav.* **44** 1713 (2012)
8. Landau L D, Lifshitz E M *The Classical Theory of Fields* (Oxford: Butterworth-Heinemann, 2000); Translated from Russian: *Teoriya Polya* (Moscow: Fizmatlit, 2014)
9. Maraner P "Rotating Michelson – Morley interferometer", http://demonstrations.wolfram.com/RotatingMichelsonMorleyInterfemeter/
10. Coisson R *Am. J. Phys.* **41** 585 (1973)
11. Malykin G B *Phys. Usp.* **43** 1229 (2000); *Usp. Fiz. Nauk* **170** 1325 (2000)
12. Maraner P *Gen. Relativ. Gravit.* **48** 1 (2016)
13. Malykin G B, Pozdnyakova V I *Phys. Usp.* **59** 719 (2016); *Usp. Fiz. Nauk* **186** 796 (2016)