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Features of the motion of spin-1/2 particles in a noncoplanar magnetic field

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Contents

Abstract. **It is shown that a necessary condition for the nonreciprocal scattering of unpolarized thermal neutrons is that the spatial distribution of the magnetic induction be noncoplanar. An experiment on neutron transmission through a system of magnetic mirrors is performed in which a nonreciprocity of 75% is reached.**

Keywords: magnetism, neutron, noncoplanar, nonreciprocity

1. Introduction

The study of the motion of spin-1/2 particles in an inhomogeneous magnetic field, being a traditional field of physics, continues to attract great attention of researchers. The problem of finding the magnetic moment distribution in a substance based on the data on neutron scattering remains as urgent as before. As is well known [1], the interaction of slow neutrons with a substance is described by the Schrödinger equation with a Pauli term. A similar equation describes the motion of conduction electrons in ferromagnets in the framework of the s – d model [2]. Thus, the basic laws that govern the motion of neutrons in substances with a nonuniform distribution of magnetic induction and the motion of electrons in conducting ferromagnets with an inhomogeneous distribution of magnetization can be considered from a common standpoint.

The properties of the motion of spin-1/2 particles are determined by the number of components of the inhomogeneous magnetic field. If the magnetic field distribution is collinear, the spin of a particle is conversed in the process of particle motion. In a noncollinear field, scattering processes take place with a change in the spin state, which leads, for example, to the Zeeman splitting of a neutron beam [3–5] and to the magnetization reversal in a ferromagnetic layer upon the transmission of a spin-polarized current through it [6, 7].

In general, the distribution of a magnetic field is noncoplanar. The noncoplanarity of a magnetic structure leads to new physical phenomena. For example, phenomena such as the existence of a persistent electric current in mesoscopic rings with a noncoplanar magnetic structure [8, 9] and a 'topological' Hall effect, which was observed in the lattices of magnetic skyrmions [10], have been predicted. In Refs [11, 12], a diode effect of electric current and a photogalvanic effect in ferromagnets with a helical magnetic structure have been described. From the standpoint of scattering theory, all the above effects are due to the manifestation of the 'nonreciprocity' of scattering of spin particles in inhomogeneous magnetic fields, i.e., due to the dependence of the scattering cross section on the interchange of the positions of the source and the receiver of particles without a change in the sign of the magnetic field [13].

We note that the conditions for the reciprocity violation in elastic scattering of spin-1/2 particles have not been established until recently, and the majority of the effects that were predicted for noncoplanar systems have not been discovered yet. In this article, we present the results of theoretical and experimental studies of the 'nonreciprocity' of scattering of thermal neutrons in systems with a noncoplanar distribution of the magnetic field.

2. Conditions necessary for the nonreciprocity of scattering

We show that a necessary condition for the nonreciprocity of scattering of particles is the noncoplanarity of the distribution of the magnetic field (also see [14]). For this, we consider the transformation of the wave function of the Schrödinger equation with a Pauli term under the rotation of the magnetic

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field. The Schrödinger equation with the magnetic field $\mathbf{B}(\mathbf{r})$ that is rotated at each point about the axis $\mathbf{n}$ through the same angle α is written as

$$\left(\frac{\hat{\mathbf{p}}^2}{2m} + \hat{V}(\mathbf{r}) + \mu\big(\hat{\boldsymbol{\sigma}}\hat{R}_{\mathbf{n},\alpha}\mathbf{B}(\mathbf{r})\big)\right)\hat{\psi}\,'(\mathbf{r}) = E\hat{\psi}\,'(\mathbf{r})\,, \tag{1}$$

where $\hat{R}_{\mathbf{n},\alpha}$ is the operator of the rotation of the vector $\mathbf{R}$ about the axis $\mathbf{n}$ through an angle α, and $\hat{\boldsymbol{\sigma}}$ is the vector of the Pauli matrices. We assume that the wave function $\hat{\psi}\,'$ in the rotated magnetic field is related to the wave function of the original Schrödinger equation $\hat{\psi}$ as

$$\hat{\psi}\,'(\mathbf{r}) = \hat{S}_{\mathbf{n},\alpha}\,\hat{\psi}(\mathbf{r})\,, \tag{2}$$

where $\hat{S}_{\mathbf{n},\alpha}$ is the operator of the rotation of a spinor about the axis $\mathbf{n}$ through an angle α. Acting by the Hermitian-conjugate operator $\hat{S}^{+}_{\mathbf{n},\alpha}$ on the left-hand and right-hand sides of Eqn (1), we obtain

$$\left(\frac{\hat{\mathbf{p}}^2}{2m} + \hat{V}(\mathbf{r}) + \mu\big(\hat{S}^{+}_{\mathbf{n},\alpha}\,\hat{\boldsymbol{\sigma}}\hat{S}_{\mathbf{n},\alpha}\,\hat{R}_{\mathbf{n},\alpha}\mathbf{B}(\mathbf{r})\big)\right)\hat{\psi}(\mathbf{r}) = E\hat{\psi}(\mathbf{r})\,. \tag{3}$$

Using the commutation relations for the Pauli matrices, we find

$$\hat{S}^{+}_{\mathbf{n},\alpha}\,\hat{\boldsymbol{\sigma}}\hat{S}_{\mathbf{n},\alpha} = \left[\cos\frac{\alpha}{2} - \mathrm{i}\sin\frac{\alpha}{2}(\mathbf{n}\,\hat{\boldsymbol{\sigma}})\right]\hat{\boldsymbol{\sigma}}\left[\cos\frac{\alpha}{2} + \mathrm{i}\sin\frac{\alpha}{2}(\mathbf{n}\,\hat{\boldsymbol{\sigma}})\right]$$
$$= \cos^2\frac{\alpha}{2}\,\hat{\boldsymbol{\sigma}} + \mathrm{i}\cos\frac{\alpha}{2}\sin\frac{\alpha}{2}\left[\hat{\boldsymbol{\sigma}}(\mathbf{n}\,\hat{\boldsymbol{\sigma}}) - (\mathbf{n}\,\hat{\boldsymbol{\sigma}})\hat{\boldsymbol{\sigma}}\right]$$
$$+ \sin^2\frac{\alpha}{2}\,(\mathbf{n}\,\hat{\boldsymbol{\sigma}})\,\hat{\boldsymbol{\sigma}}(\mathbf{n}\,\hat{\boldsymbol{\sigma}}) = \hat{\boldsymbol{\sigma}}\cos\alpha + \sin\alpha\,(\mathbf{n}\times\hat{\boldsymbol{\sigma}}) = \hat{R}_{\mathbf{n},\alpha}\hat{\boldsymbol{\sigma}}\,. \tag{4}$$

Taking into account that $\hat{R}^{+}_{\mathbf{n},\alpha} = \hat{R}^{-1}_{\mathbf{n},\alpha}$, we obtain

$$(\hat{R}_{\mathbf{n},\alpha}\,\hat{\boldsymbol{\sigma}})\big(\hat{R}_{\mathbf{n},\alpha}\mathbf{B}(\mathbf{r})\big) = \hat{\boldsymbol{\sigma}}\big(\hat{R}^{+}_{\mathbf{n},\alpha}\hat{R}_{\mathbf{n},\alpha}\mathbf{B}(\mathbf{r})\big)$$
$$= \hat{\boldsymbol{\sigma}}\big(\hat{R}^{-1}_{\mathbf{n},\alpha}\hat{R}_{\mathbf{n},\alpha}\mathbf{B}(\mathbf{r})\big) = \hat{\boldsymbol{\sigma}}\mathbf{B}(\mathbf{r})\,.$$

The validity of the hypothesis on the transformation of wave function (2) under the coherent rotation of the magnetic field is thereby proved. Thus, simultaneously rotating the magnetic field and the spinor at each point through the same angle does not change the Pauli term in the Schrödinger equation. It follows from Eqn (2) that the amplitude matrices of neutron scattering [13] under the rotation of the magnetic field are connected as

$$\hat{f}\,' = \hat{S}_{\mathbf{n},\alpha}\hat{f}\hat{S}^{+}_{\mathbf{n},\alpha}\,. \tag{5}$$

The differential scattering cross section is calculated as $\partial\sigma/\partial\Omega = \mathrm{Tr}\,[\hat{\rho}\hat{f}^{+}\hat{f}\,'']$, where $\hat{\rho}$ is the matrix of the number density of neutrons and Tr denotes the sum of the diagonal elements of the matrix that is given in brackets. In the case of an unpolarized neutron beam, the density matrix is diagonal and its nonzero elements are equal to $1/2$. Because we can perform cyclic permutations under the trace, we conclude that the differential scattering cross section of unpolarized neutrons in a magnetic field is invariant under coherent rotations of this field:

$$\frac{\partial\sigma'}{\partial\Omega} = \mathrm{Tr}\left[\hat{\rho}\hat{f}\,'^{+}\hat{f}\,'\right] = \mathrm{Tr}\left[\hat{\rho}\hat{S}_{\mathbf{n},\alpha}\hat{f}^{+}\hat{S}^{+}_{\mathbf{n},\alpha}\hat{S}_{\mathbf{n},\alpha}\hat{f}\hat{S}^{+}_{\mathbf{n},\alpha}\right]$$
$$= \mathrm{Tr}\left[\hat{\rho}\hat{f}^{+}\hat{f}\right] = \frac{\partial\sigma}{\partial\Omega}\,, \tag{6}$$

or

$$\frac{\partial\sigma\big(\mathbf{k}_0,\mathbf{k}\,',\mathbf{B}(\mathbf{r})\big)}{\partial\Omega} = \frac{\partial\sigma\big(\mathbf{k}_0,\mathbf{k}\,',\hat{R}_{\mathbf{n},\alpha}\mathbf{B}(\mathbf{r})\big)}{\partial\Omega}\,, \tag{7}$$

where $\mathbf{k}_0$ and $\mathbf{k}\,'$ are the respective wave vectors of the incident and scattered particle. On the other hand, for any interaction of a particle with a magnetic field, the reciprocity theorem

$$\frac{\partial\sigma\big(\mathbf{k}_0,\mathbf{k}\,',\mathbf{B}(\mathbf{r})\big)}{\partial\Omega} = \frac{\partial\sigma\big(-\mathbf{k}\,',-\mathbf{k}_0,-\mathbf{B}(\mathbf{r})\big)}{\partial\Omega} \tag{8}$$

is valid [13], which states that the scattering cross section is not changed under a simultaneous change in the positions of the source and the receiver or a change in the sign of the magnetic field. We note that formulas (7) and (8) are written for the scattering cross section of unpolarized neutrons. It follows from (7) and (8) that the elastic scattering of neutrons by coplanar magnetic systems has an additional symmetry. Indeed, the rotation of the magnetic field through an angle π about an axis (e.g., the axis y) that is perpendicular to the plane in which the magnetic induction vectors lie corresponds to the sign reversal of this field, $\hat{R}_{y,\pi}\mathbf{B} = -\mathbf{B}$. Then, combining the reciprocity theorem with the invariance of the differential scattering cross section under rotations, we obtain the following additional relations for scattering in a coplanar field:

$$\frac{\partial\sigma\big(\mathbf{k}_0,\mathbf{k}\,',\mathbf{B}(\mathbf{r})\big)}{\partial\Omega} = \frac{\partial\sigma\big(-\mathbf{k}\,',-\mathbf{k}_0,\mathbf{B}(\mathbf{r})\big)}{\partial\Omega}\,,$$
$$\frac{\partial\sigma\big(\mathbf{k}_0,\mathbf{k}\,',\mathbf{B}(\mathbf{r})\big)}{\partial\Omega} = \frac{\partial\sigma\big(\mathbf{k}_0,\mathbf{k}\,',-\mathbf{B}(\mathbf{r})\big)}{\partial\Omega}\,. \tag{9}$$

Relations (9) correspond to two cases of scattering. In the first case, the source and the detector interchange positions, but the sign of the magnetic field remains unaltered. In the second case, on the contrary, the source and the detector retain their positions, whereas the magnetic field is reversed at each point of space. Both equalities (9) are always valid in the case of a coplanar distribution of the magnetic field. If the distribution of the magnetic field is noncoplanar, relations (9) can be violated. We refer to the cases of such violation as nonreciprocity, and call the effects that follow from this violation the effects of nonreciprocal scattering.

3. Experiment

Of undoubted interest is experimentally proving the nonreciprocity of the transmission of neutrons through noncoplanar magnetic systems [15].

We consider two parallel magnetic mirrors placed in an external magnetic field. The magnetic moments $\mathbf{M}_{1,2}$ lie in the planes of the mirrors, and the external magnetic field $\mathbf{B}$ is perpendicular to these planes. Depending on the mutual orientation of the magnetic moments of the mirrors, both coplanar ($\mathbf{B}[\mathbf{M}_1\times\mathbf{M}_1] = 0$) and noncoplanar ($\mathbf{B}[\mathbf{M}_1\times\mathbf{M}_1] \neq 0$) distributions of the magnetic induction can be realized in this system (Fig. 1).

We assume that the magnetic mirrors are ideal polarizers. If the neutrons are incident on an ideal mirror with a magnetization $\mathbf{M}_1$, those with the magnetic moment codirected with $\mathbf{M}_1$ are totally reflected. The neutrons with the opposite magnetic moment pass through the mirror. The transmission coefficient of unpolarized neutrons through a

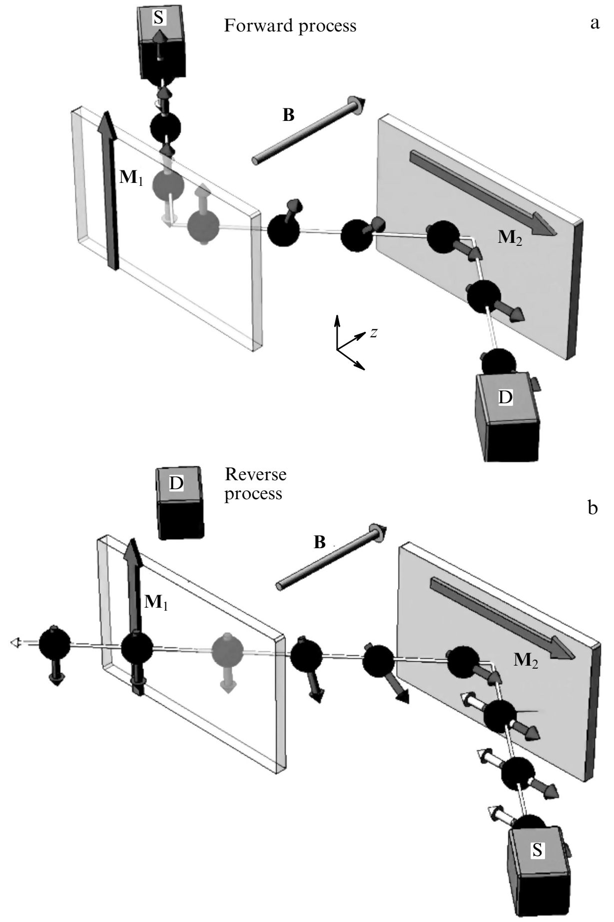

Figure 1. Schematic of a nonreciprocal cell. The unpolarized beam of neutrons is ejected by a source (S) and is registered by a detector (D). The beam is sequentially reflected from the mirrors with magnetizations $\mathbf{M}_1$ and $\mathbf{M}_2$. The spin of the neutron precesses in an external field **B** between the mirrors. When the magnetizations of the mirrors are perpendicular to each other, the transmission rates differ in the (a) forward and (b) reverse (in terms of the time) processes.

system of two mirrors placed in an external field is written as

$$I_\pm = \frac{1 + \cos(\varphi \pm \beta)}{4} \,. \tag{10}$$

The plus and minus signs in (10) correspond to the 'forward' and 'reverse' transmission of neutrons through the cell. The transmission coefficient depends on the phase $\varphi = \omega\tau$, which is gained as a result of the precession of the neutron magnetic moment in the external magnetic field B with the frequency $\omega = 2\mu_n B/\hbar$ in the time τ of its flight between the magnetic mirrors arranged such that the angle between their magnetic moments is equal to β. In accordance with the theorem proved in Section 2, in the coplanar case ($\beta = 0, \pi$), the passage of neutrons is reciprocal, $I_+(B) = I_-(B)$; and for a noncoplanar distribution of the magnetic induction, the system represents a nonreciprocal cell, with $I_+(B) \neq I_-(B)$ or $I_\pm(B) \neq I_\pm(-B)$. Figure 1 illustrates the operating principle of this nonreciprocal cell. After the reflection from the first mirror, the neutron beam becomes completely polarized parallel to $\mathbf{M}_1$.

Let the precession frequency and the time of flight of the neutron between the mirrors be such that the average magnetic moment of the neutron is rotated clockwise through $\pi/2$ such that its direction coincides with the magnetic moment of the second mirror $\mathbf{M}_2$ (Fig. 1a). In this case, the coefficient of reflection from the second mirror is equal to unity and the coefficient of transmission through the entire system is maximum (equal to 1/2). With the change in the sequence of reflections from the mirrors, the rotation of the magnetic moment through $\pi/2$ after the first reflection leads to mutually opposite orientations of the magnetic moment of the neutron and of the magnetization of the mirror $\mathbf{M}_1$ (Fig. 1b). The reflection coefficient in this case is equal to zero and the transmission is minimum. In the coplanar case, the transmission coefficients for the forward and reverse processes are identical and equal to 1/4.

For the experimental observation of the above features of the transmission of neutrons, two magnetic mirrors representing CoFe films approximately 115 nm in thickness applied onto glass substrates by magnetron sputtering were prepared; the lateral dimensions of the mirrors were 140×50 mm^2. It is known [16] that such films have a rectangular hysteresis loop with an in-plane magnetization, which was confirmed by our measurements. The residual magnetization was equal to 90–95% of the saturation magnetization; the coercivity field was ≈ 150 Oe. The mirrors were located as shown in Fig. 2. To ensure the parallelism of the mirrors, a glass plate 0.5 mm thick was pressed between them. The entire construction was located in an external magnetic field (with an induction of 10–30 Oe) oriented perpendicular to the surface of the mirrors. The neutrons consecutively reflected from the magnetic mirrors were registered by a detector. The angle of grazing incidence of the neutron beam $\alpha \approx 7$ mrad was selected between the first and second critical angles characteristic of neutrons with the wavelength of 3–6 Å, which are determined by the relation between the nuclear and magnetic potentials of the CoFe films [17]. The polarizing efficiency of the mirrors in the relevant range of wavelengths was about 80%. The experiments were conducted in the IBR-2M pulsed fast reactor at the Joint Institute for Nuclear Research, Dubna.

In the first series of experiments, the sequential order of reflection of neutrons from the mirrors was changed by

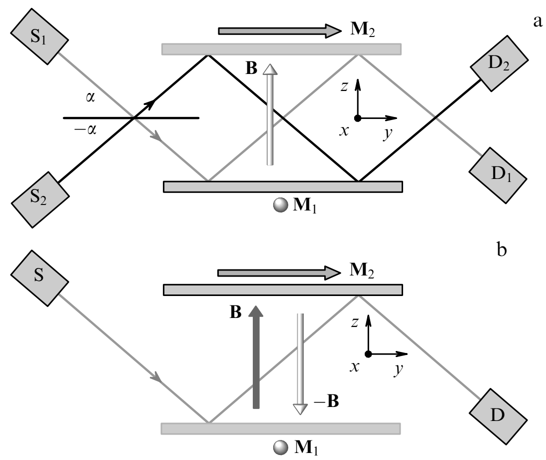

Figure 2. Schematic of measurements: (a) measurements with different angles of grazing incidence ($\alpha, -\alpha$) at a fixed external field; the neutron beams from the sources S_1 and S_2 correspond to the forward and time-reversed processes; (b) measurements in the case of reversal of the external magnetic field with the unaltered positions of the source and the detector. The neutron beam passes through the system and, depending on the sign of the external magnetic field, its passage corresponds to the forward or reverse processes.

changing the incidence angle α with $-\alpha$ (Fig. 2a). For the system in question, this process is equivalent to the interchange of the positions of the source and the detector. Indeed, the rotation of the entire system through an angle of 180° about the z axis with the subsequent rotation of the magnetic field about the same axis through the same angle maps these processes into each other. In this case, we used the invariance of the scattering cross section of neutrons under a coherent rotation (by the same angle at each point) of magnetic induction vector (7) and the 'unidimensionality' of the system (dependence of the magnetic field and nuclear potentials on the coordinate z only).

In the second series of experiments, we changed the sign of the external magnetic field (Fig. 2b). For the system under consideration, this is equivalent to changing the directions of the magnetic inductions of the mirrors and of the external field, because the change in the sign of the magnetic moments of the mirrors does not change the angle between them, Eqn (10). The experiments were carried out for both coplanar ($\mathbf{M}_1 \parallel \mathbf{M}_2$) and noncoplanar ($\mathbf{M}_1 \perp \mathbf{M}_2$) distributions of the magnetic induction.

The dependences of the intensity of the transmitted beam on the z component of the wave vector of neutrons are given in Figs 3 and 4. The common property of these dependences is their nonmonotonic, oscillating character, which is a manifestation of the precession of the magnetic moment of a neutron in the external magnetic field during its flight between the mirrors. The phase φ in formula (10) is written as

$$\varphi = \frac{2\mu_n B d m}{\hbar^2 k_{0z}}\,, \tag{11}$$

where $k_{0z} = (2\pi/\lambda)\sin\alpha$, λ and m are the wavelength and the mass of a neutron, and d is the distance between the mirrors. In the coplanar case, the extrema of the transmission coefficient are observed at $\varphi = \pi n/2$, $n = 0, \pm 1, \pm 2, \ldots$. Such oscillations were first observed in experiments on the neutron spin echo [18].

The main result of our work is the observation of nonreciprocal transmission of neutrons in the system under consideration in the case of a noncoplanar distribution of the magnetic induction. If the distribution of the induction is coplanar ($\mathbf{M}_1 \parallel \mathbf{M}_2$), the transmission coefficients for the forward and reverse processes coincide within the experimental accuracy, which is determined by the accuracy of the goniometer (≈ 0.1 mrad), the divergence of the neutron beam (≈ 0.6 mrad), and the fluctuations of the magnetic field in the gap between the mirrors (≈ 0.3 Oe day^{-1}) (Fig. 3). For the noncoplanar distribution of the magnetic induction ($\mathbf{M}_1 \perp \mathbf{M}_2$), the transmission coefficients for the forward and reverse processes ($\alpha \to -\alpha$) (Fig. 4) differ substantially; the relative difference reaches 75%. The system examined represents a nonreciprocal cell for neutrons.

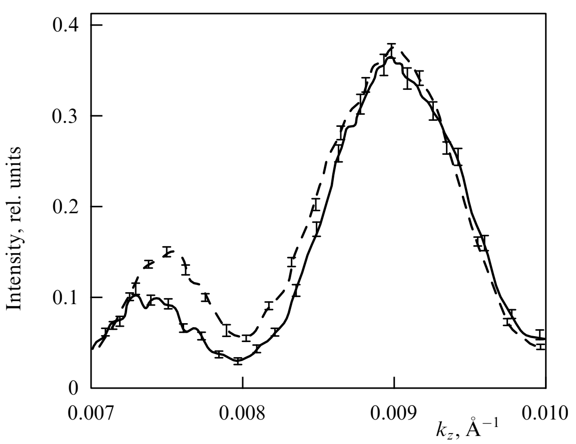

Figure 3. Experimental intensity of a neutron beam transmitted through a coplanar system depending on the z component of the wave vector ($B = 18$ Oe). The solid curve corresponds to the forward process; the dashed curve corresponds to a change in the order of the passage through the mirrors ($\alpha \to -\alpha$).

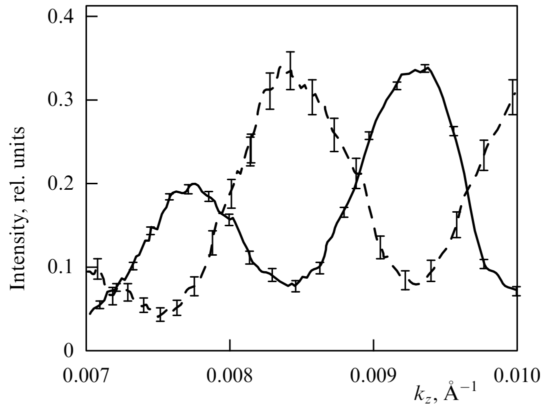

Figure 4. Experimental intensity of a neutron beam transmitted through a noncoplanar system depending on the z component of the wave vector ($B = 18$ Oe). The solid curve corresponds to the forward process; the dashed curve corresponds to a change in the order of the passage through the mirrors ($\alpha \to -\alpha$).

It is of interest to compare this cell with the nonreciprocal Faraday cell for light [19]. Both systems consist of a polarizer, a phase inverter, and an analyzer. However, the dependences of the difference in the forward and reverse transmission coefficients $\Delta I = I_+ - I_-$ on the angle β between the analyzer and polarizer are substantially different: for light, $\Delta I \sim \sin(2\beta)$, while for neutrons, $\Delta I \sim \sin\beta$.

4. Conclusion

The results of our work presented in Sections 2 and 3 can be used to create nonreciprocal elements of spintronics based on the control of the spin precession of electrons [20] and, possibly, they will serve as a stimulus for the experimental study of other nonreciprocal effects of scattering of electrons [9, 11, 12] and neutrons [21, 22] by noncoplanar magnetic systems.

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