

CONFERENCES AND SYMPOSIA

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Long-range ballistic transport mechanisms in superconducting spintronics

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Abstract. We review the mechanisms responsible for long-range Josephson transport in ballistic hybrid superconductor/ferromagnet/superconductor (SFS) structures with the exchange field modulated in either coordinate or momentum space. These mechanisms are based on the suppression of the destructive interference of electron and hole waves in a ferromagnet caused by the exchange field. The interference suppression results in a slow decay of the singlet component of the pair correlation function in a ferromagnet and an increase in the Josephson current in SFS structures.

Keywords: spintronics, superconductor–ferromagnet hybrids, Josephson effect, proximity effect, triplet superconductivity

1. Introduction

Future progress in modern information technologies will be largely driven by the creation of new (opto)electronic devices with high performance. However, until recently, new functionalities were provided mainly by the development of semiconductor-based heterostructures. At the same time, the analysis of and the search for possible new principles of operation and other advanced materials and structures are very relevant and timely. Various types of metamaterials with controlled electrodynamic parameters and nanoelectromechanical devices that provide efficient coupling of electronic and mechanical degrees of freedom look promising for applications. In particular, there is intense activity aimed at using the benefits of superconductivity, which should reduce the dissipation losses in such systems. Combining materials with opposing electron spin orders — a superconductor (S) and a ferromagnet (F) — provides very efficient ways to manipulate superfluid (dissipation-free) transport via control of the spin degree of freedom and will therefore pave the way to new types of hybrid devices of superconducting spintronics and nanoplasmonics.

A pivotal role in realizing the dissipation-free spin transport in such hybrid structures is played by the proximity effect between a superconducting and a normal conducting material [1]. The leakage of superconducting pairs into the ferromagnetic metal induces an inhomogeneous superconducting state near the SF boundary, resembling the Larkin–Ovchinnikov–Fulda–Ferrell (LOFF) state [2, 3], which means that the Cooper pairs acquire a finite center-of-mass momentum. The joint influence of the proximity effect and exchange interaction in heterogeneous SF hybrid structures implies the creation of superconducting correlations whose amplitude decays and oscillates in the ferromagnet in the direction perpendicular to the boundary [4–6]. A complete synergy between S and F subsystems turns out to be possible via the creation of long-range spin-triplet Cooper pairs [7, 8], which are generated in certain inhomogeneous magnetic configurations [7, 9–12] and/or on a carefully engineered SF interface [13, 14].

Here, we discuss a few promising proposals that provide an opportunity to generate long-range spin-triplet superconducting correlations in ballistic SF hybrids.

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2. Proximity effect and long-range triplet correlations in hybrid SF structures

A unique characteristic of the proximity effect at superconductor interfaces with a normal nonmagnetic (N) metal is closely related to the phenomenon of the Andreev reflection of quasiparticles at the SN boundary [15]. It describes correlations between particles and holes on the normal side of the interface due to penetration of pairs and provides a nonzero amplitude of the superconducting order parameter Δ in the absence of the pairing potential [16]. In ballistic ('clean') nonmagnetic metals, the spin structure of a particle–hole pair can be omitted, and the wavevector mismatch between the incoming particle (q_u) and the reflected hole (q_v) with the energy ε ($0 < \varepsilon \ll \varepsilon_{\mathrm{F}}$) measured with respect to the Fermi level $\varepsilon_{\mathrm{F}} = \hbar k_{\mathrm{F}} V_{\mathrm{F}}/2$, is very small ($q_{u,v} = k_{\mathrm{F}}\sqrt{1 \pm \varepsilon/\varepsilon_{\mathrm{F}}}$). Nevertheless, the wavevector mismatch $\delta q = q_u - q_v \simeq k_{\mathrm{F}}\varepsilon/\varepsilon_{\mathrm{F}}$ results in the phase difference $\gamma \sim \delta q L$ between the electron- and hole-like parts of the total two-component quasiparticle wave functions $\hat{\psi} = (u, v)$ gained over a path of length L. Because the measurable quantities (current density, conductance, etc.) should be calculated as averages over different trajectories, i.e., as superpositions of rapidly oscillating contributions $uv^* \sim \exp(\mathrm{i}\gamma)$ from different trajectories, the phase factor γ between the u and v parts of the wave functions $\hat{\psi}$ induces destructive trajectory interference at the characteristic dephasing length $L_{\mathrm{p}} \sim 1/\delta q = \hbar V_{\mathrm{F}}/\varepsilon$. This leads to a power-law decay of the amplitude Δ on the length scale $L_{\mathrm{p}} \sim \xi_{\mathrm{N}} = \hbar V_{\mathrm{F}}/2\pi T_{\mathrm{c}}$ in a normal metal (T_{c} is the critical temperature of the superconducting transition) [17, 18].

When the electrons of a Cooper pair encounter an interface region of a ferromagnetic metal, a different spin-dependent shift of the quasiparticle energy for Andreev particles and holes at the normal side of the SF interface appears due to the Zeeman splitting of the spin subbands by the exchange field h [19]. Here, we assume that the triplet superconducting pairing is absent and the spins in a Cooper pair ($\uparrow\downarrow - \downarrow\uparrow$) are opposite as in the usual case of singlet superconductors. Wavevectors of quasiparticles with the energy $\varepsilon \ll h \ll \varepsilon_{\mathrm{F}}$ and opposite spin projections are different. The wavevector mismatch $\delta q = q_{u\uparrow(\downarrow)} - q_{v\downarrow(\uparrow)}$ is equal to $|\delta q| \simeq 2h/\hbar V_{\mathrm{F}}$, while the sign of δq is determined by the spin structure of the wave function $\hat{\psi}$ with respect to the exchange field direction. We note that the sign of δq can be changed in two evident cases: (i) reversal of the exchange field direction ($\mathbf{h} \to -\mathbf{h}$); (ii) singlet Cooper pair scattering with a spin-flip transition of electrons. The effect of the exchange field h for a trajectory of length L leads to the phase difference $\gamma \sim \delta q L = \pm L/\xi_h$ between the u and v components of the wave function $\hat{\psi}$ [9, 20], where $\xi_h = \hbar V_{\mathrm{F}}/2h$ is a characteristic length determined by the exchange field h. Such a dephasing with a homogeneous exchange field h induces destructive trajectory interference, which leads to a power-law decay of the amplitude of the Copper pairs wave function at the length scale $L_{\mathrm{p}} \sim \xi_h = \hbar V_{\mathrm{F}}/2h$ in the ferromagnetic region. In diffusive ('dirty') structures, superconducting correlations exist at distances of about $L_{\mathrm{d}} \sim \xi_{\mathrm{f}} = \sqrt{\hbar D_{\mathrm{f}}/h}$ (D_{f} is the diffusion constant) and decay exponentially. In both 'clean' and 'dirty' cases, the decay length is extremely small and does not exceed ten nanometers even for a weakly ferromagnetic interlayer like the CuNi alloy [21].

This simple qualitative picture of strong suppression of singlet superconductivity by the homogeneous exchange field of a ferromagnet appears to be in sharp contrast to a number of recent experiments that point to an anomalously large length of decay of superconducting correlations inside the F metal. Judging from observation [22], there is a noticeable supercurrent through a thin Co nanowire half a micrometer in length, which exceeds the typical decay length of singlet superconducting correlations in cobalt L_{d} by several orders of magnitude. The giant proximity effect in ferromagnetic metals is usually attributed to the excitation of spin-triplet superconducting correlations ($\uparrow\uparrow$) and ($\downarrow\downarrow$) of electrons with aligned spin projections [7, 8] (see Ref. [23] for a review). Because these triplet Cooper pairs are not destroyed by the exchange field, the superconducting correlations decay in a ferromagnet at the same length as in a normal (nonmagnetic) metal. As a result, long-range effects arise. Superconducting correlations spread at anomalously large distances inside FS hybrids [24], and there is a strong Josephson coupling in the SFS junction with a ferromagnetic barrier when the F-layer thickness is much larger than the characteristic scale ξ_h (or ξ_{f}) [25].

The crucial ingredient for explaining the long-range effect is the mechanism of conversion of singlet Cooper pairs ($\uparrow\downarrow - \downarrow\uparrow$) generated in the superconductor into the triplet ones ($\uparrow\downarrow + \downarrow\uparrow$), ($\uparrow\uparrow$), and ($\downarrow\downarrow$) in a ferromagnetic metal. The process of generating spin-triplet Cooper pairs with aligned spins can be understood by introducing a standard parameterization of the anomalous semiclassical Green's function in a ferromagnet $f = f_{\mathrm{s}} + \mathbf{f}_{\mathrm{t}}\hat{\boldsymbol{\sigma}}$, where f_{s} is the amplitude of the singlet part, the vector $\mathbf{f}_{\mathrm{t}}$ describes the triplet part of the function, and $\hat{\boldsymbol{\sigma}} = (\sigma_x, \sigma_y, \sigma_z)$ is the Pauli matrix vector in spin space. The component of the vector $\mathbf{f}_{\mathrm{t}}$ parallel to the magnetization $\mathbf{M}$ has a zero spin projection along the exchange field (the spin quantization axis) and decays in the ferromagnet at the same length scale $\xi_{h,\mathrm{f}}$ as the singlet part f_{s}. The noncollinear orientation of the magnetization $\mathbf{M}$ and the vector $\mathbf{f}_{\mathrm{t}}$ serves as a source of long-range triplet pairs with the spin projection ± 1 [26, 27]. The smooth profile of the exchange field suppresses the long-range effect, and the spin-triplet generation turns out to be most efficient for the inhomogeneity scale of the order of $\xi_{h,\mathrm{f}}$ [10]. The optimal structures for the long-range triplet Josephson effect observation are the multilayers SF$'$FF$''$S with noncollinear magnetizations in different ferromagnetic layers [25, 28]. In such a case, the long-range Josephson current results from the propagation of the triplet superconducting correlations with aligned spins through a thick ($d_{\mathrm{F}} \gg \xi_{h,\mathrm{f}} \sim d_{\mathrm{F}'}, d_{\mathrm{F}''}$) central noncollinear domain F. Consequently, this process is called the triplet long-range effect.

3. Long-range Josephson transport through a ferromagnetic bilayer

The inhomogeneous exchange field $\mathbf{h}$ caused by the ferromagnetic domain structure can also improve the conditions of superconductivity survival in FS hybrids in the clean limit, i.e., when the electron mean free path significantly exceeds the sample size [9, 20]. The destructive phase gain γ induced by the exchange fields can be strongly suppressed or even fully canceled if the orientation of the exchange field changes along the quasiparticle trajectory [9]. Total compensation of the destructive phase gain γ and the long-range Josephson effect can be realized in a 'clean' SFS junction containing a bilayer as an F barrier (see Fig. 1a) if both ferromagnetic layers (domains) have the same thickness $d_1 = d_2 \ll \xi_{\mathrm{s}}$ and

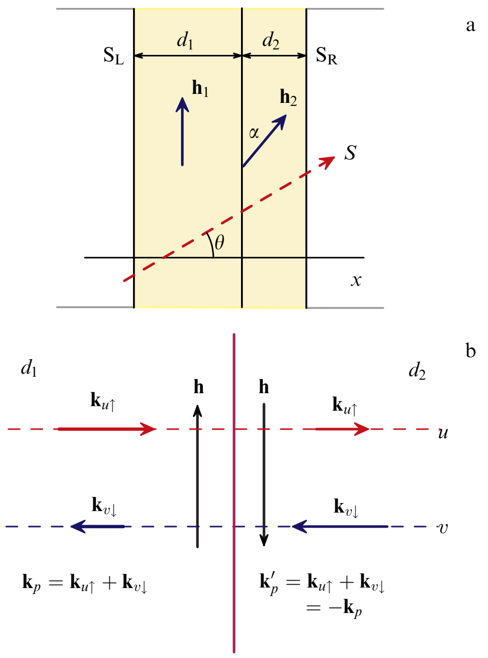

Figure 1. (a) SFS Josephson junction containing two ferromagnetic layers (domains) as a barrier. A linear quasiparticle trajectory is shown by the red dashed line. (b) The spin arrangement $\mathbf{q} = \mathbf{k}_{u\uparrow} - \mathbf{k}_{v\downarrow}$ of the wave function ψ in the ferromagnetic bilayer with opposite directions of the exchange field in the domains.

opposite orientations of the exchange field in the domains [9, 20, 29]. In this case, the phase gains $\gamma_{1,2}$ arising due to the ballistic motion across each layer are equal in magnitude but opposite in sign for the two layers: $\gamma_{1,2} = \pm d_{1,2}/\xi_h$. As a result, the total phase gain $\gamma = \gamma_1 + \gamma_2$ in the ferromagnetic bilayer must cancel, and the destructive effect of the exchange field is absent. In this case, the amplitude of superconducting correlations decay in the ferromagnet bilayer at the same characteristic length as in a normal metal. A qualitative picture of this effect is shown in Fig. 1b. In general, when $d_1 \neq d_2$ and the exchange fields $\mathbf{h}_1$ and $\mathbf{h}_2$ in the layers are equal in magnitude ($|\mathbf{h}_1| = |\mathbf{h}_2| = h$) but are rotated through an arbitrary angle α, we find the following expression for the phase shift $\gamma(\theta)$ gained in a 2D junction containing the F bilayer [29]:

$$\cos\gamma(\theta) = \cos^2\frac{\alpha}{2}\cos(\delta_1+\delta_2) + \sin^2\frac{\alpha}{2}\cos(\delta_1-\delta_2)\,, \quad (1)$$

where $\cos\theta = (\mathbf{n}, \mathbf{n}_\mathrm{F})$ is the angle between the trajectory direction and the vector normal to the FS plane, and $\delta_{1,2} = d_{1,2}/(\xi_h\cos\theta)$. The relation between the current I and the phase difference φ of a short 2D SFS Josephson junction must be calculated as a superposition of the contributions from different trajectories,

$$I(\varphi) = \sum_n I_{cn}\sin(n\varphi) = \sum_n a_n g_n \sin(n\varphi)\,,$$
$$g_n = \int_0^{\pi/2} \mathrm{d}\theta\,\cos\theta\cos\big(n\gamma(\theta)\big)\,. \quad (2)$$

The coefficients $a_{1,2}$ coincide with the coefficients of the Fourier expansion for the current–phase relation SNS junction of the same geometry, and the amplitudes of the first two harmonics at $T \approx T_\mathrm{c}$ are determined by the relations

$$a_1 = \frac{eT_\mathrm{c}}{8\hbar}N\left(\frac{\Delta(T)}{T_\mathrm{c}}\right)^2, \qquad a_2 = -\frac{eT_\mathrm{c}}{384\hbar}N\left(\frac{\Delta(T)}{T_\mathrm{c}}\right)^4, \quad (3)$$

where $\Delta(T)$ is the temperature-dependent superconducting order parameter in the electrodes and the factor N is determined by the number of transverse modes in the junction. Expression (1) allows writing the first harmonic of the current–phase relation $I(\varphi) = \sum_n I_{cn}\sin(n\varphi)$ in the convenient form

$$I_{c1} = \cos^2\frac{\alpha}{2}\,I_{c1}\left(\frac{d_1+d_2}{\xi_h}\right) + \sin^2\frac{\alpha}{2}\,I_{c1}\left(\frac{d_1-d_2}{\xi_h}\right), \quad (4)$$

where $I_{c1}(\delta)$ is the critical current of the first harmonic in the SFS junction with the F-layer thickness $d = d_1 + d_2 = \delta\xi_h$ and a homogeneous exchange field h for which the relation $\cos(\gamma(\theta)) = \delta/\cos\theta$ is satisfied. For a thick 2D junction ($d \gg \xi_h$), expression (4) describes a slow power-law decay of the critical current I_{c1} with an increase in the ferromagnetic barrier thickness ($I_{c1} \propto (d/\xi_h)^{-1/2}$), caused by the destructive trajectory interference [30]. Taking the symmetric case $d_1 = d_2$ in (4), we immediately obtain a long–range contribution to the Josephson current: $I_1^\mathrm{LR} = I_{c1}(0)\sin^2(\alpha/2)\sin\varphi$, which does not depend on the ferromagnetic bilayer thickness d.

We note that a similar problem was considered in [31] for one-dimensional constriction. In this case, the long-range Josephson effect also exists for a homogeneous magnetization of the barrier, and a composite F layer does not lead to qualitatively new effects.

It is important that the long-range contribution to supercurrent (4) survives for an arbitrary nonzero angle α between the magnetic moments of the adjacent F layers. The long-range behavior can also be observed for the second harmonic in the current–phase relation $I_{c2}^\mathrm{LR} = -a_2\sin^2(\alpha/2)$, and does not disappear even if $d_1 \neq d_2$ and $d_{1,2} \gg \xi_h$. The emergence of a long-range Josephson effect for even harmonics of the current–phase relation is in good agreement with recent theoretical findings in Refs [32, 33]. This simple example shows the ability to significantly change the decay length of pair correlations in the F barrier and, as a result, a method is proposed to easily control the transport properties of ballistic Josephson SFS systems by manipulation of the exchange field in ferromagnetic metals.

4. Long-range Josephson transport through a ferromagnetic wire with a finite spin–orbit interaction

A different mechanism of the long-range effect in mesoscopic SFS structures was proposed recently for a ballistic transport through a thin ferromagnetic wire with spatially homogeneous magnetization [29]. An effective exchange field inhomogeneity $\mathbf{h}$ along a semiclassical trajectory required for the long-range effect appears due to multiple reflections from the ferromagnet surface if spin–orbit interaction is present inside the ferromagnet. The spin–orbit interaction results in the quasiparticle momentum dependence of the exchange field $\mathbf{h} = \mathbf{h}(\mathbf{k})$ [34]. Because the normal quasiparticle reflection on the wire surface is accompanied by a change in the trajectory direction and the quasiparticle momentum $\mathbf{k}$ (see Fig. 2), the exchange field $\mathbf{h}(\mathbf{k})$ must also change its direction along the

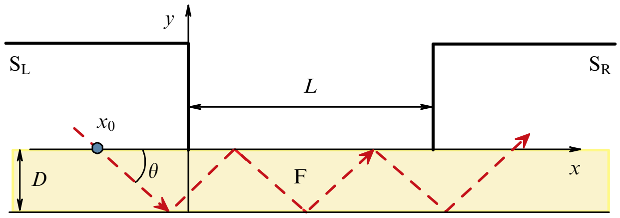

Figure 2. Schematic of the 2D overlap SFS Josephson junction containing a ferromagnetic wire of thickness D. The quasiparticle trajectory s experiencing multiple reflections from the wire surfaces is shown by the red dashed line.

quasiparticle trajectory. A similar periodic exchange field can strongly affect the phase gain γ along the trajectories and suppresses the destructive phase difference between the electron- and hole-like parts of the wave function for a large group of semiclassical trajectories, even in a single-domain ferromagnetic state. The phase gain γ can be found from the linearized Eilenberger equations [35] for the anomalous semiclassical Green's function in the ferromagnet $f = f_s + \mathbf{f}_t\hat{\boldsymbol{\sigma}}$,

$$-i\hbar V_F \frac{\partial f_s}{\partial s} + 2\mathbf{h}\mathbf{f}_t = \epsilon(k) f_s\,, \quad -i\hbar V_F \frac{\partial \mathbf{f}_t}{\partial s} + 2 f_s \mathbf{h} = \epsilon(k)\mathbf{f}_t\,, \quad (5)$$

along the trajectory s characterized by a given angle θ with respect to the wire surface. In the absence of the system anisotropy described by a polar vector, the simplest form of the resulting exchange field $\mathbf{h}$ is $\mathbf{h}(\mathbf{k}) = \mathbf{h}_0 + \beta_{SO}(\mathbf{h}_0, \mathbf{k})\mathbf{k}/k_F^2$, where $\mathbf{h}_0$ is a pseudovector determined by the ferromagnetic moment $\mathbf{M}$, β_{SO} is a constant determined by the spin–orbit interaction, and k_F is the wave number for the Fermi momentum $p_F = \hbar k_F$. The exchange field $\mathbf{h}$ varies with the period $s_\theta = 2D/\sin\theta$ on a broken trajectory experiencing multiple specular reflections from the wire surfaces.

To solve Eqns (5), we use a perturbative approach similar to the nearly free electron approximation in the band theory of solids. Applying this method, we obtain the following expression for the long-range component f_s^{LR} of the singlet part $f_s(s)$ of the anomalous Green's function f:

$$f_s^{LR} = \sum_{m=0}^{\infty} \frac{8|H_{q_m}|^2}{(\hbar V_F q_m - 2h_x)^2 + 8|H_{q_m}|^2}\,, \quad (6)$$

which is independent of the trajectory length and is therefore not destroyed by destructive interference under averaging over different trajectories. Here, $H_q = -2i\beta_{SO}h_0 \sin^2\theta\cos\theta/(Dq)$ are the amplitudes of the Fourier harmonics of the periodic function $h_y(s) = \sum_q H_q \exp(iqs)$, corresponding to the reciprocal lattice vectors $q = q_m = 2\pi(2m+1)/s_\theta$, $m = 0, \pm 1, \ldots$. Determined by the amplitude f_s^{LR}, the long-range first harmonic in the current–phase relation for $T \simeq T_c$ takes the form

$$I_1^{LR} = I_{c1}^{LR}\sin\varphi = a_1 \sin\varphi \int_0^{\pi/2} d\theta\, \cos\theta\, f_s^{LR}(s_R)\,. \quad (7)$$

Assuming the resonances to be rather narrow and not to overlap as $|H_{q_m}| \to 0$, we approximate f_s^{LR} in (6) by the sum of δ-functions and rewrite the expression for the critical current I_1^{LR} as a sum over the resonant angles θ_m determined

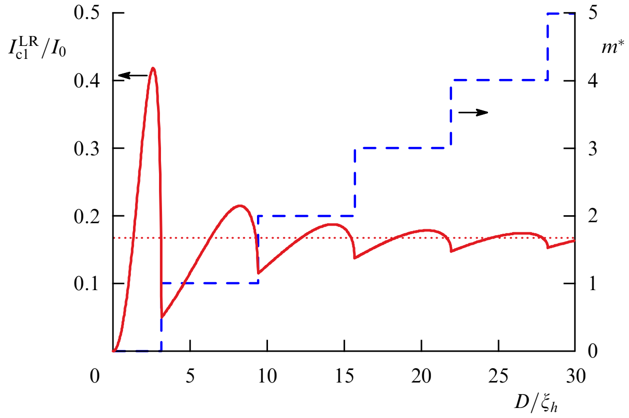

Figure 3. Dependence of the long-range first harmonic I_{c1}^{LR} (8) of the supercurrent (solid line) and the number m^* (dashed line) on the F-wire thickness D. For reference, the horizontal dotted line shows the asymptotic value of the amplitude I_{c1}^{LR} for $D \gg \xi_h$ determined by the relation $I_{c1}^{LR} \simeq a_1\sqrt{2}\beta_{SO}/3$ ($I_0 = a_1 2\sqrt{2}\beta_{SO}$).

by the relation $\sin\theta_m = 2h_x D/[\pi\hbar V_F(2m+1)]$:

$$I_{c1}^{LR} \simeq a_1 \sum_{m \geqslant m^*} \frac{\sqrt{2}\pi\hbar V_F \beta_{SO}}{h_0 D} \sin^3\theta_m \cos\theta_m\,, \quad (8)$$

where m^* is the smallest positive integer or zero satisfying the condition $2m^* + 1 \geqslant D/(\pi\xi_h)$. In the limit $D \gg \hbar V_F/(2h_x)$, the resonant angles are closely distributed in the interval $0 < \theta_m < \pi/2$, and $I_{c1}^{LR} \simeq a_1\sqrt{2}\beta_{SO}/3$.

Figure 3 shows the dependence of the amplitude of the long-range first harmonic I_{c1}^{LR} of the supercurrent on the F-wire thickness D. Oscillations of the critical current $I_{c1}^{LR}(D)$ as a function of the wire thickness have the period $\Delta D = 2\pi\xi_h$ and originate from a change in the number of resonant (Bragg-type) quasiparticle trajectories that correspond to the vanishing phase gain γ between the electron- and hole-like parts of the wave function. Certainly, the above long-range effect in the first harmonic is rather sensitive to both the SFS structure configuration and possible disorder due to nonspecular quasiparticle reflection on the wire surface. However, similarly to the case of a ferromagnetic bilayer, we expect the long-range supercurrent for higher (even) harmonics to be robust against the disorder effect and changes in the of SFS structure geometry [29].

5. Singlet Josephson transport mediated by scattering with a spin-flop transition in ballistic SFS structures

We consider one more method for suppressing destructive interference due to the effect of the exchange field of a ferromagnet, proposed in Ref. [36]. The phase difference γ between the electron- and hole-like components of the wave function on a semiclassical trajectory can be strongly suppressed by the creation of a noncollinear domain of the exchange field $\mathbf{h}$, which reverses the Cooper-pair spin arrangement by scattering (a spin-flop transition). A similar localized magnetic inhomogeneity can be created, for example, by a tip of the magnetic exchange force microscope (MExFM) or another source of strongly inhomogeneous magnetic fields, which are able to change the magnetization of a small region (of the size $\sim \xi_h$) of the ferromagnetic weak

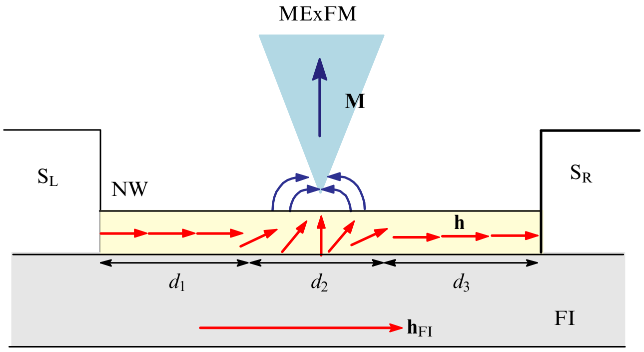

Figure 4. Schematic of the SFS constriction under consideration: a thin layer of a normal nonmagnetic metal on the surface of a ferromagnetic insulator (FI), which induces the effective exchange field $\mathbf{h}$ in the metal. The tip of the magnetic exchange force microscope (MExFM tip) with the magnetization $\mathbf{M}$ creates a noncollinear domain of the exchange field $\mathbf{h}$ located near the constriction center.

link (see Fig. 4). Created by a similar 'magnetic gate', the region in which the exchange field $\mathbf{h}$ differs from the original one is the reason for the appearance of the long-range singlet component of the Josephson current.

To elucidate the peculiarities of Cooper pair scattering with a spin-flop transition of electrons, it is convenient to introduce the new functions $f_\pm = f_\mathrm{s} \pm f_{\mathrm{t}x}$ composed of the parts of the anomalous Green's function f. The functions $f_\pm$ describe the pairs with a zero spin projection and a reversed spin arrangement: $(\uparrow\downarrow)$ and $(\downarrow\uparrow)$. It has been noted that the total momentum of a singlet Cooper pair $\hbar\mathbf{q} = \hbar\mathbf{k}_\uparrow - \hbar\mathbf{k}_\downarrow$ is not equal to zero due to the exchange splitting of the spin subbands ($|\mathbf{k}_\uparrow| > |\mathbf{k}_\downarrow|$ and $|\mathbf{q}| \sim 1/\xi_h$). The linearized Eilenberger equations for the functions $f_\pm$ are greatly simplified if the direction of the spatially homogeneous exchange field coincides with the spin quantization axis x:

$$\mp \mathrm{i}\hbar V_\mathrm{F} \frac{\partial f_\pm}{\partial s} + 2hf_\pm = 0\,. \tag{9}$$

The additional domain d_2 (the scatterer) with a noncollinear exchange field mixes ('entangles') the components f_+ and f_-. Then the resulting values of the function $f_\pm$ ($d_2 \ll d_1 \simeq d_3$) at the right electrode must be

$$f_\pm(s_\mathrm{R}) = a_\pm \exp\left[-\mathrm{i}q(s_{d_1} \pm s_{d_3})\right] + b_\pm \exp\left[+\mathrm{i}q(s_{d_1} \mp s_{d_3})\right]\,, \tag{10}$$

where $s_{d_i} = d_i/\cos\theta$, and the coefficients $a_\pm$ and $b_\pm$ depend on the scatterer parameters (the domain thickness d_2 and the exchange field rotation angle α). For $d_1 = d_3$, i.e., if the domain d_2 is placed at the weak link center, the rapidly oscillating factor in the first and the second terms vanishes. This means the emergence of the long-ranged singlet proximity effect, because the singlet part of the Green's function

$$f_\mathrm{s}(s_\mathrm{R}) = \frac{1}{2}\Big\{a_+ \exp\left[-\mathrm{i}q(s_{d_1} + s_{d_3})\right] + b_- \exp\left[+\mathrm{i}q(s_{d_1} + s_{d_3})\right]\Big\} + \frac{1}{2}(a_- + b_+) \tag{11}$$

includes the term $(a_- + b_+)/2$, where a 'fast' θ dependence disappears.

A schematic picture of Cooper pair scattering on a noncollinear exchange field domain is shown in Fig. 5. The total momentum of a singlet Cooper pair $\hbar\mathbf{q}$ is either unchanged $\hbar\mathbf{q}' = \hbar\mathbf{q}$ after the scattering (Fig. 5a), or reversed $\hbar\mathbf{q}' = -\hbar\mathbf{q}$ (Fig. 5b), depending on the scatterer parameters. In the first case ($a_- = 0, b_+ = 0$), the spin arrangement of the pair does not change with respect to the exchange field $\mathbf{h}$ ($f_\pm \to f_\pm$) and the remaining total phase gain between the electron and hole parts of the wave function $\gamma \approx (d_1 + d_3)/\xi_h$ is large, which results in a strong destructive trajectory interference of electron and hole states at the characteristic length $\sim \xi_h$. In the second case ($a_+ = 0$, $b_- = 0$, spin-flop scattering), the spin arrangement of the pair is reversed with respect to the exchange field ($f_\pm \to f_\mp$), the phase gains at the segments d_1 and d_3 differ by sign, and the total phase gain $\gamma \approx (d_1 - d_3)/\xi_h$ depends on the relative position of the scattering domain d_2. At a symmetric position of the scatterer ($d_1 \approx d_3$), the total phase gain γ in the ferromagnet goes to zero, and the destructive effect of the exchange field for the component $(a_- + b_+)/2$ of the wave function vanishes.

Therefore, the singlet part f_s of the anomalous Green's function in a ferromagnet with a magnetic exchange field inhomogeneity localized near the constriction center must decay at the same length $\xi_n = \sqrt{D_\mathrm{f}/(2\pi T_\mathrm{c})} \gg \xi_h$ as in a nonmagnetic metal. This means that long-range Josephson transport exists in the SFS hybrid under consideration. We note that the triplet long-range proximity analyzed in Refs [7, 8] is absent for this setup, but the stimulation of singlet supercurrent takes place.

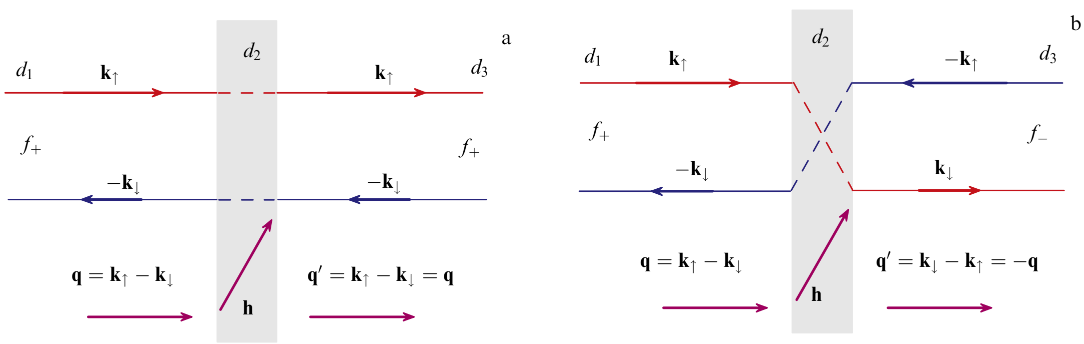

Figure 5. (a) Scattering of a singlet Cooper pair at the exchange field $\mathbf{h}$ inhomogeneity without the spin–flop transition of electrons. The spin arrangement of the pair $\mathbf{q} = \mathbf{k}_\uparrow - \mathbf{k}_\downarrow$ has not changed with respect to the exchange field $\mathbf{h}$: $\mathbf{q}' = \mathbf{q}$. (b) Scattering of a singlet Cooper pair at the exchange field $\mathbf{h}$ inhomogeneity with the spin–flop transition of electrons. The spin arrangement of the pair is reversed: $\mathbf{q}' = \mathbf{k}_\downarrow - \mathbf{k}_\uparrow = -\mathbf{q}$. The noncollinear exchange field $\mathbf{h}$ scatterer is shown in grey.

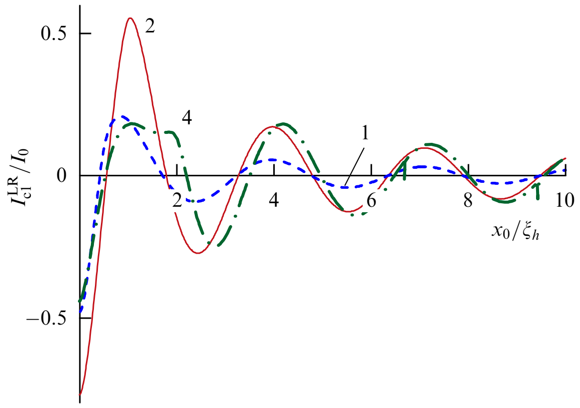

Figure 6. Dependence of the critical current I_{c1}^{LR} on the shift x_0 of the central 90°-domain d_2: $d_2 = \xi_h$ — blue dashed line; $d_2 = 2\xi_h$ — red solid line; $d_2 = 4\xi_h$ — green dashed-dotted line. Symbols near the curves denote the value d_2/ξ_h. Here, we choose $T = 0.9T_c$, $d = 50\xi_h$; $I_0 = (eT_cN/8\hbar)(\Delta/T_c)^2$.

Figure 6 shows the dependences of the maximal Josephson current I_{c1}^{LR} on the position of the central domain x_0 with respect to the weak link center for different values of the thickness of the 90°-domain d_2 with a stepwise profile of the exchange field $\mathbf{h}(x) = h\mathbf{x}_0\,[0 \leqslant x \leqslant d_1,\ d - d_3 \leqslant x \leqslant d]$; $h\mathbf{y}_0\,[d_1 \leqslant x \leqslant d_1 + d_2]$. The critical current is very sensitive to the position of the central domain d_2, and a noticeable change in both the value and the sign of the current accompanies the domain displacement. With the shift of the domain d_2 with respect to the center of the structure ($x_0 \neq 0$), a series of transitions between 0- and π-states occurs. Interestingly, at a symmetric position of the domain d_2 ($d_1 = d_3$), the long-range supercurrent is negative ($I_1^{LR} < 0$), i.e., the long-range contribution generates a π-junction. Hence, it seems to be possible to significantly change both the value and the sign of the critical current I_{c1} of the Josephson weak link, i.e., to modify the current–phase relation of the junction $I(\varphi)$ on the whole by magnetic tip displacement, which modifies the position x_0 and the properties of the induced inhomogeneity of the exchange field. The strong dependence of the critical current I_c of the SFS structure on the magnetic tip position x_0 results in the possibility of controlling the Josephson transport of SFS systems by external manipulations of the spin structure of the propagating Cooper pairs. As has been noted previously, the first harmonic vanishes at the $0-\pi$ transition ($I_{c1}^{LR} = 0$), and the second harmonic of the current–phase relation becomes dominant.

6. Conclusion

To summarize, we studied the novel mechanisms of the long-range effect in the hybrid SFS structure in the ballistic regime based on suppression of the destructive interference of electron and hole states due to the ferromagnetic exchange splitting of spin subbands. The spatial or momentum dependence of the exchange field provides an efficient mutual conversion from the spin-singlet (damped) superconducting correlations to the spin-triplet (undamped) ones. As a result, the singlet part of the Cooper-pair wave function decays slowly in the ferromagnet and the Josephson current appears to be strongly enhanced. The strong interaction between the superconducting and ferromagnetic subsystems provides very efficient ways to manipulate the superfluid (dissipation-free) transport via the control of the spin degree of freedom, and must therefore satisfy a key requirement for devices of superconducting spintronics and nanoplasmonics.

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