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V L Ginzburg's helical elliptically polarized modes and their application

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Abstract. In 1943, V L Ginzburg derived a simple and elegant solution to the polarization optics problem when a linear birefringent medium is uniformly twisted along the optical axis. In a torsion-comoving coordinate frame, mutually orthogonal polarization eigenmodes, i.e., helical modes, of such an optical medium are characterized by two ellipses with the electric field vector of light bypassed in the opposite sense. Current applications of V L Ginzburg's theory are reviewed.

Keywords: polarization modes, helical system of coordinates, history of physics

The purpose of this short science history paper is to discuss V L Ginzburg's important contribution to the field of optics and electrodynamics — his proposal [1] to use the so-called helical reference frame (HRF) twisting together with the medium to calculate radiation polarization states (RPSs) when both linear ray birefringence and circular birefringence due to the twisting of the optical medium are present. The paper also reviews the advantages of the HRF over the laboratory reference frame (LRF) in calculating RPSs and takes a look at applications and further development of the method. Because [1] is beyond the main scope of V L Ginzburg's research and because it was published during the tumultuous years of the Great Patriotic War (GPW), far from everyone involved in the field is even aware of its existence. Although the method of Ref. [1] has found extremely wide application and is currently considered classic, there are, to the author's knowledge, just a few dozen references to it, cited primarily by V L Ginzburg's disciples, his disciples' disciples, or the researchers who were in direct contact with him.

In 1943, V L Ginzburg addressed the problem of measuring mechanical stresses optically in the most general case in which transverse compression (tensile) or bending and twisting stresses are simultaneously present [1]. In this case, if a loaded body is transparent and isotropic, the mechanical stresses produce two types of ray birefringence in it: linear, due to compression/tensile stress, or bending, and circular, due to torsion stress. Describing the evolution of RPSs as the radiation travels along the optical axis of the sample under study requires solving the Riccatti differential equation — a solution which cannot be obtained in quadratures for a sample twisted arbitrarily along its optical axis [2]. V L Ginzburg obtained a simple and elegant solution not by solving the Riccatti equation but instead by moving to an HRF which has axes comoving with the optical axes of the sample being twisted. In the HRF, the two natural (or normal) mutually orthogonal polarization modes of light are two ellipses with the electric field vector wrapped around in opposite directions; importantly, in the absence of dichroism, the axes of the ellipse are mutually orthogonal and do not rotate, whereas the axes of the ellipse in the LRF undergo rotation when propagation is along the optical axis of the medium, the amount of the rotation being numerically equal to the geometric torsion of the medium [1]. Later, these modes were named 'helical elliptically polarized modes' (HEPMs) or, in short, 'helical modes'.

The question V L Ginzburg addressed arose in connection with the photoelastic study of mechanical stresses (see review [3]). The photoelasticity method is intended as a means for probing the stress distribution in structures and structural components subjected to both their own weight and external forces. The way this is done is by preparing from a transparent isotropic material a (usually reduced-scale) model of the object to be studied and placing it between two crossed linear polarizers. The input polarizer is illuminated by light, and, if there is no load, then at the output of the second, crossed polarizer, no light appears. But if the system *is* loaded, then some type of ray birefrignece occurs, resulting in the fact that at the output of the second polarizer a certain light intensity pattern can be observed, from which the localization, type, and intensity of mechanical stresses in the object can be determined.

When first discovered by D Brewster in 1816, the photoelasticity effect was not studied for mechanical torsion stresses. J C Maxwell, in 1847–1850, studied the simultaneous development of linear and circular birefringence in a transparent material subjected to various mechanical stresses, including torsion [5]. He was not, however, successful in explaining the phenomenon, and the experiments of Ref. [5] were forgotten for a long time [3]. Later, similar experiments were conducted in Russia and the USSR [6], Germany, and the USA [3]. These experiments should be the subject of a separate review. Early attempts at interpreting this phenomenon cannot be considered successful [3], and in the early 1940s, the need arose to develop a physical model of how RPSs evolve in this problem.

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V L Ginzburg's seminal work, paper [1], was submitted to *Zhurnal Tekhnicheskoi Fiziki* (*Technical Physics* journal) on 3 June 1943 — implying that it had been written in Kazan, to which the Physical Institute of the Academy of Science of the USSR had moved and from which V L Ginzburg returned to Moscow only in late 1943 [7] — and was published in March 1944. Let us have a look at the method V L Ginzburg used. Referring to his work [8], he wrote the following in Ref. [1]: *The propagation of light in an inhomogeneous anisotropic medium — which is the general case — presents a complicated picture... The ray trajectories and the polarization of both waves*[1] *are virtually impossible to find because to do this requires the solution of quite complicated differential equations*. (Below, we will follow V L Ginzburg's notation in quotations from Ref. [1]).

V L Ginsburg then continues by successively simplifying the problem using the fact that in practical cases the linear birefringence in an optical material is small compared to unity, and hence the electric field $\mathbf{E}$ and the electric displacement $\mathbf{D}$ in the beam are both along the normal to the beam. He further considers the case where the beam is along the main optical axis. He then goes on to note [1]:

The resulting oscillation $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$, *where* $\mathbf{E}_1$ *and* $\mathbf{E}_2$ *are the normal oscillations, i.e., oscillations corresponding to a certain propagation velocity, are elliptic. The ellipticity along the beam changes due to the change in the phase difference between* $\mathbf{E}_1$ *and* $\mathbf{E}_2$.

Thus, V L Ginzburg was the first to realize that in the presence of linear birefringence and torsion, the natural (normal) polarization modes are elliptic and that in the LRF the azimuths of the ellipse's axes and the ellipticity vary along the beam. For this reason, he introduced new axes, y and z, which rotate along the beam (along the x axis) and in which — unlike the unmoving axes y' and z' — the natural polarization modes retain their ellipticity along the beam. The y and z axes are the axes of the HRF. He also determined the distance after the travel along which the polarization ellipse of light returns to its position — a distance which is now known as the polarization beat distance.

In Ref. [1], V L Ginzburg paid great attention to the necessary conditions of geometric optics — conditions which validate the approximations he used. In particular, the condition

$$A\lambda \ll 1 \tag{1}$$

must hold, where, in the notation of Ref. [1], $A = 2\varphi/x$, φ is the angle of rotation over a distance x, and λ is the vacuum light wavelength. In fact, A is the circular birefringence induced by the torsion of the optical medium. Condition (1) means that the torsion of axes over a light wavelength is very small. A second limitation on the applicability of geometrical optics is given by [1]

$$R = \frac{A\lambda}{2\pi\delta n} \ll 1\,, \tag{2}$$

where $\delta n = n_y - n_z$ is the refraction index difference (related to the HRF) between the slow and fast axes of the linear birefringence of the medium. Because $2\pi\delta n/\lambda$ is the linear birefringence, the physical meaning of the parameter R is that it is the ratio of the amount of torsion-induced circular birefringence to linear birefringence, and hence condition (2) implies that this circular birefringence can be much smaller than the linear birefringence. At the same time, it is noted in Ref. [1] that for $\delta n \to 0$ the geometric optics approximation breaks down.

As pointed out in Ref. [1], the general-form solution of the problem can even be obtained for arbitrary R — provided, though, that condition (1) and two additional conditions, $(\partial n/\partial x)\,\lambda \ll 1$ and $\delta n \ll 1$, hold. This can be achieved by expanding in the small parameter λ as suggested by V L Ginzburg in Ref. [8].

Now, after the solution for the ellipticity of natural HEPMs is obtained, what remains is to find the magnitude of the elliptic birefringence of these modes. V L Ginzburg goes on to write: [1]

We will not, however, undertake such an analysis here because in the case of rectilinear propagation, which is of interest for photoelasticity... the problem allows a simple and completely comprehensive solution. The corresponding method was in fact already used by Druker and Mindlin [9].[2] *However, the fact that they use the notation of elasticity theory and our desire to highlight certain details and conclusions seem to warrant a reexamination of the problem.*

A close look reveals, however, that the study in Ref. [9] mainly focuses on the mechanical deformations of an elastic medium and, as for optics, the only issue addressed is the transfer of the elliptic light polarization in a squeezed and twisted medium. The fact that the natural polarization modes conserve their ellipticity in the HRF is not mentioned in Ref. [9]. Note also the use of the theory of 'luminiferous ether' in the calculations in Ref. [9].

V L Ginzburg used Maxwell's equations and the dielectric constant tensor ε to solve rigorously the problem of finding the natural polarization modes in a linearly and circularly birefringent optical medium. His starting point was to write two differential equations of second order in dx^2 for the electric field vector components $E_{y'}$ and $E_{z'}$ in the reference frame x, y', z', with the electric displacement components $D_{y'}$ and $D_{z'}$ entering as constant terms. He then moved to the HRF and wrote the corresponding set of second-order differential equations for E_y and E_z, making the substitution x, y, z in the process. In addition, the first derivatives of E_y and E_z with respect to dx appear in these equations. As a result, the following equation is obtained for the propagation constants of two HEPMs [1]:

$$k_{\pm} = \frac{\omega}{c}\,n_0 + \frac{\delta n_y + \delta n_z}{2} \pm \frac{\delta n_y - \delta n_z}{2}\,\sqrt{1 + R^2}\,, \tag{3}$$

where c is the vacuum light speed, $n_0 = \sqrt{\varepsilon_0}$, $\delta n_y = \delta\varepsilon_y/2\sqrt{\varepsilon_0}$, and $\delta n_z = \delta\varepsilon_z/2\sqrt{\varepsilon_0}$.

The phase difference between the radiation in the different orthogonal HEPMs passing the distance x is given by [1]

$$\Delta = \frac{2\pi}{\lambda}\,(\delta n_y - \delta n_z)\,\sqrt{1 + R^2 x}\,. \tag{4}$$

If condition (2) ($R \ll 1$) holds, then, as pointed out in Ref. [1], the field is totally 'dragged' by the rotating axes, i.e., the axes of the polarization ellipses of the orthogonal natural polarization modes exactly follow the torsion of optical axes of the medium.

In the first (1960) edition of his monograph [10], V L Ginzburg applied the results of Ref. [1] to the propagation of electromagnetic waves in a magnetized plasma. The subse-

[1] Note by the author: ordinary and extraordinary.

[2] In the bibliography of Ref. [1], this work is numbered [5].

quent development of V L Ginzburg's method is considered briefly below.

In 1968, Y A Kravtsov [11] considered the case in which, as shown in Ref. [10], the geometrical optics approximation cannot be represented in the form of independent orthogonal waves. He showed, however, that based on the results of S M Rytov [12] (quasi-isotropic approximation), the geometric-optics solution of Maxwell's equations can be obtained.

In 1972, E V Suvorov [13] considered the propagation of normal waves in a plasma with a strong magnetic field shear (an effect in which the magnetic field component perpendicular to the beam direction rotates along the light ray path) or, in other words, in the case of the helical structure of the magnetic field. E V Suvorov used the approximation of geometric optics to find normal waves in a uniformly sheared heterogeneous plasma.

Subsequently, V L Ginzburg's student V V Zheleznyakov and V V Zheleznyakov's students V V Kocharovskii and Vl V Kocharovskii published a series of studies on various applications of HEPMs (see Ref. [14] for a review). The studies examined the helical mode structure for the case of rotating anisotropy axes and an arbitrary dielectric constant tensor; showed that the helical modes are nonorthogonal even in the absence of absorption; developed a geometric optics model of helical modes; and derived general linear coupling equations explicitly taking into account the 'helical' symmetry of the electromagnetic properties of the continuous medium. Based on qualitative analysis results and a number of exact and approximate solutions of these equations, features of the linear coupling of helical modes in inhomogeneous weakly anisotropic media with nonuniformly rotating anisotropic axes were investigated together with the transfer of radiation polarization in a magnetically active medium with magnetic field shear and the linear coupling of light waves in holesteric (initially twisted) liquid crystals (LCs) and twisted single mode fibers (SMFs).

About forty years ago, the problem arose of how to describe the evolution of the degree of polarization of nonmonochromatic radiation traveling through a randomly inhomogeneous SMF and how to calculate the inhomogeneity-induced zero drift of fiber ring interferometers (FRIs). To solve this problem, the present author, along with V I Pozdnyakova and I A Shereshevskii, developed a statistical model of random inhomogeneities in a SMF, in which the coupling of orthogonal polarization modes is considered to be primarily due to the random torsions of linear birefringence axes in SMF that arise when the wire has been drawn from a preform and is not yet fully frozen [15]. Thus, different HEPMs occur on each random segment of the SMF. The model proposed in Ref. [15] — unlike the available qualitative integral models of SMF — adequately explains the known experiments on the change in the polarization conservation parameter in SMF and provides analytical expressions for the degree of polarization of nonmonochromatic radiation and zero drift in an FRI with an arbitrarily birefringent contour made of SMF. The results of these studies were published in a number of papers and are presented in monographs [16, 17].

Let us now consider the advantages of V L Ginzburg's [1] HRF over the LRF, which were first amply demonstrated in review [14] by explicitly constructing the geometric optics of helical modes and developing the theory of their linear coupling and thereby taking a major step forward in the study of the polarization features of the wave field, in contrast to the often-used Jones matrix eigenvector approach. In our paper [18], we point out that HEPMs can be described both analytically in the formalism of Jones matrix eigenvectors [19] and using the simpler and more transparent Poincaré sphere (PS) method [19–21]. These expressions are not presented here due to space limitations, but Ref. [18] suggests that expressions in the HRF are markedly simpler than in the LRF. Accordingly, the evolution of RPSs of natural HEPMs as a function of the optical length of the twisted medium on PS in the LRF is a rather complicated curve, whereas in the HRF it is a non-moving point on PS. Even for non-natural HEPMs, the evolution of RPSs on SP is much simpler in an HRF than in the LRF.

At this point, it is appropriate to mention studies on holesteric LCs [22, 23] that preceded Ref. [1]. Study [22] showed the existence of a preferred HRF but failed to expand the original linear polarization in terms of the eigenmodes of the HRF, i.e., HEPMs, and hence to realize the potential of the method of Ref. [1]. Reference [23] says nothing on the existence of a preferred HRF and does not consider the ellipticity of natural HEPMs. That is, the authors of Refs [22, 23] came close to but did not obtain expressions for HEPMs.

In 2008, V L Ginzburg told the present author that he remembered well his publication [1], that it was a simple matter for him to obtain expressions for HEPMs, and that he was, well, so very young at the time [24].

Note added in proof. The following is an interesting story which I learned from Boris Mikhailovich Bolotovskii when the paper was in the proof stage and which has a direct relation to the present work — or, more precisely, to the application of the results of V L Ginzburg's work [1].

At the very end of the 1940s, the Moscow Electric Lamp Plant was assigned to the task of mastering the production of TV kinescopes. Before that, the plant produced only incandescent bulbs and had had no experience producing kinescopes, so 97% of the kinescope bulbs broke when cooled — mainly where their necks started to widen. Vladimir L'vovich Indenbom (1924–1998), then a plant employee (a GPW participant, reserve captain, researcher at the Institute of Electrovacuum Glasses in 1950–1955 years, and M V Lomonosov Moscow State University graduate; for more on him, see Ref. [25]), took it upon himself to eliminate this problem. Well familiar with V L Ginzburg's work [1], V L Indenbom proposed that stresses in a kinescope bulb can be spotted using the polarization method of Ref. [1], which (to recall) observes mechanical stresses in an optical medium between two crossed polarizers. This innovative idea having met with a cold response at the plant, V L Indenbom went to FIAN (Lebedev Physical Institute) to consult with V L Ginzburg [3], a visit which provided him with a number of valuable suggestions and encouraged him to keep going. V L Ginzburg's

[3] There was nobody else but V L Ginzburg in 1949 whom V L Indenbom could possibly ask for consultation. In this connection, mention should be made of another expert in the matter, Lidiya Eduardovna Prokof'eva-Mikhailovskaya, associate professor at Leningrad State University (LSU) (the author of Ref. [6] and translator of paper [3] in *Uspekhi Fizicheskikh Nauk* journal, parenthetically), who lectured on optical methods at the Elasticity Theory section of the LSU Mechanics and Mathematics Department and who, in 1926–1930, led the foundation of the USSR's first laboratory on the study of stresses by the polarization optical method. L E Prokof'eva-Mikhailovskaya died of hunger in February 1942 in Leningrad [27].

authority had its effect on those in command at the plant, and Indenbom's innovative proposal was accepted. After finding out the localization of mechanical stresses in kinescope bulbs, Indenbom proposed that hot bulbs be oven-annealed — a measure which proved to have a strong stress reduction effect.

The introduction of polarization control together with the annealing treatment resulted in the fraction of defective bulbs dropping from 97% to a mere 3%. V L Ginzburg's study [1] was instrumental in arranging the mass production of black and white TV sets in the USSR. Thus, although purely theoretical, V L Ginzburg's study [1] proved to be of applied value. In 1955, V L Indenbom defended his candidate dissertation [26] and, on the recommendation of V L Ginzburg, was invited by M V Klassen-Neklyudova to the Institute of Crystallography of the Academy of Sciences of the USSR, where in 1966 he became the Chair of the Theoretical Department.

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