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On the direct detection of gravitational waves*

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Contents

Abstract. Different types of gravitational wave (GW) detectors are considered. It is noted that interferometric techniques offer the greatest prospects for GW registration due to their high sensitivity and extremely wide frequency band. Using laser interferometers, proposed as far back as 1962 in the work by M E Gertsenshtein and V I Pustovoit published in Russian (*Zh. Eksp. Teor. Fiz.*, vol. 43, p. 605, 1962) and in English translation (*Sov. Phys. JETP*, vol. 16, p. 433, 1963), it proved possible for the first time to directly detect GW emission from a merger of two black holes. It is noted that the assertion that Gertsenshtein–Pustovoit's work was unknown to some of those experts involved in direct GW detection is inconsistent with reality. The problems of high-power laser radiation affecting the electrostatic polarization of free-mass mirrors are discussed. It is shown that mirror polarization can lead to additional links with electrically conducting elements of the design resulting in the interferometer's reduced sensitivity. Some new prospects for developing high reflection structures are discussed and heat extraction problems are considered.

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1. Introduction

Direct detection of gravitational waves is a great accomplishment of basic science in recent years [1]. Gravitational waves (GWs) were predicted by Albert Einstein on the basis of his general theory of relativity (GTR) [2, 3], and their direct detection provides one more bit of evidence of the validity of its equations.

Problems pertinent to the derivation of equations describing GW propagation from the nonlinear GTR equations and to the investigation of GW emanation from various objects have been dealt with in many books, articles, and reviews, starting from the fundamental work of Einstein [2] and Einstein, Rosen [4]. These publications are readily available on the Internet (see Ref. [5] for the well-known problem of GW 'recognition').

To recall, soon after A Einstein arrived at the GTR equations in 1915, their linear approximation was developed (in 1916), which turned out to be very similar (mathematically) to the Maxwell electromagnetic field equations. This enabled Einstein to anticipate the emission of certain 'waves of gravity' presently known as gravitational waves [3]. In 1918, Einstein derived the formula for gravitational radiation intensity and showed that emission is a result of temporal variation of the quadrupole moment of the body.

The conceptual difference between gravitational wave equations and Maxwell's electromagnetic field equations lies in the fact that electromagnetic waves are emitted by a time-varying electric dipole created from charges of different signs, whereas GW emission is a result of temporal variation of the quadrupole moment of a system comprising masses with the same charge. From the quantum standpoint, this means that quanta of an electromagnetic field have spin unity, while gravitational field quanta, gravitons, possess spin 2. The difference between photon and graviton spins explains why the ratio of radiation energy flux of gravitational waves to that of electromagnetic waves for charged particles at high energies is independent of their energy, despite the fact that the sources in the equations for gravitational and electromagnetic wave emissions show a different dependence on the particle kinetic energy (see Refs [6, 7] for details, as well as review [8] and paper [9]).

Einstein showed that the power of gravitational radiation is negligible. For example, it will be only 10^{-37} W for a metal cylinder 1 meter long rotating with ultimate break speed, and for the planet Jupiter, orbiting around the Sun — just only 400 W. Therefore, the main problem of the GW registration is to create enough sensitive receivers of gravitational radiation.

The first attempts to directly detect GWs with the use of resonant antennas were undertaken by Joseph Weber at the University of Maryland in the 1960s [10, 11]. He used massive cylinders as antennas suspended by thin metal wires in Earth's gravity field. Elastic strains caused by GWs were detected by piezoelectric sensors. The antenna in Joseph Weber's pioneering experiments was a 1.2-ton aluminum cylinder ≈ 1.5 m in length and ≈ 61 cm in diameter suspended in a vacuum chamber by steel wires attached to acoustic filters to reduce the influence of seismic noises. The resonance frequency of the first longitudinal acoustic mode at room temperature was 1667 Hz, and the bandwidth about 10 Hz. For subsequent experiments, Weber deployed two antennas located 2 km apart to increase the reliability of the measurement results by means of their correlation processing. These antennas detected GW signals with a dimensionless amplitude of 10^{-15}, which corresponds to an absolute displacement close to 1.5×10^{-13} cm for a cylinder 1.5 m long, i.e., roughly equal to the size of the proton [10]. The registration system consisted of an array of piezoelectric quartz crystals (sensors of strains) placed on the surface close to the central part of the cylinder. The piezoelectric sensors converted mechanical oscillations excited in the cylinder by GWs into an electric signal. The first measurements were made in January 1965; two years later, Weber reported the first anticipated observations of gravitational waves [11].

In 1968, Weber again announced the possible detection of GWs [12] based on the data from two aluminum cylinders tuned to a frequency of ≈ 1.66 kHz and located 2 km apart. Weber argued that random coincidence of events was highly improbable [13, 14]. In 1969, he announced at last the discovery of GW emission confirmed by the large enough number of coincidences and the extremely low probability of their random character [13, 14]. By 1973, Weber had arrived at the conclusion that the excess of coincidences over the statistical average amounted to seven events per day, and that the signal peaked in the direction of the galactic center [15, 16].

These and subsequent observations by Weber aroused great interest within the scientific community. Nevertheless, later independent studies with the employment of two resonant detectors and more careful analysis of Weber's results failed to confirm his observations [17–20; see also 21]. Weber took great effort to 'prove' direct GW detection. He never recognized the fallacy of his measurements nor did he withdraw the claim of GW discovery [15, 16]. (Joseph Weber died in 2000.) A photograph of Weber's detector is presented in Fig. 1.

In spite of everything, the first experiments attracted the attention of many researchers in different countries to the problem of GW detection, giving rise to intense investigations and the design and development of Weber type resonant antennas [22–28]. Detectors based on this principle in current operation include the spherical Mini Gravitational Wave Antenna (MiniGRAIL) at the University of Leiden (Netherlands) [22], Allegro (Baton Rouge, Louisiana, USA) [23], also used by Weber in his time, Antenna Ultracriogenica Resonate per l'Indagine Gravitazionele Astronomica (AURIGA), with a detector temperature of 0.1 K [24], and Nautilus [25] in Italy, Explorer in Switzerland [26], and AGRAN in Russia [27, 28]. There is a substantial body of literature on the subject, including numerous reviews available online.

For all that, it is perfectly clear that the direct detection of gravitational waves with the aid of resonant antennas encounters many difficulties which markedly diminish the capability of detecting GWs. The main obstacle is the impossibility of recording signals in a broad waveband at a relatively high resonance frequency of the detectors themselves. It can be expected that the value of such resonant detectors as simple and cheap GW antennas will increase in the future when it would be possible to estimate the number of binary neutron stars and similar objects having a small size and large mass, and whose gravitational radiation frequency is close to the natural frequency of resonant detectors.

In 1962, M E Gertsenshtein and V L Pustovoit proposed a quite different method for the detection of gravitational

Figure 1. Joseph Weber and his first resonant gravitational wave antenna (University of Maryland museum).

waves using laser beam interferometry and Michelson type interferometers [29]. It should be emphasized that in the early 1960s many laboratories began to develop Weber type solid-state antennas and therefore tended to ignore the proposal for using laser interferometers. Reference [29] gave evidence that the efficiency of resonant techniques based on Weber type antennas is insufficient to detect GWs due to a narrow wavelength band. V L Ginzburg, my teacher, reported our study [29] at the GR3 Conference in Warsaw (July 1963). Weber, a participant in the Conference, published in August 1963 a special article [31] in response to the 'criticism' of resonance methods for GW detection.[1] The estimates of their sensitivity cited in Ref. [31] and many later publications hold for resonant or similar conditions under which the gravitational wave frequency coincides with the natural mechanical resonance frequency of a massive cylinder (around 1 kHz); however, such cases are very rare, as recent observations of real events have shown. Neither the frequencies of potential GW sources nor the directions from which gravitational waves can arrive and their polarization are known. It is these characteristic features of resonant antennas that lay behind the critique in our 1962 paper.

The basic idea of the method proposed in Ref. [29] is as follows. The antenna for direct GW detection is a laser interferometer with freely suspended mirrors playing the role of masses subjected to GW-induced forces that displace the mirrors and thereby alter the interference pattern. The change of the latter serves as a signal implying the presence of gravitational radiation. The first estimates [29] showed that the sensitivity of such antennas may be much higher than that of Weber's antennas available at that time. Moreover, laser interferometers had a quite broad bandwidth for detecting gravitational radiation. Also, exploiting the interferometers in combination with monochromatic light sources (lasers) that became available by that time (T Meiman, 1960) further increased the sensitivity of the interferometric methods proposed in Ref. [29].

It was also emphasized that the interferometer must have a maximally long baseline (i.e., arm length) to improve sensitivity, and all possible measures should be taken to reduce both the noise level and fluctuations of the refractive index of the medium in which the laser beam travels.

The first laser interferometer [32] was designed in the late 1960s by Robert Forward, a disciple of Weber's, who was trained in Weber's laboratory but thereafter joined Hughes Research Laboratories. This instrument, with its 10-m arms and mirrors attached to the simplest seismic attenuators, had a sensitivity of 2×10^{-16} Hz in a waveband of 250 Hz–25 kHz, commensurate with that of uncooled resonant sensors of that time, but its bandwidth was much wider. Later on, many laboratories began to create similar but more sophisticated laser interferometers. This work continues.

2. What are gravitational waves?

Gravitational waves are known to be perturbations or ripples in the spacetime metric. Einstein's GTR equations have the form

$$R_i^k - \frac{1}{2}\,\delta_i^k R = \frac{8\pi G}{c^4}\,T_i^k\,, \tag{1}$$

where R_i^k is the Ricci tensor related to the Riemann curvature tensor by a known expression (see book [33]), T_i^k is the energy–momentum tensor of matter, and $R=\delta_k^i R_i^k$ is the convolution of the Ricci tensor and the unit tensor. Nonlinear equations (1) define the spacetime metric or the value of the metric tensor $g_{ik}(x,t)$ setting the quadratic form of the spacetime metric $\mathrm{d}s^2 = g_{ik}\,\mathrm{d}x^i\,\mathrm{d}x^k$; G is the Newtonian constant of gravitation, and c is the speed of light in vacuum. Here, i and k take the values 0, 1, 2, 3 (t,x,y,z). Once the gravitational field is weak, the metric tensor is little different from its value for a flat space:

$$g_{ik}^{(0)} = \begin{pmatrix} -1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix};$$

then, in the presence of a GW, the metric tensor can be represented as

$$g_{ik}(r,t) = g_{ik}^{(0)} + h_{ik}(r,t)\,, \quad h_{ik}(r,t) \ll 1\,. \tag{2}$$

The components of tensor $h_{ik}(r,t)$ describe a gravitational wave. Substitution of expression (2) into Einstein's equations linearizes them and allows GW equations to be obtained from equations (1):

$$\Box\, h_i^k = \frac{8\pi G}{c^4}\,T_i^k\,, \quad \Box \equiv \frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\,. \tag{3}$$

A gravitational wave propagating in vacuum satisfies a simpler equation $\Box\, h_{ik}=0$, but these equations hold only for the choice of a proper reference frame in which

$$\frac{\partial}{\partial x^k}\left(h_i^k - \frac{1}{2}\,\delta_i^k h\right)=0\,, \quad \delta_i^k = \begin{cases}1, & i=k\\ 0, & i\neq k\end{cases}, \quad h\equiv h_k^k\,.$$

Let us choose the z-axis along the wave propagation direction and calculate components of the Ricci tensor R_{ik}. Clearly, this tensor depends only on two unrelated components of amplitude h_{ik}:

$$\begin{aligned} h_{xx}(z,t) &= -h_{yy}(z,t) \equiv h_+(\omega t - kz)\,,\\ h_{yx}(z,t) &= h_{xy}(z,t) \equiv h_\times(\omega t - kz)\,, \end{aligned} \tag{4}$$

where ω is the GW circular frequency, $k=\omega/c = 2\pi/\lambda$ is the wave number, and λ is the wavelength. This means that the choice of a proper reference frame in the case of gravitational waves allows all components of tensor h_{ik} but $h_{xx}=-h_{yy}$, $h_{yx}=h_{xy}$ to be nullified. In the presence of a GW, the spacetime metric has the form

$$\begin{aligned} \mathrm{d}s^2 = g_{ik}\,\mathrm{d}x^i\,\mathrm{d}x^k &= c^2\,\mathrm{d}t^2 - \mathrm{d}z^2 - \big(1-h_{xx}(z,t)\big)\,\mathrm{d}x^2\\ &\quad - 2h_{xy}(z,t)\,\mathrm{d}x\,\mathrm{d}y - \big(1-h_{yy}(z,t)\big)\,\mathrm{d}y^2\,. \end{aligned} \tag{5}$$

Einstein showed in paper [3] (see also Refs [17, 33]) that GW emission intensity is described by the expression

$$-\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{G}{45c^5}\left(\frac{\partial^3}{\partial t^3}\,D_{\alpha\beta}\right)^2, \tag{6}$$

[1] In his paper [31], J Weber cited our paper [29] and the talk of V L Ginzburg to Warsaw's conference [30]. Therefore, the allegations sometimes encountered in the literature about Weber's lack of knowledge concerning study [29] are untrue. Surprisingly, I have not so far found a single article written by a practising researcher in the field of GW detection, where paper [31] has been referenced.

where $D_{\alpha\beta} = \int \mu(3x^{\alpha}x^{\beta} - r^2\delta_{\alpha\beta})\,\mathrm{d}V$ is the mass quadrupole moment tensor, and the total mass is $M = \int \mu(x, y, z)\,\mathrm{d}V$. This is the general formula for low-intensity GW emission. It shows that the loss of energy by a system of masses for GW emission takes place when the motion of the masses is characterized by time-varying acceleration (e.g., a rotating sphere does not emit GW, while an ellipsoid with different values of symmetry axes does). For two masses moving in circular orbits around a common center of mass, the following equation holds instead of Eqn (6) [33]:

$$-\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{32G\omega^6(t)\,r^4(t)}{5c^5}\left(\frac{m_1 m_2}{m_1 + m_2}\right)^2. \tag{7}$$

Here, $\omega(t)$ is the GW frequency, and r is the radius vector chosen in the center of inertia of two bodies with masses m_1 and m_2. To derive Eqn (7), it was assumed that the masses of the rotating bodies do not change over time. Expression (7) integrated over all directions describes overall radiation intensity. A description of intensity distribution by polarizations and directions (of importance for concrete GW observations and movements of point masses in elliptical orbits) can be found in monograph [33] and article [34]. GW amplitudes can be expressed through derivatives of the quadrupole moment tensor (6); then, for a wave along the z-direction [33] one finds

$$h_{yx} = -\frac{2G}{3c^4 r}\frac{\partial^2}{\partial t^2}D_{yx}\,,$$
$$h_{yy} - h_{xx} = -\frac{2G}{3c^4 r}\frac{\partial^2}{\partial t^2}(D_{yy} - D_{xx})\,, \tag{8}$$

where r is the distance to the observation point. Substituting GW amplitude values (8) into the expression for the GW energy–momentum pseudotensor yields GW energy flux density along the propagation direction [17, 33]:

$$P = \frac{G}{36\pi c^5 r^2}\left[\left(\frac{1}{2}\frac{\partial^3}{\partial t^3}(D_{yy} - D_{xx})\right)^2 + \left(\frac{\partial^3}{\partial t^3}D_{yx}\right)^2\right]. \tag{9}$$

The energy flux transferred by a GW into a solid angle element is obtained by taking the product of formula (9) and $r^2\,\mathrm{d}\Omega$. The energy flux carried by GW in all directions is expressed as [17]

$$E\,[\mathrm{erg\ cm^{-2}\ s^{-1}}] = \frac{c^3 h^2 \omega^2}{32\pi G} = 1.02 \times 10^{35} h^2 \left(\frac{f}{\mathrm{Hz}}\right)^2, \tag{10}$$

(here, $h \equiv (\overline{h_{xx}^2})^{1/2}$ is the amplitude averaged over the wave period). Let us estimate, by way of example, the dimensionless GW amplitude on the assumption that the total GW radiation power $\Delta\Sigma/\Delta t$ of the system at the frequency $\omega = 2\pi f$ and $f = 50$ Hz is $\Delta\Sigma = 3M_{\odot}c^2$ during the time interval $\Delta t = 1$ s, provided that the radiation source is at a distance $R = 1.23 \times 10^{27}$ cm (1.3 billion lightyears); then, relation (10) gives

$$h = \sqrt{\frac{8G}{c^3(2\pi f)^2}\frac{1}{R^2}\left(\frac{\Delta\Sigma}{\Delta t}\right)} \approx 8.45 \times 10^{-22}\,. \tag{11}$$

Expression (11) means that in such a wave two free particles positioned normally to the wave propagation direction at a distance less than the wavelength, i.e., $\lambda = c/f = 3 \times 10^8$ cm, periodically change this distance by 8.45×10^{-22} of its total value.

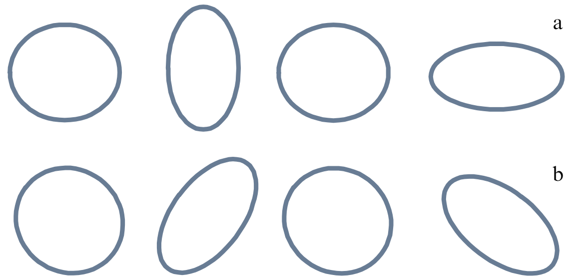

Figure 2. Deformation of a ring of test particles in the field of a gravitational wave with various polarizations: (a) h_+, and (b) $h_\times$.

Expression (5) for a change in the spacetime interval in the GW field also implies the transverse character of GW; in other words, GW accelerations and forces affect test masses only in the plane orthogonal to GW propagation, while particles along the GW propagation direction remain at rest. The relationship between tensor components $h_{xx} = -h_{yy}$ and h_{xy} determine the GW direction and polarization; by way of illustration, Fig. 2 shows the deformation of a ring of test particles in the GW field. The test mass displacement pattern suggests possible approaches to detection of gravitational radiation.

One such option implies a long enough massive elastic body positioned relative to a GW so that its largest size lies in the plane orthogonal to the GW propagation direction. In this case there are certain points of the body at which particle displacements have different signs at different time moments depending on the phase of the GW. Clearly, the displacement amplitude is largest when the GW frequency coincides with the natural mechanical vibrational mode of the elastic body being considered. This principle underlies various resonance methods of GW detection. When both the spacetime metric and components of the metric tensor in the presence of GW (5) are known, it is possible to find the volume force exerted on the elementary volume of a solid-state antenna [33] and construct equations of the theory of elasticity for solid-state antennas of any shape, viz., cylindrical, spherical, dumbbell-like, or more complicated. Cylindrical antennas are most commonly used. Evidently, their sensitivity is especially high when the natural mechanical vibrational mode of the elastic body is tuned in to the GW frequency. It is physically understandable that high-Q materials with low acoustic absorption properties are needed for such antennas known as solid-state or resonant antennas. All these issues are considered in numerous articles and reviews (see, for instance, paper [35] and references cited therein).

The second option of GW detection consists in measuring changes in the distance between two free masses traveling across a space. This method is employed in laser interferometers.

Two more approaches to GW registration are based on somewhat different physical principles. Imagine an object (or a pair of objects, e.g., a dumbbell) rotating about a fixed axis. It was shown by Braginskii, Zel'dovich, and Rudenko [36] that such a system having a frequency of rotation close to or coincident with GW frequency can acquire or release energy, depending on the relationship between the GW and the rotation phases (at close or equal frequencies).

Table 1. Main parameters of modern resonant antennas.

Parameter	Explorer	Niobe	Nautilus	Allegro	AURIGA
Temperature, K	2.6	5.0	0.13	4.2	0.25
Q-factor	1.5×10^6	2×10^6	20×10^6	1.5×10^6	3×10^6
Mechanical noise density spectrum S, Hz$^{-1/2}$	6×10^{-22}	8×10^{-22}	2×10^{-22}	6×10^{-22}	2×10^{-22}
Waveband, Hz	0.2	1	0.6	0.5	1
Noise effective temperature, mK	10	3	2	10	2
Sensitivity $h_{\min}$, Hz$^{-1/2}$	8×10^{-19}	10^{-18}	4×10^{-19}	8×10^{-19}	4×10^{-19}
SNR > 5 rate/day*	150	100	75	150	200

* SNR — signal-to-noise ratio.

Moreover, there is the so-called parametric registration method based on the equality of GW and electromagnetic wave propagation velocities [37]. As is known, a gravitational wave propagating in a vacuum is equivalent to a medium with the refractive index determined by the GW amplitude $h_{ik}(x,t)$. Then, despite the fact that the effect of an electromagnetic wave (EW) scattering or its transformation into a gravitational wave and back is very small, it can possibly be revealed at large distances under synchronism conditions for powerful optical emission. This method refers to detection of a high-frequency GW coming with strong stellar electromagnetic radiation. A distinctive feature of this effect is that the frequency of a diffracted EW by the gravitational wave originating from it contains the third harmonic of the initial eigenfrequency (see Ref. [37] for details) by virtue of parametric coupling

$$\mathrm{EW}[\omega] \Rightarrow \mathrm{GW}[2\omega] \Rightarrow$$
$$\Rightarrow \mathrm{EW}[\omega]\ \text{diffraction by}\ \mathrm{GW}[2\omega] \Rightarrow \mathrm{EW}[3\omega]\,.$$

In this case, detection of gravitational radiation reduces to the observation of the third harmonic of any strong electromagnetic signal. Notice that the maximum frequency of GW emission by massive objects cannot be higher than the inverse wave propagation time over a distance on the order of the Schwarzschild radius, i.e., $f_{\max} \leqslant c^3/(2GM)$; it equals 100 kHz for a body with a mass equivalent to the solar mass and decreases with increasing body mass.

The main noise components limiting sensitivity of resonant antennas are thermal noises from their basic mass, seismic vibrations at suspension points, and noises from measuring and recording equipment. To decrease the influence of thermal noises from the detector registration mass, it is cooled to below 1 K, which is a rather difficult technical task because the mass of resonant detectors can be 1300–2500 kg; moreover, such a low temperature has to be maintained for a rather long time (in fact, for the entire observation period). This issue was successfully resolved in many laboratories around the world. To reduce noises from measuring and recording equipment, the input circuits include cryogenically cooled low-noise amplifiers or superconducting Josephson junctions, e.g., SQUIDs. Correlation processing of observational data has been practiced since J Weber's time to enhance reliability of the results.

Such are the general approaches to the detection of gravitational waves with the use of solid-state resonant antennas. Selected characteristics of such antennas are presented in Table 1. They are considered briefly in the next section.

3. Resonant gravitational wave antennas

R P Gifford was the first to cool an antenna to low temperatures in GW experiments [38]. Antennas of this type are universally referred to as resonant GW antennas of the second generation to distinguish them from uncooled Weber type antennas of the first generation. The development of solid-state resonant antennas of the second generation began in the 1970s and ended in the 1990s with the advent of large laser interferometric GW antennas.

The Explorer cryogenic antenna was designed at CERN (Switzerland) in 1986 [25, 26] (Fig. 3). This 3-m long cylindrical detector made of a high-Q aluminum alloy had a mass of 2270 kg, operating temperature of 2.6 K, and resonance frequency of 900 Hz. Mechanical vibrations of cylinder butt-ends were measured with capacitance sensors and the signal was then amplified by SQUID-based amplifiers. The detector was placed inside a huge cryostat filled with superfluid helium to prevent damping mechanical vibrations of the cylinder by the surrounding liquid helium. Sensitivity achieved at a resonance frequency was roughly 7×10^{-22} in the 5-Hz band. The continuous work time at low temperature was limited to about 3 days, after which the cylinder had to be refilled with helium. The detector was exploited for joint observations for more than 10 years (till 2006).

A two-mode resonant cryogenic antenna to be operated at liquid-helium temperature was designed and fabricated at Stanford University in 1977. Vibrations of the 680-kg aluminum cylinder were mechanically amplified by a small

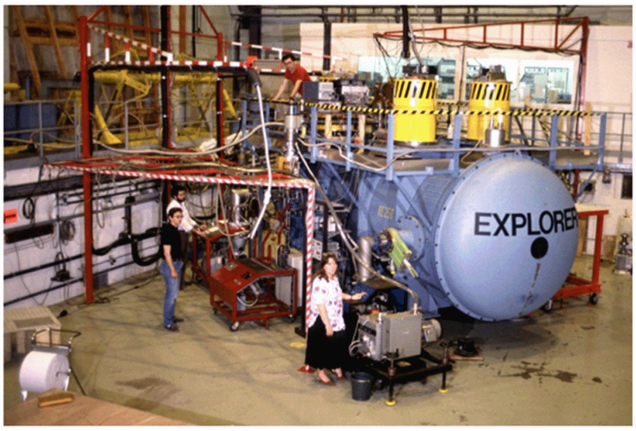

Figure 3. Explorer resonant-mass detector (CERN, Switzerland).

Figure 4. Allegro cryogenic detector at Louisiana State University (USA) without the end cap.

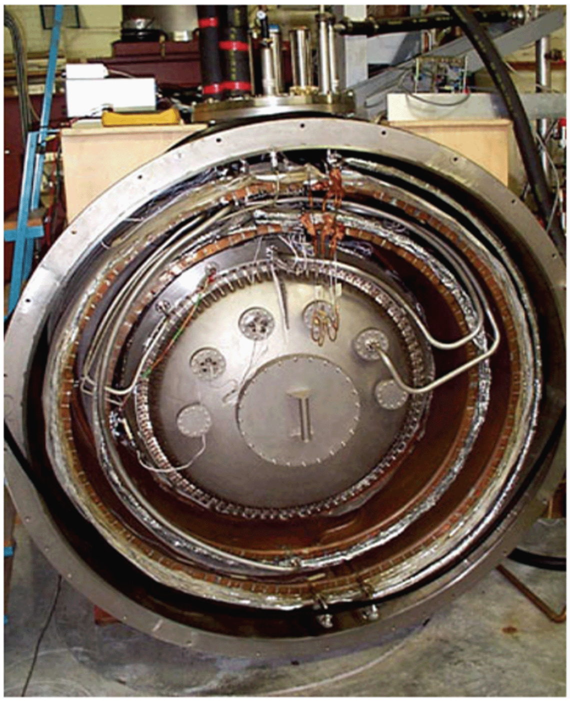

Figure 5. Niobe resonant GW antenna. Cylindrical niobium bar at the University of Western Australia, Perth.

mass attached to a butt-end with a niobium membrane. Mechanical vibrations induced currents in two coils mounted on both sides of the membrane that were measured with SQUIDs (see Refs [39, 40] for details). The detector was dismantled after the 1989 earthquake.

The Allegro cryogenic resonant GW detector designed at Louisiana State University was a 2296-kg bar of high-Q aluminum with a natural resonance frequency near 913 Hz. The cylinder was suspended in a tank with liquid helium at 4 K. Mechanical vibrations at the butt-ends were detected with SQUIDs that measured the magnetic field of the coils attached to the ends. The detector was commissioned in 1991 and decommissioned in 1998. A specific feature of Allegro was that it had a lighter mass in addition to the main heavy mass as a mechanical transformer attached to one of the ends of the cylinder with a spring type membrane so that the system had two close resonant modes with frequencies of 920.3 Hz and 896.8 Hz. This allowed the waveband in which GWs could be detected to be slightly broadened and antenna impedances and input impedances of the amplifiers to be better matched [41, 42]. The general view of a detector is shown in Fig. 4.

The resonant gravity wave antenna (Niobe) developed at the University of Western Australia in 1995 [43] (Fig. 5) consisted of a 1500-kg cylindrical niobium bar and had an operational temperature of around 5 K and natural resonance frequency of around 710 Hz. A light weight of 450 g suspended by a membrane from one end of the cylinder amplified its mechanical vibrations and slightly shifted the resonance frequency of the system. As a result, the system had two resonances at frequencies 713 Hz and 694 Hz. Vibrations were measured with superconducting microresonators, the capacity of which changed under the effect of gravitational waves. Such parametric measuring scheme differing from the ordinary linear one had the advantage of low noise levels but created the risk of various parametric instabilities and required special measures to prevent the development of parasitic instabilities.

The ultralow temperature detector Nautilus was designed at the Laboratori Nazionali di Frascati, Instituto Nazionale di Fizica Nucleare (Rome, Italy) in December 1995 [23, 44, 45] (Fig. 6a). It was operated at 0.1 K.

The AURIGA antenna (Fig. 6b) was created as the twin of Nautilus operating at a temperature of 140 mK due to the use of an ^{3}He–^{4}He mixture.

Joint studies using the dual detection technique based on Nautilus and Explorer antennas were carried out for more than 3 years (till 2006). Analysis of their results showed that the antennas were sensitive to high-energy cosmic ray showers. However, no short pulses that could be associated with GWs were recorded during the entire observation period.

K Narikawa and H Hirakawa (Tsukuba, Japan) developed an interesting 14-cm thick torsion type resonant antenna 1.65 m in length and width having a mass of 1400 kg [46]. Its fundamental mode was low-frequency ($\sim$ 60 Hz) torsional vibrations about the axis of symmetry. The antenna was cooled to the temperature of liquid helium. The antenna was designed to detect GWs from pulsar type close binaries.

The MiniGRAIL antenna with isotropic sky survey developed at the University of Leiden (Netherlands) [47] (Fig. 7) is a cryogenic spherical gravity wave antenna 68 cm in diameter made of CuAl (6%) alloy weighing 1300 kg with a resonance frequency of 2.9 kHz and a bandwidth around 230 Hz, possibly higher. The detector operates at a temperature of 20 mK. Its sensitivity is 4×10^{-21} Hz$^{-1/2}$ at a resonance frequency of 2942 Hz, and 5×10^{-20} Hz$^{-1/2}$ in the 30-Hz waveband. A similar detector is located in Sao Paulo (Brazil) to simultaneously observe GWs emitted from neutron binaries.

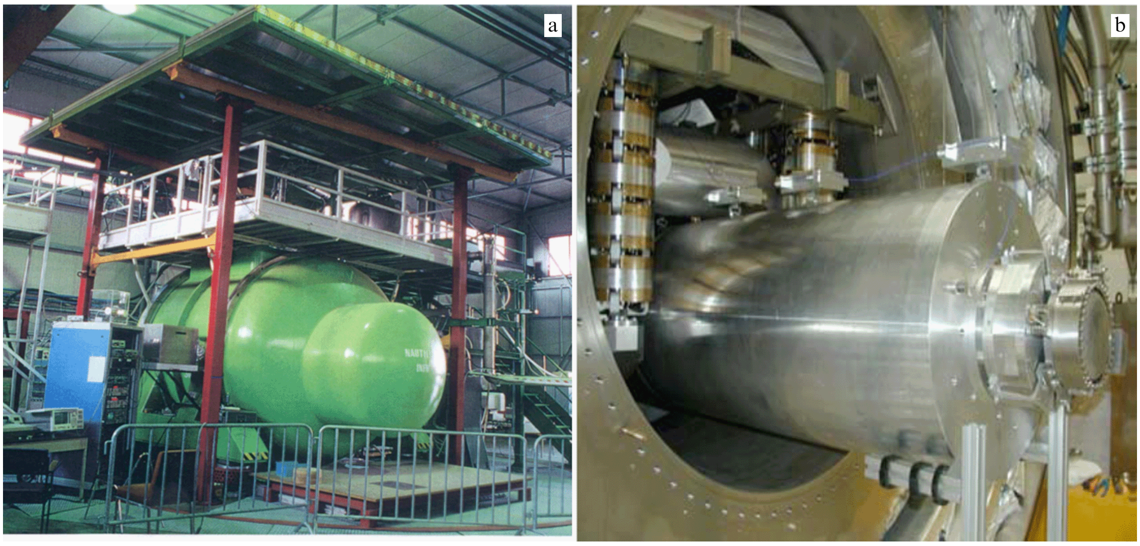

Figure 6. Resonant GW antennas Nautilus (a), and AURIGA (b).

Figure 7. MiniGRAIL spherical gravitational wave antenna with isotropic sky survey [Kamerlingh-Onnes Laboratory, University of Leiden (Netherlands)].

The main characteristics of all five resonant cylindrical antennas are presented in Table 1. Most of them were decommissioned after creation of the network of laser interferometers to be used as GW detectors, except Nautilus and AURIGA, which will continue to operate by the recommendation of the Gravitational Wave International Committee till modernization of large laser interferometers is completed. In all probability, they will no longer be used in future research or will be modified to carry out special studies aimed to search for sources of continuous gravitational radiation.

4. Laser interferometers for the detection of gravitational waves

The idea to use laser interferometers to detect GWs was put forward for the first time in the article by M E Gertsenshtein and V L Pustovoit published in 1962 [29], soon after T Maiman invented his ruby laser. The authors of Ref. [29] proposed a new method for the detection of gravitational waves from GW-induced changes of interference patterns in Michelson interferometers. As is known [17, 33], a gravitational wave in the lowest order of multipole expansion is quadrupole radiation. Therefore, gravitational radiation incident normally on the interferometer plane causes different changes to the space position of mirrors located at the ends of the arms. If the GW phase and polarization in one arm of the Michelson interferometer are such that the arm length increases, then the length of the orthogonal arm decreases; as a result, the interference pattern changes.

Evidently, the sensitivity of such antennas, depending on the changes in the laser radiation phase in the arms, increases with increasing arm length. The latter being large enough, a GW signal becomes readily measurable. Of course, the wavelength of gravitational radiation must be much larger than the arm size, which makes the method in question suitable for detecting low-frequency GWs.

It is widely believed that a gravitational wave alters space-time geometry, but an equally adequate alternative characteristic is acceptable; namely, for propagating optical waves, a gravitational wave is equivalent to a certain anisotropic medium whose dielectric permittivity and magnetic permeability depend on GW phase, direction, amplitude, and polarization. This conclusion directly follows from the Maxwell equations describing propagation of electromagnetic waves in a weak gravitational field (see book [33]), provided the size of the region of interest over which the electromagnetic waves are traveling is much smaller than the gravitational wavelength (this condition is fulfilled for Michelson interferometers). It was shown in Ref. [29] that a gravitational wave changes the phase of monochromatic light in the interferometer, and optical characteristics of light rays in the GW field can be described by the eikonal equation [33]

$$g^{ik}(x,t)\,\frac{\partial\Psi}{\partial x^i}\,\frac{\partial\Psi}{\partial x^k}=\left(\frac{\partial\Psi}{\partial x^i}\right)^2-h^{\alpha\beta}\,\frac{\partial\Psi}{\partial x^\alpha}\,\frac{\partial\Psi}{\partial x^\beta}=0\,, \qquad (12)$$

where $\Psi=\Psi(x,t)$ is the eikonal, and $h^{\alpha\beta}=h^{\alpha\beta}(x,t)$ is the GW amplitude.

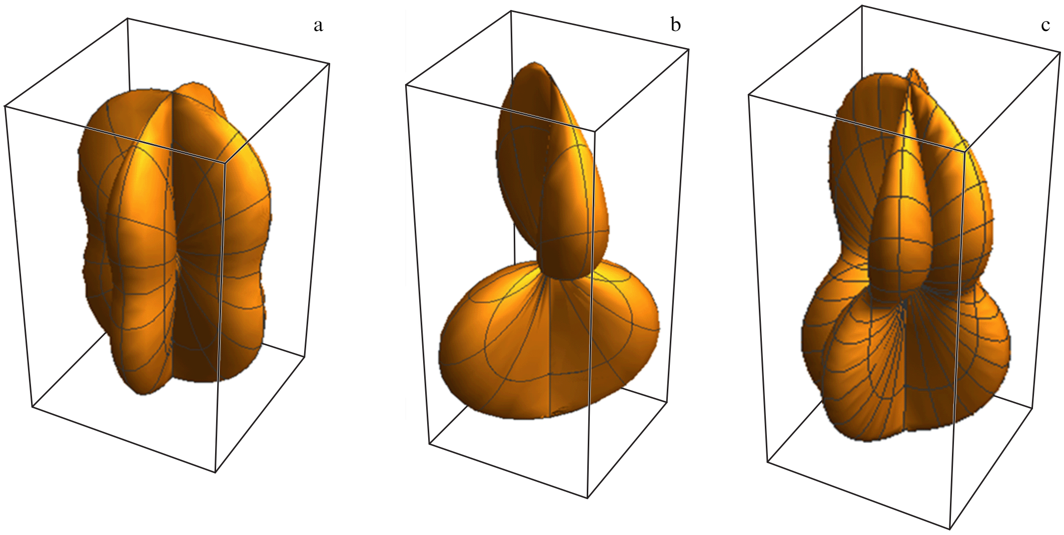

Figure 8. Directional patterns of a laser interferometric antenna for gravitational waves with different polarizations: (a) $\psi = \pi/4$, $h_+ = 0$, $h_\times = 1$, (b) $\psi = \pi/4$, $h_+ = 1$, $h_\times = 0$, and (c) $\langle h_+ \rangle = \langle h_\times \rangle = 1$ corresponding to natural GW polarization.

This is the equivalent description of a medium with the refractive index

$$n = 1 + \frac{1}{2} h_{\alpha\beta} n^\alpha n^\beta \,, \tag{13}$$

where n^α is the unit vector along the laser beam propagation.

The relative change in the interferometer arm length for the light rays propagating across a gravitational wave incident normally on the interferometer plane, i.e., in the direction of its highest sensitivity, is expressed as

$$\frac{\Delta l}{l_0} = \frac{1}{2} h_{\alpha\beta} n^\alpha n^\beta \,, \tag{14}$$

where l_0 is the unperturbed arm length; therefore, Eqn (14) gives

$$\frac{\Delta l}{l_0} = \frac{1}{l_0} \int_0^{l_0} \sqrt{g_{22}}\, \mathrm{d}x_2 - \frac{1}{l_0} \int_0^{l_0} \sqrt{g_{11}}\, \mathrm{d}x_1 \simeq \frac{1}{2} h_{22} \,. \tag{15}$$

Thus, a gravitational wave periodically changes the length of the interferometer arms. GW detectors based on laser interferometers, in contrast to resonant antennas, have a very large bandwidth, the sole limitation imposed by physical conditions being $\lambda_{\mathrm{g}} \gg l$, where λ_{g} is the gravitational wavelength, and l is the size of the interferometer arm. For example, for the Laser Interferometer Gravitational-Wave Observatory (LIGO), $l = 4$ km and $f = c/l \ll 750$ kHz; for Virgo, $f \ll 1$ MHz. The presence of Fabry–Perot resonators in each arm of these interferometers increases the time needed for the settlement of the interference pattern. Assuming the number of laser wave passes in a Fabry–Perot resonator to be $N = 100$, this time expressed as $\tau \approx lN/c$ is 1.33×10^{-3} and 10^{-3} s for the LIGO and Virgo interferometers, respectively. For resonant antennas, the characteristic time of the settlement of the interference pattern, depending on the mechanical Q-factor of the cylinder, its length, and acoustic wave velocity, is $Q/(2\pi f) \approx 0.157$ s at a fundamental mode frequency of 1 kHz and $Q = 1000$. Evidently, the time needed to settle the interference pattern with laser interferometers is much shorter than that needed to initiate mechanical resonance in a solid-state antenna. This important characteristic of the interferometric method was also emphasized in Ref. [29]. Both the signal being measured and the sensitivity of the interferometer depend on the radiation source direction; in general, the directional pattern $A(t, \theta, \varphi, \psi)$ of the antenna can be represented as [33, 48] (see also Refs [49, 50])

$$\begin{aligned} A(t, \theta, \varphi, \psi) = {} & \frac{1}{2} (1 + \cos^2 \theta) \cos (2\varphi) \big[h_+(t) \cos (2\psi) \\ & + h_\times(t) \sin (2\psi) \big] - \cos\theta \sin (2\varphi) \big[h_+(t) \sin (2\psi) \\ & - h_\times(t) \cos (2\psi) \big] \,. \end{aligned} \tag{16}$$

Here, the following frame of reference was chosen: the arms of the interferometer make up plane xy, with the x-axis directed along the bisectrix of the angle between the arms, angles θ, φ give the direction to the source, and angle ψ determines the polarization relationship (2ψ is the angle between particle oscillations under the effect of GW with polarization $h_+(t)$ and the x-axis, while the angle between particle oscillations induced by a GW with polarization $h_\times(t)$ and the x-axis is $\pi/2 - 2\psi$). The directional sensitivity diagrams for interferometric GW antennas in accordance with formula (16) are presented in Fig. 8.

Soon after construction of the first laser interferometer by R Forward and his co-workers [32, 51], these instruments began to be developed in many laboratories all over the world, such as the Massachusetts Institute of Technology [52] (R Weiss), the California Institute of Technology (Pasadena) [35, 53, 54], the Max Planck Institute (Garching, Germany) [55], Glasgow University (UK) [56], and the Institute of Space and Astronautical Science (ISAS, Tokyo, Japan) [35, 57, 58]. These first laser interferometers had rather short arms (3 m in Garching, 10 m in Glasgow, 40 m in Pasadena, 10 m and 100 m near Tokyo). However, the installation of Fabry–Perot resonators in each of the arms considerably increased the effective arm length and thereby the sensitivity of interferometers. R Drever and co-workers were the first to propose and employ Fabry–Perot resonators to increase the arm length [59].

Interferometers with a relatively short baseline were designed for two purposes: first, to detect potential sources of noise and to explore possibilities for noise reduction, and, second, to develop suspension systems allowing reflective

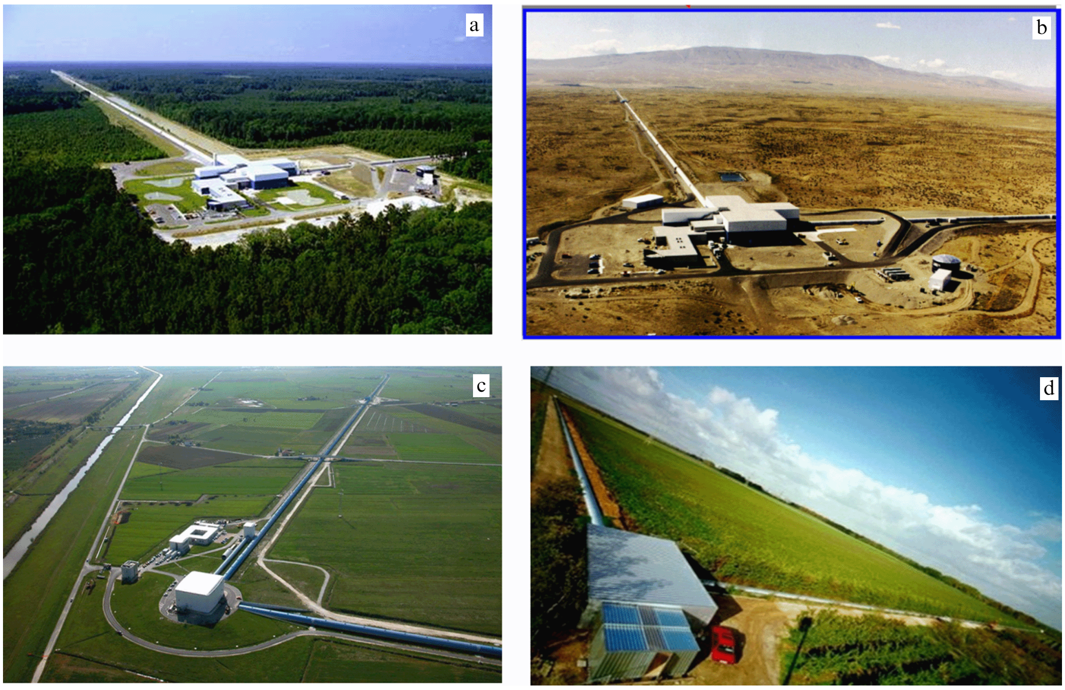

Figure 9. Laser interferometers (observatories): (a) LIGO in Livingston, (b) LIGO in Hanford, (c) Virgo, and (d) GEO-600.

mirrors of a Fabry–Perot interferometer to be moved freely at least in one direction. Moreover, a number of theoretical and technological problems had to be formulated and solved for the creation of long-arm interferometers. Investigations to accomplish these goals showed that the main noise components affecting sensitivity of laser interferometers are shot noise of photodetectors, thermal noise of reflective mirror surfaces and their suspension systems, low-frequency seismic noise, phase and frequency fluctuations of laser radiation, variations of the medium refractive index, thermal 'jitter' of laser optical elements, etc. [60–63]. Furthermore, it turned out that mirrors of interferometers accumulate charges of unknown origin on their surfaces, which hampers their free movements due to electrostatic coupling between dipole-generating charges and surrounding metal elements of the device.[2]

The knowledge and experience gained during the development and investigation of short-arm Michelson laser interferometers proved very helpful for designing unique interferometers with arms of huge sizes. The LIGO project launched in 1994 for direct GW detection consisted of two interferometers with 4-km long arms 3003 km apart, one deployed in Livingston, LA, the other in Hanford, WA. The latter facility is a dual 2-km and 4-km installation with two vacuum beam tubes in both arms.

At approximately the same time, the Virgo GW antenna with 3-km arms (Italy, near Pisa) [64, 65], GEO-600 with 600-m arms (near Hannover, Germany) [66, 67], and TAMA-300 underground GW detector (Japan, near Tokyo) with 300-m arms [35, 68] began to be constructed. The Virgo laser interferometer is a joint Italian–French project, GEO-600 is operated by a collaboration comprising the Albert Einstein Institute (Germany), Cardiff University, and Glasgow University (UK).

[2] A detailed analysis of noise sources in laser interferometers can be found in Ref. [35] and references cited therein.

These installations are shown in Fig. 9.

The KAGRA national research project (subterranean Kamioka Gravitational Wave Detector) was launched in Japan in 2010. The detector, with 3-km arms will be located at a depth of 200–500 m under the surface of Kamioka mountain (Gifu prefecture). Cryogenic mirrors will be placed in giant subterranean helium cryostats. Because laser beams incident on sapphire mirror surfaces are known to heat them, the very difficult problem of heat removal is faced by Japanese researchers. One of the options being considered is to use a superconducting cable with a high heat transfer capacity [35, 60, 69, 70].

The development of the European underground Fabry–Perot interferometer with mirrors located 10 km apart (Einstein telescope [71]) is currently underway (see also selection of related publications issued by the Max Planck Institute [72]). The Laser Interferometer Space Antenna (LISA) with one-million-kilometer arms placed in a heliostationary orbit is a joint project of NASA and the European Space Agency (ESA). Comprehensive information about the project (tasks, cooperation, scientific problems planned to be resolved, etc.) is online [72]. A special satellite was launched in December 2015 to work out details of the LISA project, including 'satellite-in-satellite' technology. The mission was successful. The project is expected to be implemented in 2029.

A number of scientific, engineering, and technological problems emerged and were resolved during development of laser interferometers. Many of them were related to the detection of such important noise sources as mirror suspension systems and reflecting surfaces, and fluctuations of laser radiation intensity. Also, the influence of laser light scattering in the residual atmosphere of beam propagation channels had to be evaluated and a low-noise photocurrent control system designed. Relevant studies yielded important results. It turned out that the surface of the mirrors for laser radiation (35–40 cm in diameter with a mass of 40 kg) must have a highly reflective coating and be suspended by quartz fibers as

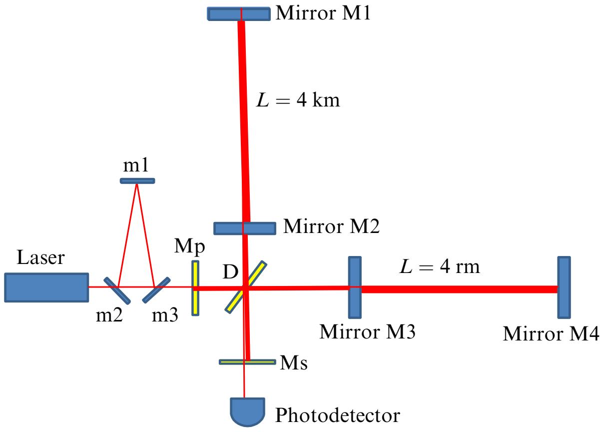

Figure 10. LIGO optical scheme.

proposed by Braginskii, rather than steel wires as in the early LIGO project. Moreover, a suspension system requires good vibration isolation from seismic noises (a few cascades of pendulum suspensions), the air along the beam propagation path must be pumped out, and laser radiation power has to be increased to diminish the relative influence of shot noises. These challenges were successfully overcome after several years of hard work, which allowed the optimal optical scheme of laser interferometers to be chosen (see Refs [35, 60] for details].

Let us consider, by way of example, the optical scheme of the LIGO laser interferometer (Fig. 10). The laser beam passes through a three-mirror Fabry–Perot resonator (mirrors m1, m2, m3) where the mode cleaning is performed before it is sent through the Mp mirror and further to the beamsplitter D, where the input beam is split into two equally intense beams that enter the interferometer arms each containing Fabry–Perot interferometers formed by mirrors M1, M2 and M3, M4, respectively. These resonators elongate the arms by a factor of n, with n being the number of beam passes in a Fabry–Perot interferometer. Mirrors M1, M2 and M3, M4 are moved freely along the beam propagation path. The two beams leaving the arms are sent back to the splitter D, interfere, and their intensity is measured by a photodetector. The additional Mp mirror together with M2 (and M3) mirrors makes up one more Fabry–Perot interferometer needed to increase the laser radiation power inside Fabry–Perot interferometers and thereby the signal-to-shot noise ratio. A supplementary semi-transparent mirror (Ms) is installed between the beamsplitter and the photoderector that serves as an additional Fabry–Perot interferometer in

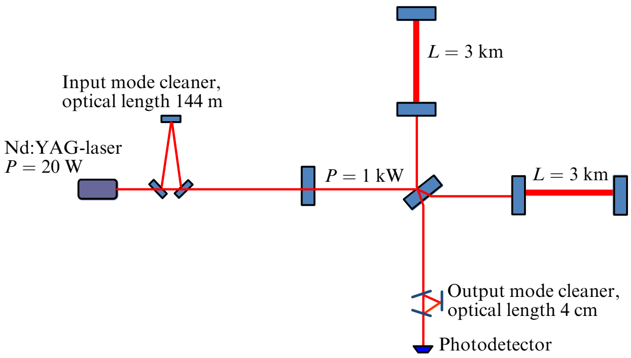

Figure 11. Virgo optical scheme.

each arm to provide further 'peaking' of the interference pattern. Mp and Ms mirrors are frequently referred to as the power recycling mirror and signal recycling mirror, respectively. The interferometer is tuned in so that diffracted beams cancel each other completely (so-called destructive interference). Such a registration channel is usually called the dark port.

The optical scheme of the Virgo interferometer (Fig. 11), like that of LIGO, is a laser *Michelson interferometer* having Fabry–Perot resonators in each arm with a distance of 3 km between the mirrors. The laser beam propagates in a high-vacuum tube 1.2 m in diameter to prevent scattering due to fluctuations of air and, especially, water vapor refractive indices. To remove water vapor, the tubes are heated to 150 °C for a few days before each measuring cycle. Virgo has an additional Fabry–Perot interferometer instead of Ms mirror to pick out the interference signal and clean it of undesirable radiation components.

To enhance the sensitivity of laser GW antennas like LIGO and Virgo, the laser radiation power in kilometer-scale Fabry–Perot resonators making up the orthogonal arms of Michelson interferometers needs to be increased. At present, the laser radiation power in Fabry–Perot resonators is around 100 kW and must be elevated to 830 kW in the future. Evidently, such high radiation powers impose rigorous requirements on the resonator mirrors that serve as free masses and the main sensitive element registering GWs. To increase the reflective coefficient, the resonator mirrors are coated with multiple quarter-wavelength layers of SiO_2 or Ti_2O_5. This unique technology was specially developed for the LIGO and Virgo projects.

The antennas described in preceding paragraphs were modified and improved over the following 10 years. For LIGO antenna, the initial sensitivity in the region of maximum values at a frequency of 150 Hz was $h \approx 10^{-21}$ $\mathrm{Hz}^{-1/2}$. Detectors of the first generation (LIGO, GEO-600, Virgo, and TAMA-300) began to be dismantled at the end of 2010 to be converted into second-generation devices, such as Advanced LIGO, GEO-HF, and Advanced Virgo, respectively. Modification of LIGO was completed by mid-2015 when the Advanced LIGO detector was put into operation, whereas it is planned to complete the Advanced Virgo modernization towards the end of 2016. The LIGO and Virgo collaborations agreed on cooperation for data processing. Frequency dependences of the sensitivity of laser interferometer antennas or GW observatories and the AURIGA resonant detector are illustrated in Fig. 12 showing that most currently operated and designed interferometric antennas have a very broad waveband, possessing a maximum sensitivity in the range of 100–150 Hz due to the attained noise level.

A detailed analysis of the optical schemes of the above interferometers can be found in numerous articles and reviews. Selected features of these installations are listed below:

• large length of the arms of Michelson interferometers (e.g., 3 km in Virgo, 4 km in Advanced LIGO, planned 10 km in the Einstein Telescope);

• large test masses (mirrors) of Fabry–Perot–Michelson interferometers (40 kg in Advanced LIGO, 200 kg in the Einstein Telescope);

• high optical power in the arms (over 100 kW to be elevated to 380 kW in Advanced LIGO, 3 MW in the Einstein Telescope);

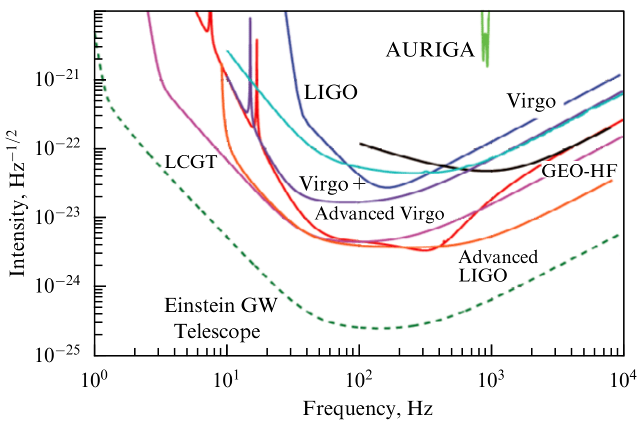

Figure 12. (Color online.) Frequency dependences of sensitivity of GW detectors; LCGT (Laser Cryogenic Gravity Telescope).

• reduction of laser light scattering and refractive index fluctuations by creating an ultrahigh vacuum (up to 10^{-8} Torr) along the entire beam propagation path (total length 8 km for LIGO with a tube 1.5 m in diameter, 6 km for Virgo with a tube 1.2 m in diameter, 6–8 km for KAGRA, and 30 km for the Einstein Telescope);

• suspension systems of test masses (mirrors) ensuring efficient insulation from seismic noises;

• cooling test masses to about 20 K and below for eliminating the heating effect of the light energy absorbed by the mirrors (KAGRA, Einstein Telescope);

• large mirrors with high-efficiency multilayer dielectric reflective coatings.

It is expected that a series of modifications will further increase the threshold sensitivity of laser interferometers to $h \approx 4 \times 10^{-23}$ or higher in a frequency range from 30 to 500 Hz. This will make possible regular detection of gravitational waves. The threshold sensitivity of the underground Einstein Telescope to be commissioned in 2029 must be even higher, i.e., $h \approx 4 \times 10^{-25}$ in a frequency range of 0.03 mHz–0.1 Hz.

5. Direct detection of gravitational waves

On 11 February 2016, American researchers announced the discovery of GWs produced from the merging of two black holes in a binary system. This occurred at an official press conference organized jointly with the American National Research Foundation supporting the project. GWs were reliably detected (CI $> 5\sigma$) on 14 September 2015 with two Advanced LIGO laser interferometers located in Livingston and Hanford, respectively. The signal received by the interferometers as reported in Ref. [1] is shown at the top of Fig. 13a. The signal looks like a chirp with quasilinear frequency modulation from 35 to 250 Hz within 0.2 s. The event was designated GW150914 after the date of its occurrence. In this way, the rules and procedures governing registration of similar events in the future were established. According to the authors, a comparison of the observed signals with those from the library of binary system collapse scenarios (over 250,000 cases) allowed the conclusion that the event was a result of the merging of two black holes (BHs) with initial masses of $36^{+5}_{-4}M_\odot$ and $29^{+4}_{-4}M_\odot$ ($M_\odot$ is the solar mass). Later on, these estimates were updated to $35^{+5}_{-3}M_\odot$ and $30^{+4}_{-3}M_\odot$ [73, 74].

It should be noted that a signal emitted by a BH binary and detected by interferometers depends not only on the BH masses but also on many independent parameters, viz. the angular momentum of each black hole, their relative location and space position relative to the BH orbit, the position of the BH orbit plane with respect to the observation line, the mutual position of the observation line and the directional pattern of the receiving antenna, and, certainly, GW radiation intensity (luminosity). Measurement of luminosity can be reduced to the relativistic problem of rotation of two BHs under conditions of strong gravitational potential when the mass velocity is commensurate with the speed of light. In conditions of great uncertainty, a number of model and approximate methods to deduce equations of motion for binary systems have to be used in the post-Newtonian approximation taking account of spin–spin and spin–orbit interactions between rotating masses [75–82] before numerical methods can be applied [83–88]. Such methods were developed for the treatment of observable data obtained in Refs [75, 76]. Nevertheless, initial characteristics of collapsing binary systems remain obscure (see Ref. [89] and references cited therein); further observations are needed to make precise measurements with the use of several high-sensitivity interferometers of the next generation.

The BH resultant mass proved to be by $3^{+0.5}_{-0.5}M_\odot$ smaller than the sum of initial BH masses, the mass deficit being attributable to GW emission. The distance to the origin of GW150914 discovered on 14 September 2015 is 1.3 billion lightyears based at a redshift of $z = 0.09^{+0.03}_{-0.04}$. The signal was first detected in Livingston and reached the Hanford interferometer $6.9^{+0.5}_{-0.4}$ milliseconds later. The observers concluded that the source of the GWs is located in the southern celestial hemisphere. The delay suggests an extraterrestrial origin of the signal.

A few months after the first detection of gravitational waves from the black hole merger event GW150914, the LIGO Observatory made another observation of gravitational waves from the collision and merging of a pair of black holes [90]. This signal, called GW151226, arrived at the Advanced LIGO detectors on 26 December 2015 and was cleaned by correlation filtering (Fig. 14). Evidently, it was a chirp signal like GW150914 with a frequency varying from 35 to 450 Hz during 55 cycles. The total duration of the noise-free signal was about 1 second. The analysis demonstrated that such a signal corresponds to the collapse of two BHs with initial masses of $14.2^{+8.3}_{-3.7}M_\odot$ and $7.5^{+3.3}_{-2.3}M_\odot$, respectively. The resultant mass of the new BH was $20.8^{+6.1}_{-1.7}M_\odot$, and the mass deficit resulting from GW emission was $1.0^{+0.1}_{-0.2}M_\odot$ [89, 90]. Reference [89] reported the following values of the remaining parameters: chirp-mass

$$M_c = \left(\frac{m_1 m_2}{(m_1 + m_2)^{1/3}}\right)^{3/5} \tag{17}$$

equaled $8.9^{+0.3}_{-0.3}M_\odot$, energy emitted in the form of GWs $1.1^{+0.1}_{-0.2}\,M_\odot = 3.3^{+0.8}_{-1.6} \times 10^{56}$ erg s^{-1}, distance from Earth to the source 440^{+180}_{-190} Mpc (around 1.36×10^{27} cm), red shift $z = 0.09^{+0.03}_{-0.04}$, GW dimensionless amplitude at the reception point $h = 3.4^{+0.7}_{-0.9} \times 10^{-22}$, lag time between signal detections by Livingston and Hanford interferometers $1.1^{+0.3}_{-0.3}$ ms (the signal reached the former installation earlier than the latter). The signal-to-noise ratio in both GW150914 and GW151226 was higher than 13, i.e., over 5σ, which corresponds to the likelihood of false detection below 10^{-7}. The sensitivity of the laser interferometers was calibrated by applying modulated

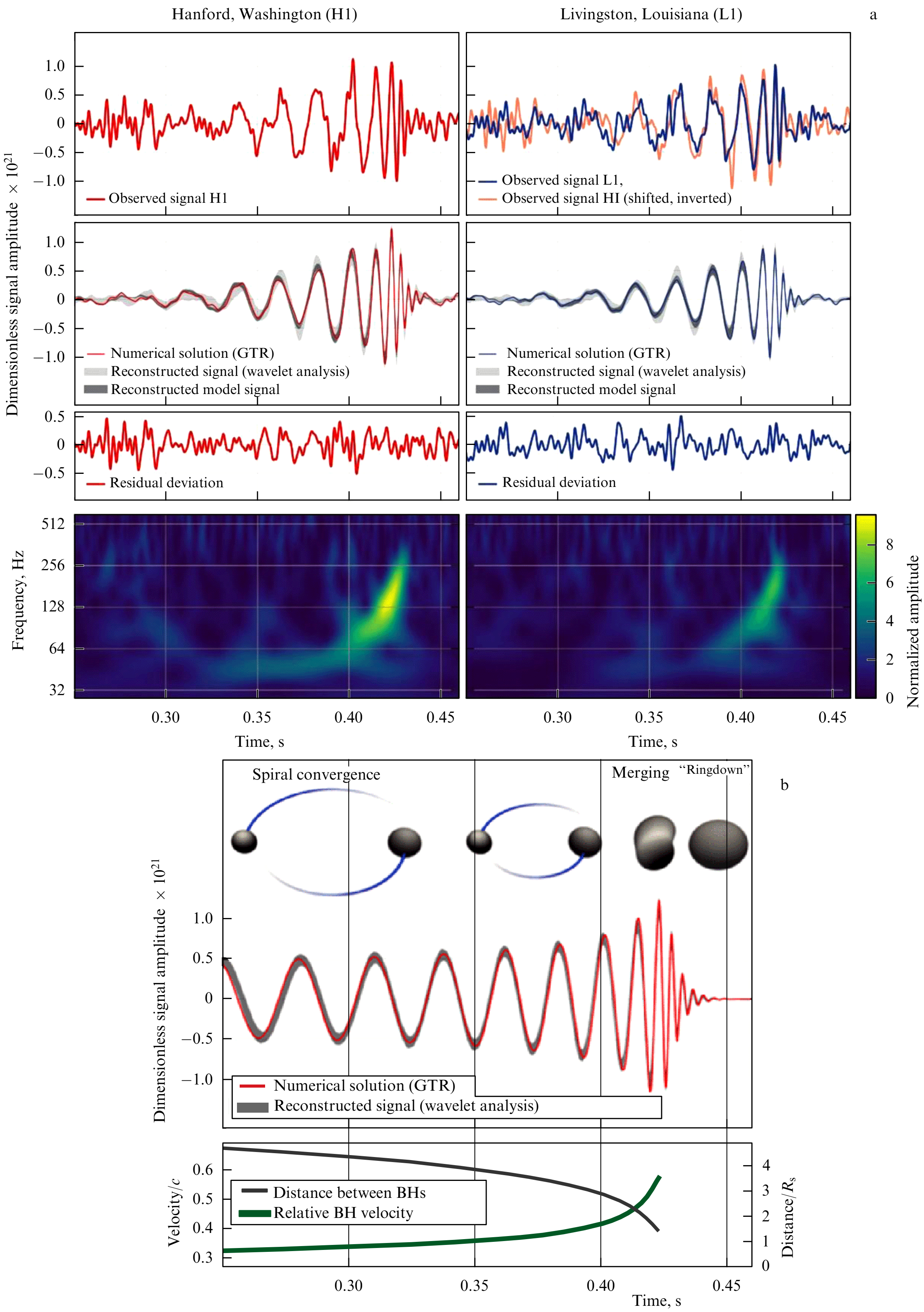

Figure 13. (Color online.) (a) GW signal detected by laser interferometers in Hanford and Livingston. (b) Reconstructed (post-processing) signal (GW150914 event).

laser radiation to the test masses (mirrors) [91]. This additional radiation exerted light pressure on the mirrors and caused their small but measurable displacement, thus giving rise to a signal in the interferometer. (One of the few examples of the practical application of light pressure effect.) The magnitude of a GW energy flux was evaluated from the

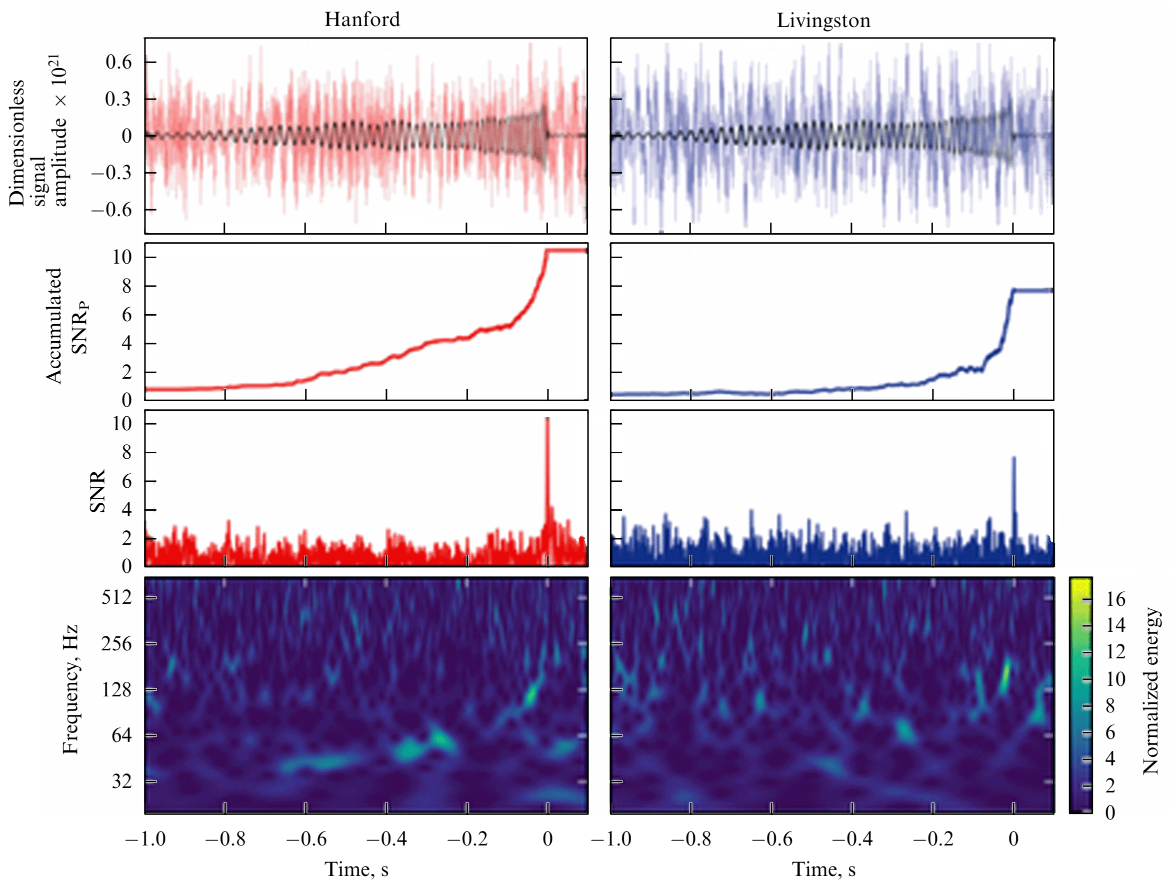

Figure 14. (Color online.) The second-detection GW signal , GW151226 event. Accumulated SNR_p — integral peak signal-to-noise ratio determined by one of the correlation filter methods.

dimensionless GW amplitude measured with the two Advanced LIGO interferometers and the distance to the gravitational radiation source deduced from the red shift value [92]. The most intense GW radiation is known to be emitted close to the moment of collapse. It is at this moment that the signals from GW150914 and GW151226 events were registered. The temporal form of the signal preceding the moment of approach for different scenarios (i.e., under different initial conditions) was deduced beforehand, and it was necessary to compare the preliminarily computed signals and those observed in the noisy environment. In fact, the problem reduced to the choice of an optimal filter by calculating the maximum value of the convolution between the signal and the optimal filter (see review [93]).

Let us consider in brief the main physical ideas pertinent to an analytical description of the merger of two black holes.

Because GW radiation is an effect of order $(v/c)^5$, where v is the characteristic velocity of mass traveling in the system, the description of a system of gravitating bodies requires construction of the Lagrange function with an accuracy up to $(v/c)^4$ [33, 94] and subsequent derivation of the equation of motion. It becomes impossible to use this approach for obtaining the equations of motion when the v/c ratio is close to unity; therefore, other methods need to be employed. It is easy to demonstrate using the Newtonian approximation to the two-body problem that the characteristic BH velocity near collapse has the form $v/c \approx \sqrt[3]{\pi G(m_1+m_2)}\, f/c$, where f is the GW frequency. This ratio is $v/c \approx 0.32$ for GW150914 at the beginning of observations when $f = 20$ Hz, and it is roughly 0.2 for GW 151226 ($f = 35$ Hz). This suggests the necessity of different methods to describe how black holes approach each other. Such methods were developed based on the successive approximation theory in which the v/c ratio appears as a small parameter and the Hamiltonian function for the two-body problem was obtained by the successive approximation method [75–88, 93] taking into consideration expenditures of energy to emit GWs at each stage of convergence; the spacetime metric corresponded to the Schwarzschild solution [17, 33]:

$$\mathrm{d}s^2 = -\left[1 - \frac{2G(m_1+m_2)}{c^2 r}\right]\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-[2G(m_1+m_2)]/(c^2 r)} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2)\,. \quad (18)$$

This approach made it possible to step outside the bounds of the Newtonian approximation [see Eqn (9)] and derive the equations of motion for the two-body problem, taking account of spin–spin and spin–orbit interactions between rotating masses. The numerical solution of these equations (at different starting masses m_1, m_2 and angular momenta) was then used to build up the library of BH collapse scenarios. The measured temporal signals were compared with the library scenarios to arrive at final conclusions and obtain concrete data about the GW150914 and GW 151226 events, making use of different models with and without BH rotation. The temporal form of the signal at the spiral convergence stage can be described as

$$h(t) = h_0 v^2(t)\, \cos\phi\big(v(t)\big)\,, \quad (19)$$

where h_0 is the signal normalization amplitude depending on merging BH masses and distance to the radiation source, as well as detector/source mutual position [93], and $\phi(v(t))$ is the phase depending on the relative velocity $v(t)$ of the merging masses.

It appears from formula (19) that the explicit time dependence of relative velocity $v(t)$ must be found to determine the form of the signal based on the following line of reasoning. Assume that the total relativistic energy of the system $E(v(t)) = E_{\text{total}}(v(t)) - Mc^2$ depends only on velocity $v(t)$ and that the system of rotating BHs expends energy only to emit GWs; then one finds [93]: $\mathrm{d}E/\mathrm{d}t = -\Upsilon_{\text{GW}}(t)$, where $\Upsilon_{\text{GW}}(v(t))$ is the GW flux carrying energy away from the system. This expression yields

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}E/\mathrm{d}t}{\mathrm{d}E/\mathrm{d}v} = -\frac{\Upsilon_{\text{GW}}\big(v(t)\big)}{\mathrm{d}E/\mathrm{d}v}\,. \tag{20}$$

The relationship for the Kepler problem ensues that $\omega^2 r^3 = GM$. The fact that $\omega(t) = \mathrm{d}\phi(t)/\mathrm{d}t$ (it is deemed that $|(\mathrm{d}v^2/\mathrm{d}t)/v^2| \ll |\mathrm{d}\phi/\mathrm{d}t|$) leads to the second equation

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{v^3(t)}{GM}\,, \quad M = m_1 + m_2\,. \tag{21}$$

At a given dependence of flux $\Upsilon_{\text{GW}}(v(t))$ and energy, the set of equations (20), (21) can be used to characterize time dependence of phase and velocity in order to describe the time dependence of the signal. To find the velocity dependence of $\Upsilon_{\text{GW}}(v(t))$ and energy, when velocities are comparable to the speed of light, the post-Newtonian approximations of GTR should be applied with a power series expansion in $v(t)/c$. The authors of Refs [75–81, 93] (see also Refs [33, 82] and references cited therein) proposed expressions for GW energy flux $\Upsilon_{\text{GW}}(v(t))$ and relativistic energy $E(v(t))$ of a system of two compact masses rotating in a quasicircular orbit. Importantly, the post-Newtonian approximations for $\Upsilon_{\text{GW}}(v(t))$ and $E(v(t))$ remove degeneracies with respect to masses and angular momenta of collapsing BHs and thereby provide for their independent definition. The expressions for $\Upsilon_{\text{GW}}(v(t))$ and $E(v(t))$ thus obtained were substituted into Eqns (20), (21) to find their numerical solutions at different values of parameters. These solutions were then utilized as a library of various correlation filters for estimating the conformity between the observed signal and the respective filter. The values of masses and angular momenta of a correlation filter that ensured the maximum correlator value were ascribed to the observed GW scenario. This analysis yielded numerical values of masses and angular momenta of rotating BHs. This is the best method for interpreting results of observations in the absence of a complete set of initial data and the impossibility of obtaining them even if the uniqueness of such information remains an unsolved problem.

It is sometimes possible to use a Newtonian approximation for rapid numerical estimations. Indeed, it follows from observational data plots that a signal emitted just before merging has a sine waveform which allows, from Eqn (7) and relationship $\omega^2 r^3 = GM$, the equation for a time-dependent change of the radius of rotation for a circular orbit to be easily derived:

$$\frac{\mathrm{d}r(t)}{\mathrm{d}t} = -\frac{64G^3 m_1 m_2 (m_1 + m_2)}{5c^5}\, r^{-3}(t)\,. \tag{22}$$

The solution of equation (22) gives the time interval $\Delta t = t_1 - t_2$ from the onset of measurements t_1 till a certain moment t_2:

$$\Delta t = \frac{5c^5}{256 G^{5/3} m_1 m_2 (m_1 + m_2)} \left(r^4(t_1) - r^4(t_2)\right). \tag{23}$$

This formula is obtained in the Newtonian approximation that is inapplicable at the beginning of merging when the masses are far apart and relative velocities low ($r \gg r_{\mathrm{g}}$, $v/c < 1$). (Certainly, the time interval in a strong BH field of gravity varies [17, 33].) The fraction of the non-Newtonian time interval in the integral time interval decreases with increasing observation time. The assumption that the observed process of BH approaching started when the distance between the BHs was much greater than the sum of their Schwarzschild radii leads to the expression following from Eqn (23):

$$M_{\mathrm{c}} = \frac{1}{M_\odot} \frac{5c^5}{256\pi^{8/3} G^{5/3}} \frac{1}{\Delta t} \left(f^{-8/3}(t_1) - f^{-8/3}(t_2)\right), \tag{24}$$

where $f(t_1)$ and $f(t_2)$ are minimum and maximum GW frequencies, respectively (to recall, the frequency of a GW in the Newtonian approximation is twice the rotation frequency), and M_{c} is the chirp mass of the BH. This simple formula allows the value of BH chirp-mass to be estimated from the known values of minimal frequency and observation time, which is very convenient for rapid assessments [95].

The analysis of observational data shows that three time domains can be distinguished: that of low frequencies in which the Newtonian description is feasible, an intermediate one where the v/c ratio constitutes a noticeable part of unity, and that of high frequencies immediately before the collapse in which $v/c \sim 1$ and the inter-BH distance is commensurate with the Schwarzschild radius. For this reason, formula (24) is applicable in the first, low-frequency domain where the motion takes place in nearly circular orbits. The measurement of two frequency values in this domain gives the chirp-mass of a BH.

Indeed, let us consider the data obtained by observations of the GW150914 and GW151226 events. The BH frequency region for the former event extends from 20 to 50 Hz and time interval Δt is slightly longer than the observation halftime, i.e., approximately 0.12 s. Substituting these values into formula (24) for the chirp-mass gives $28.23 M_\odot$, which is very close to $28.19 M_\odot$ reported in Ref. [1] (see Ref. [89]). Similarly, for GW 151226, the frequency $f(t_1) = 35$ Hz in the starting time interval $\Delta t \approx 0.85$ s; hence, the chirp-mass is $10.9 M_\odot$ or somewhat higher than $8.9^{+0.3}_{-0.3} M_\odot$ obtained in Ref. [78]. Such a discrepancy may be a result of a slightly weaker GW signal than in the GW150914 event.

It is worthwhile to note that chirp-mass values greatly depend on the exact value of frequency $f(t_1)$. For example, if $f(t_1) = 39.5$ Hz is taken instead of $f(t_1) = 35$ Hz, the chirp-mass for the same time interval Δt becomes equal to $8.9^{+0.3}_{-0.3} M_\odot$, as in Ref. [90]; hence, the importance of precise measurement of the GW period, especially at the beginning of observation. The employment of simple relations to determine chirp-masses is very helpful, since GW radiation in this time domain is not yet strong enough and the validity of the Newtonian approximation is beyond question. Both GW frequency and time interval in the strong BH gravity field are known to vary [17, 33]. However, the time interval, unlike the GW frequency, is an integral quantity, and its variations are inversely proportional to the duration of observations. Therefore, other approaches are needed as far as the high-velocity and strong gravity-field relativistic region are concerned [75–82, 93], especially when contributions from BH rotation have to be considered. Extensive theoretical studies are currently underway, focused on various models and scenarios of collapses of BHs, BH-containing neutron stars, and other objects.

The creation of third-generation interferometric gravitational wave detectors poses a number of difficult problems that remain to be solved. Increasing laser radiation power in Fabry–Perot interferometers implies the necessity of heat removal from the mirrors. Multilayer periodic coatings sprayed over their surface with different refractive indices to ensure a high reflective coefficient for a laser radiation have a thickness of $(\lambda/4)\,N$, where λ is the laser wavelength in a layer, and N is the number of layers. Laser radiation with the wavelength $\lambda = 1.06$ μm at $N = 44$ is known to be absorbed by an 11-μm thick layer. The energy absorbed by this very thin layer at a radiation power higher than 100 kW causes strains of the mirror surface [96]. This effect can be prevented by heat removal [96]. Because laser interferometers of future generations will have even higher radiation power (e.g., up to 3 MW in the Einstein Telescope) as well as bigger and heavier mirrors, the development of methods for reducing and removing heat is becoming important. In this context, consideration of certain potential approaches to developing reflective structures may be in order [97].

Sinusoidal periodic structures of a refractive index that are possible to create in a medium using an acoustic wave are known to be instrumental in obtaining a reflective index arbitrarily close to unity (in the absence of absorption) [97]. In this case, changes in the refractive index are insignificant, and a high value of the reflective coefficient can be reached by virtue of parametric coupling between incident and reflected waves on a sufficiently extended periodic structure. This relationship provides a basis for the development of various acusto-optic devices, such as spectrometers, and electronically adjustable optical filters. Also, these structures can be used as reflecting mirrors for laser interferometers [98]. Certainly, the creation of structures by an acoustic wave for the mirrors of LIGO and Virgo type interferometers is hardly possible, but such issues as the achievement of high reflective coefficients and heat removal can be successfully addressed by the application of one-dimensional photonic crystals with the periodic structure needed to change the dielectric permittivity of the medium. Naturally, absorption of laser radiation by a material is always much lower than by the film deposited onto it. Therefore, absorption in a mirror is significantly smaller than in its thin coating, given that relation $4\gamma_{\rm m} l_{\rm m} \ll \gamma_{\rm p} \lambda N$ is satisfied, where $\gamma_{\rm m}$ and $\gamma_{\rm p}$ are the absorption coefficients of the material and the film, respectively, and $l_{\rm m}$ is the length of the reflecting periodic structure in the mirror material.

Moreover, the scattering volume in the material is much bigger than that in the film; therefore, its specific heating is smaller and the conditions for a heat release into the environment are better.

One of the promising techniques for manufacturing extended periodic one-dimensional structures is the technology employed to construct Bragg reflectors built directly into the optical fiber. Another kind of technology for the same purpose is passing workpieces with a preformed large-scale periodic structure of refractive index through cylindrical rolls. Finally, a laser standing wave can be used to gradually redistribute nanoparticles with high dielectric constants contained in a hot medium and thereby to form a periodic structure retaining its spatial distribution pattern after cooling.

The following remarks may be in order as far as mirror suspension systems are concerned. Strong enough radiation incident on a mirror is known to cause its polarization and induce an electric dipole moment [95]. The appearance of the dipole moment under the effect of electrostatic induction results in emerging the additional coupling between the mirror and the remaining equipment, e.g., high-conductivity metallic structures. Such additional coupling may prove highly undesirable especially in laser beam modulation for calibrating interferometer sensitivity. The dipole moment becomes particular dangerous if the mirror acquires a charge [99, 100], the origin of which remains unknown.

6. Conclusions

The main conclusion following from direct GW detection is that it provides more proof of the validity of Einstein's GTR equations in strong fields.

Indirect evidence of the existence of GWs was obtained when Taylor [101] and Hulse [102] (see also Refs [103, 104]) published their observations on the revolution period variation of the double pulsar PSR 1913+. However, it held only for a weak gravity field where gravitational losses were very small compared with the binary own energy. Taylor and Hulse observed changes in the binary pulsar revolution period. The observations were carried out with the 300-meter radio telescope in Puerto Rico over 15 years starting in 1974. The PSR 1913+ pulsar is a compact binary whose components have similar masses, slightly bigger than the solar mass, and roughly 10-km radii, with the distance between them only several times that between the Earth and the Moon. Variations of the pulsar period do not exceed 7.5×10^{-7} s per year. Comparing these data with calculations by the GTR formula for time-dependent changes in the revolution periods following from expression (9) showed the excellent agreement between the measured time derivative of the period and the theoretical value over the 15 year interval:

$$\frac{({\rm d}T/{\rm d}t)_{\rm meas}}{({\rm d}T/{\rm d}t)_{\rm GT}} = 1.0023 \pm 0.0047\,,$$

with an error below 0.6%. This well confirmed the validity of GTR equations but only for weak gravitational fields and the relatively low velocities of massive bodies, at which the Newtonian approximation is applicable to the two-body problem.

Direct detection and observation of gravitational waves, as opposed to indirect, are of special importance, because their results confirm the validity of Einstein's GTR equations in strong fields at relativistic velocities of motion of large masses and thereby provide a solid basis for explaining processes taking place in the Universe.

Direct GW detection can be regarded as indirect proof of the existence of black holes. Once X-ray radiation and/or gamma-radiation of the accretion disk around a black hole is recorded in the nearest space domain [105], it will be possible to interpret GW150914 and GW 151226 events as the direct observation of BH collapse and a new 'window' into the Universe providing a deeper insight into its processes.

The fact that two (possibly three) events were reliably discovered within only three months of observations suggests the necessity to revise the number of BHs in the Universe, which may be substantially greater than currently believed, since yet unobservable large objects can make a noticeable contribution to what is tentatively called dark matter [105–110].

The discovery of GWs and the interpretation of observation data bring forth many new fundamental problems. Specifically, these are the analysis of time-related behavior of matter and radiation near BHs (i.e., in the region somewhat greater than or equal to gravitational radius); the construction of models for the description of matter behavior in ultrastrong gravity fields, including that of rotating masses with relativistic velocities and finding of an event horizon for these cases; investigations into the physics of interactions between relativistic vortices near event horizon, etc.

The design of laser interferometers of future generations poses new technological problems, the solution of which will promote further progress in GW science. Special emphasis should be laid on the search for and investigation of materials and technologies for the construction of highly reflective mirrors of laser interferometers and the development of methods for vibration insulation of their suspensions.

Solid-state resonant antennas can be employed in the future to detect very long gravitational waves from binary neutron or massive double star systems with a known radiation frequency of the emitted GWs.

To sum up, the success of nearly half a century of attempts to directly detect GWs opened up one more channel to access information about processes in our Universe. It should be noted that physicists never questioned the possibility of discovering GWs, which motivated researchers in many countries to continue elaborating and maintaining new projects aimed to design increasingly sophisticated and sensitive laser interferometers.

Unfortunately, our country failed to raise funds for creating a GW observatory even though many Soviet and Russian researchers pioneered the development of laser GW detectors (see, for instance, Refs [111–113] published recently in *Physics–Uspekhi* and references cited therein and the short note by Cherepashchuk [114]). Maybe there's still time to catch up?

Notes added in proof. This issue of *Physics–Uspekhi* is dedicated to the memory of V L Ginzburg on the occasion of the anniversary of his 100th birthday. Vitaly Lazarevich supervised my post-graduate studies at the Lebedev Physical Institute on selected problems of GTR that he himself formulated as an anxious and attentive teacher. Here, I would like to share with the readers some of the memories I have of discussions in person with Vitaly Lazarevich concerning research carried out under his guidance.

The first problem that Vitaly Lazarevich (VL as he was frequently nicknamed by Theoretical Department colleagues) assigned me was to determine the intensity of GWs emitted by an ultrarelativistic charged particle performing circular motion in a magnetic field in analogy with the computation of synchrotron radiation intensity of electromagnetic waves. This issue was deemed interesting for the following reasons. Electromagnetic radiation from an electron in circular motion in a magnetic field (or a proton in a collider) is known to result in a loss of energy proportional to the particle's energy squared [33], i.e., $\partial E(t)/\partial t \sim E^2(t)$. On the other hand, equations of electrodynamics (Maxwell's equations) can be represented in the general covariant four-dimensional form [33], $\Box A^i = 4\pi j^i/c$, where A^i is the vector potential, and j^i is the current. The equations for vector potential A^i and GW amplitude [see Eqn (3)] are very similar, the only difference being that the source of the electromagnetic field (the current) on the right-hand side of the equation is proportional to the particle's velocity, i.e., the square root of the energy, whereas the source in GW equation (3), i.e., the energy–momentum tensor, is proportional to the energy proper. It might seem that energy losses for GW emission by a particle moving in a circle in a magnetic field should depend on the energy exponent higher than 2 rather than the square of the energy. In this case, the particle would have an energy at which GW emission intensity exceeds the intensity of electromagnetic radiation. Such was the line of reasoning on which VL based the formulation of the problem. However, calculations encountered difficulties and yielded different results for the dependence of GW radiation energy on the energy itself.

After almost a year of intense work. I decided to complain to VL about my hardship and failure. He asked on hearing my complaint: "How much paper do you throw into the waste basket?" I kept silent not knowing what to reply. VL: "Almost 80% of my papers go to the basket. I think it is the same with yours. Don't be disappointed and continue to work." Finally, I found the mistake. In striving to have the result as soon as possible, I moved to the limit too early in my calculations; in fact, I set the velocity of the particle equal to the speed of light (a constant) and thereafter obtained different dependences of energy losses on the energy itself. However, VL was not satisfied with the final result. It turned out that the ratio of intensity of GW radiation to that of electromagnetic radiation for an ultrarelativistic particle is a constant and energy-independent quantity determined by the ratio of the particle's gravitational radius to its electromagnetic radius. The cause behind this relationship is the 'double' transversity of GW amplitude, because the mass has a charge with a single (positive) sign, in contrast to electrodynamics where there are two signs: plus and minus; therefore, radiation has a quadrupole character. After one of the regular seminars, I familiarized VL with my poor results, which took away any possibility of intense GW radiation. He said: "Come to Kapichnik tomorrow," which meant a proposal to attend L D Landau's seminar. I was somewhat surprised that he did not seem discouraged by my report. After the seminar, VL gave me a sign to stay and came up to Landau. I joined them and heard VL asking Landau: "Dau, at what energy do you think gravitational wave energy will exceed the electromagnetic energy in a synchrotron?" Landau thought for a second and replied: "It will exceed it but only at very high energies." VL paused a moment before he said: "You are wrong, Dau! You forgot that the graviton has spin two and a gravitational wave is a doubly transverse wave, as this young fellow (he pointed at me) proved." Landau looked up and said: "All right!" I understood that VL sought an opportunity to show that Landau was sometimes wrong.

Later on, VL entrusted me with the task of writing a paper. When we met again he handed me an article by Mikhail Evgen'evich Gertsenshtein on the same subject submitted to the *Journal of Experimental and Theoretical Physics* and referred to him by E M Lifshitz with a request to review it. Looking through the article, I immediately came across the same mistake that I had made in my studies: the author concluded that GW radiation intensity was proportional to the particle's energy in the fourth power. When I told this to VL, he asked me to call ME and explain the mistake to him. Also, he decided that we must jointly publish the article. In this way, I made the acquaintance of Mikhail Evgen'evich and became his co-author [6]. When we discussed the future publication, ME offered to write a joint article on the analysis of real sensitivity of Weber type GW detectors with a critique of resonant antennas. I refused the offer, bearing in mind the opinion many times expressed by VL that publishing purely critical papers without positive content is unbecoming, especially for young researchers. Instead, I proposed including the description of the interferometric method for GW detection based on the analysis of equations of electrodynamics in the presence of a static gravity field. As is shown in Ref. [33], the Maxwell equations in a gravitational field are equivalent to equations for a medium with refractive index determined by such a field. If the gravitational wavelength is much greater than the size of the interferometer arms, the GW field for light waves in the interferometer can be regarded as static and all the conditions for detecting low-frequency GW fulfilled. That is how paper [29] appeared.

VL was and remains the Teacher for most physicists in this country. His seminars with discussions of many issues (see his list of especially important and interesting problems of physics and astrophysics [115–118]), comments, and presentations greatly contributed to the formulation of a scientific world outlook in the scientific community, including but not limited to physicists. For my part, I can say that I continued discussions with VL even after I had turned to new research topics and many times acknowledged his advice and assistance in my publications. One more remarkable quality of his style as a leader of the famous theoretical school in the Physical Institute is worthy of note. VL recommended and frequently involved in discussions of scientific problems those specialists whom he believed and whose opinion he valued. For example, he referred my work on semiconductors and solid-state physics to Leonid Veniaminovich Keldysh for approval. Today, we acutely feel the lack of the scientific atmosphere of those times. It is really disappointing that geniuses are born, not often enough.

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