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Gas rotation in discharges in a longitudinal magnetic field

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Contents

Abstract. **This paper reviews theoretical and experimental work on gas rotation in discharges ignited in a longitudinal magnetic field. There is abundant evidence that such a field causes plasma-dust structures to rotate in the plane perpendicular to itself. This rotation cannot be explained solely by the dragging action of azimuthally moving ions. Drag is also exerted by the gas rotating under the moment produced by the Ampere force that arises in discharge regions with a nonuniform magnetic field near solenoid end faces and due to the narrowing of the discharge channel cross section. In a stratified discharge in a uniform magnetic field, an eddy electric current due to the noncollinearity of the plasma-density and temperature gradients brings the gas to rotation.**

Keywords: discharge in a magnetic field, gas rotation, Ampere force moment, eddy electric current in strata

1. Introduction

"A quiet and calm life, with research conveniently minimized in scope and with little or no interest in what others do is, unfortunately, a quite typical pattern for some of our research institutes." L A Artsimovich's view a half century ago admittedly remains true even today — except alas for "a quiet and calm life." A major means of information exchange — and an important factor maintaining interest in professional research in various fields of physics — are papers that are published in *UFN*'s 'Physics of Our Days' section.

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Here, we review the work concerned with the rotation of a gas in discharges ignited in dielectric tubes in a longitudinal magnetic field. We consider a weakly ionized plasma in which the electron and ion concentrations are lower than the concentration of atoms, $n_{\rm i} = n_{\rm e} = n \ll n_{\rm a}$, and the electron temperature greatly exceeds the atom and ion temperatures, $T_{\rm e} \gg T_{\rm a} = T_{\rm i}$. Without a magnetic field, the equations of motion for electrons and ions can be written out as

$$e_0 n\mathbf{E} + \nabla(nT_{\rm e}) + \frac{m_{\rm e} n}{\tau_{\rm e}}(\mathbf{v}_{\rm e} - \mathbf{v}) = 0\,, \qquad (1)$$

$$e_0 n\mathbf{E} - \frac{m_{\rm i} n}{\tau_{\rm i}}(\mathbf{v}_{\rm i} - \mathbf{v}) = 0\,. \qquad (2)$$

Here, $\mathbf{v}_{\rm e}$ and $\mathbf{v}_{\rm i}$ are the electron and ion velocities, respectively, $\mathbf{v}$ is the velocity of the neutral gas, and $\tau_{\rm e,\,i}$ are the electron (ion) mean free path times between collisions with gas atoms. The last terms in equations (1) and (2) correspond to the momentum loss of the charged particles. For a homogeneous plasma, it follows from Eqn (1) that $\mathbf{v}_{\rm e} = -b_{\rm e}\mathbf{E}$, where $b_{\rm e} = e_0\tau_{\rm e}/m_{\rm e}$ is the electron mobility. For an electron gas with a high thermal conductivity, the temperature gradient of the electrons is much lower than their concentration gradient $\nabla(nT_{\rm e}) = T_{\rm e}\nabla(n)$. Equation (1) for the current density can be written out in the form

$$\mathbf{j} = e_0 b_{\rm e} n\mathbf{E} + D_{\rm e} e_0 \nabla n\,, \qquad (3)$$

with $D_{\rm e} = T_{\rm e}\tau_{\rm e}/m_{\rm e}$ being the electron diffusion coefficient. In general, $\mathbf{j} = e_0 n(\mathbf{v}_{\rm i} - \mathbf{v}_{\rm e})$. In low-pressure discharges, bulk ionization is due to electron impacts, and the ions produced recombine at the walls; as a result, a positive radial electric field develops which smooths the electron and ion fluxes onto the nonconducting wall.

Such discharges usually exhibit scaling laws that depend weakly on the discharge current: the electron temperature is a certain function of the ratio of the longitudinal electric field strength to the gas pressure, $T_{\rm e} = f_T(E/p)$, and this ratio is itself a function of the product of the tube radius a and the pressure, $E/p = f_E(ap)$.

Mention should be made here of B N Klarfeld's paper, "Characteristics of the Positive Column of Gaseous Discharge" [1] published in the June 1941 issue of *Journal of Physics Acad. Sci. USSR*, then the only English-language scientific journal in the Soviet Union. The paper presented a detailed list of the scaling laws Klarfeld was able to establish for various gases in a wide range of discharge parameter variations. Considering the time of publication, it is no surprise that the paper remained little known.

2. Problem of gas rotation in a longitudinal magnetic field

Interest in gas rotation was initiated by Granovskii and Urazakov's work on the magnetomechanical effect. The two researchers discovered that a light rectangular mica plate suspended by a flexible string in a plasma of a vertical positive column rotates through a certain angle in a constant longitudinal magnetic field. The direction of the rotation changed when the direction of the magnetic field changed but was independent of the current direction in the discharge [2, 3]. The authors' conclusion was that the rotation of the plate has no relation to the Ampere force due to the discharge current but is an inherent property of the positive-column plasma. They called this phenomenon the magnetomechanical effect (MME) and ascribed it to the rotation of the gas under the influence of the Hall diffusion of the electrons and ions. Similar results were revealed in Ref. [4].

In a discharge plasma in a tube with nonconducting walls parallel to a uniform magnetic field of induction $\mathbf{B}$, there are no radial electric current and no Ampere force moment that could overcome the viscosity of the gas to put it in rotation. There have been ideas circulating in the literature that a gas can rotate under the influence of the radial electronic current alone or ion current alone. These ideas neglected the moment of force from the opposite sign current.

In a plasma diffusing with a radial velocity v_r, the induced electric field $\mathbf{E}_\varphi = \mathbf{v}_r \times \mathbf{B}$ creates an azimuthal current, for which the ratio of ion-to-electron azimuthal velocities is equal to the ratio of their mobilities [5]. Notice that the friction forces acting on the neutral gas are balanced and produce no rotation of the gas.

3. Role of end face effects

References [6, 7] reported applying the Doppler effect to discover that gas in a discharge in a magnetic field rotates due to the moments of the Ampere force near the solenoid end faces. Experiments [2, 3, 6, 7] employed a two-section solenoid with a total length of 60 cm, with a narrow slit in the middle for optical measurements. The discharge tubes used were longer than the solenoid. Near the solenoid's end faces, an anode-to-cathode current flows at an angle to the magnetic field vector, creating an Ampere force moment. The longitudinal current flowing from the anode intersects the magnetic flux lines and creates a force moment which rotates the plasma counterclockwise as viewed along the magnetic field. Near the cathode end face, electrons move in the direction of increasing magnetic field along the convergent magnetic lines of force, and the discharge at the entrance into the magnetic field focuses to a thin channel on the tube axis, a fact which was noted by Tonks [8] and Reikhrudel' and Spivak [9] and repeatedly since. Inside the solenoid and close to the cathode end face, the radial electric field E_r is negative. As the current channel steadily widens, the current acquires a nonzero radial component which flows to the discharge axis and creates an Ampere force moment acting clockwise. In the tube region outside of a magnetic field, the radial electric field is determined by ambipolar diffusion and is positive. Thus, there are plasma regions with the opposite signs of E_r on either side of the solenoid's cathode end face. Because the circulation of $\mathbf{E}$ along a closed path (along the discharge axis, in the radial directions, and along the walls) is zero, it follows that the difference in the radial components of $\mathbf{E}$ is compensated for by the voltage drop on the resistance to the longitudinal current. As this current flows in the magnetic field, it creates an additional Ampere force moment. Changing the direction of the discharge current $\mathbf{I}$ interchanges the cathode and anode. At the 'new' anode end face, the current interacts with the radial magnetic field component of opposite sign, with the result that the force moment $\mathbf{I} \times \mathbf{B}$ retains its direction. The method of removing end face effects at the cathode end was realized in Ref. [10]. A several-fold increase in the diameter of the tube cathode in front of the entrance to the solenoid made it possible to arrive at the cross section of the focused discharge becoming on the order of the radius of a tube with a positive radial field. The manifestation of field nonuniformities at the anode end face was removed by placing the anode inside the solenoid.

Note here that even in the absence of a longitudinal magnetic field in the positive column of a low-pressure discharge, plasma perturbations propagate considerably toward the anode. The time of the plasma escape to the walls is estimated as $\tau_d \approx a^2/5D_a$ ($D_a \approx b_i T_e/e_0$ is the ambipolar diffusion coefficient, $b_i = e_0 \tau_i/m_i$ is the ion mobility, and the number 5 appears due to the boundary conditions). Ions escape to the walls close to the places where they appear, whereas electrons drift in the electric field a distance $L = a(b_e/5b_i)(e_0 aE/T_e)$. The cofactor $(e_0 aE/T_e)$ turns out to be of order unity in inert gases, and of a few units in molecular gases. Because $b_e/b_i \approx 10^2 - 10^3$, one has $L/a \gg 1$.

4. Macroparticles in strata in a magnetic field

Current research is demonstrating growing interest in ionization phenomena — and, in particular, in the stratified positive discharge column in a magnetic field — as Coulomb systems of macroparticles in a gas-discharge plasma have become the subject of study [11–13]. In a plasma, a macroparticle is charged by electrons to the potential difference on the order of the electron temperature. For example, macroparticles of radius 1 μm at $T_e \approx 1$ eV acquire a charge of 10^3 electrons [13].

If a macroparticle resides in a discharge plasma in a longitudinal magnetic field, it acquires an azimuthal moment when colliding with rotating ions — i.e., is acted upon by what came to be known as 'ion drag force'. It is to this force that Ref. [14] attributed the magnetomechanical effect. The difference in pressure on the opposite sides of the suspension in a gas at rest is proportional to the difference in the momentum flux of incident particles with an azimuthal velocity that is small compared to the ion thermal velocity. The force acting on the suspension is unidirectional with the azimuthal ion rotation velocity. However, it was found experimentally [4] that the suspended plate deflected in the same sense as an electron moving to the wall. In this work, the magnetic field was induced by two short 15-cm-long coils, making the end face effects likely to be important.

To prevent macroparticles from falling under the influence of gravity, the experiments used layers with a positive electric potential (standing strata). The reader is referred to Refs [15–17] for basic information on the layered discharge (strata). As repeatedly shown, applying a magnetic field causes macroparticles to rotate in the plane perpendicular to the field. This rotation was associated with the action of ion drag forces [13, 18–22]. In addition to the ion drag forces, the motion of macroparticles is also influenced by the friction force from the rotating gas.

The common way to obtain standing strata reduces to locally narrowing the discharge cross section to create on the axis a much higher plasma concentration than that in the main discharge. As the electron gas expands from the region of high plasma concentration, it cools and the atoms become harder to ionize and to excite. As a result, a weakly glowing region forms, with electron diffusion as the current transport mechanism. From Eqn (3), it is seen that at a sufficient concentration gradient electrons can diffuse counter to the operating longitudinal electric field

$$\mathbf{E} = \frac{\mathbf{j}}{e_0 n b_\mathrm{e}} - \frac{T_\mathrm{e}}{e_0}\frac{\nabla n}{n}\,. \tag{4}$$

For a high plasma concentration near a channel narrowing, the first term in Eqn (4) is small, and the negative electric field is determined by electron diffusion. From Eqn (4), it is seen that the electron concentration can only decrease to a certain limit, after which a potential jump arises, necessary to maintain the current at low concentrations. The jump gives rise to a new high-ionization high-concentration plasma region, where a new stratum starts to form. In Refs [20–22], the length of standing strata was on the order of the tube diameter, its determining factor being the plasma escape rate toward the walls. In weak magnetic fields, macroparticles rotated counterclockwise relative to the vector **B**, consistent with how ion drag forces operated and with the rotation of the gas under the influence of the anode end face effect. Increasing the field strength **B** resulted in a reversal of the rotation direction, a fact which cannot be attributed to the change in the radial electric field and is due to end face effects. Figure 1 depicts the magnetic field dependence of the angular rotation frequency of macroparticles obtained in paper [22] from measurements in a horizontal plane about 4 cm apart from the channel narrowing. Close to the channel narrowing, the radial electron current produces in a magnetic field an Ampere force moment which rotates the gas clockwise. The dragging by the rotating gas accounts for the clockwise rotation of macroparticles in high magnetic fields, when the narrowing effect dominates. In addition to the end face effects, the eddy electric current also contributes to gas rotation in strata.

5. Eddy electric current

Additional to the primary discharge current, there is an eddy electric current flowing through strata, as was first pointed out by L D Tsendin [23]. The electron temperature modulation along the discharge axis coexists with a radial plasma density gradient. In regions of elevated electron temperature, the radial electric field is larger than in those with cooler electrons, giving rise to an additional along-the-discharge potential difference which causes an electron current to flow. In the region of maximum T_e, the electron gas expands opposite to the radial electric field vector, converting some of its heat energy to electrical energy which subsequently undergoes Joule dissipation.

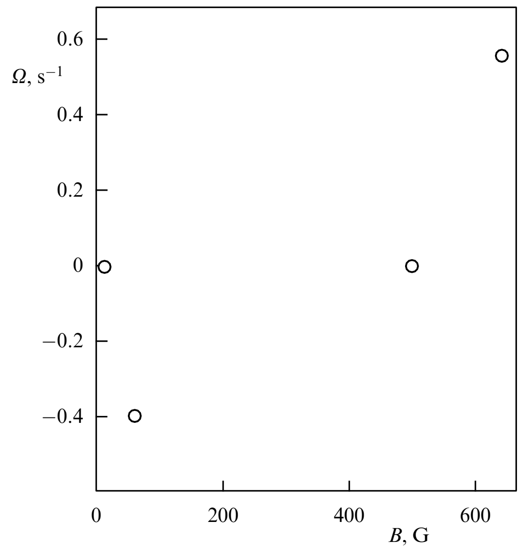

Figure 1. Angular rotation frequency vs magnetic field induction for macroparticles in plasma [22].

In a stratified positive column, the eddy current is due to the fact that the electron concentration and temperature gradients are noncollinear. This follows from equations (1) and (2) by applying the curl operator, assuming for simplicity that collision frequencies are independent of n and T_e:

$$\operatorname{rot}\mathbf{j} = -b_\mathrm{e}\nabla n \times \nabla T_\mathrm{e}\,. \tag{5}$$

The eddy current density is an order of magnitude lower than the discharge current density, so, when added to the main current, the eddy current modulates its radial profile only slightly.

6. Solving the problem of gas rotation in strata

The action of a longitudinal magnetic field on the radial component of an eddy current causes the gas to rotate about the discharge axis. In Refs [24, 25], the plasma radial concentration was taken to have a distribution in the form $n(r) = n_0 J_0(\alpha r/a)$, where α is the first root of the Bessel function. This form was first suggested by W Shockley as a means of solving the diffusion equation $D_\mathrm{a}\,\partial^2 n/\partial r^2 = zn$, where z is the ionization rate per electron (a function of the electron temperature alone), and D_a is the ambipolar diffusion coefficient. According to Shockley's boundary condition, $n(r) \to 0$ as $r \to a$ for a finite plasma flux onto a wall. The electron temperature in the strata is modulated along the discharge, $T_\mathrm{e} = T_0 + \Delta T$, where the periodic function ΔT is determined by the stratum size. Given this behavior of n and T_e, the following equation for the radial component of a current density was obtained from Eqn (5):

$$\frac{\partial^2 j_\mathrm{r}}{\partial r^2} + \frac{1}{r}\frac{\partial j_\mathrm{r}}{\partial r} - \frac{j_\mathrm{r}}{r^2} + \frac{\partial^2 j_\mathrm{r}}{\partial z^2} = \frac{\partial}{\partial z}\left[b_\mathrm{e} n_0 \frac{\partial T}{\partial z}\frac{\partial}{\partial r} J_0\!\left(\frac{\alpha r}{a}\right)\right]. \tag{6}$$

The boundary conditions used are $j_r \to 0$ for $r = 0$ and $r = a$. Multiplying throughout by $r^2\,\partial r$ and integrating over r from 0 to a yields the equation

$$\frac{\partial^2}{\partial z^2}\int_0^a j_r r^2\,\mathrm{d}r = b_e a^2 \beta\,\frac{\partial}{\partial z}\left[n_0(z)\,\frac{\partial T_e}{\partial z}\right]. \tag{7}$$

Here, the numerical factor is $\beta \cong 1.7$.

In a stratum at rest, the action of the Ampere force is balanced by the viscosity forces:

$$\eta\,\frac{\partial^2 \mathbf{v}_\varphi}{\partial r^2} + \mathbf{j}_r \times \mathbf{B} = 0\,, \tag{8}$$

with $\eta = (1/3) m_a n_a v_T \lambda$, where v_φ is the azimuthal rotation velocity of the gas, $v_T = (2T_a/m_a)^{1/2}$ is the thermal velocity of the gas, $\lambda = (n_a\sigma)^{-1}$ is the mean free path, and σ is the gas-kinetic scattering cross section for atoms.

Using Eqn (8), the equation for the azimuthal rotation velocity of the gas follows from equation (7) as [24]

$$\frac{\partial}{\partial z}\int_0^a \frac{\partial^2 v_\varphi}{\partial r^2}\,r^2\,\mathrm{d}r = \frac{\omega_e\tau_e}{\eta}\,\beta a^2 n_0\,\frac{\partial T_e}{\partial z}\,. \tag{9}$$

Having regard to the fact that v_φ vanishes on the tube's axis and walls, integration in Eqn (9) gives the following equation for the cross-section-averaged gas rotation velocity $\bar{v}_\varphi$:

$$\frac{\partial \bar{v}_\varphi}{\partial z} = \frac{1}{2}\,\beta\,\frac{\omega_e\tau_e}{m_a v_T}\,a\sigma n_0\,\frac{\partial T_e}{\partial z}\,. \tag{10}$$

Reference [21] provides evidence for gas rotation in discharge strata at rest. It was found that macroparticles falling through such strata have their paths deflecting sign-alternatingly in the azimuthal direction in the sense that they rotate with different velocities in different parts of a stratum. On the top of a stratum, macroparticles were additionally deflected counterclockwise; on the bottom, motion in the opposite direction was observed, and between, particles did not deflect at all.

Integrating Eqn (10) with due regard for the usual z dependences of n and T_e gives the absolute value of the average gas rotation velocity in the stratum:

$$\bar{v}_\varphi \cong K\omega_e\tau_e a\sigma n_0 \left(\frac{T_{e0}}{2T_a}\right)^{1/2}\left(\frac{T_{e0}}{m_a}\right)^{1/2}, \tag{11}$$

where K is a numerical coefficient of the order of unity. For the conditions of experiments [20–22], formula (11) gives the estimate $\bar{v}_\varphi \cong 3-4$ cm s^{-1}.

Unlike strata at rest, the action of the Ampere force in moving strata determines the acceleration of the gas, which is equal by the order of magnitude to $\partial v_\varphi/\partial t \approx v_\varphi/\tau_d$ (τ_d, the average plasma diffusion lifetime, depends on $\mathbf{B}$). Moving strata produce plasma density oscillations which in inert gases (except for helium) obey the experimentally established scaling law which states that the product of the frequency by the tube radius and the molecular weight of the gas ($fa\mu$) is approximately a function of the ratio of the product of the gas pressure and radius to the ionization potential (pa/φ_i) [26]. The explanation is that, for moving large-amplitude strata showing relaxation oscillations of an ionization–diffusion nature, the characteristic scale of the oscillation frequency is the inverse of the average diffusion lifetime of the plasma: $f \approx 1/\tau_d = \alpha^2 D_a/a^2$ [15].

As the magnetic field grows, the transverse ambipolar diffusion decreases by a factor of $1 + (b_i/b_e)(\omega_e\tau_e)^2$, thus decreasing the strata frequency.

The condition for a gas to rotate in moving strata has the form [25]

$$j_r B + m_a n_a\,\frac{v_\varphi}{\tau_d} = 0\,,$$

giving

$$\frac{\partial \bar{v}_\varphi}{\partial z} \cong \frac{10\beta}{\alpha^2}\,\frac{aBe_0}{m_a}\,\frac{b_e}{b_i}\,\frac{n_0(z)}{n_a}\,\frac{1}{T_e}\,\frac{\partial T_e}{\partial z}\left[1 + \frac{b_i}{b_e}(\omega_e\tau_e)^2\right]. \tag{12}$$

Equation (12) remains true if longitudinal viscosity is allowed for, provided $\partial\bar{v}_\varphi/\partial z \ll 10\bar{v}_\varphi/a$. For large-amplitude strata, integrating Eqn (12) gives the dependence for the absolute magnitude of the average velocity:

$$\bar{v}_\varphi \cong a\omega_e\,\frac{T_{e\,\max}}{T_e}\,\frac{n_{\max}}{n_a}\left[\frac{b_e}{b_i} + (\omega_e\tau_e)^2\right]. \tag{13}$$

The values of $n_{\max}$ and $T_{e\,\max}$ refer to the region where the electron concentration and temperature exhibit jumps. According to Eqn (13), the velocity of rotation is proportional to the magnetic field induction and current strength. For large rotation velocities, this proportionality breaks down due to the gas viscosity and to the transition to gasdynamic turbulence.

As an example, consider the calculation of the gas rotation velocity in running strata for a neon pressure of 0.38 Torr, a magnetic field of 500 G, a current of 0.1 A, and a stratum frequency of 9 kHz [27]. Under these conditions, the rotation velocity would be $\bar{v}_\varphi \approx 10^5$ cm s^{-1}, i.e., close to the sound speed — a speed at which gasdynamic turbulence plays an essential role. Expression (13) can only be used for low currents and low Reynolds numbers.

In a stratum moving in a magnetic field, a macroparticle will perform small azimuthal oscillations under the action of a periodic friction force from the rotating gas. Unlike standing strata, the sign-alternating rotation of the gas in moving strata does not affect the motion of the particle, i.e., the azimuthal rotation of the macroparticles is determined by the ion drag forces.

7. Conclusion

The motion of charged macroparticles in a discharge ignited in a longitudinal magnetic field is affected by the rotation of the neutral gas due to the moment of the Ampere force from the current flowing at an angle to the magnetic field. Such a moment shows its worth at the end faces of a solenoid where the longitudinal magnetic field changes; close to the discharge channel narrowings; and if an eddy current arises due to noncollinearities of the electron concentration and temperature gradients. If several factors are present at the same time, the motion of the macroparticles differs from that due to the action of ion drag forces alone. The present review does not consider how dusty structures are influenced by the electric field in a plasma with a nonuniform electron temperature, nor the action of thermophoretic forces in a nonuniformly heated gas. Until independent measurements of the gas rotation velocity are available, it cannot be estimated other than from the motion of macroparticles. It is hoped that laser spectroscopy and laser anemometry will provide a means to measure the rotation velocity of a gas and thus to experimentally verify the existing theoretical models.

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