

# Use of Cherenkov counters in experiments at accelerators for particle identification

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**Abstract.** The key properties of Cherenkov radiation are briefly discussed, together with various types of Cherenkov counters used to identify relativistic particles in accelerator studies. Specific examples of the application of Cherenkov counters in experiments are discussed. Particular attention is given to the role of Cherenkov counters in antinuclei observations, including the discovery of the antiproton.

**Keywords:** particle, radiation, counter, facility, accelerator, photon, proton, antiproton, antitritium, antihelium

## 1. Introduction

The identification of long-lived charged particles, i.e., the determination of their characteristics and, primarily, of their masses, is one of the most important and difficult tasks of experimental high-energy physics (in this article, those particles are considered long-lived that during their lifetime cover a distance sufficient for their effective registration in an experimental installation). The mass  $m$  of a particle can be determined by measuring its energy  $E$  or momentum  $p$  and velocity  $\beta = v/c$  or Lorentz factor  $\gamma = 1/(1 - \beta^2)^{1/2}$ :  $mc^2 = E/\gamma = \beta p/\gamma$ . The energy and momentum of a particle are measured with the aid of magnetic spectrometers and calorimeters, which are not dealt with in this article. Particle velocities can be determined by their time of flight along a certain base between scintillation counters. On the basis of

recent achievements in the accuracy of time measurement with the aid of scintillation detectors (several dozen picoseconds) and bases of  $\sim 10$  m, it happens to be possible to identify particles by their mass for Lorentz factors up to  $\gamma \sim 10$ . But in modern experiments, the necessity often arises of identifying particles with Lorentz factors varying within a broad range up to  $10^4$  and even more. In such cases, aid is rendered by Cherenkov counters.<sup>1</sup>

## 2. Main properties of Cherenkov radiation

Cherenkov radiation is known to originate when a particle travels in a medium with a velocity exceeding the propagation velocity of electromagnetic radiation. In this case, radiation with a frequency  $\omega$  is emitted at an angle  $\theta$  to the particle propagation direction, which, with an accuracy up to a small quantum correction ( $\sim 10^{-6}$ ), is determined by the formula

$$\cos \theta = \frac{1}{n(\omega)\beta}, \quad (1)$$

where  $n(\omega)$  is the refraction index of the medium.

The energy emitted per path length  $L$  within a unit frequency interval is expressed as

$$\frac{d^2 W}{d\omega dL} = \left(\frac{ze}{c}\right)^2 \left(1 - \frac{1}{\beta^2 n^2}\right) \omega, \quad \beta n > 1,$$

where  $ze$  is the particle charge. If dispersion of the medium is neglected, then the number of photons  $N$  emitted in the wavelength interval  $(\lambda_1, \lambda_2)$  is determined by the formula

$$N = 2\pi\alpha(ze)^2 L \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) \sin^2 \theta, \quad \lambda_1 < \lambda_2, \quad (2)$$

<sup>1</sup> Sometimes, for the identification of ultrarelativistic particles with Lorentz factors of  $\sim 10^3$ , use is made of transition radiation detectors, the consideration of which is beyond the scope of this article.

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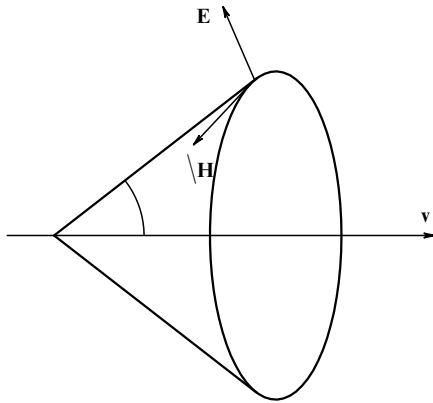


Figure 1. Polarization of Cherenkov radiation.

where  $\alpha = 1/137$  is the fine-structure constant. Cherenkov radiation is polarized: the electric vector is perpendicular to the surface of the emission cone, while the magnetic vector is directed along the tangent to this surface (Fig. 1).

### 3. Cherenkov counter

To determine particle velocities with the aid of Cherenkov detectors, use is made, as a rule, of relationship (1). The simplest of these detectors is the threshold detector which registers all particles with velocities superior to the threshold value:  $\beta_t > 1/n$ . Threshold counters, understandably, are involved for registration of the lightest particles. One possible arrangement of a threshold counter is demonstrated in Fig. 2. The Cherenkov radiation from a beam of particles moving along its axis from right to left is deflected by a flat mirror through  $90^\circ$  and collected with the aid of a conical mirror on a photoelectron multiplier (PM). Gases [air, freon ( $\text{CF}_4$ ), and others] serve as the radiators in threshold counters. By altering the pressure and sort of gas, it is possible to choose the appropriate  $\beta_t$  value.

Another type of Cherenkov detectors utilized for measuring particle velocities is the differential counter (Fig. 3), which registers Cherenkov radiation emitted in a narrow interval of angles  $\Delta\theta$  relative to the optical axis of the counter. Light collection in these detectors is usually done with the aid of spherical mirrors that focus Cherenkov light of frequency  $\omega$  onto a ring of radius

$$r(\omega) = 0.5R \tan \theta(\omega) \quad (3)$$

in the focal plane of the mirror, i.e., at a distance of  $0.5R$  from its center ( $R$  is the mirror radius). Owing to the dependence of the emission angle on frequency (chromatism of Cherenkov radiation), the ring turns out to be colored: the outer radius exhibits a blue hue, while the inner one is reddish. If a particle moves parallel to the optical axis of the counter, the center of the circle lies on it. If  $\theta_p$  and  $\varphi_p$  are the angular coordinates of the particle with respect to the axis, then  $r_0$ ,  $\varphi_0$  are the polar coordinates of the ring's center in the focal plane, which are expressed by the formulae

$$r_0 = 0.5R \tan \theta_p, \quad \varphi_0 = \varphi_p. \quad (4)$$

The average radius and slot width of a ring diaphragm placed in the focal plane determine the average velocity  $\beta$  and the velocity interval  $\Delta\beta$  of the particles registered. The light

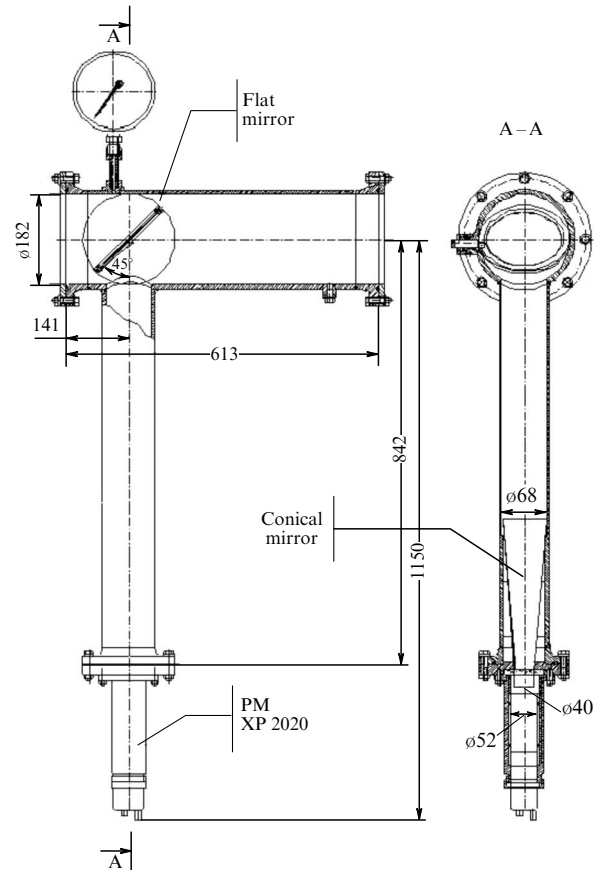


Figure 2. Arrangement of a threshold counter [1]. Dimensions are marked in millimeters.

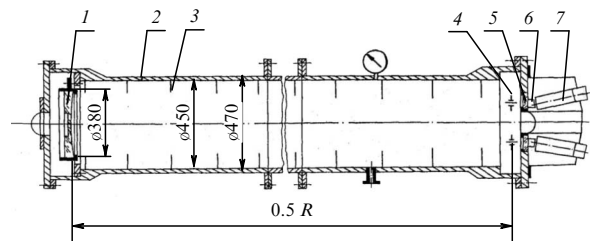


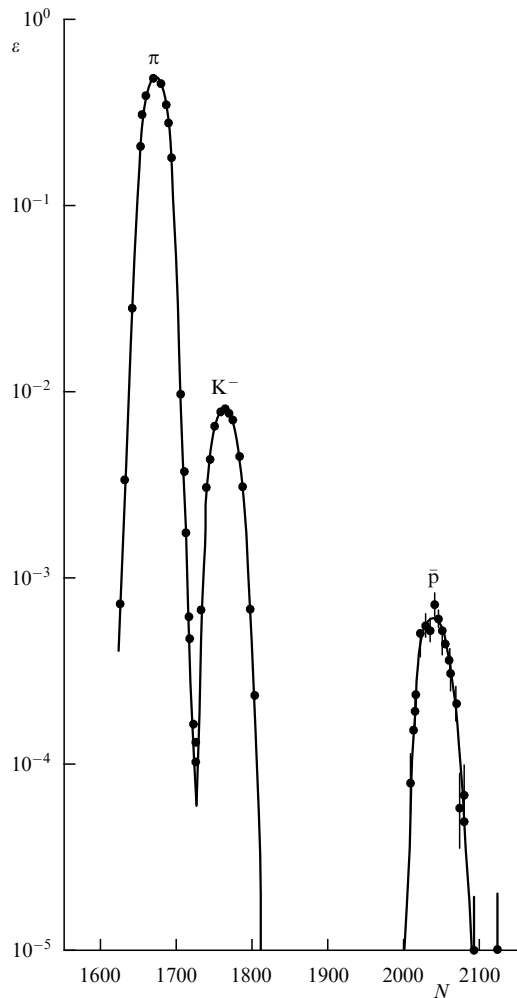
Figure 3. Typical construction of a differential counter [2]: 1 — spherical mirror of radius  $R$ , 2 — housing, 3 — blinds, 4 — ring diaphragm, 5 — quartz window, 6 — quartz prism, and 7 — PM. Dimensions are marked in millimeters.

passing through the diaphragm, the quartz window and prism hits the photomultipliers situated around the ring. The blinds fastened onto the counter housing prevent background radiation from hitting the PM. We note that, owing to the fact that the paths of Cherenkov photons are isochronous, all the light hits the PM practically at the same time, independently of the emission point.

From formula (1) it follows that the velocity resolution of a differential counter is defined by the expression

$$\frac{\Delta\beta}{\beta} = \Delta\theta \tan \theta + \frac{\Delta n}{n}. \quad (5)$$

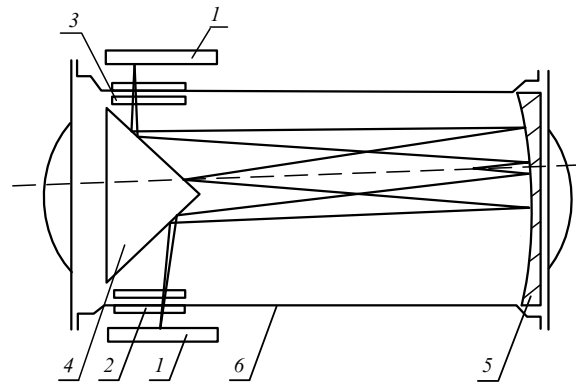
As is seen from Eqn (5), to achieve a high velocity resolution necessary for the identification of ultrarelativistic particles in modern accelerators, small  $\theta$  values must be examined, i.e.,



**Figure 4.** Dependence of 45 GeV/c negative particle counting efficiency  $\varepsilon$  of differential counter [2] with mirror radius  $R = 10.3$  m on  $N$ , where  $N$  is the number of interferometer bands, which is proportional to the helium pressure in the counter.

gas radiators should be used. Here, at the same time small values of  $\Delta n/n$  can be achieved, and the possibility arises of readily varying the refraction index of the counter radiator by changing the gas pressure  $P$  ( $n - 1 \sim P$ ). We note that the  $\theta$  value cannot be chosen arbitrarily small, since the Cherenkov radiation intensity (2) depends on it, as, consequently, does the particle registration efficiency. Sometimes, in order to reduce the value of  $\Delta n/n$ , a special conical optical device is established in front of the diaphragm [3] to compensate the chromatism of Cherenkov radiation.

Figure 4 demonstrates the dependence of the counting efficiency of negative particles in a beam of momentum 45 GeV/c on the helium pressure in the differential counter shown in Fig. 3. The radius of the mirror was 10.3 m. The Cherenkov radiation emitted at an angle of 23 mrad was registered by 12 XP 1023 PMs. The signals from three adjacent PMs were input into logical summators, the output pulses of which were input into a fourfold coincidence circuit. The velocity resolution of the counter,  $\Delta\beta$  (FWHM)/ $\beta$ , amounted to  $2.2 \times 10^{-5}$  (FWHM — full width at half maximum). Although the admixtures of kaons and antiprotons in the beam comprised less than 1% and 0.1%, respectively, their identification is quite clear owing to the high resolution and low ( $< 10^{-5}$ ) background level.



**Figure 5.** Layout of SCRR counter with HPM: 1 — HPM, 2 — glass window, 3 — cylindrical lens, 4 — conical mirror, 5 — spherical mirror, and 6 — housing.

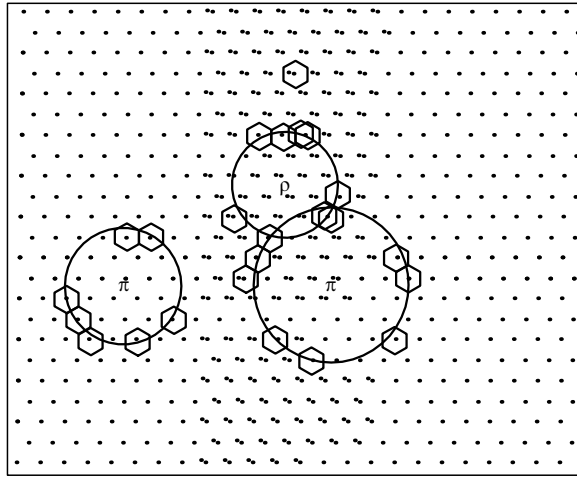
In accordance with expression (5), differential Cherenkov counters effectively register particles only at angles within a narrow interval  $\Delta\theta$  relative to the optical axis; therefore, they are applied in accelerators mainly for implementing a sole, but very important, task, namely, for the particle selection in well collimated beams. The counters involved in measuring particle velocities within a broad angular interval are of the RICH (Ring Imaging Cherenkov) type detector, and they are similar in construction to differential counters. However, RICH counters have no diaphragm, and the detectors are established in the focal plane of the spherical mirror that permit the coordinates of the photon registered to be determined. Upon retrieval of the radius and center of the Cherenkov radiation ring from the photon coordinates it is possible to determine not only the velocity, but also, in accordance with formula (4), the angular coordinates of the particle trajectory. With the aid of RICH counters, it is also possible to register several particles simultaneously (see below).

Apparently, one of the first RICH type counters was the counter MCC<sup>2</sup> [4], with original hodoscopic photoelectron multipliers (HPMs) [5] developed at the Institute for High Energy Physics (IHEP). HPMs had an extended photocathode that was put in crossed electric and magnetic fields, and the coordinate of a photon could be determined with an accuracy of better than 1 mm from the drift time of the photoelectron in these fields. The MCC counter was created for an experiment on searching for antitritium nuclei, and its application permitted the reliable registration of such nuclei for the first time. HPMs were also utilized in the SCRR<sup>3</sup> counter [6], in which light collection was effected by spherical and conical mirrors and by cylindrical lenses (Fig. 5). SCRR counters are invoked for the identification of particles in the FDAS (focusing double-arm spectrometer) at the IHEP accelerator.

Another example of RICH counters is provided by those counters designed and developed at IHEP, in which the focal plane of a spherical mirror was filled with small-sized PM-60 photomultipliers with photocathodes 10 mm in diameter. One such counter [7] with 736 PMs was developed for the Sphinx installation representing a universal wide-aperture spectrometer at IHEP, and another one [8] involving 2848 PMs has

<sup>2</sup> MCC — abbreviation of ‘Majority coincidence circuit’.

<sup>3</sup> SCRR — abbreviation. of ‘Spectrometer of Cherenkov radiation rings’.



**Figure 6.** Particle identification in one of the events registered by the Sphinx installation with the aid of a Cherenkov counter involving PM-60 photomultipliers in the focal plane. The fired photomultipliers are marked by hexagons.

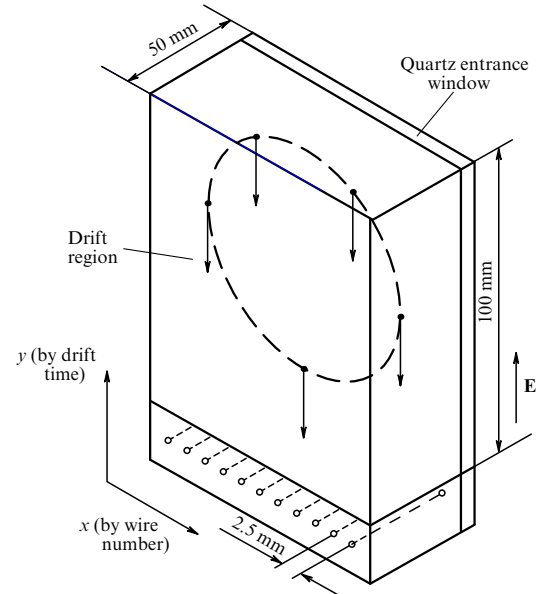
been implicated in the SELEX (SEgmented LargeE X baryon spectrometer) experiment at the Fermi National Accelerator Laboratory (Fermilab). Figure 6 illustrates identification of secondary particles by a RICH counter in one of the events registered by the Sphinx installation.

In RICH counters, use is also made of drift chambers filled with gases exhibiting low photoeffect thresholds. An example of the construction of such a chamber [9] is shown in Fig. 7. One of the largest counters [10] with drift chambers used in an experiment with the Omega spectrometer in a study of hadron photoproduction at the SPS (Super Proton Synchrotron) accelerator at CERN is sketched in Fig. 8.

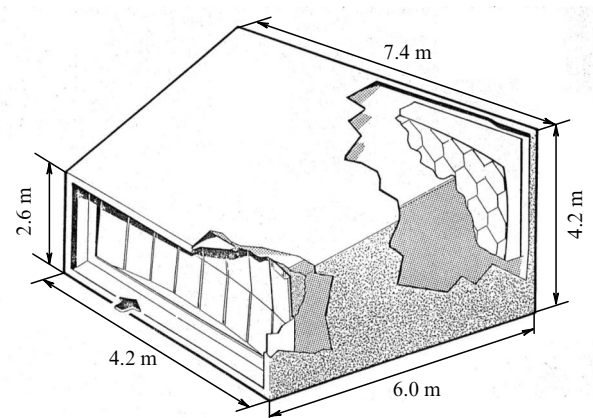
#### 4. Role of Cherenkov counters in the discovery of antinuclei

One of the most glorious episodes in the history of Cherenkov detector application is likely related to the discovery of the antiproton. After the prediction by P Dirac of the existence of an electron with a positive charge (antielectron) [11] and the discovery by Anderson [12] of the positron, a question arose concerning the existence of antinuclei, first of all, of the antiproton. The positive answer concerning the antiproton was given by Chamberlain, Segre, Wiegand, and Ypsilantis [13] in an elegant and, at the same time, surprisingly simple (from today's point of view) experiment at the Betatron in the USA, for which the first two were awarded the Nobel Prize in Physics 1959.

The experimental device [13] involved only five detectors: three scintillation counters, and two Cherenkov counters (Fig. 9). The momenta of negative-charged particles in a beam of initial energy 1.19 GeV (the velocity of antiprotons was  $\beta = 0.78$ ) from an internal copper target T were analyzed twice with the aid of magnets M1 and M2 and focused by triplets of lenses, Q1 and Q2, on scintillation counters S1 and S2, which were used to determine the mean velocity of particles by their time of flight. The threshold counter C1 with a  $C_8F_{16}O$  radiator ( $n = 1.276$ ,  $\beta_t = 0.79$ ) registered particles with masses below the proton mass. A differential counter C2 [14] with a quartz radiator permitted determining the velocity of particles of antiproton mass with a precision of



**Figure 7.** Layout of drift chamber with quartz window for registering Cherenkov radiation in RICH type counters. The  $y$  coordinate is determined from the photoelectron drift time, while the  $x$  coordinate from the number of a fired anode wire. Arrows indicate the drift directions of photoelectrons.

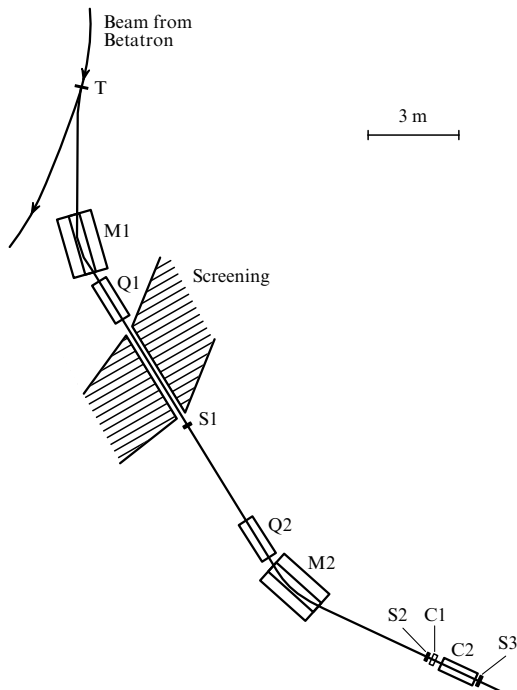


**Figure 8.** RICH type counter for WA69 experiment at the SPS accelerator of CERN.

$\Delta\beta = \pm 0.014$ . To test and to determine the characteristics of all the elements of the experimental device, protons of the same energy, 1.19 GeV, were injected into the channel.

In all, 60 candidates for antiprotons were registered during the experiment [13] against a practically zero background, although the fraction of antiprotons in the beam only amounted to  $\sim 2.5 \times 10^{-5}$ . The masses of all the candidates coincided with the proton mass with an accuracy up to 5%, and this was the main argument in favor of antiprotons actually having indeed been observed. Cherenkov detectors played an extremely important role in this experiment by providing the necessary background suppression (the threshold counter) and for reliable identification of the antiprotons by the measurement of their velocities (the differential counter).

After the antiproton observation, discoveries were made of antideuteron [15] at the AGS (alternating gradient synchrotron) accelerator in the Brookhaven National



**Figure 9.** Layout of experiment [13] wherein the antiproton was discovered: T—copper target, M1, M2—magnets, Q1, Q2—triplets of quadrupole lenses, S1, S2, S3—scintillation counters, C1—threshold Cherenkov counter, and C2—differential Cherenkov counter.

Laboratory (BNL), of the aforementioned antitritium nuclei (at the 70-GeV accelerator of IHEP), and of antihelium-3 [16] (also at the IHEP accelerator). We shall dwell on the last experiment in more detail, since Cherenkov counters were employed in it not only for determining the local particle velocity, but also for accomplishing a number of other tasks.

The layout of the experiment reported in Ref. [16] is conceptually close to the one shown in Fig. 9. But the installation was much more complex, since the expected signal/background ratio was about six orders of magnitude smaller than in the experiment of Ref. [13]. Use was made in the installation of five magnets for momentum analysis of the particle beam [the initial momentum was 10 GeV/c for particles with  $z = 1$ , and 20 GeV/c for antihelium ( $z = 2$ )], of ten spectrometric scintillation counters for charge measurements by a signal amplitude, which is proportional to  $z^2$ , of three scintillation counters for measuring the mean particle velocity along the channel, of three threshold Cherenkov counters, and of two differential Cherenkov counters situated at the beginning and the end of the installation. The task of the threshold counters consisted in suppressing the background of particles with masses smaller than the mass of antihelium, with the exception of antideuterons, the velocity of which at a momentum of 10 GeV/c was lower than the velocity of antihelium-3 nuclei with a momentum of 20 GeV/c. The threshold counters fulfilled this task excellently, providing a rejection coefficient exceeding  $10^7$  owing to a very high efficiency: 99.99996%.

The differential counters were used to resolve several problems at the same time:

- to select particles with local velocities of antihelium-3;
- to determine mean particle velocities by flight time measurements through the installation;

— to suppress the background of antideuterons, the number of which in the beam was expected to be four orders of magnitude superior to the number of antihelium-3 nuclei;

— to determine particle charges by the signal amplitude, which, as in scintillation counters, is proportional to  $z^2$  [see relationship (2)].

Owing to the good velocity resolution,  $\Delta\beta/\beta = 1.7 \times 10^{-3}$ , the high time resolution,  $2\tau = 0.7$  ns, and the low ( $< 10^{-4}$ ) background level, all the mentioned tasks were successfully fulfilled, and expectations for the Cherenkov counters turned out to be fully justified. It should be noted that the installation was occasionally readjusted for particle beams of momentum 13.3 GeV/c, which permitted calibrating the differential counters with antideuterons and controlling their stability, which turned out to be quite high.

Analysis of data from all detectors made it possible to select five events of antihelium-3 nuclei passing through the installation with a practically zero background level. Without belittling in any way the role of scintillation counters, it must be acknowledged in any case that a decisive part in the success of the experiment was played by Cherenkov detectors.

The discovery of antihelium-3 nuclei ultimately demonstrated that antimatter should exist and that, if the Universe indeed originated as a result of the Big Bang, then antiworlds should also exist. Why it has not been possible to observe them yet is one of the most surprising riddles of modern astrophysics.

To conclude the discussion of the issue of antinuclei, we shall mention one more study [17] devoted to the investigation of the production of heavy nuclei and antinuclei in pBe- and pAl-collisions at the SPS accelerator. The equipment composition of the experiment was close to that in the experiment at IHEP [16] described above. Thus, for example, use was made in the experiment of Ref. [16] of three threshold Cherenkov counters to reject the background of light particles, and of two differential counters [18, 19] involving compensation for the Cherenkov radiation chromatism. One of the results of this experiment consisted in confirmation of the existence of antihelium-3 nuclei. Antihelium-4 nuclei were first registered in the STAR (solenoidal tracker at relativistic Heavy Ion Collider) experiment [20] at the Heavy Ion Collider of the Brookhaven National Laboratory.

## 5. Application of Cherenkov counters in other experiments

Let us present several other examples of the application of Cherenkov detectors in accelerator experiments.

Owing to reliable identification of  $K^+$ -mesons with the aid of the differential counter shown in Fig. 3 and of the threshold counter [21] exhibiting high efficiency, it turned out to be possible in the experiment at the IHEP accelerator [22] to reveal for the first time an increase in the  $K^+p$ -interaction total cross sections in the energy region of 20–60 GeV. Although the fraction of  $K^+$ -mesons in the beam reached  $\sim 1\%$ , the background level under their peak in the dependence of efficiency on the working gas pressure (refraction index) never exceeded  $10^{-3}$ . Subsequent investigations confirmed this discovery and showed that the increase in total cross sections with energy is a common property of all hadron interactions at high collision energies.

In 1974, the search for new particles decaying into lepton pairs  $e^+e^-$  and  $\mu^+\mu^-$  performed with the BNL double-arm spectrometer [23]. Threshold Cherenkov counters filled with

hydrogen were utilized for the identification of electrons. A narrow peak at 3.1 GeV was found in the spectrum of effective  $e^+e^-$  masses, which was called the J-particle (at present it is known as the  $J/\psi$ -meson) and turned out to be a bound state of the new  $c$ - and  $\bar{c}$ -quarks. For this remarkable discovery, the experiment leader S Ting (together with B Richter) was awarded the Nobel Prize in Physics 1976 [24, 25].

The application of Cherenkov counters also contributed to the discovery of the  $b$ -quark at the Fermilab accelerator.

The employment of Cherenkov detectors is also widespread in searches for different sorts of exotic particles. Thus, for instance, tachyons (hypothetical particles traveling with a velocity superior to the speed of light in vacuum,  $c$ ) should emit Cherenkov light in a vacuum. However, searches for such particles with the aid of Cherenkov counters with a very low gas pressure have given no positive results yet. An interesting proposal for searching for the Dirac monopole was made by V P Zrelov (and implemented by him at the IHEP accelerator), who drew attention to two features peculiar to the Cherenkov radiation of monopoles:

— the ratio of energies emitted by a monopole with a magnetic charge  $g$  and an ordinary particle of electric charge  $e$  moving with the same speed in a medium of refraction index  $n$  is equal to  $n^2 g^2 / e^2$ , i.e., reaches  $\sim 10^4$  for a Dirac monopole of charge  $g = 68.5e$  and  $n = 1.5$ ;

— the monopole radiation polarization differs in the direction from the light polarization from a charged particle (see Fig. 1) by  $90^\circ$ .

The experimental device of Ref. [26] used in searching for the Dirac monopole consisted of eight photon detectors surrounding a quartz target-radiator placed inside the accelerator chamber. In front of the detectors there were polaroid films transmitting light with a certain polarization. Six films were oriented so as to transmit Cherenkov light from monopoles, and two from ordinary particles. During the experiment,  $7.7 \times 10^{15}$  protons were transmitted through the target-radiator, but not a single monopole candidate was revealed. The upper limit of the monopole production cross section turned out to be  $10^{-40} \text{ cm}^2$  at a confidence level of 95%.

## 6. Conclusion

Only a small number of the examples dealing with the utilization of Cherenkov detectors in accelerator experiments has been cited in this article. But even what is presented suffices to estimate their extremely important role for studies in particle physics. In the future, Cherenkov counters will doubtlessly remain one of the principal instruments employed by experimental physicists in their work.

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