Physics – Uspekhi **58** (4) 315 – 329 (2015)

REVIEWS OF TOPICAL PROBLEMS PACS numbers: 73.20.Mf, 73.43.Lp, **78.67. – n**

Magnetoexcitons in two-dimensional electronic systems

V E Bisti, A B Van'kov, A S Zhuravlev, L V Kulik

DOI: 10.3367/UFNe.0185.201504a.0337

Contents

Abstract. Experimental and theoretical research on neutral excitations in a two-dimensional electron gas in a strong magnetic field is reviewed. Methods for calculating excitation energies in the strong-field limit for integer and noninteger filling factors are considered. The effects of impurities and of the nonideality of the two-dimensional system on the excitation spectrum are examined. Experimental results that have been obtained by the method of inelastic light scattering and that lend support to the current theoretical views are presented. We also discuss possible avenues of future experimental and theoretical work.

Keywords: magnetoexcitons, magnetoplasmons, collective excitations, high magnetic fields

1. Introduction

Magnetoexcitons are neutral collective excitations that occur in a two-dimensional electronic system (2DES) in a transverse external magnetic field. The work done at the dawn of the physics of low-dimensional systems [1, 2] laid a theoretical foundation for the theory of magnetoexcitons. As time went on and higher-quality objects became available for study, this work received experimental confirmation, which in turn stimulated further theoretical advances. Both these first [1, 2] and subsequent [3, 4] studies used the strong magnetic field approximation to determine the magnetoexciton energy, reducing the multiparticle problem of excitations in a strongly interacting electronic system to a two-particle scheme.

There are a number of exact solutions for the dispersion of magnetoexcitons with relatively small integer filling factors, when the spin or orbital quantum number of the electronic system changes by unity. However, the complete spectrum of magnetoexcitons in an electronic system even with a single filled Landau spin sublevel has been calculated only partially. Calculating the energies of all possible excitations is a major theoretical problem. Correspondingly, an increase in the number of filled Landau levels greatly complicates the problem of 2DES excitations.

Allowing for dimensional effects, for the fact that only finite magnetic fields can be reached, and for the presence of a random potential in real two-dimensional systems greatly complicates the analysis; these factors are often crucial when comparing theory and experiment. Finally, even a small deviation of the filling factor from an integer value seems to alter the ground state of the electronic system so much that the theoretical approaches developed for calculating excitations for integer filling factors lose their validity.

In this review, we attempt to trace the recent development of our understanding of magnetoexcitons in two-dimensional systems, to link new theoretical ideas with our results obtained from inelastic light scattering experiments, and to identify the problems to be addressed to further develop the theory of magnetoexcitons.

2. Theory of magnetoexcitons in a two-dimensional electron gas. First-order perturbation theory. Hartree–Fock approximation. Single-mode approximation

According to Kohn's theory [5], the cyclotron resonance energy of a translationally invariant electronic system is

V E Bisti, A B Van'kov, A S Zhuravlev, L V Kulik Institute of Solid State Physics, Russian Academy of Sciences,
ul. Akademika Osipyana 2, 142432 Chernogolovka, Moscow region, Russian Federation
E-mail: bisti@issp.ac.ru; vankov@issp.ac.ru; kulik@issp.ac.ru

Received 7 July 2014, revised 21 January 2015
Uspekhi Fizicheskikh Nauk **185** (4) 337 – 352 (2015)
DOI: 10.3367/UFNr.0185.201504a.0337
Translated by E G Strel'chenko; edited by A M Semikhatov

independent of the Coulomb interaction. It follows that if the system is excited by changing the orbital quantum number by unity (magnetoplasmon, MP), then, for the zero momentum $q = 0$, the excitation energy is equal to the cyclotron resonance energy of a noninteracting electronic system. A similar statement applies to spin excitons, excitations in which the spin quantum number changes by unity while the orbital quantum number remains unchanged, which occur in electronic systems invariant under the action of a group of spin state rotations (Larmor's theorem). As a consequence, the energy of a zero-momentum exciton is equal to the Zeeman energy. Both above statements are exact, i.e., both hold independently of the value of the filling factor and of the relation between the Coulomb and cyclotron energies.

If the Landau level filling factors have integer values, then, in the strong magnetic field limit, the exciton and magnetoexciton energies for a nonzero q can be calculated by the perturbation theory in the ratio of the Coulomb energy at the interparticle distance to the cyclotron energy, $E_c/\hbar\omega_c \ll 1$ (here, $E_c = e^2/\varepsilon l_B$, l_B is the magnetic energy, $\omega_c = eB/m^*c$, m^* is the effective electron mass, and ε is the dielectric constant).

Collective excitations in a 2DES in a magnetic field were first treated theoretically [1] by modeling the magnetoexciton as a pair of charged particles, an electron and a hole, with the electron promoted to an unfilled electronic state, the hole remaining in its initial state, and the interaction of the electron with the hole and all other electrons being the Coulomb one. The study derived dispersion relations for the magnetoexciton and the spin exciton for the integer filling factor $\nu = 1$. For small momenta, $ql_B \ll 1$, the magnetoplasmon and the spin exciton have the respective linear and quadratic dispersion

$$E_{MP}(q) = \hbar\omega_c + \frac{e^2}{2\varepsilon l_B}\, ql_B\,,$$

$$E_s(q) = g\mu_B B + \frac{e^2}{4\varepsilon l_B}\,(ql_B)^2\,,$$

where g is the electron g-factor and μ_B is the Bohr magneton.

Similar results were obtained in [2] for a two-dimensional system with electrons in the conduction band and holes in the valence band.

A more detailed analysis of the magnetoexciton spectrum using the diagram formalism [3] and (equivalently) the time-dependent Hartree–Fock (TDHF) approximation [4] for excitations led to the same results.

Importantly, the problem of magnetoexcition energy admits an exact solution only for integer-filled states for which the ground state of the system is nondegenerate. In general, the magnetoexciton energy can be written in the form

$$E_{n,n'}(q) = \hbar\omega_c\delta n + g\mu_B B\delta S_z + \Delta E_{n,n'}(q)\,,$$

which includes contributions from the change in the orbital quantum number (Landau level index of the electron, n), from the change in the Zeeman energy due to electron spin flip, and from the Coulomb interaction, $\Delta E_{n,n'} \sim E_c$. The last contribution determines the excitation dispersion as a function of the generalized momentum (an integral of motion) given by

$$\mathbf{q} = \left[-\mathrm{i}\hbar(\nabla_1 + \nabla_2) - \frac{e}{c}\,(\mathbf{A}_2 - \mathbf{A}_1)\right] - \frac{e}{c}\,\mathbf{B}\times(\mathbf{r}_2 - \mathbf{r}_1), \quad (1)$$

where the indices 1 and 2 respectively refer to the negative and positive-charge particles, $\mathbf{A}_{1,2}$ are the magnetic field vector potentials, and $\mathbf{r}_{1,2}$ are the radius vectors of the particles. The effective electron-to-hole distance is proportional to the generalized momentum $\langle\Delta\mathbf{r}\rangle = l_B^2\,\mathbf{q}\times\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is the vector normal to the surface.

As noticed in Refs [3, 4], collective excitations in a two-dimensional electron gas can be considered two-dimensional only for transitions from a filled to the next Landau level (the orbital quantum number change $\delta n = 1$, $\delta S = 0$) or for spin-flip transitions with $\delta n = 0$, $\delta S = 1$. In the case of no impurities and $T = 0$, magnetoexcitations and spin excitons have an infinite lifetime, because there are no other excited 2DES states with the same momentum and spin. For cyclotron spin-flip excitations (CSFEs), $\delta n = 1$ and $\delta S = 1$, and for $\delta n > 1$ transitions, the two-particle approximation breaks down.

It should be remembered that a magnetic exciton can also undergo decays into other excitations as long as the energy, wave vector, spin projection, and orbital momentum are conserved. The interaction with other 2DES excitations affects not only the lifetime but also the dispersion relations of magnetiexcitons. We should therefore recognize that in a 2DES, a multiexciton complex exists whose energy can be determined in the framework of the two-exciton approximation. The traditional theory of magnetoexcitons usually neglects two-exciton corrections [3, 4, 6].

For an even filling factor, different-spin 2DES states are filled equally. The ground state has the total spin $S = 0$, whereas excitations with $\delta n = 1$ can be classified as singlet and triplet; we note that the singlet magnetoexciton is a Kohn magnetoplasmon, a synphase combination of two $\delta S_z = 0$ transitions. The components of triplet excitations are spin-split and form combined spin-flip excitations across the cyclotron gap. For odd filling factors, i.e., when two spin states are filled differently, the singlet–triplet classification is not appropriate. In this case, there are two types of magnetoplasmon excitations for $\delta n = 1$: synphase (Kohn type) and antiphase; combined spin-flip cyclotron excitations are also possible.

The Hamiltonian of a two-dimensional electronic system in a magnetic field has the form

$$\hat{H} = \sum\left(n + \frac{1}{2}\right)\omega_c\hat{a}^+_{\sigma nk}\hat{a}_{\sigma nk} + \hat{H}_{int}\,,$$

where $\hat{a}^+_{\sigma nk}$ and $\hat{a}_{\sigma nk}$ are the electron creation and annihilation operators for the nth Landau level, k labels degenerate states within one Landau level, and σ is the spin. In the Landau gauge, the electron wave functions in the one-particle approximation have the form

$$\phi_{n,k} = N^{-1/2}\exp(\mathrm{i}ky)\exp\left[-\frac{(x + kl_B^2)^2}{2l_B^2}\right]H_n\left(\frac{x + kl^2}{l_B}\right)$$

(N is the normalization coefficient and H_n are the Hermite polynomials). The interaction Hamiltonian in a magnetic field is determined by the matrix elements of the Coulomb interaction $V(\mathbf{r}_1 - \mathbf{r}_2)$ between these functions. The Hartree–Fock approximation introduces corresponding creation operators for excitations with a momentum q along the y axis,

$$\hat{Q}^+_Y(q) = \hat{Q}^+_{nm\sigma\sigma'}(q) = \sum_k \hat{a}^+_{\sigma',m,k+q}\hat{a}_{\sigma,n,k}\,, \quad (2)$$

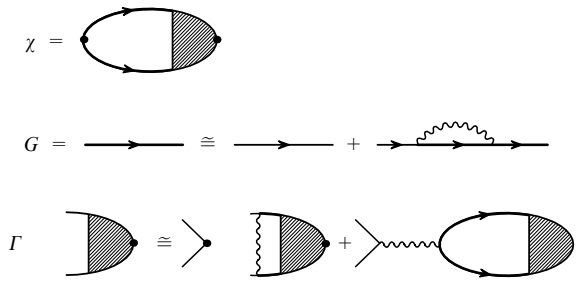

Figure 1. Charge density response function $\chi_{\sigma\sigma}(\mathbf{q},\omega)$ in the strong magnetic field approximation. $\Gamma_{\alpha\beta}(k,k',\mathbf{q},\omega)$ is the vertex part and $G_\alpha(\omega)$ is the one-particle Green's function. Thin lines with arrows: one-particle Green's functions for noninteracting particles: wavy lines: unscreened Coulomb interaction.

and the corresponding equation for the energy,

$$\left\langle \hat{Q}_Y^+(q)\left[\hat{H}\hat{Q}_{Y'}(q)\right]\right\rangle - \omega(q)\,\hat{I} = 0\,. \tag{3}$$

In the diagram formalism, the charge- and spin-density response functions $\chi_{\sigma\sigma'}(\mathbf{q},\omega)$ can be written in terms of the Green's function of an electron in a magnetic field $G_\alpha(\omega)$, where $\alpha = n_\alpha, \sigma_\alpha$ are the Landau level and spin indices, and in terms of the vertex part $\Gamma_{\alpha\beta}(k,k',q,\omega)$, which is the sum of the ladder and loop diagrams (Fig. 1); we note that the Green's function incorporates exchange-interaction corrections. In the first-order perturbation theory, the Coulomb interaction contribution to the excitation energy consists of the following three terms, corresponding to the diagrams shown in Fig. 1:

(1) a constant (momentum- and excitation energy-independent) term that represents the exchange self-energy difference between the excited electron and the hole at the Landau level from which the electron was removed, $\Sigma_{n'} - \Sigma_n$;

(2) the direct electron–hole Coulomb interaction, which depends on the excitation momentum (the second term in the diagram equation for the vertex part Γ corresponding to the inclusion of ladder diagrams);

(3) an 'annihilation' term, which is included in the random phase approximation (RPA) and which is the last term in the diagrammatic equation for Γ. This term occurs only for charge density excitations (involving no change in the spin of the system).

We note that the formalism under discussion extends to the case of several exciton branches (the governing equation becomes a matrix equation).

Figure 2 borrowed from Ref. [3] reproduces spin wave and magnetoplasmon dispersion curves for the filling factor 1 and for the filling factors 1 and 2. We note the minimum (commonly called a roton minimum) in the magnetoplasmon dispersion.

If the filling factor is noninteger, then, neglecting the interaction, the ground state of the system is degenerate. In that case, no rigorous treatment of 2DES excitations exists. Excitation spectra are calculated with the TDHF approximation [7–9], which assumes a uniform electron distribution (all configurations are of equal probability and do not correlate), and with the generalized single-mode approximation (GSMA) [6, 10, 11], in which electron–electron correlations are included using a pairwise correlation function. For an arbitrary filling, the structural factor is not known but can be obtained from numerical calculations for a number of Laughlin incompressible states, $\nu = 1/3, 1/5...$ [12, 13].

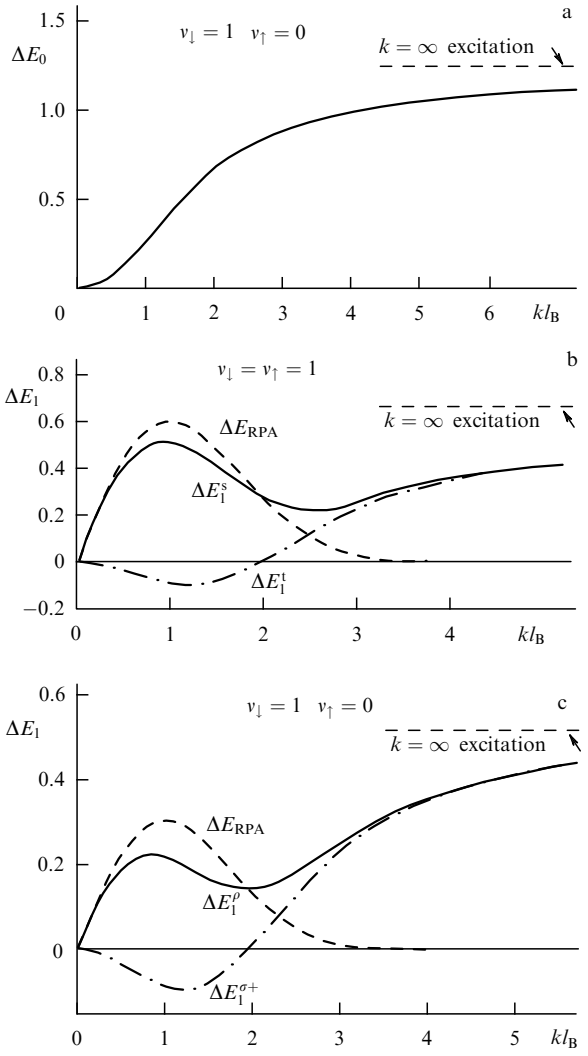

Figure 2. Collective excitation spectra for exactly solvable models: (a) for $\nu = 1$, the spin wave spectrum obtained without including the Zeeman energy is given; (b) for $\nu = 2$, magnetoplasmon TDHF (solid line) and RPA spectra (dashed line) are shown, as well as the magnetoexciton spectrum with $S_z = 0$ (dashed-dotted line); (c) for $\nu = 1$, TDHF (solid line) and RPA (dashed line) magnetoplasmon spectra are shown, as well as the $S_z = 1$ magnetoexciton spectrum; in (b) and (c), the reference point is the cyclotron energy. Energy is measured in units of $e^2/\varepsilon l_B$. Data are taken from Ref. [3].

In this review, we concentrate on single-exciton excitations, because the work reviewed uses inelastic light scattering as a key experimental tool. To make the picture more complete, however, we also note a number of studies that consider finite-density systems of interacting magntoexcitons and explore the nonideality and the phase diagram as well as the magnetoexciton condensate as the ground state [14–16].

3. Cyclotron spin-flip excitations in a quantum Hall ferromagnet. The effect of the geometric form factor on the Coulomb interaction

In this section, we examine the spectrum of magnetoexcitons in Hall ferromagnets, i.e., in even integer filling states in which the electronic system has a nonzero spin polarization. We review experimental cyclotron spin flip excitation (CSPE)

studies using inelastic light scattering and examine how the exchange interaction contributes to this type of excitation, depending on the parameters of a two-dimensional electronic system. The results obtained are compared with perturbative calculations.

For a state with one filled spin Landau sublevel ($\nu = 1$, $S = S_z = N_0/2$, where N_0 is the degree of degeneracy of the spin Landau sublevel), the lowest-energy neutral excitation of the 2DES is a spin exciton (SE), a spin-flip excitation within the lowest Landau level. Spin excitons vary from collective in the long-wavelength limit to single-particle in the short-wavelength limit ($ql_B \to \infty$). In the latter case, a spin exciton is a pair that contains a reversed-spin electron infinitely separated from a ground state hole and whose creation requires an energy equal to the sum of the Zeeman and exchange energies. The exchange contribution to the spin exciton energy increases the effective spin gap, $E_{SE}(\infty) = E_Z - \Sigma_0$, where E_Z is the Zeeman energy and $\Sigma_0 < 0$ is the exchange energy of an electron in the ground state. A suitable quantity to characterize the spin gap is the exchange-enhanced g-factor

$$g^* = \frac{E_{SE}(\infty)}{\mu_B B\,\delta S_z} = g - \left|\frac{\Sigma_0}{\mu_B B\,\delta S_z}\right|. \tag{4}$$

One of the direct methods to measure the exchange-enhanced g-factor is activation transport. It has been shown by magnetotransport and magnetocapacitance techniques that for $\nu = 1$, the activation gap depends nearly linearly on the magnetic field, i.e., the exchange-enhanced g-factor of electrons has a weak magnetic field dependence [17–20], while the energy scale of the exchange-enhanced spin splitting should be equal to that of the Coulomb interaction, $E_C = e^2/\varepsilon l_B \sim \sqrt{B}$ [1]. Moreover, the absolute value of the splitting turns out to be much less than the theoretical prediction. The obvious inconsistency between the experimental and theoretical results has triggered interest in seeking an alternative approach to the exchange interaction in a Hall ferromagnetic state.

Information on the interaction can be extracted from data on the energy of long-wavelength collective excitations. As already noted, the energy of long-wavelength magnetoplasmons and that of spin excitons do not contain a contribution from the electron–electron interaction (the Kohn–Larmor theorem). Such symmetry constraints are irrelevant for combined excitations involving a simultaneous change in the spin and orbital quantum numbers of the 2DES (see Fig. 3 for a schematic of excitations). Hence, the energy of a cyclotron spin-flip exciton ($\delta n = 1$, $\delta S_z = -1$) can be written as the sum of the cyclotron, Zeeman, and Coulomb energies:

$$E_{CSFE}(q) = \hbar\omega_c + \left|g\mu_B B\right| + \Delta(q, B)\,. \tag{5}$$

The term $\Delta(q, B)$ determines the CSFE dispersion and is equal to the difference in the energy of the multiparticle Coulomb interaction between electrons in the ground and excited states. Contributing to this energy are the term $-\Sigma_0$, which is associated with the decrease in the energy exchange with electrons at the zeroth Landau level, and the term $E_\nu^{10}(q)$, the binding energy of the magnetoexciton consisting of an electron excited to the first Landau level and a hole at the zeroth Landau level. In the Hartree–Fock approximation, the explicit expressions for these energy

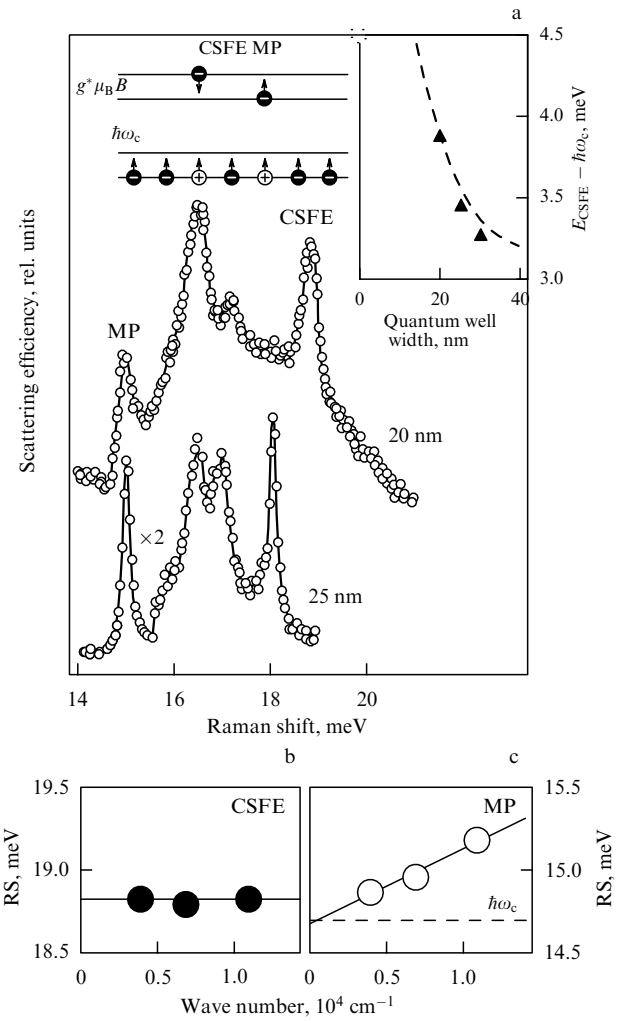

Figure 3. (a) Inelastic light scattering spectra at 8.5 T ($\nu = 1$) for quantum wells 20 nm and 25 nm in width (MP is a magnetoplasmon and CSFE is a cyclotron spin-flip magnetoexciton). Shown above the spectra are excitation schemes. Inset: the CSFE Coulomb energy $\Delta(0, B)$ for $B = 7.6$ T obtained from an experiment for three wells of different widths (solid triangles) and calculated by the Hartree–Fock method (dashed line). Panels (b) and (c) plot the Raman shift (RS) dispersion in the long-wavelength limit for the cyclotron spin-flip exciton (b) and magnetoplasmon (c).

contributions are

$$\Sigma_0(B) = -\int_0^\infty \frac{k\,\mathrm{d}k}{2\pi}\, v(k) \exp\left(-\frac{k^2 l_B^2}{2}\right), \tag{6}$$

$$E_\nu^{10}(q, B) = -\int_0^\infty \frac{k\,\mathrm{d}k}{2\pi}\, v(k)\left[1 - \frac{k^2 l_B^2}{2}\right] \times J_0(kql_B^2) \exp\left(-\frac{k^2 l_B^2}{2}\right), \tag{7}$$

where $J_0(x)$ is the Bessel function and $v(k) = 2\pi e^2/\varepsilon k$ is the Fourier component of the Coulomb potential. When calculated for an ideal two-dimensional system in the limit $q = 0$, these expressions are related by $E_\nu^{10}(0, B) = \Sigma_0/2$. We therefore conclude that the magnetoexciton binding energy $E_\nu^{10}(0)$ in a CSFE is only a partial compensation of the exchange energy loss due to a change in the spin quantum number of the 2DES; and that the resulting contribution $\Delta(0, B) = -\Sigma_0(B) + E_\nu^{10}(0, B)$ to the energy of a long-wavelength

CSFE is positive. We also note that this excitation is virtually dispersionless up to momenta $q \sim 1/l_B$.

Experimentally, long-wavelength cyclotron excitations of a two-dimensional electronic system in a $\nu = 1$ Hall ferromagnetic state were studied using inelastic light scattering from a series of GaAs/AlGaAs heterostructures containing single quantum wells of different widths and with widely different electron concentrations. The experiments studied the variation of the cyclotron excitation energy as a function of parameters such as the concentration of the two-dimensional electron gas (or equivalently, the value of the transverse magnetic field at $\nu = 1$), the quantum well width, and the excitation momentum.

Figure 3a shows typical inelastic light scattering spectra close to the cyclotron energy for $\nu = 1$. The leftmost and rightmost thin lines correspond to MP and cyclotron spin-flip excitations. The broad spectral lines between the MP and CSFE are due to the excitation of barrier impurity complexes and are discussed in Section 6. Under the experimental conditions in Fig. 3, the Zeeman energy is 0.21 meV, and its contribution to the CSFE energy is small compared to the remaining terms in expression (5). Therefore, the exchange contribution to the CSFE energy is nearly equal to the CSFE energy shift from $\hbar\omega_c$. In the experimentally accessible momentum range, the MP has a linear dispersion, while the CSFE is nearly dispersionless (Figs 3b, c).

As can be seen from a comparison of two spectra in Fig. 3a, obtained for the same magnetic field strength but for quantum wells of different widths, the exchange contributes differently to the CSFE. The decrease in $\Delta(0, B)$ with increasing the width occurs because the nonlocality of the electron wave function in the direction perpendicular to the plane of the two-dimensional channel exerts influence on the Coulomb interaction. For small B ($l/l_B \ll 1$, where l is the effective width of the quantum well), the Coulomb interaction is nearly two-dimensional. In large magnetic fields for which the effective width of the electron wave functions in the direction of growth is comparable to the characteristic electron–electron separation in the plane ($l/l_B > 1$), their interaction softens (see inset in Fig. 4a).

Hence, the effect of the quantum well width on the Coulomb interaction scale should manifest itself in the magnetic field dependences of the exchange contribution to the CSFE energy for samples with quantum wells of different widths. Figure 4 shows the measured magnetic field dependences $\Delta(0, B)$ for quantum wells with the width 20, 25, and 30 nm. We can see that for large magnetic fields, wide quantum wells show a marked decrease in the exchange energy.

The measured changes in the CSFE energy as a function of the magnetic field (see Fig. 4) and the quantum well width (Fig. 3a) agree well with predictions from the Hartree–Fock approximation, including the correction of the Coulomb interaction for a finite quantum well width, when the Fourier component of the Coulomb potential is modified by introducing the geometrical form factor

$$\overline{v}(q) = \frac{2\pi e^2}{\varepsilon q} F(q) .$$

The form factor depends on the well width and is defined by the expression

$$F(q) = \int_0^\infty \mathrm{d}z \int_0^\infty \mathrm{d}z' \, |\psi(z)|^2 |\psi(z')|^2 \exp\left(-q|z - z'|\right) , \quad (8)$$

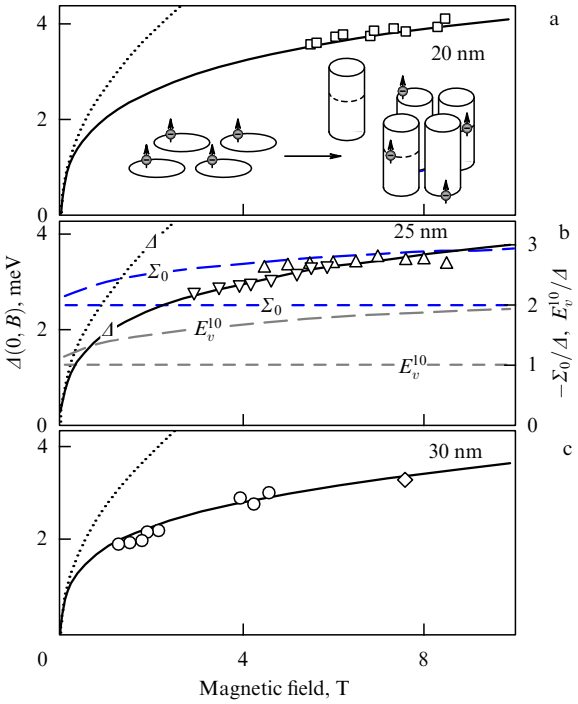

Figure 4. $\Delta(0, B)$ as a function of the magnetic field for three quantum wells of different widths: experiment (symbols) and Hartree–Fock calculations, including the geometric form factor (solid lines). Dashed lines are calculated with the form factor ignored. The inset in (a) is a schematic of the transition of the electron-electron interaction from two-dimensional in small magnetic fields to effectively three-dimensional in large magnetic fields. The right-hand scale in (b) shows the magnetic field dependences of the ratios $-\Sigma_0/\Delta(0, B)$ and $-E_v^{10}/\Delta(0, B)$ for a 25 nm quantum well (long dashed line) and for an ideal two-dimensional system (short dashed line).

where the components of the electron wave functions $\psi(z)$ are found by self-consistently solving the one-dimensional Schrödinger and Poisson equations.

The good agreement between the measured dependence $\Delta(0, B)$ and that calculated by the Hartree–Fock approximation allows using these data to estimate the exchange-enhanced g-factor in a Hall ferromagnet. To do this, the data on $\Delta(0, B)$ should be converted into the magnetic field dependence $|\Sigma_0(B)|$ using the theoretical curves for $\Sigma_0(B)$ and $E_v^{10}(0, B)$ calculated including geometrical factors (dashed lines in Fig. 4). Following this, Eqn (4) is used to obtain the magnetic field dependence of $g^*(B)$ (shown by unfilled symbols in Fig. 5b).

In low magnetic fields, the exchange-enhanced g-factor reaches huge values $|g^*| \sim 60$, an order of magnitude greater than those obtained from activation transport measurements (Fig. 5b). The differences among experiments can be accounted for by the influence of the random potential in a 2DES on the measurement results. Transport experiments measure the minimum value of the activation gap as an integral property over the entire sample, and a random potential can narrow the activation gap locally. In contrast, the formation of a long-wavelength CFSE involves only those regions in the sample where ferromagnetic order is not destroyed by a random potential. In these regions, the exchange energy reaches its *real* maximum, and, importantly, a sufficient condition for a long-wavelength CSFE to form is unperturbed ferromagnetic order on the scale of a few

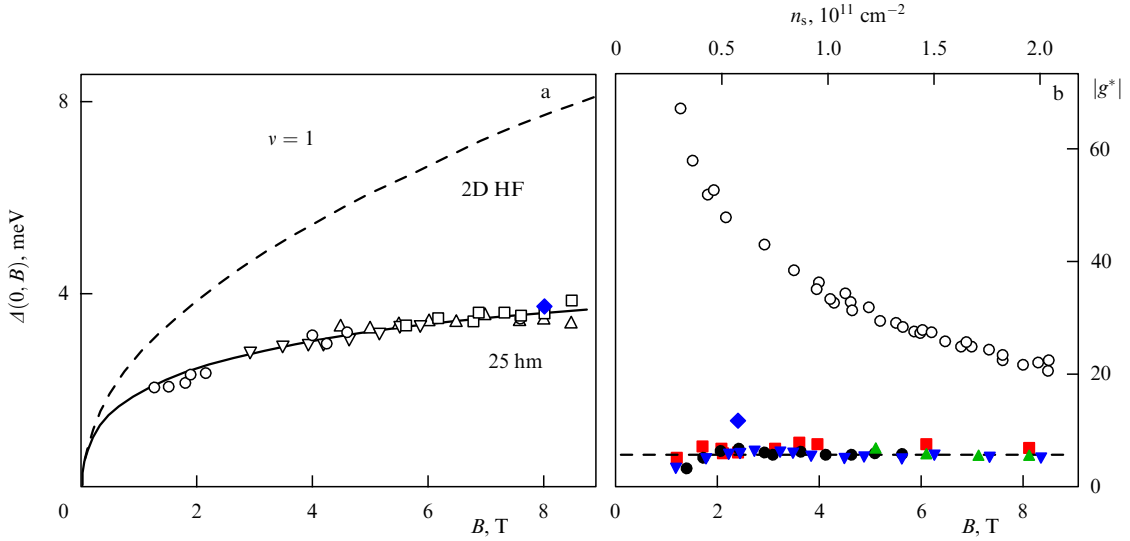

Figure. 5. (a) Magnetic field dependence of $\Delta(0, B)$ at $\nu = 1$ for a 25-nm quantum well. Experimental data are shown by symbols; solid line is calculated by the Hartree–Fock (HF) approximation including the geometric formfactor. Dashed line: a similar dependence calculated for an ideal two-dimensional case. (b) Magnetic field and electron concentration dependence of the exchange enhanced g-factor for the zeroth Landau level electrons for $\nu = 1$ calculated from experimental data for $\Delta(0, B)$. Filled symbols at the bottom: the values of the exchange-enhanced g-factor obtained in transport studies [17–20].

magnetic lengths (the size of the long-wavelength magnetoexciton).

Thus, the study of the CSFE energy in the $\nu = 1$ state of the Hall ferromagnet helped to determine the exchange energy scale in the examined system and verify the first-order perturbation theory in r_s as a means to calculate the multiparticle contribution to the energy of long-wavelength collective excitations.

In [21], an experimental and theoretical study of cyclotron spin-flip excitons in a $\nu = 3$ Hall ferromagnet was conducted. In this case, the electron system is partially spin-polarized (both spin components are filled on the zeroth Landau level and one of them on the first) and unlike the $\nu = 1$ Hall ferromagnet, the spectrum of cyclotron excitations contains two pairs of Coulomb-coupled magnetoexcitonic branches that correspond to charge-density ($\Delta S_z = 0$) and spin-density ($\Delta S_z = -1$) excitations.

Figure 6a is a schematic of two magnetoexcitonic pairs. Each of the magnetoexcitons comprising a bound pair has an identical set of quantum numbers and corresponds to a cyclotron transition of an electron from the lowest or second-lowest Landau level, such that two charge density excitations form a synphase magnetoplasmon (MP) and an antiphase plasmon (AP), and similarly, the spin-flip excitations split into a synphase and antiphase cyclotron spin-flip excitons (SF1 and SF2). A detailed discussion of the properties of the new branch of antiphase plasmons is postponed until Section 4, but some differences between the bound CSFE branches and the case of unit filling should be mentioned here. The excitation energies are determined by the Hartree–Fock method from the secular equation, whose solutions can be written as

$$E^{12}_{\delta S_z}(q) = \frac{\mathcal{E}_1(q)+\mathcal{E}_2(q)}{2} \pm \sqrt{\left(\frac{\mathcal{E}_1(q)-\mathcal{E}_2(q)}{2}\right)^2 + \Delta_{12}(q)^2}\,, \tag{9}$$

where $\mathcal{E}_1(q)$ and $\mathcal{E}_2(q)$ are the energies of single noninteracting cyclotron spin-flip excitons that include the cyclotron Zeeman and exchange Coulomb contributions, totally similar to expression (5) but including wave functions of different

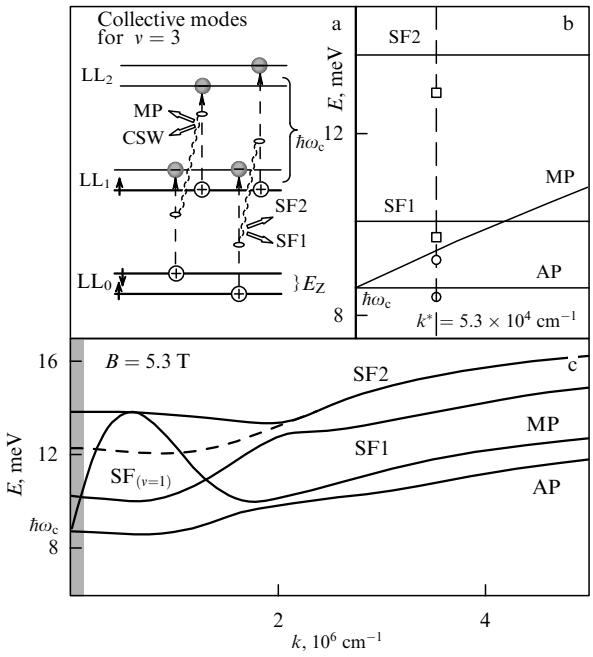

Figure 6. (a) Schematic of cyclotron excitations with $\nu = 3$. Charge excitations are a magnetoplasmon (MP) and an antiphase plasmon (AP). Cyclotron spin-flip excitons are SF1 and SF2. (b) Comparison of theoretical and experimental excitations for $B = 5.3$ T and $\nu = 3$. Squares: energies of cyclotron spin-flip excitons SF1 and SF2 measured for momentum values 5.3×10^4 cm^{-1}; circles: MP and AP energies. (c) Dispersion relation for cyclotron excitations calculated in the Hartree–Fock approximation for $B = 5.3$ T taking a finite well width into account. Mutual repulsion between the dispersion curves of two spin-flip excitons and two charge excitons is seen.

Landau levels. The energy $\Delta_{12}(q)$ gives rise to the Coulomb coupling between the two magnetoexciton branches. We note that this quantity also has a scale $\Delta_{12} \sim E_C$ in the long-wavelength limit. As a consequence, the dispersion branches of two cyclotron spin-flip excitations repel each other, and the exchange Coulomb contribution to the energy of the exciton SF2 exceeds its counterpart for a CSFE in the $\nu = 1$ Hall ferromagnet at the same field strengths (Fig. 6c).

As in the case $\nu = 1$, reasonable agreement can be seen between the predicted and measured values of the SF1 and SF2 energies (Fig. 6b) if the interaction potential is width-corrected. Data obtained for two-dimensional systems with different electron concentrations show, in agreement with the calculations in [21], that the exchange Coulomb contribution to the energy of cyclotron spin-flip excitons increases as a sub-square-root of the magnetic field

It follows from the results discussed in this section, first of all, that electron–electron correlations in a two-dimensional system can show up in the energy of long-wavelength collective excitations involving changes in both the orbital and spin quantum numbers of the system. The method of inelastic light scattering provides a tool to measure the exchange interaction in Hall ferromagnet states with integer filling factors. It is shown that they can be well described perturbatively in the one-exciton approximation. The calculated energies of cyclotron spin-flip excitons coincide with those measured experimentally if the finite quantum well width is taken into account. The Coulomb exchange corrections to the CSFE energy have a scale comparable to the exchange energy of electrons on Landau spin sublevels, $\sim e^2/\varepsilon l_B$.

4. Magnetoexcitons and quantum magnetoplasmons. Essential role of the second-order perturbation theory

In this section, we examine the key properties of magnetoexcitons and magnetoplasmons. For spin-nonpolarized states, we show that some of these properties require using the second-order perturbation theory.

In the ground-state 2DES with the filling factor $\nu = 2$, two spin sublevels of the zeroth Landau level are filled and the total spin is zero. The lowest-energy states are cyclotron excitations that involve a change by 1 in the orbital quantum number. Magnetoexcitons with $\delta n = 1$ differ in the spin δS and the spin projection on the magnetic field direction, δS_z. Zero spin occurs for a Kohn magnetoplasmon, which is a synphase oscillation of 2DES spin subsystems with the cyclotron frequency (Fig. 7).

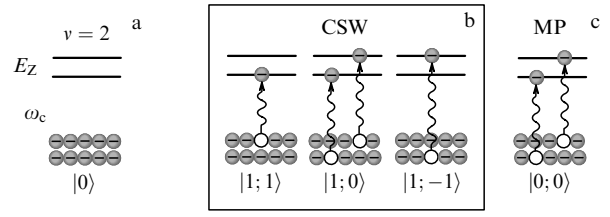

Figure 7. $\nu = 2$ ground state and lowest-energy cyclotron excitations. (a) Ground state (a zeroth Landau level is fully filled) (b) Three combined cyclotron transitions CSFE with the total spin 1. (c) Kohn (magnetoplasmon) mode (MP). At the bottom, spin quantum numbers of the excitations are shown.

Also possible are three excitations that have spin one, $\delta S = 1$, and which differ in the spin projection on the magnetic field direction, $\delta S_z = -1, 0, 1$ (spin triplet). The central ($\delta S_z = 0$) triplet component (cyclotron spin wave, CSW) represents an antiphase oscillation of the spin subsystem at the cyclotron frequency. Using the first-order perturbation theory in the Coulomb-to-cyclotron energy ratio, the Coulomb contribution to the energy of any CSFE component is zero. But because the triplet excitations are not subject to symmetry constraints similar to those in the Kohn theorem, a nonzero Coulomb correction to their energies can be expected to appear for $q = 0$. As we show in what follows, they arise only in second-order perturbative calculations and have the effect of significantly reducing the energy of cyclotron spin-flip excitations.

Inelastic light scattering was used in [22] to study cyclotron spin-flip excitations at even integer filling factors, $\nu = 2$, 4, 6. The observed spectra exhibited three lines with the splitting corresponding to the Zeeman energy E_Z. Interestingly, the position of the triplet center turned out to be red-shifted relative to the cyclotron resonance energy (see the inset in Fig. 8). Measurements of this excitation in different magnetic fields but at a fixed filling factor $\nu = 2$ have shown that the amount of the shift is virtually constant over a wide range of fields (see Fig. 8)

An analytic calculation of second-order corrections to the collective excitation energies was conducted in the exciton representation in the limit of small r_s [23]. Instead of the electron creation operators that produce one-electron states when acting on the ground state, the exciton representation involves exciton operators whose action on the ground state gives a basis of exciton states [24–30]. The major advantage here is that in the basis of exciton states, the Coulomb interaction is partially diagonalized, and only the nondiagonal part is treated as a perturbation. As a result, even the zeroth-order approximation produces a **q**-dependent Coulomb correction to the excitation energy, and the spectrum of the exciton state is nongenerate.

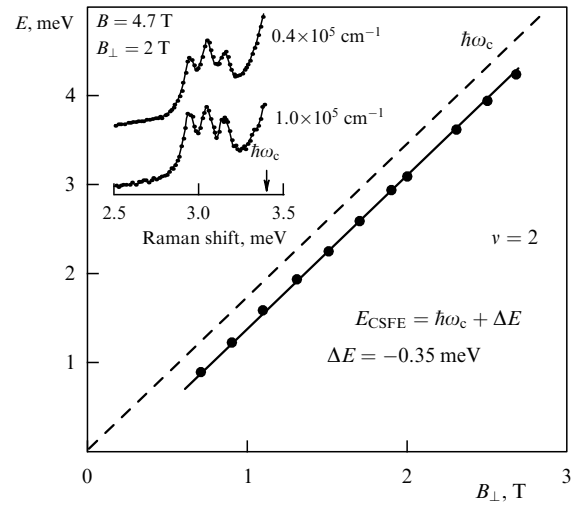

Figure 8. Energy of the central component of the CFSE triplet vs the magnetic field for $\nu = 2$. Experimental points are approximated by a straight line; the dashed line shows the cyclotron resonance energy for GaAs electrons of the effective mass $m^* = 0.067m_0$. Inset: spectra of a triplet excitation for two different momentum values; the arrows show the cyclotron resonance energy $\hbar\omega_c$.

The cyclotron excitation energy for the filling factor $\nu = 2$ and zero momentum is expressed analytically as

$$\Delta E_{\nu=2}^{\mathrm{CSW}}(0) = -\sum_{n=2}^{\infty} \frac{2-2^{2-n}}{n(n^2-1)\,n!} \int_0^{\infty} \mathrm{d}q\, q^{2n+3} V^2(q) \exp(-q^2) \quad (10)$$

in units of $2\mathrm{Ry}_{\mathrm{GaAs}} = r_s^2 \hbar\omega_c \approx 11.34$ meV [23]. In an ideal two-dimensional system, $\Delta E_{\nu=2}^{\mathrm{CSW}}(0) = (\ln 2 - 1)/2$, which is 1.74 meV in energy units. Including the finite-width correction, the numerical value measured for the experimental conditions in Fig. 8 is $\Delta E \approx -0.47$ meV, in reasonable agreement with experiment.

Antiphase plasmon excitations, which we mentioned in Section 3 and which exist for odd filling factors, allow a similar treatment. For example, for $\nu = 3$, the zeroth Landau level and the lowest-spin sublevel of the first Landau level are fully occupied in the ground state. The excitation spectrum with $\delta n = 1$ exhibits two coupled cyclotron magneotexcitons with $\delta S_z = 0$, which form a Kohn magnetoplasmon (MP) and an antiphase plasmon (AP) (see the diagram in Fig. 9). Independently of the filling, the Kohn magnetoplasmon has a linear long-wavelength dispersion due to plasma corrections. The antiphase plasmon has a structure similar to the cyclotron spin wave with even filling factors; however, at $\nu = 3$, the AP is a charge density, not a spin density, excitation.

In Ref. [31], inelastic light scattering was used to study cyclotron excitations at even and odd integer filling factors. The properties of the classical magnetoplasma mode are a matter of common knowledge and there are a variety of experimental methods to study them. As regards the antiphase excitations AP and CSW, their experimental study became possible only with the use of inelastic light scattering [22, 32, 33]. These excitations are irrelevant to the absorption of electromagnetic radiation and, therefore, have not been detected previously by the cyclotron resonance method.

Observations have been reported [34, 35] of excitations near the roton minimum for transitions from one Landau level to the next one for $\nu \geqslant 1$ in GaAs quantum wells. The experiments used the inelastic light scattering method, for which the accessible range of excitation wave vectors is $k \sim 10^5$ cm^{-1}, to be compared with significantly larger values $k \sim 10^6$ cm^{-1} at which the roton minimum exists. Similar studies of excitations near the roton minimum were carried out for the filling factor 1/3 in Ref. [36], where the nonconservation of the wave vector was ascribed to the impurity-induced broadening of Landau levels.

In Ref. [37], the excitation spectrum was investigated in the regime of the fractional quantum Hall effect for the filling factor 1/3 and large wave vectors near the roton minimum. With the method chosen to study such excitations (which used surface acoustic waves of different wavelengths to modulate the dielectric constant and employed optical methods to detect resonance absorption), the entire dispersion curve could be obtained. The presence of a short-range potential enables [34, 35] detecting excitations near the roton minimum, but it is not possible to derive the wave vector dependence on the energy. The results of the experiment in [37] were found to compare well with the theoretical roton dispersion obtained by the composite fermion method [38], indicating that the nonuniformity scale exceeds the magnetic length.

Inelastic light scattering methods, as well as other optical methods, are only useful for excitations with small momenta. In this limit, the fundamental Kohn magnetoplasma mode is classical and does not show quantum effects. In contrast, even for small momenta, the odd-ν AP mode and even-ν CSW are quantum excitations that have no classical analogs and which fundamentally require the presence of two spin subsystems.

Inelastic light scattering spectra for antiphase excitations are presented in Fig. 10, where two spectral features can be seen below the cyclotron resonance energy: a single line and a characteristic spin triplet at even and odd filing factors. It is noteworthy that although excitations with even and odd filling factors are of different natures (different spin excitations), the absolute value of the shift decreases with increasing the filling factor and falls on the same empirical curve $\Delta E^{\mathrm{CSW,\,AP}} \sim 1/\nu$ (Fig. 11).

For both even and odd filling factors, the first-order perturbation theory gives zero for the Coulomb contribution

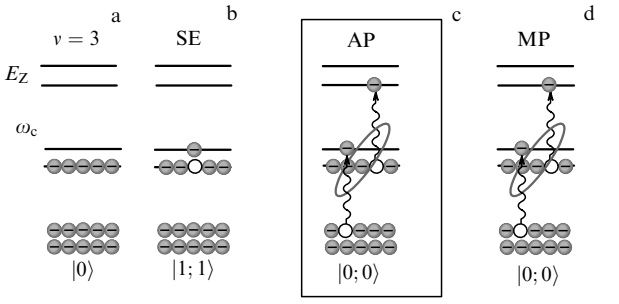

Figure 9. $\nu = 3$ ground state and lowest-energy spin and cyclotron excitations. (a) Ground state (the fully filled zeroth Landau level and the lowest-spin sublevel of the first Landau level); (b) spin exciton (SE); (c) antiphase plasma (AP) mode; (d) Kohn (magnetoplasma, MP) mode. At the bottom, spin quantum numbers S, S_z of the excitations are shown.

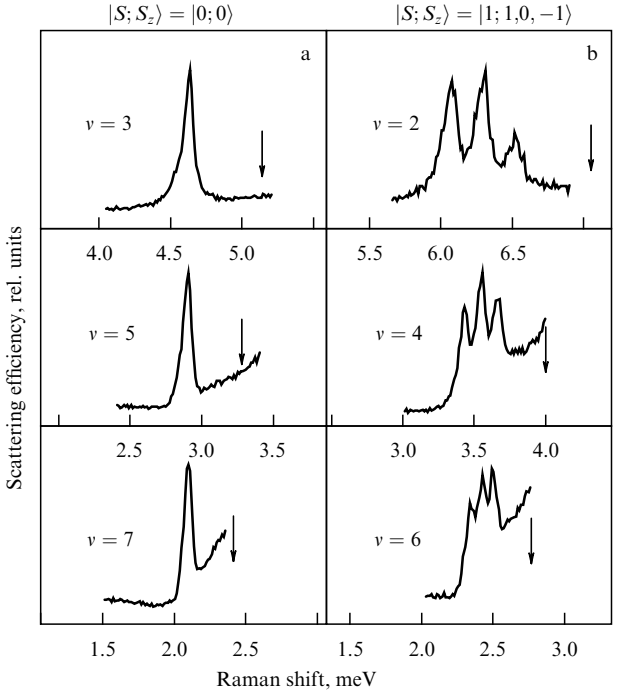

Figure 10. Inelastic light scattering spectra (a) for the odd-filling-factor antiphase magnetoplasma mode and (b) for three even-filling-factor Zeeman components of the cyclotron spin-flip excitation. Arrows indicate the position of the cyclotron energy.

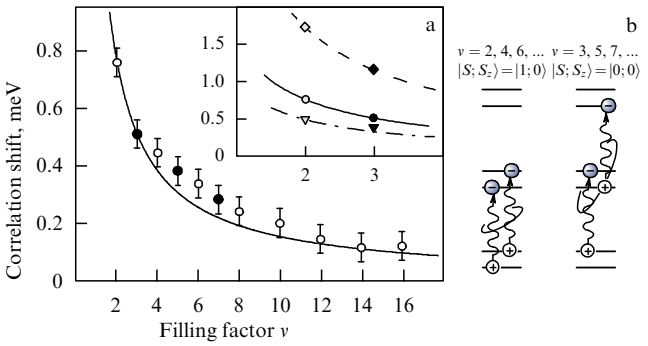

Figure 11. (a) Dependence of the correlation shift on the filling factor. Main plot: experimental data for even (open circles) and odd (filled circles) filling factors; the solid line plots $\Delta E^{\nu} \propto 1/\nu$. The inset shows calculations for filling factors 2 and 3 for an ideal two-dimensional gas (diamonds) and for a finite well width (triangles). Circles in the inset duplicate points from the main plot. (b) Schematic of transitions corresponding to antiphase excitations: a cyclotron spin wave (CSW) for even filling factors and an antiphase magnetoplasmon (AP) for odd filling factors.

to the energy of antiphase excitations AP and CSW. In both cases, however, the second-order corrections are negative, have a similar structure, and are independent of the magnetic field.

In the simplest cases $\nu = 2$ and $\nu = 3$, it proved possible to rigorously include all second-order Coulomb corrections for both types of antiphase excitations. The results obtained in the exciton representation for the Coulomb contributions to the AP energy with $\nu = 3$ are given in [31]. The resulting Coulomb contribution to the energy of the antiphase mode is given by (in units of 2Ry)

$$\Delta E^{\mathrm{AP}}_{\nu=3}(0) = -\frac{3}{2}\int_0^{\infty} q\,\mathrm{d}q V^2(q) G(q)\,, \qquad (11)$$

where $G(q)$ is the result of summation of an infinite series with diagrams of the second order in the Coulomb interaction.

In the limit of an ideal two-dimensional system, the second-order correction (in r_{s}) to the antiphase mode energy is $\Delta E^{\mathrm{AP}}_{\nu=3}(0) = -0.1044$ in units of Ry for excitons, which is about 1.18 meV. The inclusion of the finite well width through the geometrical form factor gives a good agreement with experiment, even though the parameter r_{s} cannot always be considered small under the experimental conditions (see the inset in Fig. 11a).

Thus, in the second-order perturbation theory, the energy of an antiphase magnetoplasmon and that of a cyclotron spin wave acquire negative Coulomb corrections at the zero momentum. The absolute values of the corrections are independent of the magnetic field, and their ratio is consistent with the empirical dependence $\Delta E_{\nu}(0) \propto 1/\nu$. To give an example, the predicted shifts for filling factors 3 and 2 are in the ratio $0.68 \approx 2/3$.

To summarize this section, we note the major results. Collective excitations exist whose energies at the zero momentum are determined by second-order Coulomb corrections. For even filling factors, these are cyclotron spin-flip excitations, which have three spin-split components and a negative multiparticle contribution, which shifts their energy to below the cyclotron energy. For odd filling factors $\nu \geqslant 3$, the collective excitations exhibit an antiphase plasmon in their spectra, which is of a quantum nature (hence, its other name, quantum plasmon), because its existence is fundamentally due to the existence of electron spin and because its energy is determined by the electron Rydberg constant. A theoretical approach is developed that includes the multiparticle contribution to the energy of these excitations in the second-order perturbation theory in the parameter r_{s}. The energy calculated taking the finite well width into account agrees well with the obtained experimental data.

5. Two-mode approximation. Effect of two-exciton corrections

This section is concerned with cyclotron spin-flip excitons in the fractional quantum Hall effect (QHE) regime for $\nu = 1/3$. We discuss various theoretical approaches to describing this excitation and show that the inclusion of multiparticle exciton corrections is crucial for understanding its structure.

As in the case $\nu = 1$, electron spins at $\nu = 1/3$ are aligned with the magnetic field, and there is an energy gap between the ground and excited states of the electron system. Therefore, the system with $\nu = 1/3$ is also a Hall ferromagnet. By analogy with two ferromagnetic states, the spectra of cyclotron excitations can be expected to be similar and the exchange interaction to manifest itself in the energy of a long-wavelength cyclotron spin-flip exciton.

The first observation of a $\nu = 1/3$ CSFE was made in Ref. [39] using the inelastic light scattering method. We note that in the fractional QHE (FQHE) regime, the manifestations of this excitation in inelastic light scattering spectra have only been reported for high-quality GaAs/AlGaAs heterostructures with 2DESs with a record high mobility $\sim (7-10)\times 10^6$ cm^2 V^{-1} s^{-1}. Figure 12 shows the filling factor dependence of the MP and CSFE collective excitations for a fixed magnetic field $B = 9$ T. For $\nu = 1/3$ and $\nu = 1$, the CSFE energy has local maxima, indicating that the maximum contribution comes from the exchange interaction energy. In the neighborhood of the $\nu = 1/3$ FQHE state, a CSFE spectral line is observed in the filling factor interval $\nu \in 0.25-0.41$. At the boundary of this range, the excitation energy reduces by about a half. From the experimental data for samples with different electron concentrations, the magnetic field dependence of the exchange contribution to the CSFE energy is obtained (see Fig. 13).

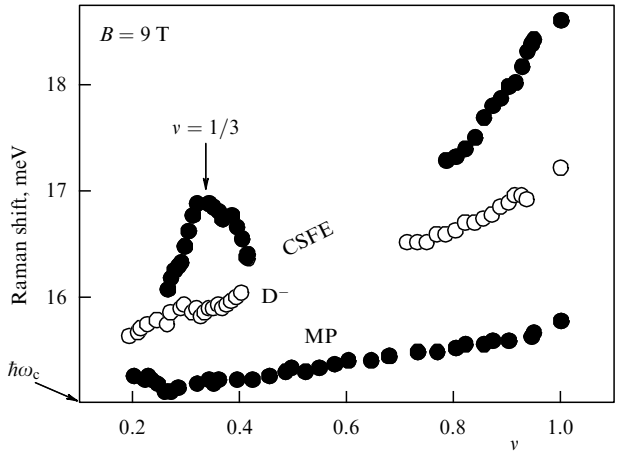

Figure 12. Energy of the collective excitations CSFE and MP (filled symbols) as a function of the filling factor ν in the magnetic field $B = 9$ T. For $\nu = 1/3$, the CSFE energy has a local maximum. Open symbols refer to D$^-$-complexes (see Section 6 below).

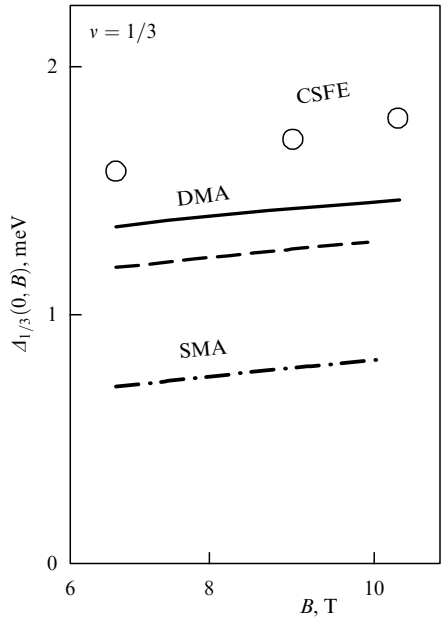

Figure 13. Exchange part of the CSFE energy as a function of the magnetic field for $\nu = 1/3$. Experimental data (circles) are compared with predictions using the double-mode approximation (DMA) (solid line) and the single-mode approximation (SMA) for the averaged density case (dashed line) and including Laughlin state correlations (dashed-dotted line).

The measured results were compared with existing theoretical models. By treating the 2DES in the Hartree–Fock approximation [3, 6, 40], the theoretical expression for the exchange contribution to the CSFE energy at $\Delta(0, B)$ can be extended to the case of an arbitrary filling factor $\nu < 1$,

$$\Delta_{\nu}^{\mathrm{HF}}(0, B) = \nu\, \Delta(0, B)\,. \tag{12}$$

In reality, however, there is a very significant difference between theoretical and experimental results. Moreover, the correspondence cannot be improved by using a generalized single-mode approximation including electron–electron correlations in the 2DES ground state [6]: indeed, this even lowers the CSFE energy compared to the Hartree–Fock approximation. Hence, both versions of the single-mode approximation greatly underestimate the exchange contribution to the CSFE energy.

One attempt to remedy this involved a theoretical approach that included multiexciton corrections to the structure of the Raman CSFE excitation and whose idea was that any collective excitation in a two-dimensional system has its own set of quantum numbers that are related to the change of the Landau level index, to the change of the electron spin projection, and to the generalized momentum $|\delta n, \delta S_z, \mathbf{q}\rangle$. For combined excitations with a set of quantum numbers such that $|\delta n| + |\delta S_z| \geqslant 2$, the basis of excited states must be augmented by superpositions of multiexciton states with the same total set δn, δS_z, and $\mathbf{q}$. The corresponding possible double-exciton states are pairs consisting of a magnetoplasmon $|\delta n = 1, \delta S_z = 0, \mathbf{q} - \mathbf{k}\rangle$ and a spin exciton $|\delta n = 0, \delta S_z = -1, \mathbf{k}\rangle$. With these double-exciton corrections, the structure of the CSFE is transformed as

$$|\mathrm{CSFE}\rangle_{\mathbf{q}} \Rightarrow |\mathrm{CSFE}\rangle_{\mathbf{q}} + \sum_{\mathbf{k}} \psi(k) |\mathrm{SE}\rangle_{\mathbf{k}} |\mathrm{MP}\rangle_{\mathbf{q}-\mathbf{k}}\,, \tag{13}$$

with $\psi(k)$ being a weight function. It turns out that if the weight function is chosen properly in a self-consistent fashion [41], this mixed state can be an eigenstate of the system Hamiltonian.

For a CSFE, the sought eigenvalue corresponds to the excitation energy that matches the experimental data much better than the single-mode approximation — a fact that highlights the significance of the multiexciton contribution to the structure of the cyclotron spin-flip exciton at $\nu = 1/3$. Similar double-exciton corrections also occur for the $\nu = 1$ QHE state [41, 42], but these do not contribute so much (about 5–7%) to the CSFE energy in the same range of the magnetic field. This correction scale is within the experimental error.

The double-mode approximation has been considered neglecting correlations in the $\nu = 1/3$ 2DES ground state. It would be more realistic — but seems as yet impossible — to apply this approach to a multielectron system including Laughlin correlations. Also left out of account are other quantum-number-preserving multiexciton combinations. Because charge-density waves within the lowest Landau level (magnetophonons) have spin and orbital quantum numbers zero, an arbitrary number of such excitations can formally be added. However, even without considering them, the above analysis shows the importance of multiexciton corrections for the filling factor $\nu = 1/3$.

6. Nonideality effects in a two-dimensional electron system. Localized magnetoexcitons (excited states of D^--complexes)

Closely related to the theme under review is the problem of electron–impurity D^--complexes in a strong magnetic field. A D^--complex is a state that binds two-dimensional electrons and a positively charged impurity, which resides either in a quantum well (in the plane of electron motion) or in the barrier. Because the impurity potential violates the translational symmetry of the system, the excitation spectrum of the complex should be affected by the presence of the Coulomb interaction energy, even in the case of a purely cyclotron or a purely spin excitation.

Of particular interest are barrier D^--complexes, in which electrons in the quantum well are spatially separated from the positive charge in the barrier [43]. This interest comes from the fact that charged impurities can be greatly reduced in concentration (to a level of $\sim 10^8$ cm^{-2}) directly in the well itself, and hence the influence of well-type D^--complexes on the properties of the two-dimensional electron gas is usually neglected. Those positively charged impurities in the AlGaAs quantum well barrier that are close to the two-dimensional plane are nearly two orders of magnitude larger in number, and they determine the optical and transport properties of a low-density ($\sim 10^{10}$ cm^{-2}) two-dimensional electron gas. Related to the same impurities is the problem of 'localized trions' or D^0X-complexes, states that consist of two electrons and a valence hole in a quantum well bound on the positive charge of the ionized impurity in the barrier [44].

The central challenge facing the experimental study of barrier D^--complexes is locating the ionized donor in the quantum well barrier. The way to do this is via the analysis of the energy of the cyclotron and intralevel excitations of D^--complexes, which is in turn determined by the Coulomb interaction between the electrons in the well and the positively charged impurity in the barrier.

The authors of [45, 46] were the first to show that the electronic excitations of barrier D^--complexes are active in inelastic light scattering. The inelastic light scattering cross section from the excitations of barrier D^--complexes may greatly exceed that from the collective excitations in 2DESs. This is the reason why those lines in inelastic light scattering spectra that are due to excitations in barrier D^--complexes can be more intense than the lines of plasma and spin excitations, even if the density of the complexes is orders of magnitude less than that of the two-dimensional electron gas.

Inelastic light scattering spectra from the cyclotron excitations of barrier D^--complexes in GaAs/AlGaAs quantum wells with high-mobility 2DESs are shown in Fig. 14 for the filling factor 1. Located between the lines of the collective excitations MP and CSFE are additional spectral features, the spin-split line S and the singular line $T+$. In Ref. [46], these were identified as the cyclotron excitations of the singlet and triplet states of barrier D^--complexes and it was shown that, unlike the CFSE energies, the Coulomb contribution to their energies is virtually independent of the well width (see Fig. 14) and scales with the magnetic field exactly as $\sim\sqrt{B}$ for the fixed filling factor $\nu = 1$ (Fig. 15).

This behavior is typical of the energy contribution from the direct Coulomb interaction between an excited electron and free electrons in accordance with the theory in [47, 48]. Also, it is shown that the excitations S and $T+$ are dispersionless, as the excitations of localized complexes are expected to be. The established properties of cyclotron-excited impurity complexes, while revealing the features of the multiparticle energy contribution, say nothing about the position of the ionized donors.

The last question was answered by studying the spectra of intralevel excitations in D^--complexes. In Ref. [49], excitations in interface D^--complexes and in high-quality GaAs/AlGaAs quantum wells were discovered and investigated using inelastic light scattering. In particular, the energies of bound states were measured and calculated for barrier complexes. Analysis of the experimental and calculated data showed that the observed excitations are due to transitions between the states of interface D^--complexes in which two electrons localized in the quantum well are bound with a charged impurity at the interface.

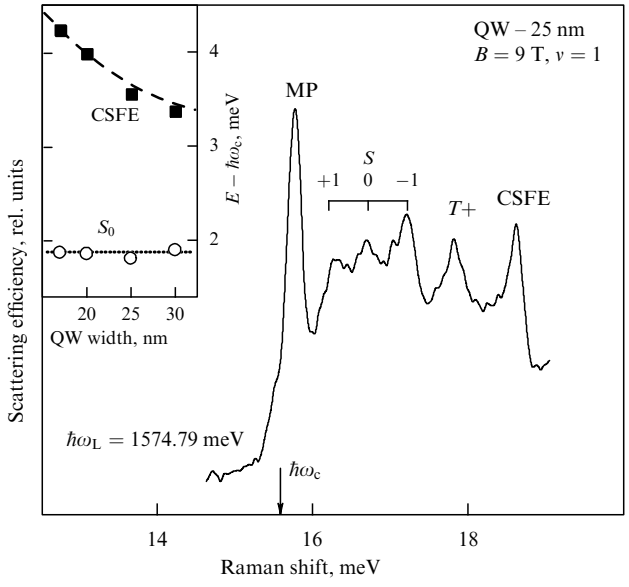

Figure 14. Inelastic light scattering spectra for the filling factor $\nu = 1$. Lines of the collective modes — magnetoplasmon (MP) and cyclotron spin-flip (CSFE) — and those of collective excitations of D^- complexes — spin-singlet (S) and triplet ($T+$) — are seen. Inset: Coulomb contributions ($E - \hbar\omega_c$) to the excitation energies of CSFE and S_0 as functions of the well width for four samples in the magnetic field $B = 8.5$ T. The CSFE energy decreases with increasing the well width; the energy of S_0 is width independent.

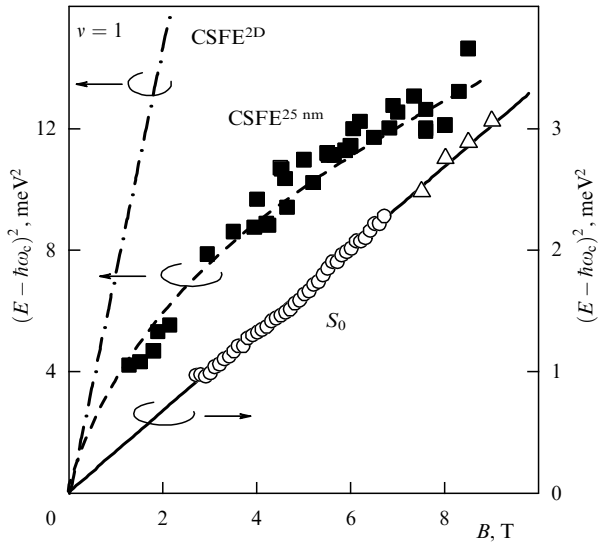

Figure 15. Squared Coulomb part of the energy of a cyclotron spin-flip exciton (filled symbols) and of the central component S_0 of cyclotron excitations of a spin-singlet D^--complex (open symbols) as a function of the magnetic field for $\nu = 1$. Dashed line: theoretical dependence of the CSFE energy on the magnetic field for a 25 nm quantum well; dashed-dotted line: the same for an ideal two-dimensional system. A straight line approximating results for line S_0 is added for clarity.

The spectra of low-energy (< 2 meV) inelastic light scattering from a low-density ($\sim 10^{10}$ cm^{-2}) two-dimensional electronic system are shown in Fig. 16 for some transverse magnetic field values. The lowest-energy line (SW) corresponds to inelastic light scattering processes that create a spin wave (SW), i.e., a long-wavelength spin exciton (SE). Line T corresponds to the intralevel excitation of interface D^--complexes. Its energy depends on the normal component of the magnetic field and has a contribution from the Zeeman splitting proportional to the total magnitude of the magnetic field. As the electron concentration increases, line T shifts toward the violet side of the spectrum and widens and then, at a certain critical concentration (4×10^{10} cm^{-2}), disappears from the spectrum (Fig. 17). The increase in the excitation energy with concentration is due to the increase in the Coulomb interaction between the excited electron and the free-electron environment around the D^--complex, similar to what occurs for cyclotron excitations in well-type [50, 51] and barrier-type [46] D^--complexes.

In the excitation spectrum models of D^--complexes, the localizing influence of a charge impurity is usually treated as a weak perturbation of the Hamiltonian of free electrons in a magnetic field [43, 52, 53]. In a strong magnetic field, the energies of D^--complexes of different momenta can be found by treating the Coulomb interaction $V(\mathbf{r}) + V(\mathbf{r}_1, Z) + V(\mathbf{r}_2, Z)$ as a perturbation (here, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, and $\mathbf{r}_1$ and $\mathbf{r}_2$ are the two-dimensional electron coordinates). The interaction between charged particles is

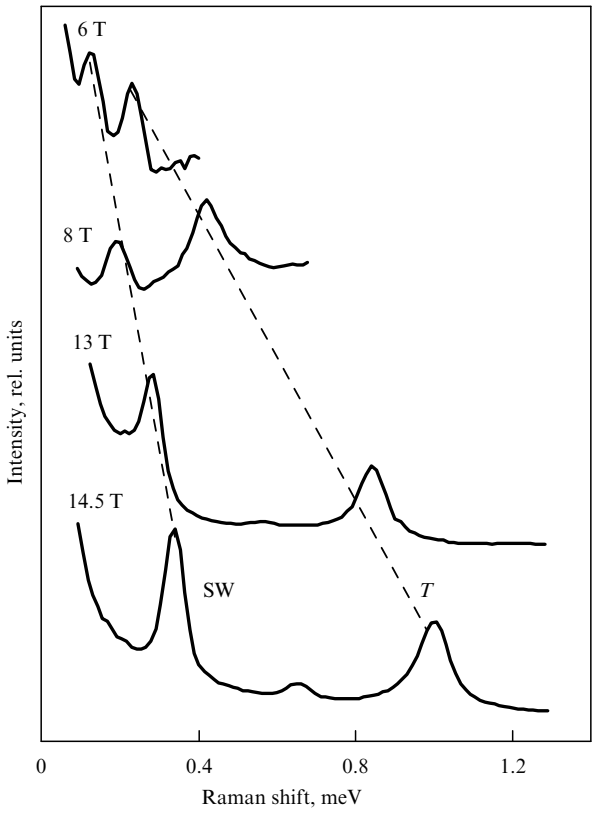

Figure 16. Inelastic light scattering spectra for various magnetic fields. Present in the spectrum are spin wave (SW) lines and the intralevel excitation of the barrier D^--complex (T).

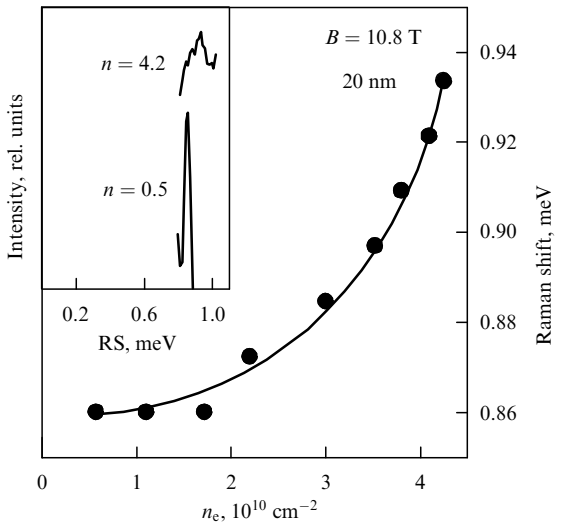

Figure 17. Measured dependence of excitation energy T of the barrier D^--complex on concentration for $B_\perp = 10.8$ T. Inset: examples of measured spectra for various electron concentrations. RS is the Raman shift.

calculated by taking the nonlocal nature of the wave functions of quasi-two-dimensional electrons in the z direction into account; the ionized donor (with positive charge 1) is at a distance Z from the electron layer, and its position in the plane is at the origin.

The wave function of a pair of electrons depends on their spin state, because the total wave function of two fermions must be antisymmetric. For the spin singlet, the total orbital momentum is $M = 0$, and the coordinate part of the wave function is symmetric and is constructed from the one-particle wave functions $\psi_m(r, \varphi)$ of the electrons on the zeroth Landau level,

$$\Psi_S = \Psi_0 = \psi_0(r_1, \varphi_1)\,\psi_0(r_2, \varphi_2)\,.$$

For the spin triplet, the total orbital momentum of the pair is $M = m_1 + m_2 = 1$, and the coordinate part of the wave function is symmetric and has the form

$$\Psi_T = \Psi_1 = \frac{1}{\sqrt{2}}\left(\psi_0(r_1, \varphi_1)\,\psi_1(r_2, \varphi_2) - \psi_1(r_1, \varphi_1)\,\psi_0(r_2, \varphi_2)\right).$$

We let $E_{Dm}(Z)$ denote the interaction energy of an electron of momentum m and a positively charged donor (D); then the perturbation theory gives

$$E_{Dm}(Z) = \left\langle \psi_m(r, \varphi)\middle| V(r, Z)\middle|\psi_m(r, \varphi)\right\rangle,$$

and the interaction energy of the electrons is given by

$$E_{I,M} = \left\langle \Psi_M(r, \varphi)\middle| V(r)\middle|\Psi_M(r, \varphi)\right\rangle.$$

Using the above results, the experimentally observed singlet–triplet energy difference can be calculated as

$$E_S - E_T = E_{I0} - E_{I1} + E_{D1}(Z) - E_{D0}(Z) + E_Z\,,$$

where $E_Z = g\mu_B B$ is the one-particle Zeeman energy.

In the magnetic field range $3 < B < 15$ T in about 20 nm quantum wells, the ground state of the barrier D^--complex is a spin triplet with the total momentum $M = 1$ ($1T$) (Fig. 18).

By varying the Z coordinate of the ionized donor in the quantum well barrier, good agreement can be obtained between the measured and calculated results, especially if the positive charge is placed at the quantum-well–barrier heterointerface. Moving the impurity by a mere 10 Å away from the heterointerface increases the excitation energy estimate by 30% relative to the experimental values. This suggests that the ionized donor is located directly at the interface between the quantum well and the barrier, presumably due to the diffusion of bulk impurities in the AlGaAs barrier during epitaxial growth (diffusion is markedly slowed down on the lattice defects of the GaAs/AlGaAs heterointerface and, moreover, impurities accumulate because the growth process ceases as molecular-beam epitaxy proceeds on the well–barrier interface).

An analysis of the line intensities of inelastic light scattering suggests that the concentration of ionized donors at the heterointerface should be at least an order of magnitude larger than the average concentration of ionized donors in the quantum well barrier. A change in the symmetry of the ground state is predicted theoretically and observed experimentally, but because the perturbation theory works poorly in this magnetic field range, the predicted and measured values of the critical magnetic field differ by 1 T. In lower magnetic fields, the excitation $0S \to 1T$ is possible, which is also observed experimentally (see Fig. 18).

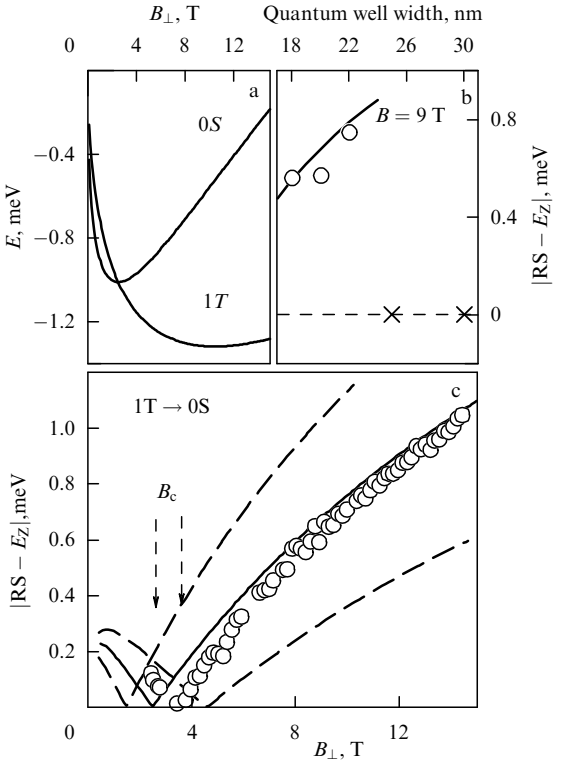

Figure 18. (a) Theoretical dependences of the lowest-energy bound states of two electrons and a positive charge on the heterointerface of a 20 nm quantum well. (b) Energy of the excitation $1T \to 0S$ of the barrier D^--complex as a function of the well width (open circles: measured, solid line: predicted). X's: quantum wells for which the line corresponding to $1T \to 0S$ is not detected. (c) Measured (points) and predicted (solid line) dependence of the energy of the excitations $1T \to 0S$ and $0S \to 1T$ of the barrier D^--complex for a 20 nm well. Arrows indicate theoretical (left) and experimental (right) magnetic fields that change the symmetry of the ground state of the D^--complex. Dashed lines, calculated energy for the excitation $1T \to 0S$ and $0S \to 1T$ for D^--complexes with a positive charge at 10 Å from both sides of the heterointerface. RS: Raman shift.

7. Problems and prospects. Slightly off-integer filling of Landau levels

While the discussion above shows that most of the excitations of a 2DES can be described with the existing approaches, fundamental questions remain concerning the excitation spectrum of a 2DSE. In particular, no success has been achieved in applying the existing theory to describing the spectrum of magnetoexcitons for a filling factor slightly deviating from an odd integer. The primary reason for this is the lack of knowledge about the nature of the ground state. The ground state is commonly considered to be a ferromagnet with charge defects in the form of additional opposite-spin electrons or ground-state holes, which is an acceptable model if the 2DES energy is large compared to the exchange energy

In GaAs/AlGaAS quantum wells, the ratio of the exchange energy to the Zeeman energy can be as large as a few hundred, with the result that for small deviations of the filling factor from odd integer values, the formation of topological ground-state spin textures, skyrmions [54–57], would be the most energetically favorable way to change the charge. The skyrmions smoothly deform the ferromagnetic order, thus forming a vortex-like spin configuration. How many electron spins are involved in this process is determined by the competition between the Coulomb energy and the Zeeman energy g. As $g \to 0$, the size of the skyrmion λ is given by

$$\left(\frac{\lambda}{l_B}\right)^3 = \left(\frac{9\pi^2}{2^8}\right)\left(\frac{l_B}{\varepsilon a_B}\right)\left(g|\ln g|\right)^{-1},$$

where l_B is the magnetic length and a_B is the Bohr radius. If the size is proportional to $g^{-1/3}$, then the number of skyrmion-forming flipped electron spins is

$$K \sim \frac{\lambda^2}{l_B^2} \sim g^{-2/3}.$$

For small g, a skyrmion has the charge e, and the number of skyrmions and their total spin both tend to infinity. For the realistic — intermediate — values of g, the skyrmion-forming flipped spins are few (0–4 for GaAs), invalidating the classical theory used for describing skyrmions [54, 56]. This fact stimulated the development of the theory of spin-texture quasiparticles (STQs) with several flipped spins [56]. Quantum calculations using the exact diagonalization for a finite number of particles, the Hartree–Fock method, and the variational method yield results identical to those obtained classically. When in the ground state, skyrmions can form a crystal lattice.

While the theory of a skyrmion crystal was first developed for two-dimensional electronic systems [58], the first experimental evidence of the formation of a skyrmion crystal lattice came from the study of thin films of the three-dimensional ferromagnet MnSi and similar compounds [59, 60]. There is no convincing evidence, however, of the existence of skyrmion crystals in a two-dimensional Hall ferromagnet with charged defects; indeed the entire theory of STQs in two-dimensional electronic systems is in doubt.

The spin polarization of an electronic system near the filling factor 1 is fully reproduced near the filling factor $\nu = 3$, where the existence of STQs is questionable, even for the zero Zeeman energy [61, 62]. The effective number of flipped electrons determined from recent optical studies of spin polarization in two-dimensional systems [64] does not agree with the previous experimental data, nor with the theoretical values determined by the relation between the Zeeman and exchange energies.

A recent study [65] showed that physical objects similar to STQs — spin textures — exist in the limit of the zero number of flipped spins. The same study suggests, based on inelastic light scattering measurements of spin excitation spectra, that a Hall ferromagnet with charge defects due to a deficiency or an excess of the electron density can have a spin-texture liquid as its ground state, whose existence was first considered in Ref. [66].

Irrespective of the value of K, new gap branches appear in the spectrum (Figs 19, 20) due to the collective precession of the electron spin in the field created by spin textures. The new excitation branches behave as if they were cyclotron excitations in a certain fictitious field proportional to the density of spin textures. Accordingly, the collective spin excitations have their 'effective mass' determined by the polarization of the electronic system and by the exchange interaction. The energy of the 'spin-cyclotron' modes tends to zero as $\nu \to 1$, and the 'cyclotron' frequency separating the spin 'Landau' levels associated with the fictitious field is given by [66]

$$\hbar\omega_c' = \frac{2E_s\,(1-\nu)}{\nu}$$

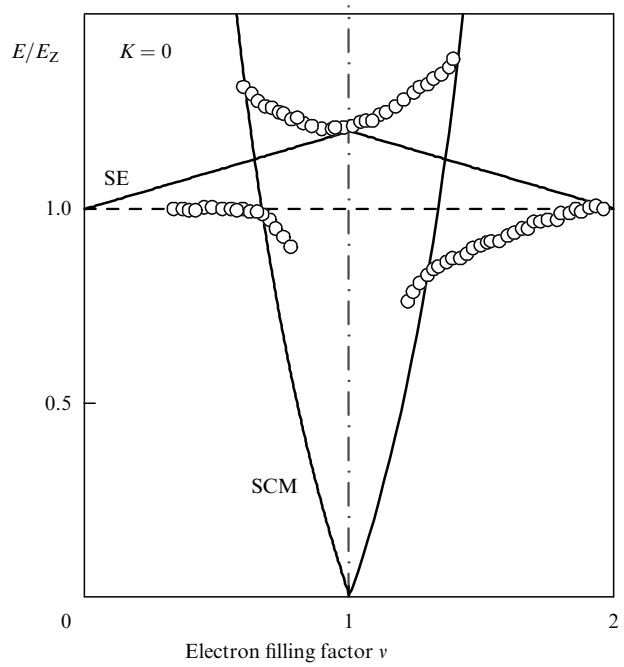

Figure 19. Energies of inelastic light scattering lines versus the electron filling factor ν measured for an expected number of flipped electrons for STQs with $K = 0$ and the generalized momentum $q = 1.5\times10^5$ cm^{-1}. Dashed line: Zeeman energy; solid lines: theoretical estimates for the energies of the 'spin-cyclotron' mode (SCM) and spin exciton (SE).

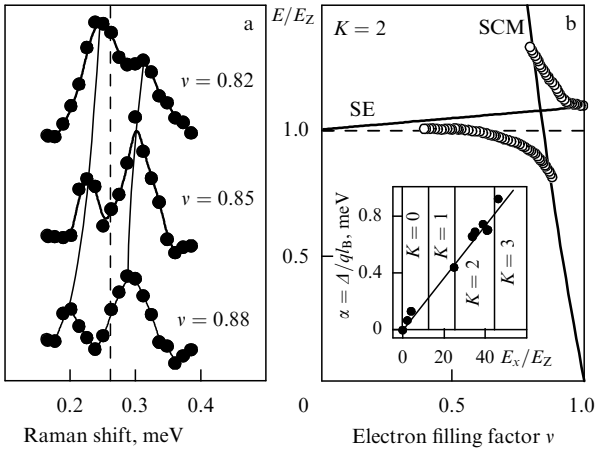

Figure 20. (a) Inelastic light scattering lines versus the electron filling factor ν measured for an expected number of flipped electrons for STQs with $K = 2$ and the generalized momentum $q=1.2\times10^5$ cm^{-1}. Dashed line: Zeeman energy. (b) Energies of inelastic light scattering lines versus the electron filling factor ν measured for the expected number of flipped spins in a $K = 2$ STQ with the generalized moment $q = 1.2\times10^5$ cm^{-1}. Solid lines: estimated energies of the 'spin-cyclotron' mode and spin exciton [65]. The inset shows the amount of mixing of the spin exciton and the spin-cyclotron mode as a function of the exchange energy (or the expected number of flipped spins in an STQ; integer values of K are denoted by solid lines); Δ is the minimum gap between the mutually repelling modes.

for $\nu \ll 1$, where $E_s = 1/4E_x$ is the formation energy of a classical skyrmion and $E_x = \sqrt{\pi/2}e^2/\varepsilon l_B$ is the exchange energy on the zeroth Landau level of electrons. When the number of skyrmions is large, $\hbar\omega_c' \gg k_B T$, the energy of spin-cyclotron modes for small momenta $qn_s^{-1/2} \ll 1$ (n_s is the density of spin textures) is approximately equal to the spin-cyclotron frequency $\hbar\omega_c'$. A surprising (and not yet understood) experimental result in Ref. [65] is that spin-cyclotron modes interact with spin excitons, producing hybrid modes at nonzero momenta in the process (see Fig. 19).

In high-quality GaAs/AlGaAs quantum wells, the effective number of flipped spins is small, and hence a consistent description of the ground state requires that the entire range of K be considered, starting from the well-developed limit $K = 0$. It was assumed that in the $K = 0$ ground state, individual holes act as spin defects. Whether they form a crystal lattice remains an open question, but anyway the ground state would have to be a collinear ferromagnet whose Goldstone mode is a spin exciton. However, the presence of an experimentally observed new spin mode with a lower-than-Zeeman energy and a position varying continuously with the filling factor suggests that the ground state of a noninteger filling two-dimensional system is a short-range ordered noncollinear ferromagnet (the observed mode is not a Goldstone one) which, parenthetically, forms even if the formation of skyrmions is not energetically favorable. The new broken-symmetry state supports an additional spin excitation branch and is not of a skyrmion type.

8. Conclusion

Presented in this paper is a review of the various theoretical and experimental aspects of the physics of magnetoexcitons in two-dimensional electronic systems. A detailed analysis is given of a wide class of excitations occurring at integer filling factors, such as quantum magnetoplasma modes, cyclotron spin-flip excitons, and spin excitons. The paper also presents experimental results on excitations that occur when the filling factor deviates from integers and whose nature is still to be explained theoretically. The positions of impurity centers in the spectrum of spin excitations are determined. Clearly, the excitations that we have reviewed and which take the spin and orbital degrees of freedom into account do not exhaust all the excitations that are possible in two-dimensional electronic systems. There is also a wide class of two-dimensional objects, graphene being an example, in which the presence of additional degrees of freedom — valleys, layers, and symmetrically connected electron and hole bands — produces a host of additional excitations. These, however, are beyond our present scope.

References

1. Bychkov Yu A, Iordanskii S V, Éliashberg G M *JETP Lett.* **33** 143 (1981); *Pis'ma Zh. Eksp. Teor. Fiz.* **33** 152 (1981)
2. Lerner I V, Lozovik Yu E *Sov. Phys. JETP* **51** 588 (1980); *Zh. Eksp. Teor. Fiz.* **78** 1167 (1980)
3. Kallin C, Halperin B I *Phys. Rev. B* **30** 5655 (1984)
4. MacDonald A H *J. Phys. C Solid State Phys.* **18** 1003 (1985)
5. Kohn W *Phys. Rev.* **123** 1242 (1961)
6. Longo J P, Kallin C *Phys. Rev. B* **47** 4429 (1993)
7. Bychkov Yu A, Martinez G *Phys. Rev. B* **66** 193312 (2002)
8. Bisti V E *JETP Lett.* **69** 584 (1999); *Pis'ma Zh. Eksp. Teor. Fiz.* **69** 543 (1999)
9. Bisti V E *JETP Lett.* **73** 21 (2001); *Pis'ma Zh. Eksp. Teor. Fiz.* **73** 25 (2001)
10. MacDonald A H, Oji H C A, Girvin S M *Phys. Rev. Lett.* **55** 2208 (1985)
11. Oji H C A, MacDonald A H *Phys. Rev. B* **33** 3810 (1986)
12. Girvin S M, MacDonald A H, Platzman P M *Phys. Rev. Lett.* **54** 581 (1985)
13. Girvin S M, MacDonald A H, Platzman P M *Phys. Rev. B* **33** 2481 (1986)

14. Lerner I V, Lozovik Yu E *JETP Lett.* **27** 467 (1978); *Pis'ma Zh. Eksp. Teor. Fiz.* **27** 497 (1978)
15. Lerner I V, Lozovik Yu E *JETP* **53** 763 (1981); *Zh. Eksp. Teor. Fiz.* **80** 1488 (1981)
16. Dzyubenko A B, Lozovik Y E *J. Phys. A Math. Gen.* **24** 415 (1991)
17. Usher A et al. *Phys. Rev. B* **41** 1129 (1990)
18. Schmeller A et al. *Phys. Rev. Lett.* **75** 4290 (1995)
19. Dolgopolov V T et al. *Phys. Rev. Lett.* **79** 729 (1997)
20. Khrapai V S et al. *Phys. Rev. B* **72** 035344 (2005)
21. Van'kov A B et al. *Phys. Rev. B* **83** 245325 (2011)
22. Kulik L V et al. *Phys. Rev. B* **72** 073304 (2005)
23. Dickmann S, Kukushkin I V *Phys. Rev. B* **71** 241310(R) (2005)
24. Dzyubenko A B, Lozovik Yu E *Sov. Phys. Solid State* **25** 874 (1983); *Fiz. Tverd. Tela* **25** 1519 (1983); *Sov. Phys. Solid State* **26** 938 (1984); *Fiz. Tverd. Tela* **26** 1540 (1984)
25. Dikman S M, Iordanskii S V *JETP* **83** 128 (1996); *Zh. Eksp. Teor. Fiz.* **110** 238 (1996)
26. Dickmann S, Levinson Y *Phys. Rev. B* **60** 7760 (1999)
27. Dickmann S *Phys. Rev. B* **65** 195310 (2002)
28. Dickmann S *Phys. Rev. B* **61** 5461 (2000)
29. Dickmann S *Phys. Rev. Lett.* **93** 206804 (2004)
30. Bychkov Yu A, Iordanskii S V *Sov. Phys. Solid State* **29** 1405 (1987); *Fiz. Tverd. Tela* **29** 2442 (1987)
31. Kulik L V et al. *Phys. Rev. B* **79** 121310(R) (2009)
32. Van'kov A B et al. *Phys. Rev. Lett.* **97** 246801 (2006)
33. Eriksson M A et al. *Phys. Rev. Lett.* **82** 2163 (1999)
34. Pinczuk A et al. *Phys. Rev. Lett.* **61** 2701 (1988)
35. Pinczuk A et al. *Phys. Rev. Lett.* **68** 3623 (1992)
36. Pinczuk A et al. *Phys. Rev. Lett.* **70** 3983 (1993)
37. Kukushkin I V et al. *Science* **324** 1044 (2009)
38. Scarola V W, Park K, Jain J K *Phys. Rev. B* **61** 13064 (2000)
39. Van'kov A B et al. *Phys. Rev. Lett.* **102** 206802 (2009)
40. Pinczuk A et al. *Phys. Rev. Lett.* **68** 3623 (1992)
41. Dickmann S, Zhilin V M *Phys. Rev. B* **78** 115302 (2008)
42. Doretto R L, Caldeira A O, Girvin S M *Phys. Rev. B* **71** 045339 (2005)
43. Fox H L, Larsen D M *Phys. Rev. B* **51** 10709 (1995)
44. Solovyev V V, Kukushkin I V *Phys. Rev. B* **79** 233306 (2009)
45. Zhuravlev A S et al. *Phys. Rev. B* **77** 155404 (2008)
46. Van'kov A B et al. *JETP Lett.* **87** 145 (2008); *Pis'ma Zh. Eksp. Teor. Fiz.* **87** 170 (2008)
47. Hawrylak P *Phys. Rev. Lett.* **72** 2943 (1994)
48. Dzyubenko A B et al. *Phys. Rev. B* **50** 4687 (1994)
49. Zhuravlev A S et al. *JETP Lett.* **92** 607 (2010); *Pis'ma Zh. Eksp. Teor. Fiz.* **92** 672 (2010)
50. Cheng J-P et al. *Phys. Rev. Lett.* **70** 489 (1993)
51. Jiang Z X, McCombe B D, Hawrylak P *Phys. Rev. Lett.* **81** 3499 (1998)
52. Dzyubenko A B, Sivachenko A Yu *Phys. Rev. B* **48** 14690 (1993)
53. Larsen D M, McCann S Y *Phys. Rev. B* **45** 3485 (1992)
54. Sondhi S L et al. *Phys. Rev. B* **47** 16419 (1993)
55. Fertig H A et al. *Phys. Rev. B* **50** 11018 (1994)
56. Abolfath M et al. *Phys. Rev. B* **56** 6795 (1997)
57. Fertig H A et al. *Phys. Rev. B* **55** 10671 (1997)
58. Brey L et al. *Phys. Rev. Lett.* **75** 2562 (1995)
59. Mühlbauer S et al. *Science* **323** 915 (2009)
60. Yu X Z et al. *Nature* **465** 901 (2010)
61. Cooper N R *Phys. Rev. B* **55** R1934 (1997)
62. Song Y-Q et al. *Phys. Rev. Lett.* **82** 2768 (1999)
63. Drozdov I K et al. *Phys. Rev. Lett.* **104** 136804 (2010)
64. Plochocka P et al. *Phys. Rev. Lett.* **102** 126806 (2009)
65. Kulik L V et al. *Phys. Rev. B* **87** 045316 (2013)
66. Rodriguez J P *Europhys. Lett.* **42** 197 (1998)