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Relation between energy conservation and dynamics
(comment on "Analytical mechanics and field theory: derivation of equations from energy conservation" by N A Vinokurov [*Phys. Usp.* **57** 593 (2014); *Usp. Fiz. Nauk* **184** 641 (2014)])

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Abstract. **Possible applications of the relation between the equation of dynamics and energy conservation are considered. It is shown that the method used in the paper by Vinokurov [*Phys. Usp.* 57 593 (2014); *Usp. Fiz. Nauk* 184 641 (2014)] should be re-examined to accurately understand the role of gyroscopic forces.**

Keywords: energy, Lagrangian, gyroscopic forces

A paper by Vinokurov [1], discussing the connection between the energy conservation law and dynamics, was published by *Physics–Uspekhi* in the section "Methodological notes." The author's opinion on the usefulness of discussing questions pertaining to the justification of the mathematical apparatus of physics is undoubtedly correct, but also invites a question on the completeness of the proposed approach.

The postulate on the validity of the energy conservation law is based on centuries-long experience, and its validity is beyond any doubt.

It is appropriate to consider this law as applied to a system subject to the action of external forces, with the work done by those forces modifying the energy:

$$\mathrm{d}E(q, v) = Q\,\mathrm{d}q \equiv Q_i\,\mathrm{d}q_i\,.$$

It is assumed that the energy depends on the generalized coordinates $q \equiv \{q_i\}$ and generalized velocities $v \equiv \{v_i = \mathrm{d}q_i/\mathrm{d}t_i\}$, but is independent of time, the preceding system history, and higher time derivatives. Each (generalized) velocity can be computed knowing either the proper time t_i of this degree of freedom or the common time t of the frame of reference. Summation over repeated indices is understood wherever this makes sense; the indices are suppressed (for example, in dot products).

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For an arbitrary function $L(q, v)$ of coordinates and velocities, we have

$$\mathrm{d}L(q, v) = \frac{\partial L}{\partial q}\,\mathrm{d}q + \frac{\partial L}{\partial v}\,\mathrm{d}v\,,$$

but

$$\frac{\partial L}{\partial v}\,\mathrm{d}v = \mathrm{d}\left(v\,\frac{\partial L}{\partial v}\right) - \mathrm{d}q\,\frac{\mathrm{d}(\partial L/\partial v)}{\mathrm{d}t}\,.$$

Hence, it follows that

$$\mathrm{d}\left(v\,\frac{\partial L}{\partial v} - L\right) = \left(\frac{\mathrm{d}(\partial L/\partial v)}{\mathrm{d}t} - \frac{\partial L}{\partial q}\right)\mathrm{d}q\,.$$

Because the change in energy contains differentials of coordinates and does not contain differentials of velocities, the energy conservation law reduces to two conditions:

(a) there exists a Lagrangian function L related to the energy E by

$$v_i\,\frac{\partial L}{\partial v_i} - L = E(q, v)\,; \tag{1}$$

(b) a system of equations for the system evolution holds in the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial v_i}\right) - \frac{\partial L}{\partial q_i} = Q_i\,. \tag{2}$$

The combination of these conditions can be viewed from different standpoints.

(1) If we presume that the explicit expression for the energy $E(q, v)$, obtained from experiment or any other considerations, is reliable enough, then relation (1) is an equation for the Lagrangian function, while Eqns (2) are constructed on the basis of a solution of this equation. However, the general solution of Eqn (1) contains a solution of the homogeneous equation

$$v_i\,\frac{\partial L}{\partial v_i} - L = 0\,,$$

which has the form $a_i(q)v_i$ and determines the gyroscopic forces

$$F_i = v_i \left(\frac{\partial a_i}{\partial q_j} - \frac{\partial a_j}{\partial q_i} \right) ,$$

which do not influence the value of the energy. And yet, these forces enter the equations of motion and hence influence the system evolution; they cannot be found from the energy conservation law alone, however.

(2) If we depart from the reliably established motion laws, for example, Newton's dynamic laws or Maxwell's equations in the form of Lagrange equations, then the energy is defined uniquely (in nonrelativistic theory, up to the choice of the reference point). The terms linear in velocities in the Lagrangian function are defined by the characteristics of gyroscopic forces entering the dynamic laws.

We note that relation (1) has the form of the Legendre transformation, which allows passing to Hamilton's equations in a standard manner.

Thus, determining the dynamics on the basis of the energy conservation law hinges on additional analysis not connected to the energy conservation law and the introduction of gyroscopic forces, while the approach based on using the dynamical law as primary determines the energy uniquely, defines the conditions for energy conservation, and offers the possibility of using both Lagrangian and Hamiltonian forms of the equations of motion.

References

1. Vinokurov N A *Phys. Usp.* **57** 593 (2014); *Usp. Fiz. Nauk* **184** 641 (2014)