Physics – Uspekhi **58** (12) 1223 – 1224 (2015)

LETTERS TO THE EDITORS

PACS numbers: **05.45.–a**, **45.40.–f**

On the validity of the nonholonomic model of the rattleback

S P Kuznetsov

DOI: 10.3367/UFNe.0185.201512h.1342

Abstract. In connection with the problem of a convex-shaped solid body on a rough horizontal plane (the rattleback or Celtic stone), the paper discusses the validity of the nonholonomic model which postulates that the contact point has zero velocity and, hence, friction performs no mechanical work. While abstract, this model is undoubtedly constructive, similar to many idealizations commonly used in science. Despite its energy-conserving nature, the model does not obey Liouville's theorem on phase volume conservation, thus allowing the occurrence in the phase space of objects characteristic of dissipative dynamics (attractors) and thereby leading to phenomena like the spontaneous reversal of rotations. Nonholonomic models, intermediate between conservative and dissipative systems, should take their deserved place in the general picture of the modern theory of dynamical systems.

Keywords: rattleback, solid body, nonholonomic model, attractor

The dissatisfaction expressed in comments by V F Zhuravlev [1] with respect to the nonholonomic mechanical model of rattleback dynamics, used in Refs [2, 3] published by *UFN* (*Phys. Usp.*), is partly understandable, for this model indeed substantially roughens and oversimplifies the description against the real physical system. Nevertheless, it would be fundamentally inapt and methodologically incorrect to reject this approach altogether. One needs simply to clearly realize which goals are being pursued and what level of understanding can be achieved on its basis. In our opinion, the advantage of the nonholonomic model is precisely that it picks up the most essential aspects in order to explain the paradoxical behavior of rattlebacks, including the reverse effect. This model offers the possibility of treating the object dynamics in the framework of dynamical system theory, which, owing to its deeply elaborated character and wide range of applications, warrants an important place in modern scientific thinking [4–6].

On the other hand, in making the problem far more specific, for instance, with respect to the law of friction, as is proposed in the comment by Zhuravlev [1], one creates models which are more complex and far less transparent [7]. Given an appropriate set of functional characteristics and parameters this would, seemingly, make a more quantitatively precise description feasible, but then, obviously, the generality would be lost and every particular rattleback would require a separate solution, and conclusions drawn on the presence of one phenomenon or another, for example, reverse effect, would be related solely to it.

In various branches of physics, nobody questions the fruitfulness of concepts idealizing the properties of real physical systems, such as a material point particle, an absolutely rigid body, an ideal gas, or an ideal fluid [8]. The circumstance that one may easily run into contradictions with physical reality by applying the respective idealized concepts too literally does not present a hindrance (the concepts must certainly be made known and interpreted, which is, understandably, the scientist's responsibility). One can be reminded, for example, of the d'Alambert–Euler paradox on the absence of reaction force exerted on bodies moving uniformly in an ideal fluid [8, 9] or the Ehrenfest paradox occurring if one attempts to use the idea of a rigid body in the framework of special relativity theory [10]. However, by omitting details related to concrete problems we always gain the possibility of a much broader view of the problem at hand and of drawing conclusions related to a broad spectrum of situations.

Similarly to the notions mentioned above, an idealized abstraction is also the basic concept of a dynamical system considered as an object whose instantaneous state is determined by a finite set of variables specifying the point position in the phase space and for which one knows the operator of evolution, as a rule, to unambiguously determine the state at any subsequent moment of time based on the initial state [4–6]. The abstraction is introduced by the basic idea of isolating the object from the rest of the world.

If we continue, among dynamical systems one distinguishes between conservative (Hamiltonian) and dissipative systems. The idea of a conservative system is also the result of abstraction, for dissipation is intrinsic to all systems in the real world because the motions being considered interact with the ambient world and because of the microscopic structure of the physical system proper (atoms and molecules). On the other hand, the dissipation, if understood simply as the loss of system's mechanical energy or as contraction for the volume of a cloud of points representing an ensemble of systems in the phase space in the process of dynamic evolution, throws overboard the fundamental coupling to fluctuations. Yet namely they lie at the heart of the origin of dissipation and, if one is willing to stay consistent to the end, break, in principle, the possibility of representing the object state as given by a finite set of variables. (This issue proves to be of fundamental importance if one turns to the problem of an adequate description of dissipation with account for quantum effects.)

S P Kuznetsov Udmurt State University,
ul. Universitetskaya 1, 426034 Izhevsk, Udmurtiya, Russian Federation;
Saratov Branch of Kotel'nikov Institute of Radio Engineering
and Electronics, Russian Academy of Sciences,
ul. Zelenaya 38, 410019 Saratov, Russian Federation
E-mail: spkuz@rambler.ru

Received 8 October 2015
Uspekhi Fizicheskikh Nauk **185** (12) 1342 – 1344 (2015)
DOI: 10.3367/UFNr.0185.201512h.1342
Translated by M V Tsaplina; edited by A Radzig

Needless to say, the idea of the nonholonomic constraint [11–14] is an abstraction, too. The richness and fruitfulness of this abstraction can hardly be substantiated by just contemplating, yet they get support through the entire history of research and development, as well as through the accumulated baggage of knowledge and 'tools' of nonholonomic mechanics which has been contributed by many outstanding scientists. Finding a place of involved dynamical phenomena intrinsic to such a system as a rattleback, in this picture is a task that is both natural and necessary [2, 3, 14, 15].

From the standpoint of the dynamical system theory, nonholonomic mechanics is interesting because it introduces a class of objects occupying an intermediate position between the conservative and dissipative systems of traditional treatment. The nonholonomic model of rattlebacks belongs to this class. On the one hand, this system is conservative in the sense of the preservation of mechanical energy; on the other, the phase volume is not conserved in the process of evolution with time, which makes possible the presence of objects in the phase space which are characteristic for dissipative dynamics — attractors, including simple (attracting fixed points, limit cycles) and complex (strange attractors) ones.

The main intriguing feature of rattleback motion (the reverse effect) is in clear correspondence with the behavior of the nonholonomic model, namely, with the coexistence in phase space of a pair of invariant sets — an attractor and a repeller, of which only the attractor corresponds to the observed motion of a physical object in the intermediate asymptotics, when the object moves sufficiently far away from strongly unstable states, yet still stays far from the decay owing to friction.

The nonholonomic model of rattlebacks demonstrates [3] diverse periodic and chaotic regimes, attractors, including Lorenz ones [6, 16, 17], the transition to chaos according to the traditional Feigenbaum scenario [6, 18]. This represents substantial interest, both as an illustration of possible phenomena in the framework of nonholonomic mechanics and as a concrete contributing example to the ideas and techniques of contemporary nonlinear dynamics. Caution with respect to the applicability of the nonholonomic model for a quantitative description of the problem is laid out in a sufficiently clear form in the conclusions of our work [3].

The strength of a theoretical concept lies not only and not so much in the fact that it gives a possibility of quantitatively describing concrete physical systems, but also in its ability to create a base for the description and interpretation of a broad range of phenomena from common positions, even though one is bound to qualitative considerations. Nonholonomic mechanics, undoubtedly, deserves attention and research effort, for it proposes precisely such a broad base, in particular, for solution to various tasks, for example, in the mechanics of vehicles, robotics, etc. [19–21].

The author is indebted to the Russian Science Foundation for the support of the work through grant No. 15-12-20035.

References

1. Zhuravlev V F *Phys. Usp.* **58** 1218 (2015); *Usp. Fiz. Nauk* **185** 1337 (2015)
2. Borisov A V, Mamaev I S *Phys. Usp.* **46** 393 (2003); *Usp. Fiz. Nauk* **173** 407 (2003)
3. Borisov A V, Kazakov A O, Kuznetsov S P *Phys. Usp.* **57** 453 (2014); *Usp. Fiz. Nauk* **184** 493 (2014)
4. Loskutov A *Phys. Usp.* **53** 1257 (2010); *Usp. Fiz. Nauk* **180** 1305 (2010)
5. Afraimovich V S et al. *Regul. Chaotic Dyn.* **19** (4) 435 (2014)
6. Kuznetsov S P *Dinamicheskii Khaos* (Dynamical Chaos) (Moscow: Fizmatlit, 2006)
7. Zhuravlev V Ph, Klimov D M *Mech. Solids* **43** 320 (2008); *Izv. Ross. Akad. Nauk Mekh. Tverd. Tela* (3) 8 (2008)
8. Feynman R P, Leighton R B, Sands M *The Feynman Lectures on Physics* (Reading, Mass.: Addison-Wesley Publ. Co., 1963); Translated into Russian: *Feinmanovskie Lektsii po Fizike* (Moscow: Mir, 1967)
9. Lamb H *Hydrodynamics* (Cambridge: The Univ. Press, 1930); Translated into Russian: *Gidrodinamika* (Moscow–Leningrad: OGIZ, 1947)
10. Sokolovskii Iu I *The Special Theory of Relativity* (Delhi: Hindustan Publ. Corp., 1962), Translated from 1st Russian Ed.; *Teoriya Otnositel'nosti v Elementarnom Izlozhenii* 2nd ed. (Moscow: Nauka, 1964)
11. Hertz H *Prinzipien der Mechanik in Neuem Zusammenhange Dargestellt* (Leipzig: Johann Ambtosius Barth, 1894); Translated into English: *The Principles of Mechanics Presented in a New Form* (Mineola, N.Y.: Dover Publ., 2003); Translated into Russian: *Printsipy Mekhaniki, Izlozhennye v Novoi Svyazi* (Exec. Ed. I I Artobolevskii) (Moscow: Izd. AN SSSR, 1959)
12. Neimark Ju I, Fufaev N A *Dynamics of Nonholonomic Systems* (Providence, R.I.: American Mathematical Society, 1972); Translated from Russian: *Dinamika Negolonomnykh Sistem* (Moscow: Nauka, 1967)
13. Mei F *Appl. Mech. Rev.* **53** (11) 283 (2000)
14. Borisov A V, Mamaev I S, Bizyaev I A *Regul. Chaotic Dyn.* **18** (3) 277 (2013)
15. Borisov A V, Kazakov A O, Sataev I R *Regul. Chaotic Dyn.* **19** (6) 718 (2014)
16. Lorenz E N *J. Atmos. Sci.* **20** (2) 130 (1963)
17. Gonchenko A S, Gonchenko S V, Kazakov A O *Regul. Chaotic Dyn.* **18** (5) 521 (2013)
18. Feigenbaum M J *J. Stat. Phys.* **19** (1) 25 (1978)
19. Baillieul J, Sastry S S, Sussmann H J (Eds) *Essays on Mathematical Robotics* (The IMA Volumes in Mathematics and its Applications, Vol. 104) (New York: Springer-Verlag, 2012)
20. Becker F et al. *Regul. Chaotic Dyn.* **18** (1 – 2) 63 (2013)
21. Svinin M, Morinaga A, Yamamoto M *Regul. Chaotic Dyn.* **18** (1 – 2) 126 (2013)