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Quantum-electrodynamic cascades in intense laser fields

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Abstract. **It is shown that in an intense laser field, along with cascades similar to extensive air showers, self-sustaining field-energized cascades can develop. For intensities of 10^{24} W cm^{-2} or higher, such cascades can even be initiated by a particle at rest in the focal area of a tightly focused laser pulse. The cascade appearance effect can considerably alter the progression of any process occurring in a high-intensity laser field. At very high intensities, the evolvement of such cascades can lead to the depletion of the laser field. This paper presents a design of an experiment to observe these two cascade types simultaneously already in next-generation laser facilities.**

Keywords: quantum-electrodynamic cascades, ultrahigh-intensity lasers, high-energy beams

1. Introduction

The invention in the latter half of the 1980s (see Ref. [1]) and subsequent development of chirped-pulse amplification (CPA) technology have made possible petawatt laser facilities capable of generating laser pulses with a peak intensity of up to 10^{22} W cm^{-2} [2]. As part of the European Extreme Light Infrastructure (ELI) project [3], three 10^{24} W cm^{-2} lasers will be launched as early as 2016. A long-term development goal planned both by ELI and the Exawatt Center for Extreme Light Studies (XCELS) projects at the RAS Institute of Applied Physics (Nizhny Novgorod) [4] is the creation of ultrahigh-power 200-petawatt lasers that will generate femtosecond pulses with an intensity of 10^{26} W cm^{-2} or even higher. These achievements are expected to widely expand the scope of experimental research. Among the high-priority fundamental research avenues are the laser-driven-plasma acceleration of electrons and ions, high-brightness γ- and hard X-ray sources, photonuclear physics (including laser-controlled nuclear reactions), and ultrarelativistic plasma dynamics under extreme conditions (‘laboratory astrophysics’). A wide range of application possibilities, from tabletop particle accelerators to materials science and hadron therapy, are the subject of intense current discussion.

Of obvious interest is the principal possibility of experimentally studying quantum electrodynamics in an intense external field, including those issues as vacuum polarization effects and the creation of electron–positron pairs by the field; birefringence in a vacuum; harmonic generation, and self-focusing and mutual focusing (defocusing) effects in a vacuum. Since the early 1960s, dozens of calculations have been performed on fundamental quantum-electrodynamic processes (in particular, the probabilities of a photon emission and absorption, of an electron–positron pair creation by a photon, of a one-photon pair annihilation, of a photon splitting, and of polarization and mass operators in a constant field, and in the field of a plane monochromatic wave). In all these processes, the external field was treated nonperturbatively. A summary of the work done prior to 1985 is given in Refs [5, 6] along with a detailed list of references. To date, the only experiments to test the effects of nonlinear quantum electrodynamics in a strong field were done at the Stanford Linear Accelerator Center (SLAC) in 1996–1999. These studies concerned the effects of the scattering of a 10^{18} W cm^{-2} laser pulse off a 46.6 GeV electron beam (nonlinear Compton effect) [7]. Also observed in this work were events of the creation of electron–positron pairs by hard photons emerged through the nonlinear Compton effect [8]. (For details of these experiments, see Ref. [9]).

With the laser intensities of order 10^{22} W cm^{-2} already available and intensities of about 10^{26} W cm^{-2} and above expected within the ELI and XCELS projects, unique possibilities appear for both moving to a higher technological level in examining the quantum electrodynamic effects already known and observing totally novel nonlinear vacuum effects not yet experimentally revealed.

Importantly, when repeating the SLAC experiments for projected ultrahigh radiation intensities, the collisions of high-energy particles with a laser pulse will result not in a mere few individual pair creation events but rather in showers — or cascades — of them [10, 11]. Such cascades of successive events with the emission of hard photons, in turn producing electron–positron pairs, represent a chain reaction that proceeds until the charged particles lose all of their energy and is reminiscent of extensive air showers produced by cosmic rays. These cascades will be referred to here as S type or Shower type cascades. When planning experiments on next-generation laser facilities, a simulation of the relevant processes under this new regime is desirable. It should be noted that at sufficiently high (but still practical) laser field intensities, obtaining accurate results can require taking into account the full set of radiative corrections arising from inserting an unlimited number of polarization loops, because the perturbation theory expansion parameter may, in this case, exceed unity [12].

Recent work [13, 14] has used simple estimates to predict that, for a laser pulse intensity of 10^{24} W cm^{-2} or more, ‘spontaneous’ showers initiated by initially slow charged particles should be expected to appear in electromagnetic fields. The role of the laser field in this case is twofold. Besides presenting a target for high-energy particles, it also acts as a continuously operating accelerator which restores the radiative energy loss of fast charged particles. As a result, the cascade develops in a self-sustained manner and its multiplicity is already limited not by the initial energy of the seed particle but rather by the pulse duration or by the time it takes all charged particles to be fully pushed out of the laser field focus due to the ponderomotive effect. In this respect, the self-sustained quantum-electrodynamic cascade is reminiscent of the avalanche breakdown in an insulator placed in a strong electric field. We will call such cascades A type, with A standing for avalanche. The predicted existence of A cascades implies a radical change in our understanding of how intense laser fields interact with matter. The formation of avalanches is necessary to take into account when describing

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interaction effects at laser intensities in excess of 10^{24} W cm^{-2}, and when planning appropriate experiments.

2. A cascades

This section presents the theory of A type cascades. We assume that at the initial time a charged seed particle (electron) is at rest at the center of the laser pulse focal area. The particle–field interaction is determined by two parameters, ξ and χ_e (see, for example, Refs [5, 6]),

$$\xi = \frac{eE}{m\omega} \sim \frac{eE\lambda}{m} \sim \frac{\lambda}{l_C}\frac{E}{E_S}\,, \qquad \chi_e = \gamma\,\frac{E_\perp}{E_S}\,, \tag{1}$$

where λ and ω are the laser wavelength and frequency, respectively, $\gamma = 1/(1-v^2)^{1/2}$ is the particle's Lorentz factor, $l_C = 1/m = 3.86\times10^{-11}$ cm is the Compton length, $E_\perp$ is the field component perpendicular to the particle's velocity, and $E_S = m^2/e = 1.32\times10^{16}$ V cm^{-1} is the characteristic quantum-electrodynamical field.[1] The parameter ξ is independent of the Planck constant $\hbar$ and represents the classical nonlinearity parameter. For $\xi \ll 1$, perturbation theory with respect to the external field is valid, and for $\xi \gtrsim 1$, the interaction of the particle with the external field becomes multiphotonic. The parameter χ_e is a dynamical quantum parameter responsible for the magnitude of nonlinear quantum effects and can be considered as a measure of the external field strength in the rest frame of the charge particle ($\gamma = 1$). In the case of the field of a plane monochromatic wave, ξ and χ_e are relativistic invariant and gauge-invariant parameters:

$$\xi^2 = -\frac{e^2\langle A^\mu A_\mu\rangle}{m^2}\,, \qquad \chi_e^2 = \frac{e^2\big\langle (F_{\mu\nu}p^\nu)^2\big\rangle}{m^6}\,, \tag{2}$$

where $F_{\mu\nu}$ is the electromagnetic field tensor. The interaction of a photon with a laser field is characterized by the dynamic quantum parameter χ_γ defined in the same way as for an electron in formula (1), but with $\gamma = \varepsilon_\gamma/m$, where ε_γ is the photon energy.

A quantum-electrodynamic cascade comprises a chain of elementary processes, each of which consists of the emission of a photon by an electron (positron), accompanied by the photocreation of a pair (Fig. 1). The formation length of these processes is determined by the parameter ξ: $l_f \sim \lambda/\xi$ [5, 6]. We assume that the laser pulse peak intensity E_0 should at least satisfy the inequality $E_0 \gtrsim 10^{-3}E_S$, which corresponds to $\xi \gtrsim 10^3$ or to the laser intensity $I \gtrsim 5\times10^{23}$ W cm^{-2}. For such fields, one obtains $l_f \lesssim 10^{-3}\lambda$, and the field can be considered constant when calculating the probabilities of the elementary processes involved (locally constant field approximation). Moreover, the formation of a cascade can involve only particles with the parameters $\chi_{e,\gamma} \gtrsim 1$, which is a necessary condition for the charged particle to emit a hard photon capable afterwards of creating an electron–positron pair (for $\chi_\gamma \ll 1$, the pair creation probability is exponentially small). In our case, this means that the parameters $\chi_{e,\gamma}$ are much larger in value than the pure field invariants nondimensionalized by normalizing to the characteristic field E_S, and hence the external field can be considered as a constant crossed field [5, 6]. The probabilities of elementary

[1] Throughout the paper, we use the natural system of units in which $\hbar = c = 1$.

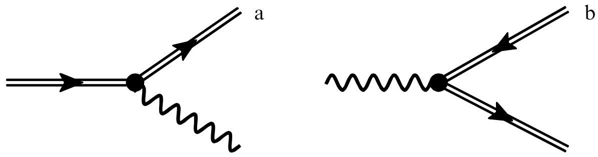

Figure 1. First-order quantum-electrodynamical processes in an external field: (a) photon emission, (b) pair creation by a photon. Rightward (leftward) double arrows correspond to electrons (positrons) interacting with the external field, and wavy lines to photons.

processes in a constant crossed field were first calculated by A I Nikishov and V I Ritus [5, 6]. For $\chi_{e,\gamma} \gtrsim 1$, one can use the expressions

$$W_{e,\gamma} \sim \frac{\alpha m^2}{\varepsilon_{e,\gamma}}\,\chi_{e,\gamma}^{2/3}\,, \qquad \chi_{e,\gamma} \gg 1 \tag{3}$$

to estimate the probabilities of elementary processes. In our problem, $W_{e,\gamma}$ determine the electron and photon lifetimes with respect to the emission of a hard photon and the creation of pair, respectively:

$$t_{e,\gamma} \sim \frac{1}{W_{e,\gamma}}\,. \tag{4}$$

For particles with $\chi_{e,\gamma} \sim 1$, the lifetimes $t_{e,\gamma} \sim (1/\alpha m)(E_0/E_S)$ turn out to be a factor of $1/\alpha$ larger than the process formation times $t_f = l_f$, suggesting, for estimation purposes, the following cascade scenario. The external field accelerates the initially-at-rest particle such that its parameter χ_e increases to $\chi_e \sim 1$, and in time $\sim t_e$ the particle emits a hard photon which then creates a pair, whereas the particle whose parameter χ_e decreased by χ_γ due to recoil is again accelerated within the same time to $\chi_e \sim 1$ and emits a hard photon again, etc. The same scenario takes place for those cascade branches generated by secondary particles. Clearly, this scheme can only occur if the acceleration time defined as $t_{acc}\,d\chi/dt \sim 1$ is less or on the order of the times $t_{e,\gamma}$ which, in turn, should be much less than the time t_{esc} the particle remains in the pulse focal area, i.e., much less than either the pulse duration or the time it takes for the particle to be pushed out from the area by the ponderomotive force. Notice that because the particle cannot emit a hard photon within time $t < t_{acc}$, it can be viewed as moving along a classical trajectory between photon emission events. Thus, the cascade can evolve if the time hierarchy

$$t_{acc} \lesssim t_{e,\gamma} \ll t_{esc} \tag{5}$$

is obeyed.

The electron mean free path is much greater than the formation length of an emission process; therefore, when estimating t_{acc}, the field can no longer be considered constant and crossed, and some reasonable model should be employed, which allows solving the particle's classical equation of motion. It should be noted that the phenomenon of cascade evolution takes place only in fields for which the parameter χ_e is not an integral of motion. In particular, an A cascade does not occur in the field of a plane wave or in a constant field. We will apply a model of a uniform purely electric field rotating with the frequency ω of the laser pulse. Such a field is set up at the antinodes of a circularly polarized standing monochromatic wave. The electron equation of motion in a rotating electric field, $\dot{\mathbf{p}}(t) = e\mathbf{E}(t)$, is readily solved to give the following expression for $\chi_e(t)$ in the limit of $\omega t \ll 1$:

$$\chi_e(t) \approx \frac{E}{E_S}\sqrt{1+\frac{p_\perp^2}{m^2}} = \frac{E}{E_S}\sqrt{1+\frac{\xi^2}{4}(\omega t)^4}\,. \tag{6}$$

Because ξ is very large, $\xi(\omega t)^2$ can exceed unity even if $\omega t \ll 1$, and then $\chi_e(t) \sim \xi(E/E_S)(\omega t)^2$. This gives an estimate for the acceleration time

$$t_{\text{acc}} \sim \frac{1}{m}\frac{E_S}{E}\sqrt{\frac{m}{\omega}}\,. \tag{7}$$

An extremely important point to note is that the rotating electric field not only accelerates the particle but also bends its trajectory, which is exactly the reason why the particle acquires a momentum component transverse to the field.

Using the solutions to the equations of motion for ε_e and χ_e, it is a simple matter to obtain the time dependence of probability in Eqn (3). Then, from Eqn (4) it is found that

$$t_e \sim \frac{1}{m}\left(\frac{E_S}{\alpha^3 E}\right)^{1/4}\sqrt{\frac{m}{\omega}}\,, \tag{8}$$

and the left inequality in Eqn (5) reduces to the condition [15]

$$E \gtrsim E_* = \alpha E_S\,. \tag{9}$$

Here, E_* is a new characteristic field determining the threshold for the development of a cascade in the field of an optical laser. The work done by such a field on the electron over its mean free path is on the order of m. The intensity corresponding to the field E_* is $I_* = 2.5\times10^{25}$ W cm^{-2}. As for the right inequality in Eqn (5), it is quite conservative for optical frequencies and for field strengths of order E_*.

Presented in Fig. 2 are the results of numerical simulations, illustrating the evolution of the dynamic quantum parameter $\chi(t)$ along the trajectory of a particle in fields of different configurations [15]. It is seen that, while the behavior of $\chi(t)$ is case-specific in detail, the basic qualitative

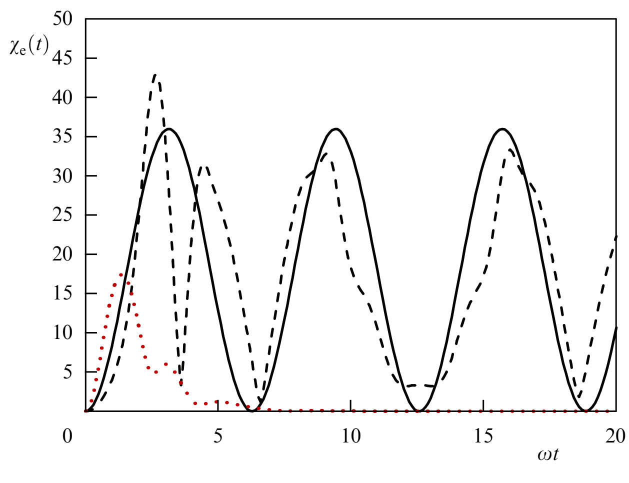

Figure 2. Evolution of the dynamic quantum parameter $\chi_e(t)$ along the particle trajectory for $\xi = 3\times10^3$, and $\omega = 1$ eV in three cases: head-on collision of two elliptically polarized plane waves (solid line); right-angle collision of two linearly polarized plane waves (dashed line) and single strongly focused e-polarized pulse (dotted line).

characteristics, notably the oscillation period and amplitude, are similar to an order of magnitude. A point of special importance is that this similarity occurs for small times, $\omega t < 1$, suggesting that the estimates obtained for a uniformly rotating electric field also hold, at least qualitatively, for any field that accelerates charged particles.

The effect of the emergence of an A type cascade can change markedly the course of any process occurring in an ultraintense laser field. As an example, consider the initiation of a cascade by electron–positron pairs created by a laser field from a vacuum [15]. Pair creation from a vacuum by a classical electromagnetic field is apparently the most interesting of all nonlinear vacuum effects. The long-held belief was that this effect can only be observed if the external electric field strength is close to the quantum-electrodynamic critical value E_S. Such a field, first introduced by F Sauter [16, 17], does work m on the electron over a distance equal to the electron Compton length. However, as previously shown by the authors of Refs [18–20], for pair creation by focused laser pulses to be observable, the peak strength should be $E_0 < E_S$. The reason is that the so-called Schwinger exponential $\exp(-\pi E_S/E_0)$ defines the pair creation probability in the Compton volume $V_C \sim l_C^3$ and in Compton time $T_C \sim l_C$. For $E_0 < E_S$, the pair creation probability in the Compton 4-volume is exponentially small. However, if the field is induced in a 4-region whose volume VT greatly exceeds the Compton volume, then the pair creation probability acquires a large preexponential, which is equal by an order of magnitude to $VT/(V_C T_C)$. For an optical 1μm, 10-fs laser pulse focused to the diffraction limit, this ratio reaches a value on the order of 10^{25}, making the preexponential large enough to compensate for the small value of the Schwinger exponential for $E_0 < E_S$. Therefore, pair creation can even be observed for peak laser pulse strengths two orders of magnitude less than E_S and thus will be accessible for observation in the next-generation facilities currently under discussion in the framework of the ELI and XCELS projects.

Assuming that the focal area initially contains a single at-rest electron–positron pair created, for example, by an external photon, the total number of pairs created during cascade evolution per laser shot is estimated as [15]

$$N_e \sim \exp\left(\frac{t_{\text{esc}}}{t_e}\right) \sim \exp\left(\pi\alpha\mu^{1/4}\sqrt{\frac{m}{\omega}}\right), \tag{10}$$

where $\mu = E/E_*$ (see the solid line in Fig. 3).The number N_e naturally increases with increasing peak voltage, until ultimately the created pairs become so large in number that their energy can become equal to that of the laser pulse itself. Under the assumption of the laser pulse being focused to the diffraction limit — so that the pair creation focal area has a volume on the order of $(\lambda/2)^3$ — the total energy of the field can be estimated as $W \sim (E^2/4\pi)(\lambda/2)^3$. Thus, the maximum number of the field-produced pairs cannot be larger than

$$N_{e,\max} = \frac{W}{2\varepsilon_e} \sim \alpha\mu^{5/4}\left(\frac{m}{\omega}\right)^{5/2}. \tag{11}$$

This quantity is presented by the dashed line in Fig. 3. It is seen that the energy of the created pairs becomes equal to the laser pulse energy when $\mu \approx 10$, which in the case of two colliding circularly polarized waves corresponds to the total intensity $I \approx 6\times10^{26}$ W cm^{-2}.

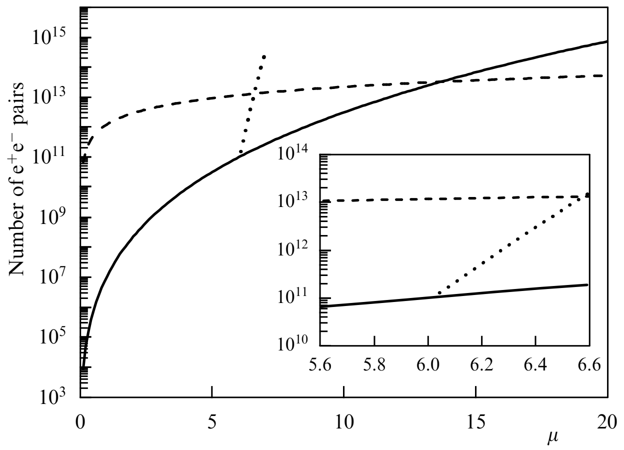

Figure 3. Number of created pairs as a function of $\mu = E/E_*$. Solid line: the number of pairs, N_e, created in the single-particle-initiated cascade process. Dotted line: the number of pairs in cascades generated by pairs created from a vacuum by the field of two colliding, circularly polarized 10-fs laser pulses. The intersection point of the two lines corresponds to the threshold value of μ at which pairs start to be created spontaneously from the vacuum. The dashed line shows the maximum possible number of pairs, $N_{e,\max}$, determined by the laser pulse energy. Laser frequency is $\omega = 1$ eV. Inset: a magnified view of the intersection region.

Estimate (11) was obtained under the assumption that the cascade is initiated by a single pair. In our case, however, the threshold intensity for pair creation is $I_{th} \approx 2.3 \times 10^{26}$ W cm^{-2} [19]. Because of its very sharp exponential dependence on intensity, the number N_e of created pairs at $I \approx 6 \times 10^{26}$ W cm^{-2} can reach the value of $N_e \approx 6 \times 10^8$, which means that the laser pulse breaks up much earlier than predicted by estimate (11). The field dependence of N_e taking into account spontaneously created pairs is shown by a dashed line in Fig. 2. The branching point of the solid and dotted lines corresponds to the threshold value of $\mu = \mu_{th}$ which, according to Ref. [19], is $\mu_{th} \approx 6$ for the field set up by two colliding, circularly polarized pulses. We see that N_e reaches its maximum possible value at $\mu \approx 6.6$, which corresponds to the colliding pulse intensity of $I \approx 2.7 \times 10^{26}$ W cm^{-2}. Note that, because in this case there is no pair creation for $\mu < \mu_{th}$, the portion of the solid line up to the branching point remains unrealized in the absence of a primary pair.

Earlier work [18, 19] indicates that the pair creation effect by an ultrastrong laser field causes the pulse depletion and sets a natural limit on the achievable laser intensity. We see now that the evolution of an A cascade initiated by a primary particle is yet another destruction mechanism for a laser pulse. Notice that if the process takes place in a vacuum and if the intensity is sufficient for the field to start the pair creation, then evolution of the cascade becomes a catalyzer for the destruction of the laser pulse. Our estimates above strongly support Niels Bohr's suggestion of more than 80 years ago [21] that the field creating electron–positron pairs cannot reach an intensity on the order of E_S.

Certainly, the results presented above are just estimates. A more rigorous approach should lean upon a self-consistent system of kinetic equations. The theory of S type cascades is by now well developed, especially in the context of extensive air showers (EASs) generated in the atmosphere by cosmic rays [22]. Cascades in an external magnetic field are very similar to EASs and are also well studied [23, 24]. In the case of a laser field, A type cascades are governed by the so-called cascade equations

$$\frac{\partial f_\pm(\mathbf{r}, \mathbf{p}_e, t)}{\partial t} \pm e\big(\mathbf{E}(\mathbf{r}, t) + \mathbf{v}_e \times \mathbf{H}(\mathbf{r}, t)\big)\, \frac{\partial f_\pm(\mathbf{r}, \mathbf{p}_e, t)}{\partial \mathbf{p}_e}$$
$$= \int w_{rad}(\mathbf{p}_e + \mathbf{p}_\gamma \to \mathbf{p}_\gamma)\, f_\pm(\mathbf{r}, \mathbf{p}_e + \mathbf{p}_\gamma, t)\, d^3p_\gamma$$
$$- W_{rad}(\mathbf{p}_e)\, f_\pm(\mathbf{r}, \mathbf{p}_e, t) + \int w_{cr}(\mathbf{p}_\gamma \to \mathbf{p}_e)\, f_\gamma(\mathbf{r}, \mathbf{p}_\gamma, t)\, d^3p_\gamma\,, \tag{12a}$$

$$\frac{\partial f_\gamma(\mathbf{r}, \mathbf{p}_\gamma, t)}{\partial t} = \int w_{rad}(\mathbf{p}_e \to \mathbf{p}_\gamma)\big[f_+(\mathbf{r}, \mathbf{p}_e, t)$$
$$+ f_-(\mathbf{r}, \mathbf{p}_e, t)\big]\, d^3p_e - W_{cr}(\mathbf{p}_\gamma)\, f_\gamma(\mathbf{r}, \mathbf{p}_\gamma, t)\,, \tag{12b}$$

where $f_\pm(\mathbf{r}, \mathbf{p}_e, t)$ and $f_\gamma(\mathbf{r}, \mathbf{p}_\gamma, t)$ are the distribution functions of positrons, electrons and photons, respectively, $\mathbf{v}_e = \mathbf{p}_e/\varepsilon_e$ and $\varepsilon_e = \sqrt{p_e^2 + m^2}$ are the velocities and energies of electrons and positrons, $dW_{rad}(\mathbf{p}_e \to \mathbf{p}_\gamma) = w_{rad}(\mathbf{p}_e \to \mathbf{p}_\gamma)\, d^3p_\gamma$ and $dW_{cr}(\mathbf{p}_\gamma \to \mathbf{p}_e) = w_{cr}(\mathbf{p}_\gamma \to \mathbf{p}_e)\, d^3p_e$ are the differential probability of photon emission (Fig. 1a) and that of the creation of a pair by a photon (Fig. 1b) in an external electromagnetic field, and $W_{rad}(\mathbf{p}_e)$ and $W_{cr}(\mathbf{p}_\gamma)$ are the corresponding total probabilities of these processes [25]. Equations (12a) and (12b) differ from the standard equations for EASs [26, 27] only in containing on the left-hand side of Eqn (12a) a second term accounting for the action of the Lorentz force on the charged particle. Notice that the classical radiation reaction force has already been accounted for by two terms on the right-hand side of Eqn (12b) and need not be additionally introduced (for more on this, see Refs [25, 28, 29]).

Reference [25] uses equations (12a) and (12b) to model A type cascades in a uniform, uniformly rotating electric field in the framework of the assumptions formulated above. Here, we will limit ourselves to comparing the calculated growth increments of the number of pairs with the earlier estimates of Ref. [15]. From Fig. 4 it is seen that the agreement is very good.

Reference [30] applies the particle-in-cell (PIC) method to introduce plasma effects. Taking account of an acceleration mechanism for a self-sustained cascade requires a two-dimensional or even three-dimensional analysis and thus greatly complicates the problem, whereas in the case of S cascades no such complication arises. Reference [30] considered the field of two colliding linearly polarized laser

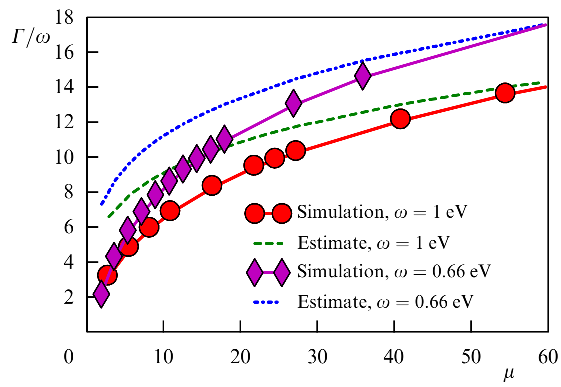

Figure 4. Increment $\Gamma = d \ln N_e/dt$ as a function of the dimensionless field strength μ for two field frequencies $\omega = 1$ eV and $\omega = 0.66$ eV. Dashed and dotted lines correspond to Γ estimated with Eqn (10).

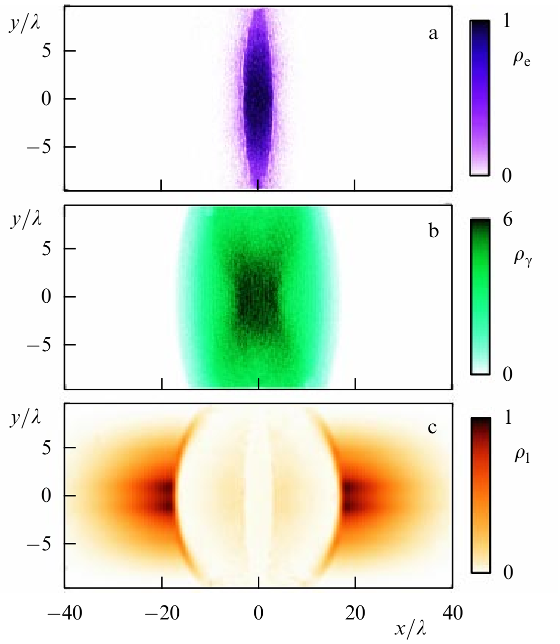

Figure 5. Normalized electron density $\rho_e = n_e/\xi n_{cr}$ (a), normalized photon density $\rho_\gamma = n_\gamma/\xi n_{cr}$ (b), and laser field intensity normalized to the initial intensity maximum ρ_l (c) at time $t = 25.5\lambda/c$ for the collision of two linearly polarized laser pulses. The positron and electron density distributions are about the same.

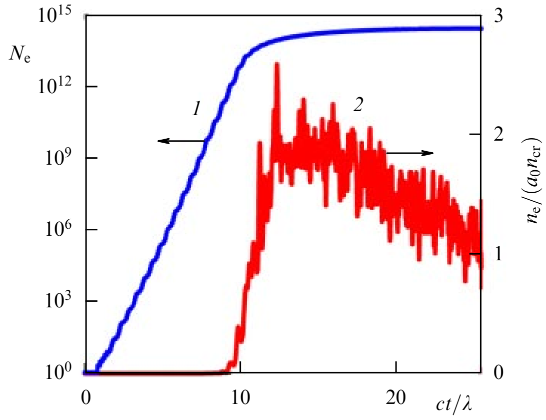

Figure 6. Number N_e of electrons created in a cascade (line *1*) and the electron–positron plasma density normalized to the critical relativistic density (line *2*) as functions of time.

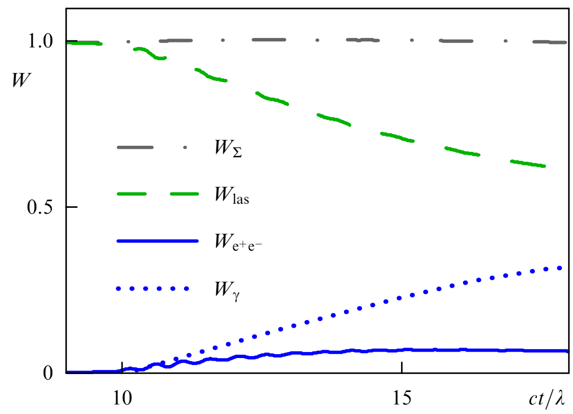

Figure 7. Electron and positron energies (solid line), photon energy (dotted line), laser field energy (dashed line), and the total energy of the entire system (dashed–dotted line) as functions of time. All energies are normalized to the initial laser field energy.

pulses which were assumed to have a Gaussian envelope and propagated along the x-axis; the field components at time $t = 0$ were taken in the form

$$E_y, H_z = E_0 \exp\left(-\frac{y^2}{\sigma_r^2}\right) \sin(\omega x - \phi) \times \left\{\exp\left[-\frac{(x+x_0)^2}{\sigma_x^2}\right] \pm \exp\left[-\frac{(x-x_0)^2}{\sigma_x^2}\right]\right\}, \quad (13)$$

where E_0 is the electric field amplitude of one of the pulses, $2x_0$ is the initial pulse separation, and ϕ is the phase shift. Calculations were made for the following values of the parameters: $\sigma_x = 125/\omega$, $\sigma_r = 40/\omega$, $x_0 = \sigma_x/2$, $\phi = 0.8\pi$, and $\xi = 1.2 \times 10^3$, which for $\lambda = 0.8$ μm corresponds to the peak intensity of $I = 3 \times 10^{24}$ W cm^{-2}, the focusing radius of 10 μm, and the pulse duration of 100 fs. The cascade was initiated by a single electron at rest at the coordinate origin.

The calculated results provide qualitative support to the cascade evolution picture described above. As expected, the multiplicity first increases exponentially, but when the plasma reaches the density ξn_{cr} and, hence, ceases to be optically transparent, the absorption begins and the laser field is rapidly 'depleted' at the pulse trailing edges, with the number of pairs reaching saturation (Figs 5 and 6). Figure 7 shows the energy evolution of the laser field and created particles. It is seen that about half of the energy of the laser pulse is lost to absorption by the emerging plasma of electrons, positrons, and photons, being subsequently re-emitted in the form of γ quanta.

3. S cascades. Collapse and revival of a cascade

We now proceed to the problem of the collisions of ultrarelativistic electrons with the field of two colliding circularly polarized, focused laser pulses of frequency ω and duration $\tau \gg 1/\omega$ [31]. The pulses are assumed to be polarized such that in the focus of the standing wave the field has zero magnetic component and can be considered as a uniformly rotating electric field. As in Section 2, we assume that $\xi \gg 1$ and the field strengths $E, H \ll E_S$.

Let us first estimate the duration τ_S of an S cascade. The initial value of the parameter χ for the ultrarelativistic primary electron can be estimated as $\chi_0 \sim (\varepsilon_0/mc^2)(E_0/E_S)$, where E_0 is the peak strength of the combined electric field of the two colliding pulses in the focal area, and ε_0 is the initial electron energy. As a result of each event, each of the particles involved converts into two particles, so that the multiplicity of the cascade can be estimated in the same way as in Ref. [23],

$$2^n = \frac{\chi_0}{\chi_f}, \quad (14)$$

where n is the number of generations of secondary particles, and χ_f is the dynamic quantum parameter of the final electrons. For τ_S, we have $\tau_S \sim t_e n$, where $t_e \sim W_e^{-1}(\varepsilon_0, \chi_0)$ is the mean lifetime of the initial electron with respect to the emission of a hard photon. Finally, we obtain

$$\tau_S \sim \tau_C \frac{\varepsilon_0}{\alpha m c^2} \chi_0^{-2/3} \log_2\left(\frac{\chi_0}{\chi_f}\right). \qquad (15)$$

The initial electrons are assumed to be ultrarelativistic, so that $\chi_0 > 1$. Clearly, an S type cascade decays (collapses), when $\chi_f < 1$. Let us take $\chi_f \sim 0.1$. For this χ_f, the S cascade cannot continue to evolve because the photon lifetime with respect to pair creation, $t_\gamma \sim W_\gamma^{-1} \sim \exp(8/3\chi_f)$, becomes exponentially large.

We are interested in the case in which $n > 1$ and the time τ_S is less than the pulse duration τ_L. For this, the following conditions must be satisfied:

$$t_{e,\gamma} < \tau_C \frac{\varepsilon_0}{\alpha m c^2} \chi_0^{-2/3} \log_2(10\chi_0) < \tau_L. \qquad (16)$$

Let $\varepsilon_0 = 3$ GeV and $E_0 = 3.2 \times 10^{-3} E_S$. Then, $\chi_0 \approx 20$ and the left inequality in Eqn (16) is satisfied due to the large value of the logarithm: $\log_2(10\chi_0) \approx 8$. The right inequality in Eqn (16) is fairly conservative for $\tau_L \gtrsim 10$ fs.

As mentioned in Section 2, the rotating electric field causes a charged particle to acquire a very large momentum perpendicular to the pulse propagation direction, so that the particle's dynamic quantum parameter χ increases by a large amount, $\Delta\chi \sim 1$, in a short time interval, $t_{acc} \sim (E_S/E_0)\sqrt{mc^2/(\hbar\omega)}\,\tau_C$, $\omega t_{acc} \ll 1$. If the particle trajectory is bent so much that the particle can radiate a sufficiently hard transverse photon that is capable, in turn, of producing a pair, then an A type cascade arises. In our case, the initial particle possesses a large longitudinal momentum and, hence, a large χ. Therefore, a highly bent trajectory is a necessary condition for the evolution of an A cascade when an ultrarelativistic electron collides with a laser pulse.

Let us look at this last point in more detail. The only reason why a particle can change its longitudinal momentum $\mathbf{p}_\parallel(t)$ in the field of two colliding pulses is the emission of photons, and the characteristic time for this change is $t_\parallel \sim \tau_S \lesssim \tau_L$. The change in the transverse component $\mathbf{p}_\perp(t)$ is determined by the rotating electric field $\mathbf{E}(t) = \{E_0\cos(\omega t), E_0 \sin(\omega t), 0\}$:

$$\dot{\mathbf{p}}_\perp(t) = e\mathbf{E}(t),$$

and its characteristic time of alteration is estimated as $t_\perp \sim 1/\omega$. We assume that the charged particle enters the focal area at time $t = 0$ with a zero transverse momentum, $\mathbf{p}_\perp(0) = 0$. Then, taking into account that $t_\parallel/t_\perp \sim \omega\tau_L \gg 1$, we obtain the relation

$$\chi(t) \approx \sqrt{\chi_\parallel^2(t) + \left(\Delta\chi_\perp(t)\right)^2}, \qquad (17)$$

which holds over the time interval $\Delta t \lesssim 1/\omega$. Here, $\chi_\parallel = \chi(0) = (E_0/E_S)(1 + p_\parallel^2/m^2)^{1/2}$ has the meaning of a dynamical parameter in the absence of an accelerating field, and $\chi_\perp$ increases according to Eqn (6) by an amount $\Delta\chi_\perp \sim 1$ in a time $t_{acc} \ll 1/\omega$. As a result, it follows from Eqn (17) that a marked trajectory bending can occur and an A type cascade can start only when $\chi_\parallel \sim 1$. Hence, the A cascade starts operation with a delay with respect to the S cascade, the delay time τ_R being estimated from Eqn (15) where we need to set $\chi_f = 1$. The duration τ_A of the A cascade is found to be $\tau_A \sim \tau_L - \tau_R$. Clearly, this requires that the conditions $t_{acc} \ll t_{e,\gamma} < \tau_A$ be satisfied. It is easily

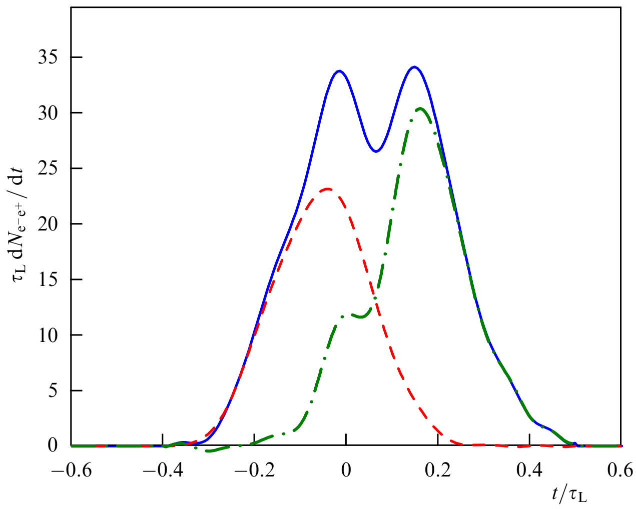

Figure 8. Pair creation rate as a function of time. Solid line: simulation including the Lorentz force; dashed line: simulation without the Lorentz force; dashed–dotted line: the difference between the two. Initial parameters: $E_0 = 3.2 \times 10^{-3} E_S$, and $\varepsilon_0 = 3$ GeV.

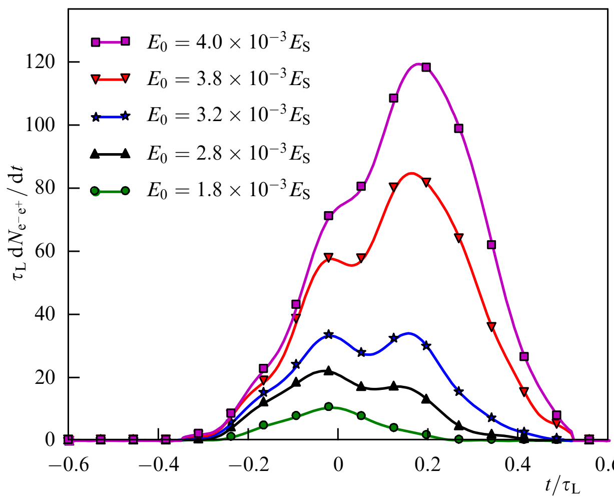

Figure 9. Pair creation rate as a function of time for different values of E_0 and an initial electron energy of $\varepsilon_0 = 3$ GeV.

verified that this is indeed the case for our assumed values of ε_0, E_0, and τ_L.

In Ref. [31], the process under discussion was investigated by Monte Carlo simulations using the realistic three-dimensional model proposed in Ref. [32] for describing the field of a focused laser pulse. Figures 8–10 present the results. All distributions are normalized in the assumption of the cascade generated by a single electron. The time dependence of the pair creation rate $dN_{e^-e^+}(t)/dt$ is plotted in Fig. 8 for $E_0 = 3.2 \times 10^{-3} E_S$, $\varepsilon_0 = 3$ GeV. If we assume that the Lorentz force is 'switched off' and the electrons are not accelerated by the field, then the electrons lose energy only by emitting photons, and only an S cascade evolves. The pair creation rate obtained without considering the Lorentz force is shown by the dashed line. The cascade starts to develop immediately as the electron enters the laser field and decays in a time $\tau_S \approx 0.6\tau_L$. The solid line marks the total pair creation rate. It is seen that at the initial stage the solid line coincides with the dashed line, which means that initially only the

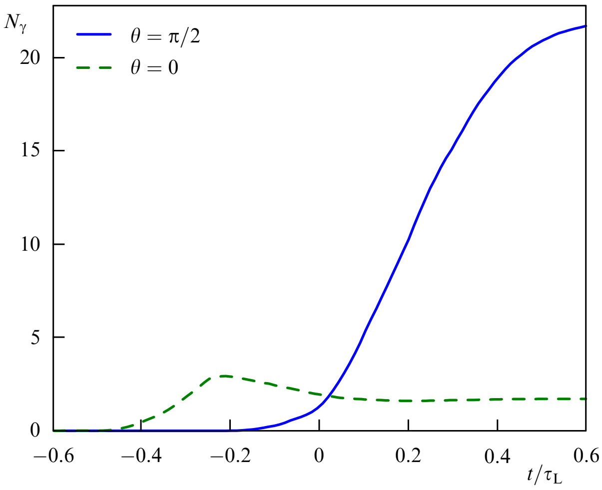

Figure 10. Total number of photons emitted in the direction of the initial electron beam $\theta \in [0, 0{,}1]$ rad (dashed line) and in the perpendicular direction $\theta \in [\pi/2 - 0{,}05, \pi/2 + 0{,}05]$ rad (solid curve) as a function of time; $E_0 = 3.2 \times 10^{-3} E_S$, and $\varepsilon_0 = 3$ eV.

S cascade develops. In full accord with our estimates, in a time of about $0.3\tau_L$ the total pair creation rate starts to exceed the value calculated for the case of no Lorentz force. And finally, a second peak appears in the solid line, which signals the 'revival' of the cascade process due to the development of the A cascade. In Fig. 8, the dashed–dotted line presenting the difference between the two lines corresponds to the A type cascade which is developing due to the field-driven acceleration of electrons in the direction perpendicular to the laser pulse propagation. Figure 9 exhibits pair creation rates for different values of E_0. It is seen that the 'revival' of the A type cascade (as indicated by the second peak) occurs only in a sufficiently strong field. For the chosen laser pulse and electron beam parameters, this occurs for $E_0 > 2.8 \times 10^{-3} E_S$.

The angular distribution of photons is another indication of the cascade revival. An ultrarelativistic electron that generates an S type cascade emits photons along its pathway. An A type cascade develops only on appearing fast particles accelerated in the transverse direction. These electrons emit photons mostly in the same direction. Figure 10 plots the time dependence of the total number of photons N_γ emitted along and perpendicular to the electron beam. The number of photons emitted in the direction $\theta = 0$ first increases for $t < 0$ and then starts to decrease due to the 'collapse' of the S cascade, even though some of the previously emitted photons continue to create pairs even for $t > 0$. At the same time, number of photons emitted in the direction $\theta = \pi/2$ start to increase considerably in number for $t \geqslant 0$. The estimate for the decay of an S cascade and that for its transformation to an A cascade agree well with the results presented in Fig. 8. The large difference in the number of photons emitted in these two directions is explained by the different numbers of particles involved in the cascades: whereas the S cascade is initiated by a single electron, a large number of secondary particles contribute to the development of an A cascade.

4. Conclusion

Quantum electrodynamics, by now a well-worked out theory, is fairly well verified experimentally — but not for strong fields, for which thus far no direct experimental work has been done but which are of special interest because of the essentially nonperturbative quantum effects they give rise to. Of these, one of the most important is the creation of electron–positron pairs from a vacuum. Furthermore, in strong fields this effect can become dominant and, in particular, can prevent the field strength from reaching the characteristic QED value E_S. In light of the ELI and XCELS projects, prospects are emerging for the employment of the optical laser for the comprehensive verification of our views on quantum electrodynamics in a strong external field.

Recent work [13, 14, 25, 30] has revealed another effect that changes considerably the nature of the interaction of radiation with matter under extreme intensities, namely A type quantum-electrodynamical cascades. Unlike 'ordinary' S type cascades, A cascades obtain their energy directly from the laser field and, therefore, also prevent the reaching of E_S. Accounting for quantum-electrodynamical cascades requires considerable modernization of the program codes which are utilized for calculating laser plasma dynamics in ultraintense fields and without which planning necessary experiments is impossible. An experimental scheme proposed in Ref. [31] enables the realization and study of the two types of cascades within the framework of a single experiment at an intensity of order $I \sim 5 \times 10^{24}$ W cm^{-2}. This scheme transforms the S cascade into an A cascade and allows both of them to be distinctly identified both in terms of the time of onset and the angular distribution of the emitted hard photons. The expectation is that the next-generation laser facilities now under construction will allow reaching this level of field intensity in the near future.

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