

METHODOLOGICAL NOTES

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Sagnac effect in ring lasers and ring resonators. How does the refractive index of the optical medium influence the sensitivity to rotation?

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Contents

Abstract. **The Sagnac effect in a ring laser (RL) results in a frequency difference of counterpropagating waves that is proportional to the RL angular rotation rate. We address the question of how an optical medium filling the whole RL or a part of it influences the frequency difference of counterpropagating waves. While the formulas for this difference in a rotating RL abound in the literature, there is no agreement among them as to whether the medium increases or decreases this difference or indeed leaves it unchanged. Nor do the available (and often contradictory) experimental data fully clarify the situation. Because the Sagnac effect is a special relativity effect, the relativistic velocity addition law is used here to calculate the frequency difference of counterpropagating waves in an RL. When a homogeneous optical medium fills the entire perimeter of the resonator of a rotating RL, we show that the frequency difference of counterpropagating waves is inversely proportional to the refractive index of the medium. The results obtained can also be used to calculate the difference between the resonant frequencies of counterpropagating waves in rotating ring resonators in the presence of an optical medium.**

1. Introduction

In 1913, Sagnac [1, 2; see also 3–5] discovered an optical effect in which one of two counterpropagating waves in a rotating ring interferometer (RI) acquires a phase shift relative to the other, the shift being directly proportional to the angular rotation velocity, the area covered by the interferometer, and the wave frequency.

If we use an inertial reference frame (IRF) at rest (i.e., a laboratory IRF), it can be shown (as we did in Refs [4, 5]) that the Sagnac effect is a consequence of the relativistic velocity addition law applied to the propagation velocities of counterpropagating light waves (or, most generally, any waves whatsoever) and the linear rotation velocity. It was shown in [4, 6] that in a corotating noninertial reference frame, the Sagnac effect can be considered as resulting from the difference between counterpropagating waves, in the exact way time is retarded by the nonrelativistic (Newtonian) scalar potential of the gravitational field (the potential that is responsible for the centripetal acceleration and the Coriolis acceleration). We note that the relativistic nature of the Sagnac effect as viewed in an IFR was recognized in Refs [7–9] (see also Ref. [3]).

The Sagnac effect finds application in optical rotation sensors: in ring interferometers, including widely known fiber ring interferometers (FRIs), ring lasers (RLs), and ring resonators (RRs) [10]. We here show that the Sagnac effect manifests itself somewhat differently in RLs and RRs than it does in RIs.

Ring laser gyroscopy is one of the key areas in modern optical Sagnac gyroscopy [10]. Ring lasers, apart from being useful in solving gyroscopic and navigation problems, facilitate detecting some of the fundamental effects of general relativity and some geodesic phenomena [11–14] and can be used in a range of other practical applications.

The ring gas laser (RGL) concept proposed in 1962 by Rosenthal [15] was first implemented in 1963 by Macek and Davis [16]. However, despite the passage of over fifty years since then, one unsolved problem remains: it is not precisely clear how the sensitivity of an RL to rotation depends on the

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refractive index $n = \sqrt{\varepsilon\mu}$ (with ε being the dielectric constant and μ the magnetic permeability) of the optical medium that fills the resonator (fully or partially).

As discussed in Section 2, the literature provides about a dozen very different expressions for the effect of the optical medium on the RL sensitivity to rotation, their predictions ranging from an increase to no change to a decrease, thus leaving it unclear whether the empty part of the resonator (if present) should reasonably be filled with an optical medium.

Passive ring resonators, in which rotation produces a resonance frequency difference between counterpropagating waves, face a similar situation. There are also devices known as recirculation RLs (RRLs), which are essentially RLs below the generation threshold, i.e., where amplification does not compensate the loss. This leads to the possibility of optical resonances with a high quality factor and hence with a high sensitivity to rotation. However, the sensitivity versus n problem has not been explored sufficiently in the literature.

The reason for this is historical. The first RLs were the ring gas lasers, with the optical medium filling only a small portion of the perimeter of the optical resonator: the optical (so-called Brewster) window of RGL discharge tubes was no more than 1–2 mm thick, and the sensitivity versus n question was of purely academic interest. Moreover, already in the second half of the 1960s, so-called ring laser single blocks were developed, with a total internal reflection prism as a reflecting element. Contained within a single block resonator was a gas mix of helium and neon at a low pressure (≈ 1 mm of mercury) for which $n - 1 \ll 1$, and a very small part of the perimeter of the optical resonator was filled with glass consisting of total internal reflection prisms.

Early solid ring lasers (SSRLs) were built almost simultaneously with RGLs, but had at the time only a tiny part of their resonators filled with an optical medium. SSRLs have found wide application over the last fifteen to twenty years [17]. In addition, fifteen years ago, semiconductor ring lasers (SRLs) were developed. Current so-called monolithic SRLs [17] have their entire resonator perimeter filled with an optical medium with $n \approx 1.8$. For an active SRL, $n \approx 3.5-4.0$, but an active element in the SRL resonator is relatively small in size (about 1.5–2.0 mm), and most of the resonator perimeter is usually filled with a single-mode fiber (SMF) light guide ($n \approx 1.45$). Active research is presently being carried out to explore ring microresonators (RMRs) and ring microlasers (RMLs) [17], whose resonators are also filled with an optical medium.

The question of the effect of the refractive index n on the rotation sensitivity is especially acute for the currently studied ring rotation sensors in the super-high frequency (SHF) range (wavelength $\lambda \approx 2$ cm) that are analogs of RLs and RRs [18]. For electromagnetic waves in the optical and near-infrared (IR) spectral regions, $n \approx 1.5-2.0$, but for the SHF range, a ring resonator can be filled with a $\mu \approx 100$ ferrite or an $\varepsilon \approx 1000$ dielectric, which corresponds to $n \approx 10-30$.

The aim of this paper is to provide a rigorous derivation of expressions (as functions of the refractive index n of the optical medium filling an RL resonator and the RR) for the Sagnac-effect-related difference in the generation frequency between counterpropagating waves in an RL and in the resonance frequency in an RR. In discussing this question, both a laboratory (nonmoving) inertial reference frame and a reference frame rotating with an RL or an RR are used. We do not consider the effect of dispersion (including the nonlinear dispersion due to the saturation of the optical medium) on the generation frequency shift and 'extension' [19] in an RL. Nor do we consider the mutual capture of the counterpropagating frequencies in an RL. The results in Refs [4, 6] are used in our derivations.

2. Literature review: various expressions for the effect of the refractive index of an optical medium on the rotation frequency of ring lasers and resonators

If an RL resonator or an RR is not filled with an optical medium, then the difference in the resonance frequencies ν^+ and ν^- of the counterpropagating waves, $\Delta\nu = \nu^- - \nu^+$, is given by [20]

$$\Delta\nu = \frac{4S\Omega}{\bar{\lambda}_0 L}\,, \tag{1}$$

where Ω is the angular rotation frequency, $\bar{\lambda}_0 = (\lambda_0^+ + \lambda_0^-)/2$ is the average light wavelength of counterpropagating waves (see Section 3 for a discussion on the value of $\bar{\lambda}_0$), $\lambda_0^\pm$ are the wavelengths of counterpropogating resonance frequency waves in an RL in the vacuum, S is the area covered by the RL (more precisely, the projection of the area onto the plane perpendicular to the angular velocity $\mathbf{\Omega}$), and L is the RL perimeter.

Although a real laser always contains an optical medium, Eqn (1) does approximate the frequency difference of the RL-generated counterpropagating waves if the medium satisfies the condition $n - 1 \ll 1$ (which is the case for the RGL) and (or) if it occupies a negligibly small portion of the RL resonator perimeter (optical windows of gas discharge tubes or total internal reflection prisms).

The literature suggests different expressions for the resonance frequencies of counterpropagating RL waves and for the frequency difference of RL-generated counterpropagating waves in the cases where the resonator is fully or partially filled with an optical medium. To make the comparison more convenient, we reduce all the expressions to the form corresponding to a resonator fully filled with an optical medium with a refractive index n. As noted in Section 1, we neglect the effect of the refractive index dispersion $\mathrm{d}n/\mathrm{d}\lambda$.

The expressions given in the literature are as follows:

$$\Delta\nu = \frac{4S\Omega}{n^2\bar{\lambda}_0 L} \tag{2}$$

(Refs [21–26]),

$$\Delta\nu = \frac{4S\Omega}{\mu\varepsilon\bar{\lambda}_0 L} \tag{3}$$

(Ref. [27]) [because $n = \sqrt{\mu\varepsilon}$ expression (3) reduces to (2)],

$$\Delta\nu = \frac{4S\Omega}{n\bar{\lambda}_0 L} \tag{4}$$

(Refs [4, 18, 28–53]), and

$$\Delta\nu = \frac{4S\sqrt{\mu/\varepsilon}\,\Omega}{\bar{\lambda}_0 L} \tag{5}$$

(Refs [54, 55]). If $\mu = 1$, which is the case for most optical materials, Eqn (5) reduces to Eqn (4).

According to the results in Refs [17, 56–75], the rotation sensitivity of RLs and RRs does not depend on n at all. In other words, in the presence of an optical medium in the RL resonator or in an RL, the resonance frequency difference between counterpropagting RL waves and the frequency difference between RL-generated counterpropagating waves, $\Delta\nu$, are given by expressions identical to Eqn (1).

In [76–80], $\Delta\nu$ is expressed as

$$\Delta\nu = \frac{4Sn\Omega}{\lambda_0 L}\,. \tag{6}$$

From the results in Ref. [81], it follows that the rotation sensitivity of a microresonator is proportional to

$$\frac{4Sn^2}{\lambda_0 L}(1-n^2)\Omega\,. \tag{7}$$

Thus, it follows from the results in Refs [21–27] that the rotation sensitivity of the RL and RR is inversely proportional to n^2; from the results in Refs [4, 18, 28–53], it follows that the sensitivity is inversely proportional to n; according to Refs [54, 55], the sensitivity is proportional to $\sqrt{\mu/\varepsilon}$; according to Refs [17, 56–75], the sensitivity does not depend on n at all; based on the results in Refs [76–80], the sensitivity is proportional to n; the results in Ref. [81] suggest that the sensitivity is proportional to $n^2(1-n^2)$. Expression (7) is obviously wrong because it implies that as $n \to 1$, the rotation frequency of an RL and an RR tends to zero, which is inconsistent with reality.

We note that for an SMF-based many-coil RR or an RL, S is the sum of the areas of all the coils, and L is the sum of the lengths of all the coils. Clearly, for a given coil radius, $\Delta\nu$ is independent of the number of coils, because S and L in Eqns (1)–(7) are proportional to the number of coils [10].

Thus, we have surveyed 63 papers published between 1964 and 2012 that give seven different expressions, Eqns (1)–(7), for the resonance frequency difference between counterpropagating waves in a rotating ring resonator (and, correspondingly, for the generation frequency difference between counterpropagating waves in a rotation ring laser) in the presence of an optical medium. In fact, there are many more such expressions: as noted above, we confine ourselves for simplicity to the case of an RL resonator or an RL fully filled with an optical medium. As it happens, however, different papers give different versions of the same expression in those cases where only a part of the RR resonator or of an RL is filled. For example, Refs [33, 34, 37, 41] discuss versions of expressions (4) and (6) for a resonator partially filled with several optical media with different n; these versions are presented in Refs [28–30].

We note that the most serious errors are found in Ref. [77]. The extremely cumbersome expressions obtained there suggest that $\Delta\nu$ depends not only on the refractive index of the optical medium on the path of the wave but also on that of the optical medium filling the entire area S within the perimeter of the RL resonator (or a part of that area). But this implies — contrary to common sense — that $\Delta\nu$ depends on the part of the optical medium that is far from the beam trajectories of counterpropagating waves in the RL. For this reason, we do not reproduce the results in [77] here.

It is surprising that for decades, hardly anyone noticed the significant differences among Eqns (1)–(7). The only exception was given by Refs [33, 34, 37, 41], pointing to significant differences among Eqns (1)–(7). Occasionally, expressions for the resonance frequency difference between RL or RR counterpropaging waves in the presence of an optical medium vary even from one work to another by the same author. For example, papers [65], [27], and [28] by Heer present the respective results in Eqns (1), (3), and (4). Sunada and Harayama give Eqn (2) in Refs [22, 23] and Eqn (4) in Ref. [48]. Duraev gives Eqn (4) in Ref. [49] and Eqn (1) in Ref. [73]. And finally, Scheuer and coworkers give Eqn (4) in Ref. [53], Eqn (2) in Ref. [24], and Eqn (7) in Ref. [81].

3. Resonance frequency difference between counterpropagating waves in a ring resonator and the generation frequency difference in a ring laser in an inertial laboratory reference frame

To calculate the resonance frequency difference between the counterparopagating waves in a rotating RR and the generation frequency difference of counterpropagating RL waves as a function of the refractive index n, we need the eigenfrequencies of the ring laser. While this is obviously so for an RR, in the case of an RL this is only true in the absence of linear or nonlinear dispersion of n and hence when the frequencies of counterpropagating waves are not extended or captured. In this case, the generation frequencies of an RL are exactly identical to the RR resonance frequencies.

We assume for simplicity that the resonator has a circular shape with radius R and that it rotates with an angular frequency Ω and is centered at the rotation center. With this assumption, there is no need in what follows to integrate over the linear rotation velocity $R\Omega$, which, for $R \neq \text{const}$, is different for different parts of the resonator. As shown already by Michelson [82], the magnitude of the Sagnac effect is practically independent of both the shape of the closed optical path and of its location relative to the rotation center. The results in Ref. [82] are valid for $R\Omega \ll c$ [4, 30]. We simplify the discussion by restricting ourselves to a single-plane RL mode and assuming that the RL generates only one longitudinal mode for each opposite direction. We also assume that the angular rotation velocity is constant, $\Omega = \text{const}$.

The length of the RL, independent of whether it rotates, should be an integer multiple of light wavelengths. The natural wavelengths corresponding to the resonances of a rotating hollow ring resonator are determined by the condition

$$\frac{L}{\lambda_0} = N\,, \tag{8}$$

where $L = 2\pi R$ is the ring resonator perimeter, λ_0 is the vacuum wavelength of the Nth light wave, and the integer N labels the longitudinal modes. Then $\nu = c/\lambda_0$ is the frequency of the Nth longitudinal mode, where c is the speed of light in the vacuum. For a nonrotating RL, the two counterpropagating waves have the same values of N and λ_0. As Eqn (8) suggests, if the RR perimeter L changes sufficiently little, the light wavelength λ_0 changes such that N remains unchanged: $N = \text{const}$. Such a change in L may be due to the resonator being mechanically or thermally acted upon, and in the case we are considering, it is rotation — and hence the Sagnac effect — that causes an effective change in the optical path lengths of counterpropagating waves. If the change in L is somewhat larger, the wavelength of the Nth longitudinal mode goes out of that region of the amplification line of the active element where amplification exceeds losses. The Nth longitudinal modes then cease to be generated, and the $(N+1)$th or $(N-1)$th modes are generated instead. For this change in the mode number to occur, the generation frequency ν should change by an amount close to the ring resonator mode spacing c/L. However, for realistic angular rotation frequencies, the Sagnac-related change in the generation frequency is orders of magnitude less than c/L, and we assume in what follows that N is conserved for both counterpropagating waves as rotation proceeds.

The natural wavelengths corresponding to the resonances of a rotating RR filled with an optical medium are given by

$$\frac{L}{\lambda} = N = \text{const}\,, \tag{9}$$

where $\lambda = \lambda_0/n$ is the light wavelength in the filling medium.

In the presence of rotation, the counterpropagating waves have different wavelengths. Under condition (9), the expression for the natural light wavelengths of a rotating RR is obtained as [4]

$$\lambda_0^{\pm} = \frac{L^{\pm} n_{\text{eff}}^{\pm}}{N} = \frac{L^{\pm} n_{\text{eff}}^{\pm}}{Ln}\,\lambda_0\,, \tag{10}$$

where $L^{\pm}$ and $n_{\text{eff}}^{\pm}$ are the effective resonator lengths and effective refractive indices for counterpropagating waves in the presence of rotation, where the respective superscripts plus and minus correspond to the waves whose directions are along or opposite to the rotation direction.

We write the expressions for the lengths $L^{\pm}$ in a nonmoving (laboratory) inertial reference frame, where the special theory of relativity is certainly valid [4]:

$$L^{\pm} = 2\pi R \pm R\Omega t^{\pm}\,, \tag{11}$$

where $t^{\pm}$ are the times it takes the counterpropagating waves to pass through the resonator [4]:

$$t^{\pm} = \frac{2\pi R\big(1 \pm (c/n)R\Omega/c^2\big)}{(c/n)(1 - R^2\Omega^2/c^2)}\,. \tag{12}$$

We next define the effective refractive indices for the counterpropagating waves $n_{\text{eff}}^{\pm}$ as the ratio of the speed of light c in the vacuum to the phase velocity of counterpropagating waves $v^{\pm}$: $n_{\text{eff}}^{\pm} = c/v^{\pm}$. The phase velocities of the waves $v^{\pm}$ are in turn the ratios of the effective resonator length for the waves $L^{\pm}$ to the propagation times of the waves in the presence of rotation $t^{\pm}$, $v^{\pm} = L^{\pm}/t^{\pm}$. Then, using Eqns (10)–(12), we can write

$$\lambda_0^{\pm} = \frac{ct^{\pm}}{Ln}\,\lambda_0\,. \tag{13}$$

Thus, we have eliminated the need to calculate $L^{\pm}$ and $n_{\text{eff}}^{\pm}$, and the times $t^{\pm}$ were calculated previously in [4]. The wavelength difference between the counterpropagating waves ($L = 2\pi R$) is expressed as

$$\Delta\lambda_0 = \lambda_0^{+} - \lambda_0^{-} = \frac{4\pi R^2\Omega}{Lnc}\,\lambda_0 = \frac{2R\Omega}{nc}\,\lambda_0\,. \tag{14}$$

The resonance frequency difference between counterpropagating waves of a rotating RR and, correspondingly, the generation frequency difference between counterpropagating waves of an RL has the form

$$\Delta\nu = \frac{4S\Omega}{Ln\lambda_0}\,. \tag{15}$$

It only remains to find the relation between the light wavelength in the vacuum in the absence of rotation, λ_0, and the average light wavelength of counterpropagating waves in the vacuum in the presence of rotation, $\bar{\lambda}_0$, which enters Eqns (1)–(7):

$$\bar{\lambda}_0 = \frac{\lambda_0^{+} - \lambda_0^{-}}{2} = \lambda_0\left(1 + \frac{R\Omega n}{c}\right) + \frac{1}{2}\left(1 - \frac{R\Omega n}{c}\right) \equiv \lambda_0\,. \tag{16}$$

Thus, we have shown that the correct expression is Eqn (4), and therefore the resonance frequency difference between counterpropagating waves of a rotating RR fully filled with an optical medium, and hence the generation frequency difference of counterpropagating waves in an RL, are inversely proportional to the refractive index n of the optical medium. Because the area of a circular resonator is $S = \pi R^2$ and its perimeter is $L = 2\pi R$, we have

$$\Delta\nu = \frac{2R\Omega}{n\lambda_0}\,. \tag{17}$$

If the optical medium fills only a part of the perimeter of a ring resonator l (i.e., the segment $L - l$ is unfilled), then, using our proposed method and noting that $\Delta t = t^{+} - t^{-}$ is independent of the presence of an optical medium, we obtain, after simple manipulations,

$$\Delta\nu = \frac{4S\Omega}{\big[ln + (L - l)\big]\lambda_0}\,. \tag{18}$$

For $l = L$, Eqn (18) is identical to Eqn (15).

4. Resonance frequency difference between counterpropagating waves in a ring resonator and the generation frequency difference in a corotating reference frame

Because the observer rotates with the RR or RL as he measures the angular velocity, it is of interest to look at the problem in the corotating reference frame. As shown in Refs [4, 6], for fixed-phase points of counterpropagating waves in an IRF, K'_{in}, coinciding with its equivalent noninertial rotating reference frame, the time propagation (with the nonrelativistic, Newtonian, scalar potential of gravitational field that describes the centripetal acceleration and the Coriolis acceleration) is given by

$$t^{\pm}_{K'_{\text{in}}} = t\sqrt{1 - \frac{\Omega^2R^2}{2c^2} \mp \frac{2\Omega Rv}{c^2}}\,, \tag{19}$$

where $t = 4\pi R/(v^{+} + v^{-})$ is the time it takes for the counterpropagating waves to travel along the ring from the beamsplitter to their meeting point in the nonmoving (laboratory) reference frame, $v^{\pm} = (v \pm R\Omega)/(1 \pm vR\Omega/c^2)$ [4], $v = c/n$. It is easy to show that $t \approx 2\pi Rn/c$.

Expanding $t^{\pm}_{K'_{\text{in}}}$ in the small parameter $2\Omega Rv/(c^2) \ll 1$ and neglecting the effect of the gravitational potential corresponding to the centrifugal acceleration,

$$\frac{\Omega^2 R^2}{2c^2} \ll \frac{2\Omega R v}{c^2}\,,$$

we obtain an approximate expression for the travel times for fixed-phase points of the counterpropagating waves, or for the reading of the clock whose velocity is equal to the velocity of these points,

$$t^{\pm}_{K'_{\rm in}} \simeq t\left(1 \mp \frac{\Omega R}{cn}\right). \tag{20}$$

It follows from Eqn (20) that for a wave superscripted with a plus sign, the time is retarded; hence, the frequency decreases and the light wavelength increases by the factor $[1 + R\Omega/(cn)]$. For the minus counterpart, the time is accelerated, and hence the frequency increases, and the light wavelength decreases. Then

$$\lambda_0^{\pm} = \left(1 \pm \frac{R\Omega}{cn}\right)\lambda_0\,, \tag{21}$$

whence it follows that

$$\Delta\lambda_0 = \lambda_0^+ - \lambda_0^- = \frac{2R\Omega}{nc}\,\lambda_0\,. \tag{22}$$

Equation (22) is identical to Eqn (14). Thus, we have obtained the same results for the laboratory IRF and the corotating reference frame.

5. Analysis of experimental results

We now discuss whether Eqn (4) fits the experimental results. The only study on this was published about forty years ago by Privalov and Filatov [33], who experimented on an RGL resonator with a perimeter $L = 0.85$ m, into which two quartz plates 40 mm thick were introduced at Brewster's angle to the beam. The experiment measured the frequency difference of the counterpropagating waves $\Delta\nu$ in a rotating RGL both in the absence and in the presence of the plates, with the change in the area and perimeter of the resonator due to light refraction in the plates taken into account. As a result, the correctness of Eqn (4) was established to a high degree of accuracy.

There was no follow-up to the work in Ref. [33]. Currently, results from measurements of the frequency difference of counterpropagating waves in a rotating SRL with a single-mode fiber resonator are available. A shortcoming of these measurements is that the SRL resonator, unlike the RGL, does not allow removing the optical medium to perform control measurements. In [48, 51, 68–70], $\Delta\nu$ was measured at known values of Ω, R, λ_0, and n and the expression that fitted the data best was determined.

The results in Refs [48, 51] confirm those in Ref. [33] in identifying Eqn (4) as the correct expression. According to Refs [68–70], the correct expression is Eqn (1). In addition, Ref. [69] shows that $\Delta\nu$ is independent of the number of guide winding coils in the SRL resonator. We note that in the SRLs used in Refs [68–70], the ring fiber resonator had a complex structure and consisted of two coupled fiber rings, one basic and the other auxiliary; the latter could be varied in radius, thus changing the effective area of the former (to a degree dependent on the strength of the coupling).

The experimental results in Refs [33, 48, 51] contradict those in Refs [68–70]. It seems worthwhile to conduct measurements for the RGL similar to those inf Ref. [33] — but in the case where the two optical plates in the laser resonator mirror are replaced by two long glass or quartz slabs, which, combined, make up half or more of the mirror perimeter and hence considerably affect the generation frequency difference $\Delta\nu$ of counterpropagating waves. But this is exactly the setting used by Pogany when verifying that the phase difference of counterpropagating waves at the output of an RI is independent of the presence of an optical medium therein [83]. Because the perimeter of the RGL resonator cannot be fully filled with an optical medium, it follows that Eqn (18) should be used in processing measurement data.

6. Conclusion

In the course of more than fifty years since the ring gas laser was introduced, thousands of papers have been published on the operation of RLs and RRs (the latter of which have not yet found wide practical application). However, only a few dozen papers, often with erroneous results, can be found in the literature on the influence of the refractive index of the optical mode on the rotation sensitivity of these devices. The corrected results obtained in a series of papers [4, 18, 28–31] between 1964 and 2012 remained virtually unnoticed against the many erroneous publications.

An analysis of all the mistakes made would increase the size of this paper unacceptably, and we therefore confine ourselves to pointing out that some incorrect results are so in a conceptual sense, often due to incorrect formulation of Maxwell's equations in a rotating reference frame in the presence of an optical medium; some arise from gross mistakes in computation; and some are given with no derivation at all or with a reference to a third, often erroneous, paper.

We conclude by summarizing our results.

1. In the presence of rotation, the resonance frequency difference between counterpropagating waves in an RR and the generation frequency difference between counterpropagating waves in an RL are inversely proportional to the refractive index n of the optical medium that fills the resonator — but only in the case of complete filling. In the partial filling case, a more complicated dependence applies. If there is free space in an RR and an RL, it does not make sense to fill it with an optical medium. On the contrary, as shown in Ref. [4], in an RI — and in particular, in an FRI — in the presence of rotation, the phase difference of a counterpropagating wave at the output of the interferometer is independent of n.

2. In the presence of rotation, the resonance frequency difference between counterpropagating waves in an RR and the generation frequency difference between counterpropagating waves in an RL are proportional to the area-to-perimeter ratio of the resonator, and the phase difference of counterpropagating waves in an RI — and in particular, in an FRI — is proportional to the area of the interferometer and is independent of the length of its perimeter. In particular, in an SRL with a multicoil fiber circuit, the frequency difference of counterpropagating waves is independent of the number of coils at a fixed coil radius. By contrast, in an FRI, the phase difference of counterpropagating waves is proportional to the number of coils.

Interestingly, in the late 19th century, the optical two-mirror Fabry–Perot resonator was developed [84], with a resolution up to $\Delta\lambda/\lambda \sim 10^{-6}$. Early in the twentieth century, the interference fringe resolution was about a 0.01 band, i.e., four orders of magnitude worse. If Sagnac had conducted his experiments [1, 2] not with an RI but with a three- or four-mirror ring resonator, he could have significantly increased the accuracy of measuring the angular rotation velocity.

When this paper was already in print, an experiment confirming the correct expression (4) was reported [85].

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