

METHODOLOGICAL NOTES

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The Hubble flow: an observer's perspective

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Abstract. This paper discusses some aspects of cosmological expansion as viewed by an observer. It identifies the rate of change of the angular distance v_{Θ} with respect to the observer's clock as a crucial quantity in properly understanding the expansion. Using this quantity and the angular distance, in addition to the traditional approach to visualization, provides a more vivid picture of the cosmological expansion and is key for adequately visualizing it.

1. Introduction

How do we imagine the expansion of the Universe? The traditional image used both in popular literature and textbooks, and even in the specialized literature, is a 'bird's-eye view', or 'God's view', when we look at the space, so to say, from outside. For example, we imagine our expanding Universe as an inflated ball or a stretchable plane. Moreover, it is convenient to present all points of this ball or plane as if they were observed simultaneously, i.e., within this representation, we perceive the whole picture 'as it is now'. Thus, we usually not only represent the Universe from 'outside', but also 'see' all its points at the same time.

This image is good (and maybe useful for understanding), but a real observer can never see such a picture, even theoretically. So how would the expansion of the Universe be viewed by an 'inside' observer?

Imagine that we can conduct observations with an arbitrarily high accuracy or during a long enough time interval to be able to measure a change in the characteristics of remote objects due to their recession. How can we better represent the results, directly reflecting the expansion of the Universe? For example, it may be necessary to visualize a realistic (3D) modeling (say, for a planetarium) of the picture the observer would see in the expanding Universe. Which parameters are preferential for this task? In particular, which velocity should we use to describe the recession of galaxies?

In this paper, we show that the velocity related to the so-called angular distance d_{Θ} is one of the most illustrative quantities. We discuss some properties of this velocity, as well as its behavior in universes with different parameters. This approach appears to substantially complement the traditional illustration ('God's picture') and allows creating a more adequate image of the expanding Universe. This is important, because many cosmological phenomena are not so visual and at first glance contradict common sense, including very 'advanced' views (see, e.g., an interesting discussion on the admissibility of superluminal motions of the Hubble flow and emerging misconceptions in [1], as well in [2] and the references therein). Insufficient visualization, in particular for students, makes it difficult to qualitatively understand the cosmological expansion and related phenomena. It is this difficulty that we attempt to overcome here.

2. Distances in cosmology

Different distances introduced in cosmology are the basic element in standard cosmological textbooks (see, e.g., [3, 4]). In this section, we briefly summarize the basic concepts related to distances in cosmology, because they are essential below.

As an example, we consider a Friedmann cosmological model. It should be recalled that a Friedmann universe suggests a particular choice of the time coordinate, with the corresponding spatial cross sections being homogeneous, and it is natural to use this cosmic time.

The flat Friedmann metric is given by

$$ds^2 = c^2\,dt^2 - a^2(t)\,dl^2\,, \tag{1}$$

where dl is the elementary length in the space with constant curvature, c is the cosmic time, and t is a scale factor. The second term in the right-hand side of (1) reflects the existence of the Hubble flow: with the peculiar velocities ignored, remote objects recede from each other as the scale factor $a(t)$ increases if their comoving coordinates do not change.

In defining comoving coordinates, it is natural to introduce a spherical reference system centered at the

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observer: then the distances and velocities of the Hubble flow defined below depend solely on the radial coordinate $a(t)$. We restrict ourselves to considering only those models in which the Universe is filled with barotropic matter (i.e., with the density depending only on one parameter, the pressure) of one sort, which allows us to derive all necessary formulas in a closed (and fairly simple) form. Indeed, in a universe filled with matter with the equation of state

$$p = w\rho c^2\,, \tag{2}$$

where p is the matter pressure, ρ is the matter density, and w is a constant parameter, the homogeneous solution of the Friedmann equations takes the power-law form $a \sim t^{1/\alpha}$, where $\alpha = 3(w+1)/2$. With the Hubble constant $H = \dot{a}/a = 1/(\alpha t)$ and $t \propto a^{\alpha}$, we can use the redshift definition $1 + z(t) = a(t_0)/a(t)$, where t_0 is the present-day time, to express the Hubble parameter in terms of the redshift: $H = H_0(1+z)^{\alpha}$, where H_0 is the present-day value of the Hubble constant.

Using the light propagation law $\mathrm{d}s^2 = 0$, we obtain the standard expression for the comoving coordinate of an object observed with a redshift z:

$$\chi = \frac{c}{a(t_0)\,H_0}\int_0^z \frac{\mathrm{d}z}{H(z)} = \frac{c}{a(t_0)\,H_0}\,\frac{1}{1-\alpha}\left[(1+z)^{1-\alpha} - 1\right]. \tag{3}$$

For models with $\alpha > 1$ corresponding to decelerating expansion, the integral in (3) converges as $z \to \infty$, which, as is well known, leads to the appearance of a particle horizon (particles with higher χ cannot be observed at the present time). The particle horizon is absent if $\alpha < 1$, but the event horizon emerges with the comoving coordinate given by

$$\chi_{\mathrm{e.h}} = \frac{c}{a(t_0)\,H_0}\int_{-1}^{0} \frac{\mathrm{d}z}{H(z)}\,, \tag{4}$$

which it is the coordinate the ray emitted at the present time can reach over infinite time. Accordingly, events that occurred starting from the current cosmic time in objects with $\chi > \chi_{\mathrm{e.h}}$ will never be accessible for the observer.

It is instructive to compare the cosmological event horizon crossed by an object in an accelerating expanding universe with another 'archetypal' process in general relativity (GR) — the fall of an object into a Schwarzschild black hole (where different times can be introduced: the time of an infinitely remote observer or the proper time of the freely falling object, but there is no special time for the picture 'as a whole'). Observing the Universe from 'God's position', one can 'see' the event horizon at $\chi = \chi_{\mathrm{e.h}}$ and make statements like the following one: if the ΛCDM model with the current cosmological parameters is true, then galaxies with $z > 1.8$ lie outside the event horizon. But such a statement makes absolutely no sense when describing the fall of a body into a black hole. The observer that has no access to the whole picture of the Universe 'now' does not see any horizon, and the comoving coordinate $\chi = \chi_{\mathrm{e.h}}$ for him is not special in any way. In this sense, the process $r \to r_g$ of freely falling into a black hole, which theoretically takes infinite time, is analogous to the process $z \to \infty$. Of course, the above statement can be made meaningful without invoking the moment 'now' of the cosmic time by 'swapping' the source and the observer. Namely, the above statement means that the signal sent by 'us' now never reaches that galaxy (see paper [5] for more details).

We turn to describing distances in two cases, the first of which is figuratively called 'God's view' and the second, 'an observer's view'.

By definition, the proper distance to an object is $d = a\chi$. Depending on which distance to the object is of interest — the one that the object has 'now' (i.e., simultaneously with us in cosmic time) or the one the object had at the moment when it emitted the light we are now receiving — we should use either the present-day value of the scale factor $a(t_0)$ or its value at the time of the light emission $a(t_{\mathrm{em}})$.

The proper distance to an object 'now' can be written as

$$d = \frac{c}{(1-\alpha)\,H_0}\left[(1+z)^{1-\alpha} - 1\right]. \tag{5}$$

The distance defined in this way is useful from the standpoint of creating a more or less comprehensible observer's image of the universe in which the observer lives. Such a distance increases monotonically with the redshift and tends to a finite limit for $\alpha > 1$ and $z \to \infty$, which is in agreement with the intuitive notion of a finite distance to the particle horizon.

But we are now interested in the other definition of the distance, because it reflects the way the observer perceives the world. The proper distance to the source at the moment of emission $d(t_{\mathrm{em}})$ coincides with the angular distance d_{Θ}, well-known in astrophysics (and directly observed in some important cases), describing the change in the angular size of a fixed-size object as the redshift increases. For barotropic matter in general,

$$d_{\Theta} = \frac{c}{H_0}\,\frac{1}{1-\alpha}\,\frac{(1+z)^{1-\alpha} - 1}{1+z}\,. \tag{6}$$

The angular distance, unlike the proper-motion distance (see below), depends on the redshift nonmonotonically in most of the realistic cosmological models (actually, putting aside exotic models, only in the de Sitter world, where $\alpha = 0$, does the function $d_{\Theta}(z)$ not have a local maximum). For example, for a dust universe (in which $p = 0$, i.e., $\alpha = 3/2$), $d_{\Theta}(z)$ reaches a maximum at a fairly small redshift $z = 5/4$. Therefore, at first glance, this function can hardly be regarded as the most logical measure of distance. However, this distance is very interesting for observations. In particular, the proof that our Universe is nearly flat, derived from CMB observations, relies on the determination of the angular distance to the last-scattering surface. And strange though this would seem at first glance, spots on the last-scattering surface ($z \sim 1100$) that we see on CMB temperature maps were located at the moment of emission of the signal observed now at the same distance (about 13 Mpc; the precise value depends on the model parameters) as some nearby galaxy with $z \approx 0.003$ (it is clear that the emission moment of the signal now observed occurred much later in cosmic time for the galaxy).[1]

Because light ray trajectories in a flat world are straight lines, the increase in the angular distance with redshift reflects exactly this fact: high-redshift objects at the moment of emission were closer than low-redshift objects (for sufficiently high z) (Fig. 1). Naturally, the transition to z as the 'distance measure' immediately restores the familiar picture. However, if we wish to make sense of distances to remote objects in the picture of the Universe directly viewed by an observer, the angular distance is the best choice.

[1] Many numerical estimates can be conveniently made using online cosmological calculators, for example, Ned Wright's calculator (http://www.astro.ucla.edu/~wright/CosmoCalc.html).

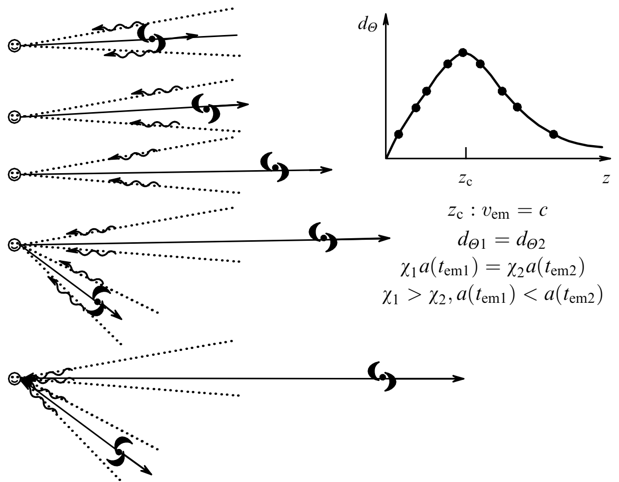

Figure 1. Schematics showing how pairs of objects with the same angular distance but different comoving coordinates form pairs relative to the maximum d_Θ as a function of z. The maximum corresponds to the redshift at which the velocity at the time of emission is equal to the speed of light. Light rays from a more distant galaxy first recede from the observer and only then start approaching him, with the angle between the rays kept fixed.

To conclude this section, for completeness, we briefly discuss the photometric distance d_{ph} and the proper-motion distance d_{pm}.

The photometric distance, which is very popular in observational cosmology, is defined as

$$d_{ph} = \left(\frac{L}{4\pi f}\right)^{1/2} = a^2(t_0)\,\frac{\chi}{a(t_{em})}\,, \tag{7}$$

where L is the luminosity of the source and f is the received radiation flux. We note that the photometric distance tends to infinity at the event horizon (if it exists), but this is not due to the distance as such but occurs because d_{ph}, according to its definition, increases with decreasing the emission intensity due to redshift.

The proper-motion distance is remarkable because it coincides with the proper distance at the moment of observation: $d_{pm} = a(t_0)\,\chi$. Presently, from the observational standpoint, there are no good methods for its determination. However, they can become available from studies of jets from remote sources.

We note that the photometric distance is frequently used in modern observational cosmology because it operates with so-called 'standard candles' (i.e., astronomical sources with a known luminosity) and not because of theoretical advantages of this distance. The angular distance has so far played a minor role, but the situation can change if a 'standard ruler' arises. Some time ago, it was proposed that the characteristic scale of baryonic acoustic oscillations could be used as a standard ruler [6]. Some papers have already used this approach [7–9]. Using data on the spatial distribution of a large number of quasars or galaxies (up to several dozen or hundred thousand, respectively), the authors of those papers were able to estimate the main cosmological parameters.

We stress that the principal feature of the angular distance in our consideration is not related to a certain level of astrophysical knowledge, but follows from the fact that the angular distance coincides with the fundamental theoretical object: the proper distance at the moment of emission of the signal observed now.

The different distances are related as follows:

$$d_\Theta = a(t_{em})\,\chi = \frac{d_{pm}}{1+z} = \frac{d_{ph}}{(1+z)^2}\,. \tag{8}$$

3. Velocities in cosmology

When addressing the velocity of the Hubble flow, we must note that the appearance of different velocities used in different situations is a general property of GR. The free fall into a Schwarzschild black hole provides a classic example. Viewed by an observer at infinity, this fall first appears to be accelerating, and then decelerating, due to gravitational time retardation. The transition to the decelerating motion occurs after the coordinate velocity reaches the maximum value $2c/(3\sqrt{3})$ (for a fall with zero initial velocity) [10]. Naturally, the velocity such introduced, which is equal to the ratio of the distance passed by the object to the time interval measured at infinity, is important only when describing the apparent picture of the fall into a black hole and is completely useless when attempting to describe processes near the black hole itself (for example, the accretion of matter onto it).

The Hubble flow velocity in the traditional treatment, which implicitly assumes the proper distance at the present time, is expressed as

$$\dot{d} = \dot{a}\chi\,, \tag{9}$$

because the comoving coordinate remains fixed in the course of expansion of the Universe (here, we ignore peculiar velocities by setting $\dot{\chi} = 0$).

Using Eqns (3) and (9), it is straightforward to obtain that in a universe filled with barotropic matter, the 'present-day' velocity has the form

$$v_{now} = \frac{c}{1-\alpha}\left[(1+z)^{1-\alpha} - 1\right], \tag{10}$$

while the velocity at the moment of emission is

$$v_{em} = \frac{c}{1-\alpha}\left[1 - (1+z)^{\alpha-1}\right]. \tag{11}$$

We recall some quantities related to the velocities introduced above. It is these parameters that are typically used in standard illustrations of the expansion of the Universe (we note that many subtle points are well presented in [11]). We see that for a decelerating universe ($\alpha > 1$), the asymptotic expansion velocity at the moment of light emission at z tending to infinity is arbitrarily large (which is natural because the light was emitted at a time close to the Big Bang, when the time derivative of the scale factor was arbitrarily large). For an accelerating universe ($\alpha < 1$), this velocity for $z \to \infty$ tends to a finite limit exceeding c (except for the de Sitter model, where this limit is exactly c).

More interesting is the asymptotic recession velocity 'now' (in cosmic time), which in different models can be either larger or smaller than the velocity of light [2] as $z \to \infty$. The border case is $w = 1/3$ ($\alpha = 2$, a universe filled with radiation). Then $H = H_0(1+z)^2$, and, accordingly, $v_{em} = cz$ and $v_{now} = cz/(1+z)$. It hence follows that v_{now} tends to c at $z \to \infty$. For a matter-dominated universe ($w = 0$, $\alpha = 3/2$),

[2] We note that when speaking about the speed of light here, we assume $c \approx 300{,}000$ km s^{-1}. If some galaxy has a recession velocity exceeding the speed of slight, a photon emitted by a source in this galaxy would recede from the source in the galaxy with the speed c, while its recession velocity from us would be higher than that of the galaxy.

$v_{\rm now}$ tends to $2c$, and the value $v_{\rm now} = c$ is reached at some finite z (this is the so-called Hubble sphere, $R_c = c/H$). On the contrary, if $w = 1$ ($\alpha = 3$), then $v_{\rm now}(\infty) = c/2$ and $v_{\rm now}$ does not reach the speed of light at any z. The general formula for the limit as $z \to \infty$ is quite simple: $v_{\rm now}(z = \infty) = c/(\alpha - 1)$. For $\alpha < 1$, the last formula is inapplicable because there is no finite limit of the velocity considered, and it increases without bound as $z \to \infty$.

At first glance, it seems that the introduced velocities solve the problem: the velocity 'now' corresponds to God's view, where all points in the universe can instantly be 'seen' at a given cosmic time, whereas the velocity $v_{\rm em}$ should correspond to the observer's view. But the validity of the last statement is questionable if we recall that in deriving the formula for $v_{\rm em}$, the same cosmic time was used, which, we noted, cannot be directly measured by the observer: the light signals emitted by an object over some finite time interval of cosmic time will reach the observer over a longer time interval, leading to a smaller apparent velocity of the Hubble flow. Therefore, $v_{\rm em}$ also corresponds to the view of God, who travels in time and, being present at the moment of emission and, as usual, grasping the universe 'as a whole and instantly', can see the object and the observer receding with the velocity $v_{\rm em}$. But what does a real observer see?

We note that posing such a question (when we generally disregard cosmic time and consider only values associated with the observer) starts being relevant in relation to the possible forthcoming discovery of redshift change with time. The corresponding formula describing the change of z in the observer's time interval has the form (see, e.g., [12])

$$\frac{{\rm d}z}{{\rm d}t} = H_0\left[1 + z - (1+z)^\alpha\right]. \qquad (12)$$

Using (12) and taking into account that $\dot{H}/H^2 = -\alpha$, we find the time derivative of the proper distance at the emission moment:

$$\frac{{\rm d}d_{\rm em}}{{\rm d}t} \equiv \tilde{v}_{\rm em} = \frac{{\rm d}d_{\rm em}}{{\rm d}H}\frac{{\rm d}H}{{\rm d}t} + \frac{{\rm d}d_{\rm em}}{{\rm d}z}\frac{{\rm d}z}{{\rm d}t} = \frac{c}{1-\alpha}\,\frac{1-(1+z)^{\alpha-1}}{1+z}. \qquad (13)$$

Velocity (13), which is purported to be the Hubble flow velocity directly measured by an observer in an expanding universe using its proper time, does not in general coincide with any of the velocities discussed above. The velocity of expansion at the emission moment, as determined with respect to cosmic time, differs from (13) by the factor $1 + z$, which reflects the difference in time intervals at the object at the time of emission and at the observer at the time of detection. Essentially, this is the rate of change in the angular distance, and we therefore refer to it as v_Θ ($v_\Theta \equiv \tilde{v}_{\rm em}$) in what follows.

The velocity v_Θ has totally different asymptotic forms at large z. First of all, it is easy to see that for models with an event horizon ($\alpha < 1$), v_Θ vanishes as $z \to \infty$, having passed through a maximum. In particular, in the de Sitter world, the velocity v_Θ has the maximum $c/4$ at $z = 1$. The situation with the particle horizon is more complicated. Unexpectedly, radiation-dominated models once again play a special role here. For $w < 1/3$, the velocity still passes through a maximum and vanishes at the particle horizon. The maximum disappears at $w = 1/3$ (the velocity at the particle horizon is equal to c). For more stiff equations of state, the velocity tends to infinity with increasing z.

The velocity reaches a maximum (if it exists) at

$$z_{\rm m} = (2-\alpha)^{1/(1-\alpha)} - 1\,. \qquad (14)$$

The location of the maximum smoothly increases from $z_{\rm m} = 1$ for the de Sitter world to infinity for $w = 1/3$, passing through $z_{\rm m} = 3$ in the important particular case of a dust universe. The velocity at the maximum smoothly increases from $c/4$ (de Sitter world) to c (radiation-dominated universe). The case $\alpha = 1$ must be treated separately in all formulas. For such a universe (the Milne model), the scale factor linearly increases with time, the velocities $v_{\rm em}$ and $v_{\rm now}$ coincide, and v_Θ reaches the maximum $c/{\rm e}$ at $1 + z_{\rm m} = {\rm e}$, where e is the base of the natural logarithm.

Curiously, just some 50 years ago, the velocity introduced in such a way was purported to play the role of a characteristic of the Hubble flow that always remains subluminal, reaching c only at the horizon in the limit of a universe filled with matter with an ultrarelativistic equation of state. Such an equation of state was believed to be the stiffest one (see, e.g., the classic textbook by Landau and Lifshitz [13]). It is fair to say that the only 'nonexotic' example of matter with a stiffer equation of state is a scalar field — an object that continues to be a theoretical construction. Nevertheless, because superluminal velocities of the Hubble flow are possible in GR and the special case of matter with $p = \rho/3$ plays no special role in modern physics, we do not insist on some deeper meaning of the above asymptotic forms and simply note them as a curious fact.

Figures 2–5 illustrate the behavior of the three velocities considered above as a function of redshift for cosmologically interesting equations of state.

In the de Sitter world ($\alpha = 0$), the universe expands with acceleration, and hence the velocity $v_{\rm em}$ at the moment of emission is always smaller than the velocity 'now' $v_{\rm now}$ (see Fig. 2). As noted above, the first velocity tends to c as $z \to \infty$, and the second increases without bound, passing through c at $z = 1$. As regards the apparent velocity of expansion v_Θ, it reaches the maximum $(1/4)\,c$ at $z = 1$, and decreases to zero as the redshift increases further.

In a dust universe, the expansion is decelerating, and hence always $v_{\rm em} > v_{\rm now}$ (see Fig. 3). The first of the velocities increases without bound, while the second tends to the asymptotic value $2c$ when approaching the particle horizon.

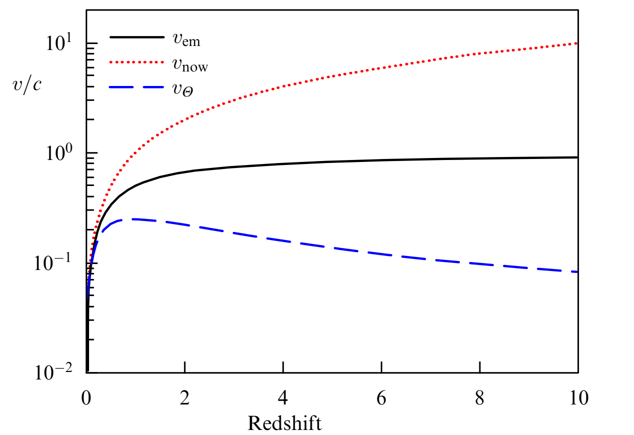

Figure 2. Velocities $v_{\rm em}$, $v_{\rm now}$, and v_Θ as functions of the redshift z in the case $\alpha = 0$ (de Sitter world). The velocities $v_{\rm em}$, $v_{\rm now}$, and v_Θ are respectively shown by the solid, dotted, and dashed lines.

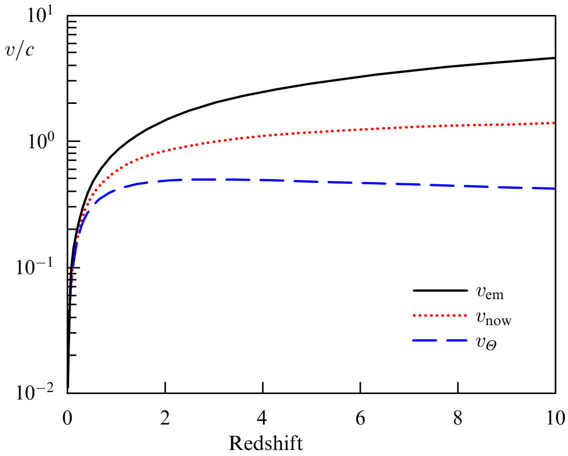

Figure 3. The same as in Fig. 1 for $\alpha = 3/2$ (dust universe).

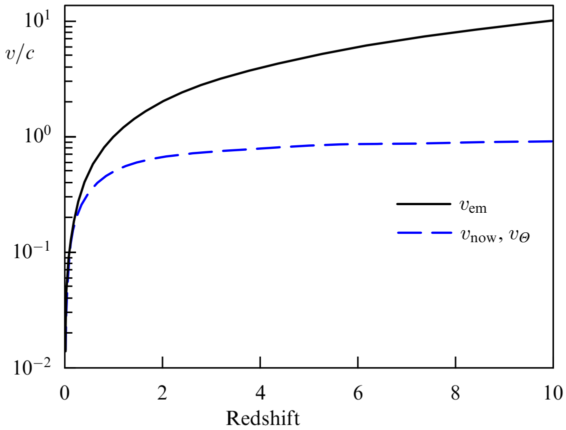

Figure 4. The same as in Fig. 1 for $\alpha = 2$ (radiation-dominated universe). The solid curve shows v_{em}. In this case, the velocity v_{now} coincides with v_Θ (dashed curve).

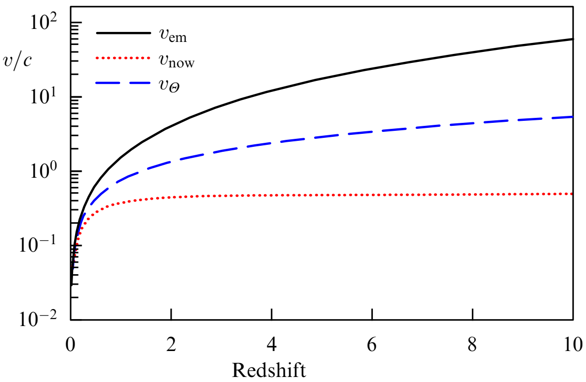

Figure 5. The same as in Fig. 1 for $\alpha = 3$ (matter with the stiffest equation of state).

The apparent velocity v_Θ reaches half the speed of light at $z = 3$ and then decreases. At this redshift, $v_{\mathrm{now}} = c$. The above formulas suggest that this is not a pure coincidence. Namely, in a single-component Friedmann universe with barotropic matter, $v_{\mathrm{now}} = c$ exactly at the same z where v_Θ reaches a maximum (this occurs when $(1+z)^{1-\alpha} = 2 - \alpha$).

The case of a radiation-dominated universe ($\alpha = 2$) is special (see Fig. 4). It corresponds to the minimal value of α at which v_Θ has no maximum as a function of z. Moreover, in such a universe, both velocities are identically equal to $cz/(1+z)$. As regards v_{em}, it becomes superluminal at z above unity and monotonically increases to infinity (it has a very simple dependence: $v_{\mathrm{em}} = cz$).

The reader is encouraged to study the interesting 'symmetry' in the behavior of the velocities in the range $0 \leqslant \alpha \leqslant 2$ by replacing $\alpha \to 2 - \alpha$.

Finally, for a universe filled with matter with the stiffest equation of state (see Fig. 5), both v_{em} and v_Θ monotonically increase without bound as z increases. On the contrary, the velocity 'now' always remains subluminal.

4. From an observer's viewpoint...

Clearly, our treatment of the apparent velocity of the Hubble flow is not free from shortcomings. From the very beginning, we relied on the notion of the proper distance, which is by no means an observable quantity. For a rigorous definition of this quantity, a chain of observers is required, with each observer measuring the distance to the neighboring one at a given cosmic time, after which the measurements of all observers are summed up. Because the realization of such a 'cosmic conspiracy', according to Steven Weinberg, relates to the category of non-science fiction, alternative ways must be found. We use the coincidence of the proper distance at the moment of emission (it is this distance that we consider meaningful when discussing the apparent picture by the observer who does not calculate the object location but 'sees' it) with the theoretically measurable angular distance. In this sense, it is may be reasonable to believe that the observer does measure the proper distance to the object at the time of emission. Then v_Θ is the most meaningful characteristic of the apparent velocity of the Hubble flow (if we try to give any meaning at all to the 'velocity of expansion viewed by an observer').

Of course, the visual picture from the observer's point of view is not needed in solving many problems. For example, only the redshift is commonly used in practice as a measure of distance to the object. This is sufficient to make all necessary calculations. Therefore, many experts believe that the discussion of different velocities and distances in cosmology is unnecessary and can only muddle the issue. For example, the opinion given in [1] (although not shared by the author himself) is that the recession velocity is unphysical because it cannot be directly measured. However, in our opinion, simple images with a clear physical interpretation allow effectively using intuition when carrying out research. The observable quantities that reflect important features of cosmological models deserve careful analysis.

It is interesting to consider how maximal angular distances and velocities as a function of z are related in different models. The maximum velocity can occur at lower or higher z, depending on the parameters, unlike the angular distance maximum (see Fig. 6, which is calculated for a dust universe). However, the common property is that in accelerating universes, the velocity maximum occurs at lower c than the maximum of d_Θ, whereas in decelerating universes, the situation is the opposite. The border case is the Milne universe with $a \sim t$, in which both maxima coincide at $z = \mathrm{e} - 1 \approx 1.71$. Interestingly, the angular distance reaches a maximum when $v_{\mathrm{em}} = c$ [for a single-component universe, this happens at $(1+z)^{\alpha-1} = \alpha$ (see Fig. 1)].

We note that the change in the angular distance corresponds to our psychological perception of a receding object, according to which the object is receding if its size

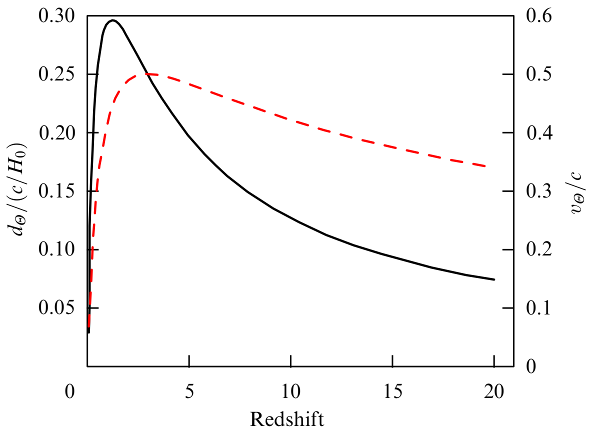

Figure 6. The angular distance (solid curve) and velocity (dashed curve) versus redshift for a dust universe ($w = 0$). The angular distance is in units of c/H_0 (modern Hubble radius), on the left vertical axis. The velocity is in units of the speed of light, on the right vertical axis.

decreases, where, of course, we mean the angular size. Therefore, when constructing an accurate visualization of the expansion of the Universe as viewed by a terrestrial observer, we must reproduce, first of all, exactly the change in the angular distance.

In our visualization (imagine it on a planetarium's dome), let the universe be filled (say, up to $z \approx 10$) with identical galaxies of constant size. More remote galaxies appear weaker and reddened. The angular size of the galaxies behaves as shown in Fig. 6: starting from some distance, more remote galaxies appear to have a larger angular size (their angular distance decreases). Dynamically, more remote galaxies are weaker and redder. However, the main effect of the 'recession' is due to the decrease in the angular size of all galaxies, and the rate of the size decrease also decreases starting from some distance (see Fig. 6 and the schematic picture in Fig. 7). It is essential that, in agreement with the intuitive notion of the event horizon, the dynamics are 'frozen' for the most remote (and the reddest) objects.

From the standpoint of real observations, the first results of the direct registration of the expansion dynamics are apparently related to measurements of $\dot{z}$ using ultra-stable spectrographs with new-generation telescopes (see, e.g., review [12]). In addition, there is hope that the GAIA

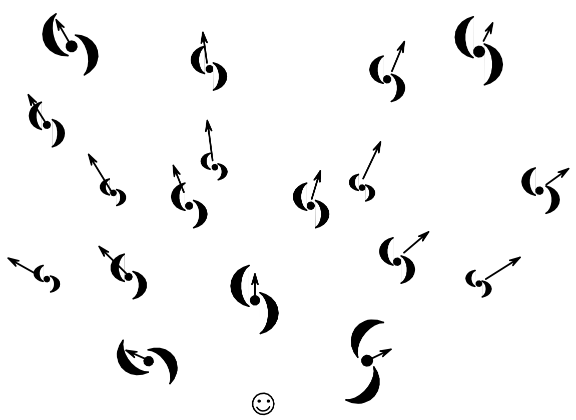

Figure 7. Diagram showing how the angular size of a standard galaxy changes with distance and how the velocity related to the angular distance changes. Both quantities behave nonmonotonically and can reach a maximum at different redshifts.

(Global Astrometric Interferometer for Astrophysics) satellite will be able to detect the decrease in size of gravitationally bound objects with time due to the cosmological expansion [14].

5. Conclusion

In anticipation of the direct experimental discovery of the time dependence of quantities characterizing the Hubble expansion, it is appropriate to recall once again which observables can be directly measured and which ones, although important for the understanding and accurate description of the Universe's expansion, represent purely theoretical constructions. That was the main goal of this paper.

Our main points can be summarized as follows.

• We wish to determine the quantities that maximally correspond to the distance and velocity of the Hubble flow intuitively perceived by an observer in the expanding universe.

• The proper distance is special in the theory and does not depend on the current technical capabilities or astrophysical knowledge.

• Because we see an object as it was when it emitted the light received by us, it seems natural to consider the distance at the moment of emission (this distance is more 'seen' than calculated by the observer) as a characteristic of the source.

• The proper distance, measured at the moment of emission of the signal being currently received, is calculated using the same formula as for the angular distance. In addition, the angular distance itself and the behavior of its derivative correspond to both the psychological perception of a receding object and the intuitively expected behavior of objects 'at the horizon'. Correspondingly, it is the angular distance and its time derivative that are the most natural characteristics of the Hubble flow from the observer's standpoint.

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