CONFERENCES AND SYMPOSIA

100th ANNIVERSARY OF THE BIRTH OF B M PONTECORVO

Bruno Pontecorvo and the neutrino

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Contents

Abstract. This paper commemorates the 100th anniversary of the birth of the great scientist and neutrino researcher Bruno Pontecorvo. His major contributions are reviewed, including the radiochemical method of neutrino detection, the idea of the $\nu - e$ universality of the weak interaction, and the proposal of an accelerator experiment to prove that ν_e and ν_μ are different particles. Pontecorvo's fundamental idea of neutrino masses, mixing, and oscillations is discussed in detail, as is the development of this idea by Pontecorvo and Gribov and Pontecorvo and the author.

1. Introduction

Bruno Pontecorvo was born on 22 August 1913 in Pisa (Marina di Pisa). His father was the owner of a textile factory, which was founded by Pellegrino Pontecorvo, Bruno's grandfather.

After the war, the factory was closed for many years, and the building was not used. Presently, the Pisa department of the INFN is in the building of the Pontecorvo factory. The square before the building is called Largo Bruno Pontecorvo.

There were 8 children in the family: 5 boys and 3 girls. All of them were very successful. Three of the boys became famous: biologist Guido, movie director Gillo, and physicist Bruno.

Bruno entered the engineering faculty of Pisa University, but after two years he decided to study physics. His elder

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brother Giudo, who knew about the existence of Fermi's group in Rome, suggested that he go to Rome. Bruno passed the exam, which was taken by Fermi and Rasetti, and was accepted in the third year of the Faculty of Physics and Mathematics at the University of Rome. Thus, Bruno Pontecorvo started his scientific work in 1932 in Rome as a student of Enrico Fermi. He later became a member of Fermi's group. He was the youngest 'ragazzo di Via Panisperna'.

Bruno took part in many experiments of Fermi's group. One, performed by Amaldi and Pontecorvo, led to the discovery of the effect of slow neutrons, the most important discovery made by the group. The effect of slow neutrons opened the door to a variety of applications of neutrons (reactors, isotopes for medicine, atomic bombs, etc.). For the discovery of the effect of slow neutrons, Fermi was awarded a Nobel Prize.

In 1936, Bruno received an award from the Italian Ministry of Education and went to Paris to work in Joliot-Curie's group. In Paris, he studied nuclear isomers — metastable nuclear states with large spins. He carried out the first experiments on the observation of conversion electrons in decays of isomers, on the production of nuclear isomers in the process of irradiation of nuclei by high-energy γ-quanta, etc. For the study of nuclear isomerism, Bruno received the Curie–Carnegie prize. Fermi congratulated Bruno on his excellent results. Bruno was very happy and proud of Fermi's congratulation (as he wrote in his autobiography, he had thought that Fermi, who usually called him a great champion, respected him only as an expert in tennis).

From 1940 till 1942, Pontecorvo worked in a private oil company in Oklahoma (USA). He developed and applied a method of neutron well logging for oil exploration. This was the first practical application of the effect of slow neutrons. Pontecorvo's method of neutron well logging is widely used nowadays.

In 1943, Pontecorvo was invited to take part in the Anglo–Canadian Uranium Project in Canada. He was the scientific

leader of the project of a research reactor that was built in 1947 and was the first nuclear reactor outside the USA.

In Canada, Pontecorvo started research in elementary particle physics. Soon after the famous Fermi paper on the theory of β-decay [1] (1934), Bethe and Pierls [2] estimated the cross section of the interaction of a neutrino with a nucleus. They showed that the cross section was extremely small ($\sigma < 10^{-44}$ sm^2). After that paper, for many years, the neutrino was considered an 'undetectable particle'.

Pontecorvo was the first to challenge this opinion. In 1946, he proposed the radiochemical method of neutrino detection [3]. The method was based on the observation of the decay of a daughter nucleus produced in the reaction $\nu + (A, Z) \to e^- + (A, Z + 1)$. He discussed the reaction

$$\nu + {}^{37}\text{Cl} \to e^- + {}^{37}\text{Ar} \tag{1}$$

in detail He considered the method of neutrino detection based on the observation of Cl–Ar reaction (1) as promising for the following reasons:

• C_2Cl_4 is a cheap, nonflammable liquid;

• ^{37}Ar nuclei are unstable (due to K-capture) with a suitable half-life (34.8 days);

• A few atoms of ^{37}Ar (a rare gas) produced during the exposition time can be extracted from a large detector.

Pontecorvo's Cl$-$Ar method was used by R Davis in his first, pioneering experiment on the detection of solar neutrinos [4] (see [5]), for which Davis was awarded the Nobel Prize in 2002.

The radiochemical method of neutrino detection based on the observation of the reaction

$$\nu + {}^{71}\text{Ga} \to e^- + {}^{71}\text{Ge}, \tag{2}$$

proposed by Kuzmin [6], was used in the GALLEX (GALLium EXperiment)–GNO [7, 8] and SAGE (Soviet-American Gallium Experiment) [9] solar neutrino experiments in which the ν_e from all thermonuclear reactions on the Sun, including neutrinos from the main reaction $p + p \to n + p + e^+ + \nu_e$, were detected.

In the seminal Chalk River paper (1946) [3], Pontecorvo identified the following possible intensive sources of neutrinos:

• the Sun;

• reactors;

• radioactive materials produced in reactors.

In Canada in 1948, Pontecorvo and his collaborators invented a low-background proportional counter [10] that allowed them to detect very rare events. This counter was crucial for detecting solar neutrinos in the Homestake, GALLEX–GNO, and SAGE experiments.

After the famous Conversi, Pancini, and Piccioni experiment (1947) [11], from which it followed that muons weakly interact with nuclei, Pontecorvo and Hincks [12, 13] performed a series of experiments on the investigation of the fundamental properties of muons. They proved that

• the charged particle emitted in μ-decay is the electron;

• muons decay into three particles;

• muons do not decay into electrons and γ-quanta.

Bruno Pontecorvo was the first to focus on the strong analogy between the weak interaction of the electron and the muon [14]. He suggested that the muon is a particle with spin 1/2 and, in the process of muon capture by a nucleus, a neutrino is emitted.

Pontecorvo compared the probabilities of the processes

$$\mu^- + (A, Z) \to \nu + (A, Z - 1),$$

$$e^- + (A, Z) \to \nu + (A, Z - 1),$$
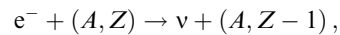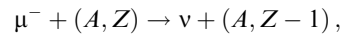
and found that the constants characterizing these two processes were of the same order of magnitude. Based on this observation, Pontecorvo came to the conclusion that a "fundamental analogy between β-processes and processes of the absorption of muons" exists. Thus, in 1947, Pontecorvo came to the idea of the existence of a universal weak interaction that included e$-$ν and μ$-$ν pairs. Later, the idea of μ$-$e universality was put forward by Puppi [15], Klein [16], and Young and Tiomno [17].

In 1950, Pontecorvo and his family (wife and three sons) moved from England to the USSR. He started working in Dubna, where, at that time, the largest accelerator in the world was located (460 MeV, later 680 MeV). Pontecorvo and his group performed experiments on the production of π^0 in neutron–proton and neutron–nuclei collisions, on pion–nucleon scattering, and others.

Pontecorvo always thought about the neutrino. In the late 1950s, a project of a meson factory in Dubna was prepared (unfortunately, the project has not been realized). In connection with this project, Pontecorvo thought about the feasibility of neutrino experiments with neutrinos from decays of pions and kaons produced at high-intensity accelerators. He came to the conclusion that experiments with accelerator neutrinos are possible [18] (Markov [19] and Schwartz [20] independently came to the same conclusion).

Pontecorvo always, starting from his time in Canada, had in mind that muon and electron neutrinos could be different particles. When he realized that experiments with high-energy accelerator neutrinos are feasible, he understood that such experiments give us the best, model-independent possibility of answering the question of whether ν_μ and ν_e are the same or different particles [18]. Pontecorvo's proposal was realized in the famous Brookhaven experiment [21] (1962). It was proved that $\nu_e \neq \nu_\mu$. In 1988, Lederman, Schwartz, and Steinberger were awarded the Nobel Prize for "the discovery of the muon neutrino leading to classification of particles in families."

2. First ideas of neutrino masses, mixing, and oscillations

We now come to the most important idea of Bruno Pontecorvo, that of neutrino masses, mixing, and oscillations, which created a new field of neutrino research and a new era in neutrino physics. He proposed the idea of neutrino oscillations in 1957–1958 [22, 23] and developed it over many years.

Pontecorvo was impressed by the possibility of $K^0 \leftrightarrows \bar{K}^0$ oscillations suggested by Gell-Mann and Pais [24]. The phenomenon of $K^0 \leftrightarrows \bar{K}^0$ oscillations is based on the following facts:

(1) K^0 and $\bar{K}^0$ are particles with the respective strangeness 1 and -1. The strangeness is conserved in strong interactions;

(2) weak interactions, in which strangeness is not conserved, induce transitions between K^0 and $\bar{K}^0$;

(3) states of K^0 and $\bar{K}^0$ produced in the processes of strong interaction are superpositions ('mixtures') of states of K_1^0 and K_2^0, particles with definite masses and widths.

In [22], Pontecorvo put forth the following question: "...do there exist other 'mixed' neutral particles (not necessa-

rily elementary ones) which are not identical to corresponding antiparticles and for which particle–antiparticle transitions are not strictly forbidden." He came to the conclusion that such a system could be the muonium ($\mu^+ + e^-$) and anti-muonium ($\mu^- + e^+$).

At that time, it was not known that ν_e and ν_μ are different particles. If they were the same particles, transitions ($\mu^+ + e^-) \to (\mu^- + e^+$) would be allowed and induced (in the second order of the perturbation theory) by the same interaction that is responsible for μ-decay. In [22], Pontecorvo considered $\mu^+ + e^- \leftrightarrows \mu^- + e^+$ oscillations in some detail.

It is worth noting that modern experiments on the search for muonium–antimuonium transitions (see [25]) are considered a sensitive way of obtaining information about an interaction in which flavor lepton numbers are changed by 2.

In [22], Pontecorvo made the following remark about the neutrino: "If the theory of the two-component neutrino was not valid (which is hardly probable at present) and if the conservation law for neutrino charge was not valid, neutrino $\to$ antineutrino transitions in a vacuum would, in principle, be possible." As is well known, according to the two-component neutrino theory [26–28], the neutrino is massless and for one neutrino type, only the left-handed neutrino ν_L and the right-handed antineutrino $\bar{\nu}_R$ exist. Transitions between them are forbidden by the conservation of angular momentum.

A rumor helped Pontecorvo realize the idea of neutrino oscillations in the case of one neutrino. In 1957, Davis performed an experiment on the search for ^{37}Ar production in the process of interaction of reactor antineutrinos with ^{37}Cl [29]. A rumor reached Pontecorvo that Davis had observed such events. He suggested that these 'events' could be due to transitions of reactor antineutrinos into right-handed neutrinos on the way from the reactor to the detector and published the first paper dedicated to neutrino oscillations [23] (1958).

This was a very courageous idea. We stress again that only one type of neutrino was known in 1957–1958. Pontecorvo assumed that there were transitions $\bar{\nu}_R \to \nu_R$ (and $\nu_L \to \bar{\nu}_L$). Thus, he had to assume not only the lepton number nonconservation but also the existence, in addition to the standard right-handed antineutrino $\bar{\nu}_R$ and left-handed neutrino ν_L (quanta of the left-handed field $\nu_L(x)$), of a right-handed neutrino ν_R and a left-handed antineutrino $\bar{\nu}_L$ (quanta of a right-handed field $\nu_R(x)$).

According to the two-component neutrino theory, which was perfectly confirmed by an experiment on the measurement of the neutrino helicity [30], only the $\nu_L(x)$ field enters the weak-interaction Lagrangian. Thus, from the standpoint of this theory, ν_R and $\bar{\nu}_L$ are noninteracting 'sterile' particles.

To explain Davis's events, Pontecorvo had to assume that "a certain fraction of particles (ν_R) can induce the $Cl-Ar$ reaction." Later, when such anomalous events disappeared and only an upper bound for the cross section of the reaction $\bar{\nu} + {}^{37}Cl \to e^- + {}^{37}Ar$ was found in Davis's experiment, Pontecorvo understood that there was no need for such an assumption. The terminology 'sterile neutrino', which is standard nowadays, was introduced by him in the next paper on neutrino oscillations [31].

In the very first paper on neutrino oscillations, Pontecorvo pointed out that in the experiment of Reines and Cowan [32–34], a deficit of antineutrino events could be observed. He wrote in [23]: "The cross section of the process $\bar{\nu} + p \to e + n$ with $\bar{\nu}$ from the reactor must be smaller than expected. This is because the neutral lepton beam, which at the source is capable of inducing the reaction, changes its composition on the way from the reactor to the detector."

Starting from paper [23], all his life Bruno believed in the existence of neutrino oscillations. He wrote: "Effects of transformation of a neutrino into an antineutrino and vice versa may be unobservable in the laboratory but will certainly occur, at least, on an astronomical scale."

3. Second Pontecorvo's paper on neutrino oscillations (1967)

The next paper on neutrino oscillations was written by Pontecorvo in 1967 [31]. At that time, the phenomenological $V-A$ theory was established, $K^0 \leftrightarrows \bar{K}^0$ oscillations were observed, and it was proved that (at least) two types of neutrinos, ν_e and ν_μ, existed in nature. In [31], Pontecorvo wrote: "If the lepton charge is not an exactly conserved quantum number, and the neutrino mass is different from zero, oscillations similar to those in K^0 beams become possible in neutrino beams." Pontecorvo considered oscillations between active neutrinos $\nu_\mu \leftrightarrows \nu_e$ and between active and sterile neutrinos $\nu_\mu \leftrightarrows \bar{\nu}_{eL}$, etc.

In the 1967 paper, Pontecorvo for the first time discussed the effect of neutrino oscillations of solar neutrinos: "From an observational point of view, the ideal object is the Sun. If the oscillation length is smaller than the radius of the Sun region effectively producing neutrinos, direct oscillations will be smeared out and unobservable.... The only effect on Earth's surface would be that the flux of observable Sun neutrinos must be half the total neutrino flux."

At that time, Davis prepared his famous solar neutrino experiment. When the first results of the experiment were obtained in 1970 [35], it turned out that the detected flux of the solar neutrinos was about a third to a half of the predicted flux. This result created the so-called solar neutrino problem. It was soon commonly accepted that, among different explanations of the problem, the oscillations of solar neutrinos is the most natural one. Thus, Bruno Pontecorvo anticipated the solar neutrino problem.

4. Gribov–Pontecorvo's paper on neutrino oscillations (1969)

Gribov and Pontecorvo [36] considered a scheme of neutrino mixing and oscillations with four neutrino and antineutrino states: the left-handed neutrinos ν_e and ν_μ and right-handed antineutrinos $\bar{\nu}_e$ and $\bar{\nu}_\mu$. They assumed that there are no sterile neutrino states.

It was assumed in [36] that in addition to the standard $V-A$ charged lepton current

$$j_\alpha = 2(\bar{\nu}_{eL}\gamma_\alpha e_L + \bar{\nu}_{\mu L}\gamma_\alpha \mu_L), \tag{3}$$

the total Lagrangian also involves an effective interaction that violates L_e and L_μ. After the diagonalization of the most general effective Lagrangian of this type, the following mixing relations were found:

$$\nu_{eL} = \cos\theta\, \nu_{1L} + \sin\theta\, \nu_{2L}, \quad \nu_{\mu L} = -\sin\theta\, \nu_{1L} + \cos\theta\, \nu_{2L}. \tag{4}$$

Here, $\nu_{1,2}$ are fields of the Majorana neutrinos with masses $m_{1,2}$ and θ is the mixing angle. Neutrino masses and the

mixing angle are determined by the parameters of the effective Lagrangian.

The authors obtained the following expression for the $\nu_e \to \nu_e$ survival probability in the vacuum (in the modern notation),

$$P(\nu_e \to \nu_e) = 1 - \frac{1}{2}\sin^2 2\theta \left(1 - \cos \frac{\Delta m^2 L}{2E}\right), \qquad (5)$$

and applied the developed formalism to solar neutrino oscillations. They considered the possibility of the maximal mixing $\theta = \pi/4$ as the most simple and attractive one. In this case, the averaged observed flux of the solar neutrinos is equal to 1/2 of the predicted flux.

5. General phenomenological theory of neutrino mixing and oscillations (Dubna, 1975–1987)

The work by Pontecorvo and myself on neutrino masses, mixing, and oscillations started in 1975 [37, 38]. The first paper was based on the idea of the quark–lepton analogy.

It was established at that time that the charged current of quarks has the form (in the case of four quarks)

$$j_\alpha^{\text{CC(quark)}}(x) = 2\big(u_L(x)\,\gamma_\alpha d_L^c(x) + \bar{c}_L(x)\,\gamma_\alpha s_L^c(x)\big), \qquad (6)$$

where $d_L^c(x)$ and $s_L^c(x)$ are Cabibbo–GIM mixtures of d and s quarks with masses m_d and m_s, and θ_C is the Cabibbo angle:

$$d_L^c(x) = \cos \theta_C d_L(x) + \sin \theta_C s_L(x),$$

$$s_L^c = -\sin \theta_C d_L(x) + \cos \theta_C s_L(x). \qquad (7)$$

The lepton charged current

$$j_\alpha^{\text{CC(lept)}}(x) = 2\big(\bar{\nu}_{eL}(x)\,\gamma_\alpha e_L(x) + \bar{\nu}_{\mu L}(x)\,\gamma_\alpha \mu_L(x)\big), \qquad (8)$$

has the same form as the quark charged current (the same coefficients and left-handed components of the fields). In order to make the analogy between quarks and leptons complete, it was natural to assume that $\nu_{eL}(x)$ and $\nu_{\mu L}(x)$ are also mixed fields:

$$\nu_{eL}(x) = \cos \theta\, \nu_{1L}(x) + \sin \theta\, \nu_{2L}(x),$$

$$\nu_{\mu L}(x) = -\sin \theta\, \nu_{1L}(x) + \cos \theta\, \nu_{2L}(x). \qquad (9)$$

Here, $\nu_1(x)$ and $\nu_2(x)$ are Dirac fields of neutrinos with masses m_1 and m_2, and θ is the leptonic mixing angle. We wrote in [37]: "...in our scheme, ν_1 and ν_2 are just as leptons and quarks (which, maybe, is an attractive feature), while in the Gribov–Pontecorvo scheme the two neutrinos have a special position among the other fundamental particles."

If mixing (9) takes place, the total lepton number $L = L_e + L_\mu$ is conserved, and neutrinos with definite masses ν_i ($i = 1, 2$) differ from antineutrinos $\bar{\nu}_i$ by the lepton number ($L(\nu_i) = -L(\bar{\nu}_i) = 1$).

After the great success of the two-component theory, in 1975, there was still a general belief that neutrinos are massless particles. It is obvious that in this case, mixing (9) has no physical meaning. Our main arguments for neutrino masses at that time were as follows:

(1) there is no principle (like gauge invariance in the case of γ-quanta) that requires the masses of neutrinos to be equal to zero;

(2) in the framework of the two-component neutrino theory, the masslessness of the neutrino was an argument in favor of the left-handed neutrino field. It turned out, however, that left-handed components of all fields enter the weak Hamiltonian (the $V-A$ theory). It was more natural after that to consider the neutrino not as a special massless particle but as a particle with some mass.

In [37], we discussed a possible value of the mixing angle θ. We argued that

• there is no reason for the lepton and Cabibbo mixing angles to be the same;

• "the special values of the mixing angles $\theta = 0$ and $\theta = \pi/4$ (maximum mixing) are of the greatest interest."

The probabilities of transitions $\nu_l \to \nu_{l'}$ are the same in the scheme of the mixing of two Majorana neutrinos and in the scheme of the mixing of two Dirac neutrinos.

In the paper "Again on neutrino oscillations" [39], we considered the most general neutrino mixing. In accordance with modern gauge theories, we started to characterize neutrino mixing by the neutrino mass term. Three types of neutrino mass terms are possible.[1]

5.1 Left-handed Majorana mass term
The standard CC and NC Lagrangians of the interaction of leptons with $W^\pm$ and Z^0 bosons have the form

$$\mathcal{L}_I^{\text{CC}} = -\frac{g}{2\sqrt{2}}\, j_\alpha^{\text{CC}} W^\alpha + \text{h.c.}, \quad \mathcal{L}_I^{\text{NC}} = -\frac{g}{2\cos \theta_w}\, j_\alpha^{\text{NC}} Z^\alpha. \qquad (10)$$

Here, g is the SU(2) gauge coupling constant, θ_w is the weak angle, and the leptonic charged current j_α^{CC} and neutrino neutral current j_α^{NC} are given by the expressions

$$j_\alpha^{\text{CC}} = 2\sum_l \bar{\nu}_{lL}\gamma_\alpha l_L, \quad j_\alpha^{\text{NC}} = \sum_l \bar{\nu}_{lL}\gamma_\alpha \nu_{lL}. \qquad (11)$$

We assume that the following neutrino mass term also enters the total Lagrangian:

$$\mathcal{L}_L^M = -\frac{1}{2}\sum_{l',l} \bar{\nu}_{l'L}\big(M_L^M\big)_{l'l}(\nu_{lL})^c + \text{h.c.} \qquad (12)$$

Here,

$$\nu_{lL}^c = C\bar{\nu}_{lL}^T,$$

where C is the charge conjugation matrix, which satisfies the conditions

$$C\gamma_\alpha^T C^{-1} = -\gamma_\alpha, \quad C^T = -C,$$

and M^M is a complex symmetric matrix.

After the standard diagonalization of mass term (12), we obtain the expression

$$\mathcal{L}_L^M = -\frac{1}{2}\sum_i m_i \bar{\nu}_i \nu_i, \qquad (13)$$

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[1] We follow reviews [40, 41].

where v_i is the field of a neutrino with the mass m_i, which satisfies the Majorana condition

$$v_i = v_i^c = C\bar{v}_i^T \,. \tag{14}$$

The flavor field v_{lL} is connected with the Majorana fields v_{iL} by the mixing relation

$$v_{lL} = \sum_i U_{li} v_{iL} \,. \tag{15}$$

In the case of a Majorana mass term, there are no conserved lepton numbers. Hence, Majorana neutrinos are truly neutral particles: they carry neither electric charge nor lepton numbers. In other words, Majorana neutrinos and antineutrinos are identical particles, $v_i \equiv \bar{v}$. The scheme with the mass term $\mathcal{L}_L^M$ is a generalization of the scheme considered in [36].

5.2 Dirac mass term
We now assume that in addition to the standard CC and NC Lagrangians (10), the following neutrino mass term enters the total Lagrangian:

$$\mathcal{L}^D = -\sum_{l',l} \bar{v}_{l'L} M_{l'l}^D v_{lR} + \text{h.c.} \,, \tag{16}$$

where M^D is a complex 3×3 matrix.

After the standard diagonalization of M^D, we find the mixing relation

$$v_{lL} = \sum_i U_{li} v_{iL} \,, \tag{17}$$

where U is a unitary 3×3 mixing matrix and v_i is the field of a Dirac neutrino with the mass m_i.

The mass term $\mathcal{L}^D$ preserves the total lepton number L (which is the same for $(v_e; e)$, $(v_\mu; \mu)$, and $(v_\tau; \tau)$). The Dirac neutrino v_i and antineutrino $\bar{v}_i$ have the same mass m_i and differ by the lepton number ($L(v_i) = 1$ and $L(\bar{v}_i) = -1$). The scheme with the mass term $\mathcal{L}^D$ is a generalization of the scheme considered in [33, 38].

5.3 Dirac and Majorana mass term
In [39], we considered the most general Dirac and Majorana mass term

$$\mathcal{L}^{D+M} = \mathcal{L}_L^M + \mathcal{L}^D + \mathcal{L}_R^M \,. \tag{18}$$

Here, $\mathcal{L}_L^M$ is the left-handed Majorana mass term (12), $\mathcal{L}^D$ is the Dirac mass term (16), and the right-handed Majorana mass term $\mathcal{L}_R^M$ is given by

$$\mathcal{L}_R^M = -\frac{1}{2} \sum_{l',l} (\overline{v_{l'R}})^c (M_R^M)_{l'l} v_{lR} + \text{h.c.} \,, \tag{19}$$

where M_R^M is a 3×3 complex symmetric matrix.

After the diagonalization of mass term (19), we find the mixing relations

$$v_{lL} = \sum_{i=1}^6 U_{li} v_{iL} \,, \quad (v_{lR})^c = \sum_{i=1}^6 U_{\bar{l}i} v_{iL} \,, \tag{20}$$

where, U is a unitary 6×6 matrix (the first three rows of U correspond to the indexes $l = e, \mu, \tau$, and the last three rows, to the indexes $\bar{l} = \bar{e}, \bar{\mu}, \bar{\tau}$), and v_i is the field of a Majorana neutrino with the mass m_i.

Thus, in the general case of the Dirac and Majorana mass term, flavor neutrino fields v_{lL} are unitary combinations of the left-handed components of six Majorana fields with definite masses. The same left-handed components of six Majorana fields with definite masses are unitarily connected with the conjugate sterile right-handed fields $(v_{lR})^c$, which do not enter the Lagrangian of the standard electroweak interaction.

In the case of the Dirac or Majorana mass term, due to mixing, only transitions between flavor neutrinos $v_l \rightarrow v_{l'}$ are possible. In the case of the Dirac and Majorana mass term, not only transitions between flavor neutrinos but also transitions between flavor and sterile neutrinos ($v_l \rightarrow \bar{v}_{l'}$) are possible.

In 1977, Pontecorvo and I wrote the first review on neutrino oscillations [40], in which we summarized the situation with neutrino masses, mixing, and oscillations at the time when dedicated experiments on the search for neutrino oscillations had not yet started. This review attracted the attention of many physicists to the problem of neutrino mass and oscillations.

5.4 Neutrino oscillations in the vacuum
In the preceding subsections, we considered possible mixings of neutrino fields. What are the states of flavor neutrinos v_e, v_μ, and v_τ (and flavor antineutrinos $\bar{v}_e$, $\bar{v}_\mu$, and $\bar{v}_\tau$) that are produced in weak decays, captured in neutrino processes, etc.? By definition, a muon neutrino is a particle that is produced together with μ^+ in the decay $\pi^+ \rightarrow \mu^+ + v_\mu$; a particle that produces e^+ in the process $\bar{v}_e + p \rightarrow e^+ + n$ is the electron antineutrino, etc. This definition is unambiguous if neutrino mass-squared differences can be neglected in matrix elements of neutrino production (and absorption) processes. In this case, the matrix element of a decay in which v_l is produced is given by the Standard Model matrix element (with mass-squared differences equal to zero) and independently of the production process, the state of the flavor neutrino v_l ($l = e, \mu, \tau$) is given by

$$|v_l\rangle = \sum_i U_{li}^* |v_i\rangle \,. \tag{21}$$

Here, $|v_i\rangle$ is a neutrino state with the momentum $\mathbf{p}$ and the energy

$$E_i = \sqrt{p^2 + m_i^2} \simeq E + \frac{m_i^2}{2E} \,, \tag{22}$$

where $E = p$ is the energy of a neutrino at $m_i \rightarrow 0$. Thus, in the mixing of neutrinos with small mass-squared differences, a flavor neutrino state is a coherent superposition of (Dirac or Majorana) neutrino states with definite masses. In [40], we formulated the coherence condition

$$L_{ik} \gtrsim a \,, \tag{23}$$

where $L_{ik} = 4\pi E/|\Delta m_{ik}^2|$ ($i \neq k$) is the oscillation length ($\Delta m_{ik}^2 = m_k^2 - m_i^2$) and a is a quantum mechanical size of the neutrino source.

We note that for mass-squared differences determined from the data of modern neutrino oscillation experiments,

$$\Delta m_{12}^2 = 7.54^{+0.26}_{-0.22} \times 10^{-5} \text{ eV}^2 \,, \quad \Delta m_{23}^2 = 2.43^{-0.06}_{+0.1} \times 10^{-3} \text{ eV}^2 \,, \tag{24}$$

and neutrino energies $E \geqslant 1$ MeV, condition (23) is definitely satisfied.

Relation (21) is basic for the phenomenon of neutrino oscillations. In accordance with QFT, we assume that the evolution of states is determined by the Schrödinger equation

$$\mathrm{i}\,\frac{\partial\left|\Psi(t)\right\rangle}{\partial t} = H_0\left|\Psi(t)\right\rangle,\qquad(25)$$

where H_0 is the free Hamiltonian. From (25), it follows that if a flavor neutrino ν_l is produced at $t = 0$, then the neutrino state at the time t is

$$\left|\nu_l\right\rangle_t = \exp\left(-\mathrm{i}H_0 t\right)\left|\nu_l\right\rangle = \sum_i U_{li}^* \exp\left(-\mathrm{i}E_i t\right)\left|i\right\rangle.\qquad(26)$$

It is important that the phase factors in (26) are different for different mass components. Hence, the flavor content of the final state $\left|\nu_l\right\rangle_t$ differs from the initial one.

Neutrinos are detected through the observation of CC and NC weak processes. Decomposing the state $\left|\nu_l\right\rangle_t$ with respect to a full system of neutrino states $\left|\nu_{l'}\right\rangle$, we have

$$\left|\nu_l\right\rangle_t = \sum_{l'}\left|\nu_{l'}\right\rangle\left[\left(\sum_{i=1}^{3} U_{l'i}\exp\left(-\mathrm{i}E_i t\right)\right)U_{li}^*\right].\qquad(27)$$

Hence, the probability of the transition $\nu_l \to \nu_{l'}$ during the time t is given by

$$P(\nu_l \to \nu_{l'}) = \left|\delta_{l'l} + \sum_{i\geqslant 2} U_{l'i}U_{li}^*\left[\exp\left(-\mathrm{i}\Delta m_{il}^2\,\frac{L}{2E}\right) - 1\right]\right|^2,\qquad(28)$$

where, $L \approx t$ is the distance between the neutrino source and the neutrino detector.

Analogously, for the probability of the transition $\bar{\nu}_l \to \bar{\nu}_{l'}$, we find

$$P(\bar{\nu}_l \to \bar{\nu}_{l'}) = \left|\delta_{l'l} + \sum_{i\geqslant 2} U_{l'i}^*U_{li}\left[\exp\left(-\mathrm{i}\Delta m_{il}^2\,\frac{L}{2E}\right) - 1\right]\right|^2.\qquad(29)$$

Expressions (28) and (29) became the standard expressions for the transition probability. They are commonly used in the analysis of data of experiments on the investigation of neutrino oscillations.

We know that three neutrino flavors exist in nature. If the number of neutrinos with definite masses is also equal to three (there are no sterile neutrino states), then neutrino transition probabilities depend on two mass-squared differences, Δm_{12}^2 and Δm_{23}^2, and on parameters that characterize the 3×3 unitary mixing matrix (three angles and one phase).

It follows from an analysis of the experimental data that $\Delta m_{12}^2 \ll \Delta m_{23}^2$, and one of the mixing angles (θ_{13}) is small. It is easy to show (see, e.g., [42]) that in the leading approximation, oscillations observed in atmospheric and accelerator neutrino experiments are two-neutrino $\nu_\mu \to \nu_\tau$ oscillations.

For the probability of ν_μ to survive, we find the following expression from (28):

$$P(\nu_\mu \to \nu_\mu) = 1 - \frac{1}{2}\sin^2(2\theta_{23})\left(1 - \cos\frac{\Delta m_{23}^2 L}{2E}\right).\qquad(30)$$

In the leading approximation, the observed disappearance of the $\bar{\nu}_e$ in the KamLAND reactor experiment is due to $\bar{\nu}_e \to \bar{\nu}_{\mu,\tau}$ transitions. The survival probability is then given by

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2}\sin^2(2\theta_{12})\left(1 - \cos\frac{\Delta m_{12}^2 L}{2E}\right).\qquad(31)$$

In 1998, after many years of heroic efforts, oscillations of atmospheric neutrinos were discovered in the Super-Kamiokande experiment [43, 44]. This was the beginning of the golden years of neutrino oscillations. In 2001, it was proved in a model: independent way in the SNO experiment [45] that the solar neutrino deficit is due to neutrino oscillations. In 2002, oscillations of reactor neutrinos were discovered in the KamLAND reactor experiment [46]. Several recent accelerator [47–51] and reactor [52–54] neutrino oscillation experiments have confirmed this discovery.

The discovery of neutrino oscillations was a great triumph of the ideas of Pontecorvo, who came to the idea of neutrino oscillations at a time when common opinion favored massless neutrinos and no neutrino oscillations, and who pursued and developed the idea of massive, mixed, and oscillating neutrinos for many years.

From our standpoint, the history of neutrino oscillations is an illustration of the importance of analogy in physics. It is also an illustration of the importance of courageous new ideas that are not always in agreement with general opinion.

6. Conclusion

Small neutrino masses cannot be naturally explained in the framework of the standard Higgs mechanism of mass generation. Their explanation requires a new physics beyond the Standard Model. Many models have been proposed. The most plausible and viable mechanism of the generation of neutrino masses is the seesaw mechanism [56–59], which connects the smallness of neutrino masses with a violation of the lepton number at a large scale.

In the most general form, the seesaw mechanism was formulated in the framework of the effective Lagrangian approach [60]. The only dimension-5 effective Lagrangian is a lepton-number-violating $\mathrm{SU}(2) \times \mathrm{U}(1)$-invariant product of two lepton doublets and two Higgs doublets. After spontaneous violation of the electroweak symmetry, this effective Lagrangian generates a Majorana mass term of the type considered first by Gribov and Pontecorvo [36]. In this approach, the scale of neutrino masses is determined by the parameter v^2/Λ, where $v = (\sqrt{2}G_\mathrm{F})^{-1/2} \approx 246$ GeV is the parameter that characterizes electroweak breaking and Λ characterizes the scale of a new physics. For $m_i \approx 10^{-1}$ eV, we have $\Lambda \approx 10^{15}$ GeV.

From investigations of solar neutrinos in numerous solar neutrino experiments (Homestake [4, 5], GALLEX-GNO [7, 8], SAGE [9], Super-Kamiokande [61], SNO [45], and Borexino [62]), it follows that the disappearance of the solar ν_e is not only due to neutrino masses and mixing but also due to coherent scattering of neutrinos in matter (the MSW effect [63, 64]).

In the minimal scheme of neutrino mixing with three flavors and three massive neutrinos, the unitary mixing matrix is characterized by three mixing angles θ_{12}, θ_{23}, and θ_{13}, and a *CP* phase δ. From an analysis of data of neutrino oscillation experiments, it was found that in the very first approximation,

$$\sin\theta_{12} \approx \frac{1}{\sqrt{3}},\quad \sin\theta_{23} \approx \frac{1}{\sqrt{2}},\quad \sin\theta_{13} \approx 0,\qquad(32)$$

and the unitary matrix U has a tri-bimaximal form. These findings led to numerous papers in which possibilities of broken discrete symmetries in the lepton sector were thoroughly investigated (see, e.g., review [65]).

Pioneering papers by Bruno Pontecorvo on neutrino masses, mixing, and oscillations opened a new field of research. To date, neutrino oscillations have been discovered and four neutrino oscillation parameters have been determined with accuracies ranging from 3% to 15%. The study of neutrino oscillations has revealed several fundamental problems that require further investigation. The major problems are:

(1) Are the ν_i neutrinos with definite masses Majorana or Dirac particles? This question can be answered via observation of the lepton-number-violating neutrinoless double β-decay of some even–even nuclei.

(2) Is the neutrino mass spectrum normal or inverted? The existing neutrino oscillation data do not allow distinguishing between the following two possibilities for the neutrino mass spectrum:
- normal spectrum $m_1 < m_2 < m_3$, $\Delta m_{12}^2 \ll \Delta m_{23}^2$
- inverted spectrum $m_3 < m_1 < m_2$, $\Delta m_{12}^2 \ll |\Delta m_{13}^2|$.

(3) What are the absolute values of the neutrino masses?

(4) What is the value of the CP phase δ, the last unknown parameter of the neutrino mixing matrix?

(5) Are there transitions of flavor neutrinos ν_e, ν_μ, and ν_τ into sterile states?[2]

Many ongoing neutrino experiments and those in preparation aim to resolve these challenging issues.

Independently of Pontecorvo, in 1962, Maki, Nakagawa, and Sakata [67] came to the idea of neutrino masses and mixing. Their arguments were based on the Nagoya model, in which neutrinos were considered constituents of baryons. In [67], the possibility of the transition ('virtual transmutation') $\nu_\mu \rightarrow \nu_e$ was considered. To acknowledge the pioneering ideas of Pontecorvo and Maki, Nakagawa, and Sakata, the neutrino mixing matrix is usually called the PMNS matrix.

Bruno Pontecorvo was one of the first to understand the importance of neutrinos for elementary particle physics and astrophysics. He felt and understood neutrinos probably better than anybody else in the world. Starting from his time in Canada, he thought about neutrinos all his life. He was never confined by narrow theoretical frameworks. He was completely open-minded, without any prejudices, very courageous, and had very good intuition and scientific taste.

Bruno Pontecorvo was a very bright, wise, exceptionally interesting, and very friendly personality. People liked him and he had many friends in Italy, Russia, France, Canada, and many other countries. He participated in many conferences, seminars, and discussions. His clear laconic questions and remarks were very important for clarifying many problems.

The name Bruno Pontecorvo, who was the founder and father of modern neutrino physics, will be forever connected with the neutrino. He will remain with us in our memory and our hearts as a great outstanding physicist, as a man of great impact and humanity.

[2] Some indications in favor of sterile neutrinos exist at present (see, e.g., [66]).

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