

100th ANNIVERSARY OF THE BIRTH OF Ya B ZELDOVICH

PACS numbers: 07.55.Db, **41.20.–q**, **91.25.–r**, 96.12.Hg, 96.60.Hv

Dynamos: from an astrophysical model to laboratory experiments

D D Sokoloff, R A Stepanov, P G Frick

DOI: 10.3367/UFNe.0184.201403g.0313

Contents

Abstract. **Magnetic field generation and evolution in celestial bodies — the subject matter of the theory of the dynamo — held Ya B Zeldovich's interest for years. Over the time since then, the study of the dynamo process has developed from a part of astrophysics and geophysics to a self-contained domain of physics, with the possibility of laboratory dynamo physics experiments. We give some theoretical background and discuss laboratory dynamo experiments (including those conducted in Russia), as well as their impact on dynamo theory and its astrophysical applications.**

1. Introduction. What is a dynamo?

Investigations into magnetism are usually associated with the elucidation of spin ordering, quantum solid-state physics, and other fields of physical research that left a notable mark on 20th century science. It is these issues that in this country are included in the programs of conferences on magnetism, in scientific classification systems, and in topics of dissertation committees and foundations. These doubtlessly interesting subjects need to be supplemented by other magnetic phenomena, such as magnetic fields of Earth and some other planets where their existence is implied based on data collected by space missions. The Sun is known to exhibit an 11-year cycle of magnetic activity (see, for instance, Ref. [1]). Similar cycles have been discovered in many other stars [2]. When E Fermi pondered cosmic ray distribution (see Refs [3] and [4]), he came to the conclusion that the Milky Way (our Galaxy) makes up a giant magnet. Later on, radio astronomers discovered magnetic fields in certain spiral galaxies [5]. In all these cases, the spatial scale of magnetic fields proved to be commensurate with the size of the respective celestial bodies, and the energy sufficient to enable these magnetic fields to play a prominent role, at least in some physical processes proceeding in these bodies. Evidently, studies of the origin and evolution of cosmic magnetic fields are of great cognitive and practical interest.

Quasistationary and large-scale magnetic fields in the highly rarefied interstellar medium of galaxies or in the hot stellar plasma can hardly be associated with any quantum or relativistic process. The relation of Earth's magnetism problems to classical physics is a bit less obvious. This accounts for the idea of a huge ferromagnetic body inside Earth generating the geomagnetic field that arises from time to time at the periphery of scientific thought. For example, it was discussed by Jonathan Swift in the third part of *Gulliver's Travels*, which is especially rich in paradoxical scientific ideas. However, the scientific community flatly dismisses such speculations on the grounds that the temperature in Earth's interior is too high, and ferromagnetism disappears above the Curie point. For this reason, ferromagnetic phenomena are considered only in so far as they help to explain some important but local peculiarities of the geomagnetic field, such as the Kursk Magnetic Anomaly, whose existence is attributable to huge ore deposits.

D D Sokoloff Department of Physics,
Lomonosov Moscow State University,
Leninskie Gory 1, 119991 Moscow, Russian Federation
Tel. +7 (495) 939 10 33, +7 (945) 939 23 46
E-mail: sokoloff.dd@gmail.com
R A Stepanov Institute of Continuous Media Mechanics,
Ural Branch of the Russian Academy of Sciences,
ul. Akademika Koroleva 1, 614013 Perm, Russian Federation
E-mail: rodion@icmm.ru
Perm National Research Polytechnic University,
Komsomol'skii prosp. 29, 614990 Perm, Russian Federation
P G Frick Institute of Continuous Media Mechanics,
Ural Branch of the Russian Academy of Sciences,
ul. Akademika Koroleva 1, 614013 Perm, Russian Federation
E-mail: frick@icmm.ru

Received 1 October 2013
Uspekhi Fizicheskikh Nauk **184** (3) 313 – 335 (2014)
DOI: 10.3367/UFNr.0184.201403g.0313
Translated by Yu V Morozov; edited by A Radzig

Away back in 1919, J Larmor [6] dismissed lame attempts to associate cosmic magnetism with unknown fundamental interactions and concluded that electromagnetic induction is actually the sole cause behind this phenomenon. That was the time of the automotive industry boom, when enthusiasts did not hesitate to give the name of a car engine part to a football team or a subway station, let alone to the Larmor concept of dynamo.

Larmor's idea proved rather difficult to realize scientifically. Common sense suggests that the generation of a magnetic field by the dynamo mechanism must involve some degree of instability leading, say, to the formation of the Sun's magnetic field. The instability increment therewith must be complex, with its real part characterizing the growth of the magnetic field at the linear stage of its evolution, subject to suppression in the course of time by certain nonlinear effects, and the imaginary part determining the duration of the cycle. Of course, this scheme disregards some details related to the description of situations in which the magnetic field is introduced into a medium by an external source.

The problem consists in the fact that electromagnetic induction gives rise to a negative feedback between the initial and new magnetic fields as stated by the Lenz rule (hopefully still studied in a secondary school). The Lenz rule naturally interferes with the dynamo process. It can be obviated by considering, in electrotechnical terms, electromagnetic induction in two coupled circuits, one of which induces a magnetic field in the other and vice versa, which results in the emergence of positive feedback and instability being sought.

Concrete realization of this idea proved a rather difficult task that required much time and effort by physicists and astronomers; the history of understanding the relationship between various aspects of the problem is most confusing and sometimes reminiscent of a detective novel. Evidently, a dynamo phenomenon may occur only in a highly conducting medium in which inductive effects prevail over dissipative ones. In such a case, however, the magnetic field is frozen (or almost frozen) into the medium so that magnetic lines flow together with it. How then does instability leading to exponential growth of the magnetic field develop, i.e., how can two magnetic lines appear instead of one?

This question arose after the first sustainable dynamo models had been proposed. Setting chronology aside, it should be mentioned that the answer to this question given by Ya B Zeldovich is one of the most important contributions to the development of the dynamo concept. Let us imagine a magnetic loop (Fig. 1) in plane (x, y) stretched to twice its extent along the x-axis by an incompressible flow. In this case, the magnetic field strength is doubled and the cross section reduced by half, while the magnetic flux persists. Thereafter, the flow changes; it no longer stretches the loop but instead removes it from the plane, twists, and folds into a figure-eight whose halves overlap. Clearly, the magnetic flux through the area across the two halves is doubled. The procedure then repeats many times, thus leading to exponential growth of the magnetic field.

Such is the famous figure-eight loop of Zeldovich or the stretch-twist-fold process. As legend (which Ya B never contradicted) has it, he first demonstrated this scheme at a conference in Krakow using a trouser belt borrowed from a colleague as a model of the magnetic loop. Zeldovich was by no means an absent-minded, oddish character careless with his ideas. But in the 1970s, he may have thought that a statement at an international conference was akin to publication (the modern press law actually upholds such a view). Anyway, Zeldovich never disclosed his model in a special article. Moreover, he appears to have experienced euphoria when visiting Krakow, it being a big event for him since it was unusual to be allowed to go abroad, even for a thrice Hero of Socialist Labor. Of course, the model was considered in many subsequent papers (probably for the first time in review [7])[1] including those written by Ya B and his co-workers (he did not like to call them 'pupils'), but Zeldovich's priority is not disputed [13].

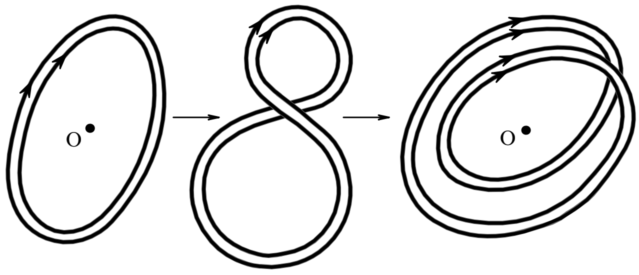

Figure 1. Zeldovich figure-eight magnetic loop stretched, turned over, and folded onto itself with the doubled magnetic flux. Magnetic diffusion is needed to smooth the magnetic field in the place where the loop was twisted.

The figure-eight loop concept helps to understand possible realizations of the dynamo mechanism. First, it may be assumed that magnetic lines are not completely frozen in the flowing medium but can slightly separate from it. In other words, one may try to construct a dynamo in which the magnetic field growth rate essentially depends on the magnetic diffusion coefficient, and magnetic lines can close on themselves as the dynamo operates. A similar picture of magnetic field line evolution was postulated by Alfvén [14], who certainly did not know about the Zeldovich figure-eight loop. In his scheme (see Fig. 2), the magnetic loop bends so that its two distant parts approach each other, close on themselves, and the loop breaks down into two stretched and overlapping halves, which results in the doubled magnetic flux.

Another example of a magnetic diffusion-dependent dynamo was described by Ponomarenko [15] in the work where the magnetic field grew exponentially in a rotating conductive jet surrounded by a conducting medium. This classical priority study has numerous implications (see Section 4 below). Surprisingly, the author's biographical details remain virtually unknown, barring the fact that he was affiliated with the N V Pushkov Institute of Terrestrial Magnetism, Ionosphere, and Radiowave Propagation (IZMIRAN), Russian Academy of Sciences.

Ponomarenko's scheme requires rather specific flows nontypical of celestial bodies and, therefore, did not find wide application in astrophysics. Moreover, such slow process as magnetic diffusion does not allow observing variations in the Sun's magnetic field with an 11-year period negligible on astronomical scales. Hence, the contraposition between slow dynamo associated with magnetic diffusion and the fast dynamo in which magnetic diffusion is needed only to slightly smooth the magnetic field increased under the effect of the figure-eight loop. The contraposition between fast and

[1] The dynamo problem and related issues were discussed many times in *Physics Uspekhi* in the articles by Zeldovich (see, for instance, reviews [8, 9]), his co-workers [10, 11], and colleagues (see, e.g., Ref. [12]).

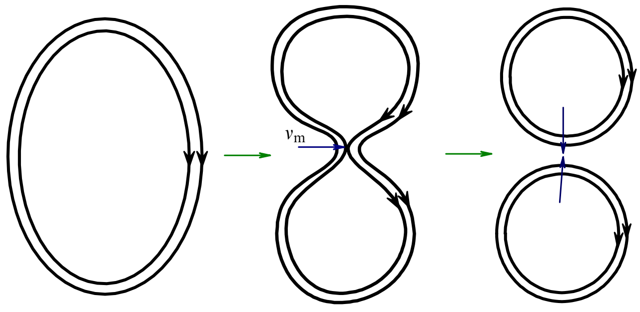

Figure 2. Alfvén process: the magnetic loop stretches, flattens, and breaks under the effect of magnetic diffusion. The resulting loops overlap. The magnetic flux is doubled, but the process governed by weak diffusion is slow.

slow dynamos in terms of the role played by magnetic diffusion also originated in the group of researchers headed by Zeldovich. At that time, it became possible to write articles for foreign scientific journals and even books in the English language, and the English terms 'fast' and 'slow' were coined, although it appears that the word 'quick' would have been a more appropriate term in the former case. Nevertheless, scientific English adopted the original terminology.

One more idea needs to be introduced into the notion of a dynamo that is historically a result of hard effort. It is not necessary to choose carefully the proper velocity fields; instead, suffice it to consider random, say, turbulent or convective flows in which the magnetic configuration of interest sooner or later emerges by the play of chance given it is not forbidden by the properties of symmetry of a random velocity field. Naturally, the magnetic field growing in a random flow is random, too. One should decide in which sense the growth of the field may be interesting: either in the sense of average magnetic energy growth or in the sense of growth of a given magnetic field realization (growth with probability 1 in the language of probability calculus). The possibility of studying random physical fields not only in terms of average values but also in the framework of individual realizations entered into physics in the first place with the theory of Mott–Anderson localization and simultaneously with the dynamo theory developed by Zeldovich and his co-workers [16]. As mentioned in the Preface to the just cited book, Ya B worked it over even on the day of his death. Unfortunately, the book has not been translated into Russian and remains virtually unknown to readers in this country.

Here, we again face a dilemma, because loops can be folded into figure eights without specifying the direction of turn, i.e., by equiprobably turning them to the right or to the left. In such a case, we hardly have a nonzero mean magnetic field and can only speak about the growth of the mean magnetic energy. In fact, this idea was put forward in 1967 by Kazantsev [17], who derived the equation for the magnetic field correlation function in a reflective-symmetric flow (to recall, this we describe the developments in a logical but not chronological order; Kazantsev had formulated his equation ten years before Zeldovich proposed his model but had not thought of applying it to the dynamo theory). This equation, currently known as the Kazantsev equation, does have solutions growing exponentially for low enough dissipation. Notice that the same equation was independently derived, even if somewhat later, by the American physicists Kraichnan and Nagarajan [18], but the priority of the Russian inventor is internationally recognized.

A mean, i.e., large-scale, magnetic field of special interest for astronomy can be obtained by adding one loop to another, giving preference to one of their orientations. In other words, it is necessary to break the reflective symmetry of the flow. E N Parker, a well-known American astronomer, was the first to understand it in 1955 and employ the idea in constructing a simple but viable model of the solar cycle [19] based on a series of conjectures that proved correct. These ideas turned into a physical theory after the historic paper by Steenbeck, Krause, and Rädler [20] was published independently 10 years after Parker's article. M Steenbeck was President of the GDR Academy of Sciences after he had worked for a long time in Sukhumi against his own free will. He left interesting memoirs that were translated into Russian during the period of Perestroika but got lost in the literature flow of that time.

That was how the notion of the α-effect evolved as the one parametrizing mirror asymmetry of convection or turbulence in the description of the mean magnetic field. The electromotive force (EMF) $\mathcal{E}$ is somehow parametrized by magnetic field $\mathbf{B}$, with $\mathcal{E}$ usually being normal to $\mathbf{B}$. Symmetry analysis of Maxwell equations (reduced for a quasistationary magnetic field to a single equation, i.e., the induction equation) shows that the magnetic field readily grows if $\mathcal{E}$ has a component parallel to $\mathbf{B}$. However, $\mathcal{E}$ is an ordinary vector, while $\mathbf{B}$ is a pseudovector; therefore, the pseudoscalar α is needed in order to write out that

$$\boldsymbol{\mathcal{E}} = \alpha \mathbf{B} + \ldots \, . \tag{1}$$

This pseudoscalar α can be found taking into account the mirror asymmetry of the flow. Its physical source is, for example, the action of the Coriolis force on the vortices either emerging and expanding or submerging and contracting in stratified rotating turbulence. In principle, this picture fits well into the standard arguments leading to Maxwell equations for the medium. May be it is better to say that the derivation of Maxwell equations can be regarded as the first example of the derivation of mean field equations in a random medium.

Had Maxwell thought about mirror-asymmetric (chiral) media when he derived his equations, he would have arrived at the mean field equations of Steenbeck and co-authors. Anyway, parametrization of α via the angular speed and density gradient proposed by F Krause in his graduate thesis (which he did not care to publish and the results of which have been used by specialists for a few decades) conforms with the standard estimates of medium parameters included in well-known textbooks on electrodynamics.

The α-effect plays a key role in astrophysical dynamos. It is therefore appropriate to present the simplest formula by which this effect is calculated in a specified flow of a well-conducting fluid:

$$\alpha = \frac{\tau \langle \mathbf{v} \cdot \operatorname{rot} \mathbf{v} \rangle}{3} \, , \tag{2}$$

where the angle brackets denote averaging over the ensemble of turbulent vortices, τ is memory time, and $\mathbf{v}$ is the root-mean-square rate of turbulent pulsations. Clearly, the α-effect is determined by flow helicity density. In other words, helicity and other phenomena associated with the break of mirror symmetry unessential in classical domains of physics acquire importance not only in a microcosm but also for the macrophenomena studied in the dynamo theory. It is worth mentioning here that the α-effect, besides solution (2), may take place in a flow with strictly zero helicity [21].

As is clear from the foregoing, a dynamo cannot origin in very simple flows. According to many statements (so-called antidynamo theorems), there are flows that cannot maintain a magnetic field. By way of example, Zeldovich [22] argued that a velocity field in which current lines are located in the system of parallel planes cannot produce a dynamo.

At the same time, Zeldovich and co-workers [23] emphasized the possibility of exponential growth of a magnetic field in certain two-dimensional flows. It is easy to verify that a uniform magnetic field parallel to the x-axis grows exponentially in a divergence-free velocity field with $v_x = Cx,\ v_y = -Cy,\ v_z = 0$. This means that Zeldovich's assertion, like any other theorem, is correct only under certain conditions specifying the notion of a dynamo. This notion implies the generation of magnetic fields unrelated to the work of current sources resided on infinity. In one of Zeldovich's studies dated 1956 [22], this condition is presented as the requirement for the existence of finite integrals of various combinations of magnetic and velocity fields. It is obviously violated in the above example. To recall, the tone of the 1956 article suggested no formal mathematical analysis of the notion of a dynamo. Such an analysis was undertaken later by Arnol'd et al. [24], who showed that the functional space in which the dynamo problem must be formulated is not a standard space of functional analysis (one of the Lebesgue or Sobolev spaces) but is determined by the fact that a certain triple integral expressing the magnetic moment converges in the sense of the principal value. We are unaware of special mathematical work on the theory of operators in such a space, although they might be of interest for physical applications. Nonetheless, the energy estimates made by Zeldovich in the metrics of this space allow completion up to formal mathematical statements. It should also be noted that Yakov Borisovich was interested in such methods for proving dissipation theorems the entire time after his earliest publications [25].

Importantly, the formalization of the notion of a dynamo adopted by Zeldovich [22] is not unique. For example, it is possible to consider the generation of a magnetic field in the spatially homogeneous random flow of a conducting fluid and understand the growth of the magnetic field in the sense of the growth of its mean or root-mean-square value. In such an understanding, a dynamo proves to be possible, as was certainly known to Zeldovich but not considered in his theorem [26].

It should be emphasized that the dynamo mechanism constitutes only one aspect of the cosmic magnetism problem — how does a magnetic field arise in the presence of a conducting medium motion? The question concerning how this magnetic field affects motion is of secondary importance in this context. Another interesting aspect is the origin of movements and the influence of a magnetic field on this process. Russian physicists greatly contributed to the elucidation of this problem as well. Suffice it to mention that one of the main relevant mechanisms, famous magnetorotational instability, was discovered by E P Velikhov [27] (see also paper [28] and fundamental review [29]).

To sum up, we can say that dynamo research has revealed interesting physics. The advent of the new field of physical studies definitely unrelated either to the theory of relativity or to quantum mechanics, that is at the same time not a burdensome relic of the past, appears to illustrate the current differentiation of formerly unified science into a system of relatively self-contained disciplines exchanging ideas to one another but avoiding hierarchy references. Up to a certain time, dynamo research had not been in the focus of physics, being regarded as a part of astronomy or geophysics; accordingly, criteria adopted in these sciences were employed. Suffice it to say that theses on paleomagnetism, i.e., studies on the evolution of the geomagnetic field during Earth's geological history, are still defended under a general heading of 'Solid Earth Physics', even though it is universally known that this evolution is a result of the processes in Earth's liquid outer core. However, the paleomagnetic history of Earth is written in ferromagnetic constituents of its solid shells, and this fact, rather than the physics of the process, determines the formal structure of classifiers.

We believe that the peripheral position of the dynamo among other physical processes was due to the difficulty of its experimental investigation and to the fact that observational approaches adopted in astronomy and paleomagnetology do not fit so well into the main line of physical research intrinsically connected with experiment.

It turns out that dynamo experiments are extremely difficult to conduct. Physics spent a very long time to get to them, starting from the pioneer research of Soviet and German scientists carried out in Salaspils near Riga, Latvia, away back in the 1960s. According to the principle of division of labor adopted in Soviet scientific policy and that now looks ill-considered, Latvia was chosen as the most suitable place for dynamo research. A flurry of dynamo experiments was done at the turn of the new millennium, when the dynamo phenomenon was first observed in the laboratories in both Salaspils and Karlsruhe (Germany). These early findings gave rise to the new domain of physics concerned with experimental investigations into the dynamo phenomenon. In the hard times after the collapse of the Soviet Union, Russian physicists managed to get into the swing of things and find their own level in joint efforts.

Of course, the genetic relationship between the dynamo, astronomy, and geophysics remains quite apparent. For this reason, we consider results of dynamo experiments, alongside models of dynamos in celestial bodies.

2. Theory

An indispensable stage in the solution to any dynamo-related problem is the consideration of the equation for magnetic field induction in the flowing conducting medium:

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \tag{3}$$

where η is constant magnetic viscosity (media with variable magnetic viscosity can be considered if appropriate), and $\mathbf{U}$ is the velocity. Anyway, dynamo studies involve the solution of this equation in one form or another. The idea to transform it so as to describe the behavior of a large-scale field taking into account only statistical properties of small-scale fields took many decades to be realized. In what follows, we summarize the results of these studies.

Following the basic Reynolds approach to the description of turbulence, one should move to quantities $\overline{\mathbf{B}}$ and $\overline{\mathbf{U}}$, which are average in terms of realization. The dynamo theory usually deals with fields averaged over spatial and temporal scales bigger than turbulence scales (scale separation hypothesis implying that the scales of a large-scale magnetic field are by far greater than the maximum scales of turbulent flows).

Quantities $\mathbf{b} = \mathbf{B} - \overline{\mathbf{B}}$ and $\mathbf{u} = \mathbf{U} - \overline{\mathbf{U}}$ are regarded as fluctuations. Further on, the Reynolds averaging procedure is applied to equation (3) to obtain the mean field induction equation in the following form

$$\partial_t \overline{\mathbf{B}} = \nabla \times (\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \boldsymbol{\mathcal{E}}) - \eta \nabla^2 \overline{\mathbf{B}}, \tag{4}$$

where $\boldsymbol{\mathcal{E}}$ is the mean electromotive force caused by pulsations of magnetic and velocity fields:

$$\boldsymbol{\mathcal{E}} = \overline{\mathbf{u} \times \mathbf{b}}. \tag{5}$$

Then, the equation of magnetic field pulsations is considered:

$$\partial_t \mathbf{b} = \nabla \times \left(\overline{\mathbf{U}} \times \mathbf{b} + \mathbf{u} \times \overline{\mathbf{B}} + (\mathbf{u} \times \mathbf{b})'\right) + \eta \nabla^2 \mathbf{b}, \tag{6}$$

where $(\mathbf{u} \times \mathbf{b})' = \mathbf{u} \times \mathbf{b} - \overline{\mathbf{u} \times \mathbf{b}}$. It immediately exposes the problem of closure, since Eqn (6) contains the second-order term in fluctuations. In fact, determination of the analytical dependence of $\boldsymbol{\mathcal{E}}$ on the mean fields $\overline{\mathbf{B}}$ and $\overline{\mathbf{U}}$ and the statistical properties of fluctuations $\mathbf{b}$ and $\mathbf{u}$ is the cornerstone of the mean-field dynamo theory applied to astrophysical objects.

Analysis of the enhancement mechanisms of $\mathbf{b}$ fluctuations presented in the form of three items under the rotor operator in Eqn (6) permits taking the next useful step, when considering two constituents of $\mathbf{b}$, viz. $\mathbf{b}_0$ fluctuations independent of the presence of $\overline{\mathbf{B}}$, and $\mathbf{b}^{\overline{\mathbf{B}}}$ fluctuations resulting only from generation of $\overline{\mathbf{B}}$ and disappearing after $\overline{\mathbf{B}}$ decays. Their contributions to $\boldsymbol{\mathcal{E}}$ can be separated as

$$\boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{E}}^{(0)} + \boldsymbol{\mathcal{E}}^{\overline{\mathbf{B}}}. \tag{7}$$

Component $\boldsymbol{\mathcal{E}}^{(0)}$ emerges in the presence of external sources supporting background magnetohydrodynamic (MHD) turbulence and independent of $\overline{\mathbf{B}}$. The action of $\boldsymbol{\mathcal{E}}^{(0)}$ can facilitate generation of the initial magnetic field but comes to be usually disregarded as $\overline{\mathbf{B}}$ increases and the contribution of $\overline{\mathbf{B}}$-unrelated pulsations becomes insignificant. The $\boldsymbol{\mathcal{E}}^{\overline{\mathbf{B}}}$ component includes all mechanisms of generation due to pulsations caused by the growth of $\overline{\mathbf{B}}$. At the beginning, this growth occurs under conditions of purely hydrodynamic turbulence maintained by certain sources. Thereafter, when the magnetic and kinetic energies become comparable, the dynamic aspect lying in the fact that an analogous component $\mathbf{u}^{\overline{\mathbf{B}}}$ arises in pulsations of the velocity field, needs to be taken into consideration. The quantity $\boldsymbol{\mathcal{E}}^{\overline{\mathbf{B}}}$ is the main and best-understood component. Historically, the described resolution of the turbulence contribution to the dynamo process is a result of the development of a system of assumptions and conjectures necessary to solve equation (6) and the analogous equation for $\mathbf{u}$ that is not presented here but can be found in most publications on this subject. The separation of contributions in the form (7) is convenient in that it allows subsequent, rather transparent parametrization of $\boldsymbol{\mathcal{E}}$ via fields $\overline{\mathbf{B}}$, $\overline{\mathbf{U}}$ and their gradients, as well as statistical characteristics of $\mathbf{b}$ and $\mathbf{u}$.

2.1 Structure of electromotive force $\boldsymbol{\mathcal{E}}^{\overline{\mathbf{B}}}$

For a given point in space and time, $\boldsymbol{\mathcal{E}}$ may depend not only on the $\overline{\mathbf{U}}$, $\mathbf{u}$, and $\overline{\mathbf{B}}$ values at this point but also on their behavior in its vicinity. The first step is to assume that $\overline{\mathbf{B}}$ slowly changes in the neighborhood of the point, so that $\boldsymbol{\mathcal{E}}$ linearly depends on vector $\overline{\mathbf{B}}$ components and its spatial derivatives. In other words, $\boldsymbol{\mathcal{E}}$ can be represented in the form

$$\boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{A}} \cdot \overline{\mathbf{B}} + \boldsymbol{\mathcal{B}} : \nabla \overline{\mathbf{B}}, \tag{8}$$

where the tensors $\boldsymbol{\mathcal{A}}$ and $\boldsymbol{\mathcal{B}}$ of the second and third order, respectively, are the average quantities now depending only on $\overline{\mathbf{U}}$ and $\mathbf{u}$. Hereinafter, operation $(\cdot)$ stands for contraction over a single index, and $(:)$ means contraction over two indices. Equation (8) resolved into symmetric and skew-symmetric parts is equivalent to writing in the vector form [30, 31]

$$\boldsymbol{\mathcal{E}} = -\boldsymbol{\alpha} \cdot \overline{\mathbf{B}} - \boldsymbol{\gamma} \times \overline{\mathbf{B}} - \boldsymbol{\beta} \cdot (\nabla \times \overline{\mathbf{B}})$$
$$- \boldsymbol{\delta} \times (\nabla \times \overline{\mathbf{B}}) - \boldsymbol{\kappa} : (\nabla \overline{\mathbf{B}})^{(s)}, \tag{9}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are symmetric second-rank tensors, $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$ are vectors, and $\boldsymbol{\kappa}$ is the third-rank tensor; they all depend on $\overline{\mathbf{U}}$ and $\mathbf{u}$ alone. Finally, $(\nabla \overline{\mathbf{B}})^{(s)}$ is the symmetric part of the tensor of the $\overline{\mathbf{B}}$ gradient, i.e., $(\nabla \overline{\mathbf{B}})^{(s)} = 1/2(\nabla \overline{\mathbf{B}} + \nabla \overline{\mathbf{B}}^{\mathrm{T}})$.

The term with $\boldsymbol{\alpha}$ in equation (9) describes the α-effect anisotropic in the general case, while the term with γ corresponds to the effective transfer of $\overline{\mathbf{B}}$ due to turbulence that is sometimes referred to as turbulent diamagnetism. It was first described by Zeldovich in the aforementioned paper [22]. Terms with $\boldsymbol{\beta}$ and $\boldsymbol{\delta}$ can be interpreted as turbulent diffusion effects that are also anisotropic in the general case. The term with $\boldsymbol{\kappa}$ plays no distinguished role but is needed to complete the resolution. The formulas for $\boldsymbol{\alpha}$, $\boldsymbol{\gamma}$, $\boldsymbol{\beta}$, $\boldsymbol{\delta}$, and $\boldsymbol{\kappa}$ expressed in terms of tensors $\boldsymbol{\mathcal{A}}$ and $\boldsymbol{\mathcal{B}}$ can be found in Ref. [32].

Further constructions are based on successive consideration, from the simplest case of homogeneous isotropic mirror-reflective symmetric turbulence at $\overline{\mathbf{U}} = 0$ to the most general case discussed in detail below. Let us consider turbulent $\mathbf{u}$ pulsations characterized by broken mirror symmetry, implying the nonzero value of a pseudoscalar quantity, helicity $\overline{\mathbf{u} \cdot \nabla \times \mathbf{u}}$, and by the presence of inhomogeneous energy of turbulent pulsations, defined by vector $\mathbf{g} = (\nabla \overline{u^2})/\overline{u^2}$, where $u = |\mathbf{u}|$. The appearance of anisotropic and mirror-asymmetric turbulence may be due to the action of a Coriolis force determined by general rotation $\boldsymbol{\Omega}$ and of the mean velocity field gradient $\nabla \overline{\mathbf{U}}$. For convenience, $\nabla \overline{\mathbf{U}}$ is resolved into symmetric and antisymmetric parts. The former is the strain rate tensor $\mathbf{D} = 1/2(\nabla \overline{\mathbf{U}} + \nabla \overline{\mathbf{U}}^{\mathrm{T}})$ describing the character of the shear motion component in the vicinity of the point being considered. Incompressibility condition $\nabla \cdot \overline{\mathbf{U}} = 0$ ensures the validity of expression $\mathrm{tr}(\mathbf{D}) = 0$. The antisymmetric part describes solid-state rotation in the neighborhood of the point of interest and is defined by vector $\mathbf{W} = \nabla \times \overline{\mathbf{U}}$.

The structure of $\boldsymbol{\mathcal{E}}$ is deduced from symmetry considerations for both complete MHD equations and Eqns (4) and (6). Given that fields $\mathbf{B}$, $\mathbf{U}$, and $\boldsymbol{\Omega}$ satisfy these equations, they are just as well satisfied by fields $\mathbf{B}'$, $\mathbf{U}'$, $\boldsymbol{\Omega}'$ derived from the initial equations by the rotation with respect to a certain axis passing through point $\mathbf{x} = \mathbf{0}$. Similarly, these equations are satisfied by the fields $\mathbf{B}' = -\mathbf{B}$, $\mathbf{U}' = -\mathbf{U}$, $\boldsymbol{\Omega}' = -\boldsymbol{\Omega}$ created from the respective initial ones as a result of reflection relative to a certain plane containing, for example, point $\mathbf{x} = \mathbf{0}$. Rotation invariance leads to the conclusion that tensors $\boldsymbol{\mathcal{A}}$ and $\boldsymbol{\mathcal{B}}$, arising in formula (8), as well as $\boldsymbol{\alpha}$, $\boldsymbol{\gamma}$, $\boldsymbol{\beta}$, $\boldsymbol{\delta}$, and $\boldsymbol{\kappa}$ may contain only elements of symmetric second-rank unit tensor δ_{ij} (Kronecker delta symbol), skew-symmetric third-rank unit tensor ϵ_{ijk} (Levi-Civita symbol), vectors $\mathbf{g}$, $\boldsymbol{\Omega}$, $\mathbf{W}$, and tensor $\mathbf{D}$. Note also that $\boldsymbol{\mathcal{A}}$ and $\boldsymbol{\mathcal{B}}$ must be pseudotensors, which means that $\boldsymbol{\alpha}$, $\boldsymbol{\delta}$, and $\boldsymbol{\kappa}$ are pseudoquantities, while $\boldsymbol{\gamma}$, $\boldsymbol{\beta}$ are usual (polar) quantities. The general form of $\boldsymbol{\mathcal{E}}^{\overline{\mathbf{B}}}$ includes the above constructive elements as follows:

$$
\begin{aligned}
\boldsymbol{\mathcal{E}}^{\overline{\mathbf{B}}} = & -\alpha^{(0)}\overline{\mathbf{B}} - \beta^{(0)}\nabla\times\overline{\mathbf{B}} \\
& - (\delta^{(\Omega)}\boldsymbol{\Omega} + \delta^{(W)}\mathbf{W})\times(\nabla\times\overline{\mathbf{B}}) \\
& - (\kappa^{(\Omega)}\boldsymbol{\Omega} + \kappa^{(W)}\mathbf{W})\cdot(\nabla\overline{\mathbf{B}})^{(s)} \\
& - \beta^{(D)}\mathbf{D}\cdot(\nabla\times\overline{\mathbf{B}}) - \kappa^{(D)}\hat{\boldsymbol{\kappa}}(\mathbf{D})\cdot(\nabla\overline{\mathbf{B}})^{(s)} \\
& - \alpha_1^{(\Omega)}(\mathbf{g}\cdot\boldsymbol{\Omega})\,\overline{\mathbf{B}} - \alpha_2^{(\Omega)}\big((\boldsymbol{\Omega}\cdot\overline{\mathbf{B}})\,\mathbf{g} + (\mathbf{g}\cdot\overline{\mathbf{B}})\,\boldsymbol{\Omega}\big) \\
& - \alpha_1^{(W)}(\mathbf{g}\cdot\mathbf{W})\,\overline{\mathbf{B}} - \alpha_2^{(W)}\big((\mathbf{W}\cdot\overline{\mathbf{B}})\,\mathbf{g} + (\mathbf{g}\cdot\overline{\mathbf{B}})\,\mathbf{W}\big) \\
& - \alpha^{(D)}\hat{\boldsymbol{\alpha}}(\mathbf{g},\mathbf{D})\cdot\overline{\mathbf{B}} \\
& - \big(\gamma^{(0)}\mathbf{g} + \gamma^{(\Omega)}\mathbf{g}\times\boldsymbol{\Omega} + \gamma^{(W)}\mathbf{g}\times\mathbf{W} + \gamma^{(D)}\mathbf{g}\cdot\mathbf{D}\big)\times\overline{\mathbf{B}}\,,
\end{aligned} \tag{10}
$$

where $\hat{\boldsymbol{\kappa}}(\mathbf{D})$ is the third-rank tensor defined as $\hat{\kappa}_{ijk} = \epsilon_{ijl}D_{lk} + \epsilon_{ikl}D_{lj}$, and $\hat{\boldsymbol{\alpha}}(\mathbf{g},\mathbf{D})$ is the symmetric tensor given by the expression $\hat{\alpha}_{ij} = (\epsilon_{ilm}D_{lj} + \epsilon_{jlm}D_{li})g_m$.

The first row of formula (10) contains classical transport coefficients $\alpha^{(0)}$ and $\beta^{(0)}$ responsible for generation and dissipation effects in homogeneous isotropic mirror-asymmetric turbulence. The terms responsible for the general rotation and local vorticity effects are grouped together in the second and third rows. Worthy of special mention is the term with $\delta^{(\Omega)}$ standing for the occurrence of an $\boldsymbol{\Omega}\times\mathbf{J}$-effect in rotating turbulence, discovered by Rädler[2] in 1969 [33]. This effect coupled to differential rotation can cause self-excitation of a magnetic field [34]. A shift of the mean velocity field characteristic of most flows in astrophysical objects is responsible for effects in the fourth row of formula (10). Although dynamo models including such a shift had been in use for a long time, its specific role in the formation of large-scale fields was confirmed only recently (see paper [35]), showing that shift effects can excite a large-scale field in helical turbulence at high Reynolds numbers.

The remaining terms appear as a result of energy inhomogeneity of turbulent pulsations in the $\mathbf{g}$ direction. Worthy of special mention is the term with $\alpha_1^{(\Omega)}$. It can be seen that the arising EMF acts in direction $\overline{\mathbf{B}}$ in analogy with the classical α-effect. In this case, however, reflective symmetry is broken by rotation. Figure 3 schematically illustrates the mechanism of this effect and demonstrates the analogy with δ-effect mentioned above as the $\boldsymbol{\Omega}\times\mathbf{J}$-effect. For the sake of symmetry, we consider four excitations, two directed upwards and two downwards, each pair undergoing both convergent and divergent motion. The Coriolis force causes the excitations to rotate in the direction of general rotation given by $\boldsymbol{\Omega}$, and in the opposite direction. It should be emphasized that the resulting system of vortices has zero general vorticity and zero general helicity. Each excitation draws out a loop from the initially horizontal magnetic field[3] and turns it so that the emerging constituent of EMF is directed parallel to the field. Importantly, convergent and divergent vortices give rise to different EMFs due to different loop areas. Given the identical strengths of two lines of force and excitation intensities, the total effect will be zero, as expected. If the vertical magnetic field gradient, as in the $\delta^{(\Omega)}$-effect, or the intensity gradient of turbulent pulsations, as in the $\alpha^{(\Omega)}$-effect, is taken into account, the contribution from the vortex in the bottom left corner exceeds that from its counterpart in the top right corner, the result being the nonzero average.

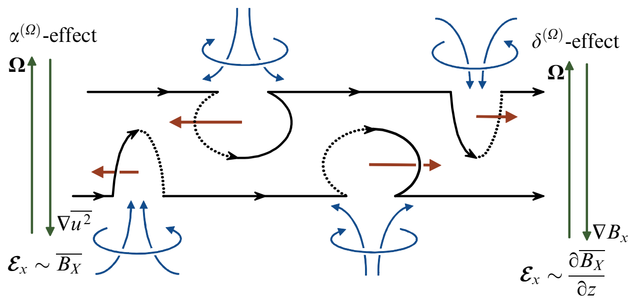

Figure 3. Schematic illustration of the appearance of $\alpha_1^{(\Omega)}$- and $\delta^{(\Omega)}$-effects (designed jointly with K-H Rädler).

The fairly well-known term with $\gamma^{(0)}$, the so-called effect of turbulent diamagnetism, was discovered by Zeldovich in 1956 [22] and later investigated by Weiss [36]. This effect by itself does not participate in the dynamo cycle but can substantially lower the generation threshold. For example, in the frame of an $\alpha-\Omega$-dynamo in a galactic disk, turbulent magnetism is known to push the magnetic field out of the galactic halo into the disk and thereby enhance the effectiveness of generation [37]. Generally speaking, $\mathbf{W}$ and $\boldsymbol{\Omega}$ enter formula (10) in a similar manner, but the respective transport coefficients may differ.

2.2 Structure of electromotive force $\boldsymbol{\mathcal{E}}^{(0)}$

It follows from the idea of $\boldsymbol{\mathcal{E}}$ resolution into constituent components (7) that $\boldsymbol{\mathcal{E}}^{(0)}$ can depend only on $\mathbf{u}$ and $\overline{\mathbf{U}}$. If such an EMF does exist, $\overline{\mathbf{B}}$ can infinitely grow from absolute zero, even in the absence of a contribution from $\boldsymbol{\mathcal{E}}^{\overline{\mathbf{B}}}$. In this case, background turbulence plays the role of an external 'battery' enhancing the mean field $\overline{\mathbf{B}}$. A more plausible hypothesis is that $\boldsymbol{\mathcal{E}}^{(0)}$ serves as a source of priming fields that are further picked up by the mechanisms involving $\boldsymbol{\mathcal{E}}^{\overline{\mathbf{B}}}$. This alternative is of special interest for explaining the evolution of magnetic fields in young galaxies [38].

Let us consider the general form of $\boldsymbol{\mathcal{E}}^{(0)}$ containing both well-known and recently added constituents [39]:

$$
\begin{aligned}
\boldsymbol{\mathcal{E}}^{(0)} = & \, c^{(U)}\overline{\mathbf{U}} + c^{(W)}\mathbf{W} + c^{(\Omega)}\boldsymbol{\Omega} \\
& + c^{(g)}\mathbf{g} + c^{(gU)}\mathbf{g}\times\overline{\mathbf{U}} + c^{(g\Omega)}\mathbf{g}\times\boldsymbol{\Omega}\,.
\end{aligned} \tag{11}
$$

The first three terms are inherent in homogeneous isotropic turbulence, whereas the remaining ones emerge in the presence of inhomogeneity caused by the energy gradient $\mathbf{g}$ of turbulent pulsations. Moreover, mirror symmetry must be broken for pseudoscalars $c^{(W)}$, $c^{(\Omega)}$, and $c^{(gU)}$ to exist. The effect produced by the action of the second and third terms is frequently called 'cross-helical' or the Yoshizawa effect [40, 41]. Its appearance in the so-called ABS flow in the context of a dynamo (Archontis dynamo) was demonstrated in Ref. [42]. A systematic presentation of the dynamo effect due to cross helicity with numerous illustrative examples can be found in Ref. [43].

2.3 Approaches to the calculation of transport coefficients

The second-order correlation approximation (SOCA) is the most popular approach to the calculation of transport coefficients entering expressions (10) and (11). It was developed along with mean-field electrodynamics in Ref. [44]

[2] It is sometimes called the Rädler effect.

[3] An analogy can be drawn to Parker's loop, but his scheme requires the prevalence of vortices with single-sign helicity to generate the mean field.

and employed in studying many problems. The name of the method indicates that highest-order correlators are neglected, i.e., the contribution from $(\mathbf{u}\times\mathbf{b})'$ to equation (6) is regarded to be small compared with that from other terms. This suggestion restricts the applicability of the results to many astrophysical problems. The relevant applicability criterion is formulated by introducing Reynolds numbers and the Strouhal number, defined through the characteristic velocity pulsation amplitude u_c, length λ_c, and velocity field fluctuation time τ_c:

$$\mathrm{Re}^{\,t} = \frac{u_c\lambda_c}{\nu}\,, \quad \mathrm{Rm}^{\,t} = \frac{u_c\lambda_c}{\eta}\,, \quad \mathrm{St} = \frac{u_c\tau_c}{\lambda_c}\,.$$

For a turbulent velocity field, u_c has the sense of root-mean-square pulsations, while λ_c and τ_c have the sense of correlation length and time, respectively. The dimensionless parameter $q = \lambda_c^2/(\eta\tau_c) = \mathrm{Rm}/\mathrm{St}$ is the ratio of diffusion time on scale λ_c to turbulence correlation time τ_c. Limit $q\to\infty$ corresponds to the passage to the case of high conductivity, and limit $q\to 0$ to low conductivity. The inequality $|\overline{\mathbf{B}}|\gg|\mathbf{b}|$ is a sufficient condition for SOCA applicability, equivalent to condition $\mathrm{Rm}^{\,t}\ll 1$ for $q\lesssim 1$ or $\mathrm{St}\ll 1$ for $q\gtrsim 1$. These conditions can be combined as

$$\min\,(\mathrm{Rm}^{\,t}, \mathrm{St}) \ll 1\,. \qquad (12)$$

The equation for $\mathbf{u}$, analogous to Eqn (6), also contains second-order pulsations neglected in SOCA. Therefore, separation by analogy of high- and low-viscosity cases in parameter $q = \lambda_c^2/(\nu\tau_c)$ yields an additional applicability condition

$$\min\,(\mathrm{Re}^{\,t}, \mathrm{St}) \ll 1\,. \qquad (13)$$

The validity of these conditions became a subject of extensive research in connection with the development of a new numerical approach — the test field method (see Section 7.1). Surprisingly, the formulas for transport coefficients (see below) proved valid under less severe restrictions, namely, when condition $\mathrm{Rm}^{\,t}\ll 10$ was fulfilled for $q\lesssim 1$ [45, 46].

Finding coefficients in the expression for turbulent EMF (8) reduces to the solution of equation (6). For the case of $\overline{\mathbf{U}} = 0$, the solution takes the form

$$\boldsymbol{\mathcal{A}}_{ip} = \left(\epsilon_{ijn}\delta_{lp} - \epsilon_{ijp}\delta_{ln}\right)\int_V\int_0^\infty G(\xi,\tau)\,\frac{\partial Q_{jn}(\mathbf{x},t;\boldsymbol{\xi},-\tau)}{\partial\xi_l}\,\mathrm{d}^3\xi\,\mathrm{d}\tau\,, \qquad (14)$$

$$\boldsymbol{\mathcal{B}}_{ipq} = -\left(\epsilon_{ijn}\delta_{lp} - \epsilon_{ijp}\delta_{ln}\right) \times\int_V\int_0^\infty \frac{\partial G(\xi,\tau)}{\partial\xi}\,\frac{\xi_l\,\xi_q}{\xi}\,Q_{jn}(\mathbf{x},t;\boldsymbol{\xi},-\tau)\,\mathrm{d}^3\xi\,\mathrm{d}\tau\,,$$

where

$$G(x,t) = (4\pi\eta t)^{-3/2}\exp\left(-\frac{x^2}{4\eta t}\right) \qquad (15)$$

is a Green function, and

$$Q_{ij}(\mathbf{x},t;\boldsymbol{\xi},\tau) = \left\langle u_i(\mathbf{x},t)\,u_j(\mathbf{x}+\boldsymbol{\xi},t+\tau)\right\rangle \qquad (16)$$

is the two-point correlation tensor of turbulent pulsations characterizing properties of background hydrodynamic turbulence. In the simplest case of homogeneous and isotropic turbulence and $q\to 0$, the expressions for the coefficients are written down as

$$\alpha^{(0)} = -\frac{1}{3}\int_0^\infty\Big\langle\mathbf{u}(\mathbf{x},t)\cdot\big(\nabla\times\mathbf{u}(\mathbf{x},t-\tau)\big)\Big\rangle\,\mathrm{d}\tau = -\frac{1}{3}\big\langle\mathbf{u}\cdot(\nabla\times\mathbf{u})\big\rangle\,\tau^{(\alpha)}\,,$$
$$\beta^{(0)} = \frac{1}{3}\int_0^\infty\big\langle\mathbf{u}(\mathbf{x},t)\cdot\mathbf{u}(\mathbf{x},t-\tau)\big\rangle\,\mathrm{d}\tau = \frac{1}{3}\langle\mathbf{u}^2\rangle\,\tau^{(\beta)}\,, \qquad (17)$$

where $\tau^{(\alpha)}$ and $\tau^{(\beta)}$ are usually estimated through τ_c.

The so-called τ-approach proposed recently includes in consideration the correlations of higher-order pulsations, expressing them through the second order terms. Detailed critical comparative analysis shows that far from all SOCA results are reproduced using the τ-approach even if conditions (12) are fulfilled. Therefore, caution is needed when applying it [47].

3. Why is a laboratory dynamo experiment so difficult to conduct?

The natural desire of researchers to examine the effect of a hydromagnetic dynamo in the laboratory arose simultaneously with the advent of the first realistic models of the cosmic dynamo in the mid-20th century. The dramatic history of dynamo experiments knows two successful realizations of MHD-dynamo effects achieved at the turn of the new century with an interval of only one month after a long period of competitive research conducted by two large groups, one based in Riga [48, 49] and the other in Karlsruhe [50]. Over the almost 15 years since then, only one new example of the generation of a large-scale magnetic field by a flowing metal has been published [von Kármán sodium (VKS) dynamo experiment in Cadarache, France [51, 52] (see detailed discussion below)], despite the hard efforts of many research teams. However, some specialists are skeptical about the results of this experiment and do not consider them as a hydrodynamic dynamo in the pure form, because it became apparent only after ferromagnetic details were introduced into the flow. Strictly speaking, none of the three dynamo experiments can be regarded as a direct analog of a natural dynamo, since solid metallic details play an important role in them (electroconducting walls in the Riga device, intricately shaped guiding tubes in Karlsruhe, and ferromagnetic components in Cadarache). This, however, does not diminish their role in the development of MHD dynamo science. It should be emphasized that a geodynamo involves participation of mantle–solid core borders regarded as hard walls in most models. Further events are discussed in the order of occurrence.

To begin with, the major source of problems related to laboratory dynamos is that the MHD dynamo constitutes a critical phenomenon feasible only after the threshold value of the magnetic Reynolds number, Rm^*, is reached. The typical values of this quantity for the known dynamos amount to a few dozen. The magnetic Reynolds number $\mathrm{Rm} = UL/\eta$ (U is the characteristic velocity, L is the characteristic size, and η is the magnetic diffusion coefficient) defines the ratio of the generation term to the dissipation one in the equation for magnetic field induction; it is related to the ordinary (hydrodynamic) Reynolds number $\mathrm{Re} = UL/\nu$ (ν is the kinematic viscosity) through the magnetic Prandtl number $\mathrm{Pm} = \nu/\eta$:

$$\mathrm{Rm} = \mathrm{Re}\,\mathrm{Pm}\,.$$

To recall, the huge sizes and velocities of cosmic objects deliberately account for supercritical values of the Reynolds numbers. For example, Earth's core thickness $H \approx 2\times10^6$ m, thickness of the Sun's convective shell $H \approx 2\times10^8$ m, and characteristic velocity of convective flows $U \approx 10^3$ m s^{-1}.

The main difficulty encountered under experimental conditions is small values of the magnetic Prandtl number characteristic of laboratory fluids (high magnetic diffusion sometimes called magnetic viscosity). Because experimentalists are interested only in liquid media for which magnetic permeability is close to unity, magnetic diffusion there depends on metal electrical conductivity. Sodium, with its high conductivity, low density, and low melting temperature (98 °C), is considered to be the metal of choice for dynamo experiments. It is also particularly remarkable that dynamically sodium behaves like water (having the same density and viscosity), which makes it possible to employ a water prototype of a dynamo device when studying sodium flow dynamics. Experiments with sodium do pose certain problems arising from its high chemical activity, but there is thus far no alternative metal for the purpose. Thus, liquid sodium at about 100 °C has the Prandtl number $\mathrm{Pm} \approx 10^{-5}$ (temperature must be close to the melting point to avoid a loss of conductivity with its further increasing). This means that the critical values of the magnetic Reynolds number can be reached only in a flow with the hydrodynamic Reynolds number $\mathrm{Re} \geqslant 10^6$. Therefore, the sodium flow velocity must be as high as tens of meters per second, even if the characteristic size $L \approx 1$ m, bearing in mind that a topologically simple flow cannot ensure the dynamo effect in principle.

The above estimates allow the conclusion that a laminar dynamo is impracticable. As far as turbulent flows in conducting liquids are concerned, the following classification of realizable dynamos can be proposed (Fig. 4) based on the Reynolds concept of mean and pulsation fields (**U** and **u** for velocity, **B** and **b** for magnetic field): quasilaminar dynamos, mean-field dynamos, and small-scale turbulent dynamos. The first category includes dynamos in which the generation of a mean (large-scale) magnetic field **B** is totally determined by the structure of the mean velocity field **U**. Turbulent velocity pulsations **u** excite small-scale pulsations **b** of the magnetic field but interfere with the generation of the large-scale magnetic field (see Section 2). The structure of the mean velocity field can also play here an important role (the key factor in many cosmic dynamos is differential rotation of the conductive liquid medium in a cosmic body). Alternatively, it can make no appreciable contribution to the dynamo process. Special attention should be given to small-scale turbulent dynamos believed to be realized in practically any turbulent flow, provided the magnetic Reynolds number is sufficiently high.

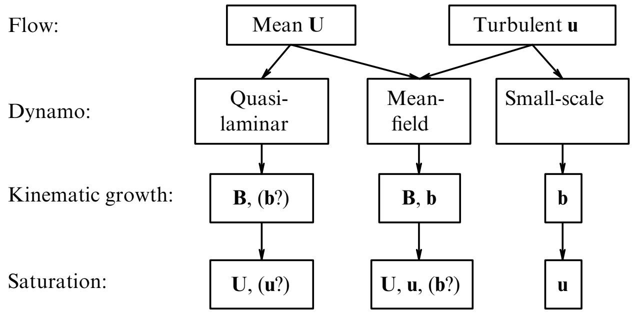

Figure 4. Classification of realizable dynamos.

The bottom row in Fig. 4 refers us to one more important aspect of the dynamo process, namely, the saturation mechanism. A growing magnetic field has an inverse effect on the velocity field, reducing in some way or other the effectiveness of generation. Both dynamic equilibrium and oscillatory regimes are possible. Very exotic examples of self-killing dynamos are also known [53]. They show their worth when a newly created magnetic field modifies the flow so that the conditions for generation can no longer be reproduced. In the case of a quasilaminar dynamo, saturation proceeds mainly due to the influence of the magnetic field being generated on the mean velocity field. Various saturation scenarios in mean-field dynamos may affect both the mean-field structure and small-scale turbulence. There are no mean fields in small-scale dynamos, and saturation also occurs at the level of pulsations alone.

4. Dynamo experiments

The history of the construction of prototype flows capable of generating magnetic fields dates back to the so-called 'unipolar dynamo' described by Bullard [54] in the form of an electrically conducting disk rotating in a coil-generated magnetic field. The potential difference emerges between the disk axis and periphery, and it is fed to the coil. Evidently, sufficiently rapid rotation of the disk may create a stable magnetic field. Thus, Bullard proposed an electrodynamic rather than hydromagnetic dynamo. The rotating disk may be regarded as a vortex, but the dynamo cycle cannot be completed without implication of a coil generating the desired magnetic field. Such a 'semihydrodynamic dynamo' was experimentally realized by Bourgoin et al. [55] in Lyon, France in 2006. The authors named their construction as Bullard–Kármán dynamo and used the von Kármán flow instead of a copper disk. Placed in a cylinder between two disks having an opposite sense of rotation, the flow generated, by virtue of differential rotation, an azimuthal field in the presence of the imposed axial field. The resulting azimuthal field was fed into an amplifier with the outlet connected to the coils generating the axial field required. Clearly, the effectiveness of such a dynamo is easy to enhance by increasing the magnification factor of the amplifier.

In 1958, Herzenberg [56] designed a dynamo consisting of two rotating spheres having nonparallel rotation axes and placed in a motionless electroconducting medium. The device was analogous to a hydrodynamic system of two intense nonparallel vortices in a conducting fluid. This idea was further developed in many theoretical publications (see, for instance, paper [57] and references cited therein). Moreover, it was realized experimentally in a purely 'solid-state' version by Lowes and Wilkinson [58], who constructed a device with two rotating cylinders surrounded by a ferromagnetic bulky material and observed the effect of magnetic field generation.

The following scheme of a solid dynamo proposed by Ponomarenko [15] proved crucial for dynamo experiments and gave rise to a new class of 'screw dynamos' [59, 60]. Ponomarenko considered a screw motion of an infinite cylinder in an infinite conducting medium and showed that such motion can lead to the generation of a magnetic field in the form of a twisted (bifibrillar) string. Notice that this scheme is characterized by a low critical value of the magnetic Reynolds number (17.7) found from the rod radius and translational motion velocity [61]. Two key factors are of importance in the Ponomarenko dynamo, viz. twisting (screw-like motion of the conducting rod) and shifting at the cylinder–conducting medium boundary.

A natural extension of Ponomarenko's scheme consists in a passage from the cylinder to a twisted fluid jet. Transition to a viscous fluid jet smooths the velocity jump at the boundary and hightens the generation threshold. Induction of a magnetic field in screw-like flows with real (not solid-state) velocity profiles has been discussed by a number of authors [62–64]. Reference [62] considered the flow of a conducting fluid in a gap between two coaxial cylinders, with the inner one being in translational and rotational motions. Ideally conducting and absolutely nonconducting cylinders were analyzed. In both cases, the critical values of Rm found from the gap thickness and the inner cylinder velocity turned out to be very similar (129 and 125, respectively). The authors of Ref. [65] reported results on a screw fluid flow in an ideally conducting tube. The velocity profile was described by smooth functions (Bessel functions), and the critical number was on the order of 40.

It was a screw dynamo (quasilaminar from the standpoint of practical realization) that was chosen as the basis for a laboratory dynamo at the Institute of Physics, Latvian Academy of Sciences, away back in the 1980s. The first facility was constructed jointly with the D V Efremov Research Institute of Electrophysical Apparatuses (NIIEFA). The pump propelled liquid sodium into a cylinder 140 mm in diameter through a diverter, in which the flow was twisted. The cylinder was placed in a bigger cylinder 500 mm in diameter, also filled with sodium. The unit was put into operation in 1987, but attempts to generate a magnetic field failed [66]. In the latter half of 1990s, the group of G Gerbeth (Dresden, Germany) joined the researchers in Riga, and the dynamo experiment was continued. The possibility of decreasing the critical value of the magnetic Reynolds number was thoroughly reconsidered. First, a cylinder 25 cm in diameter with a screw-like sodium flow realized was enclosed not only by a channel for sodium counterflow but also by a circular layer of motionless sodium (see Fig. 5). The backflow of sodium kept the field being excited in the work zone of the channel and thereby allowed producing an effect in a channel of finite length (instability developing in the Ponomarenko problem is not global and occurs only in an infinite cylinder). Second, the researchers analyzed several options for rib-stiffening the channel in order to obtain the optimal mean velocity profile. All the options were analyzed in Ref. [64]. Calculation of field excitation conditions in such a flow for its different profile, taking into account the finite length of the cylinder, gave values from 15 to 30.

The magnetic field generation regime was first achieved in Riga in November 1999 [48], when the magnetic field mode with the azimuthal number $m = 1$ traveling upflow, as expected in the Ponomarenko dynamo, was recorded. The back action effect of the magnetic field being excited on the sodium flow and the onset of saturation were studied in a subsequent series of experiments [49].

An alternative scheme of the laboratory screw dynamo was proposed at the Institute of Continuous Media Mechan-

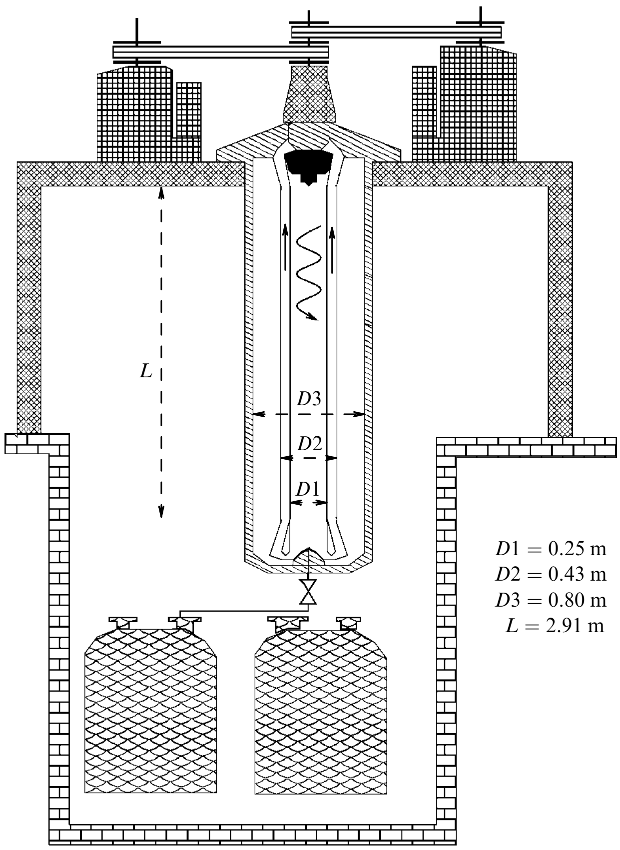

Figure 5. Schematic of the Riga experimental unit [48].

ics, Ural Branch of the Russian Academy of Sciences, in Perm [67]. The central idea of the experiment was to obtain the desired screw flow in a closed toroidal channel. The flow was created by inertial forces (the channel being accelerated and stopped abruptly) and acquired a screw-like character under the effect of stationary (with respect to the channel) diverters inside the channel (Fig. 6). An important advantage of this scheme is a substantial reduction in the sodium volume needed for the experiment (some 100 kg instead of the few metric tons used in Riga), a decrease in the engine power (by an order of magnitude), and the absence of sealings. The disadvantage lies in the fact that only a pulsed (nonstationary) screw flow that maintains the supercritical regime within a short time interval (around 1 s) can be realized.

The possibility itself of creating a screw flow of desired intensity in the torus is far from evident, because the azimuthal (along the channel) momentum hampers rotation of the 'liquid screw' along the curvilinear channel. The curvature of a channel is given by the aspect ratio: $\Gamma = R/r$, where R is the channel axial circle radius, and r is the cross section radius. The possibility of creating a large-scale screw flow was demonstrated in a transparent water-filled channel having the aspect ratio $\Gamma = 3.81$ and with a single diverter [67]. Relatively large turbulence spheres used for visualization traced screw trajectories in the flow. The dynamics of a nonstationary flow generated by abrupt deceleration of the rotating toroidal channel with 1–4 diverters inside it were studied in Ref. [68]. The serious difficulties encountered in generating screw flows in a toroidal channel are illustrated by the fact that they were obtained in the channels with the aspect ratio $2.75 \leqslant \Gamma \leqslant 3.89$ [69–71] but not in the one with $\Gamma = 2.25$ [72].

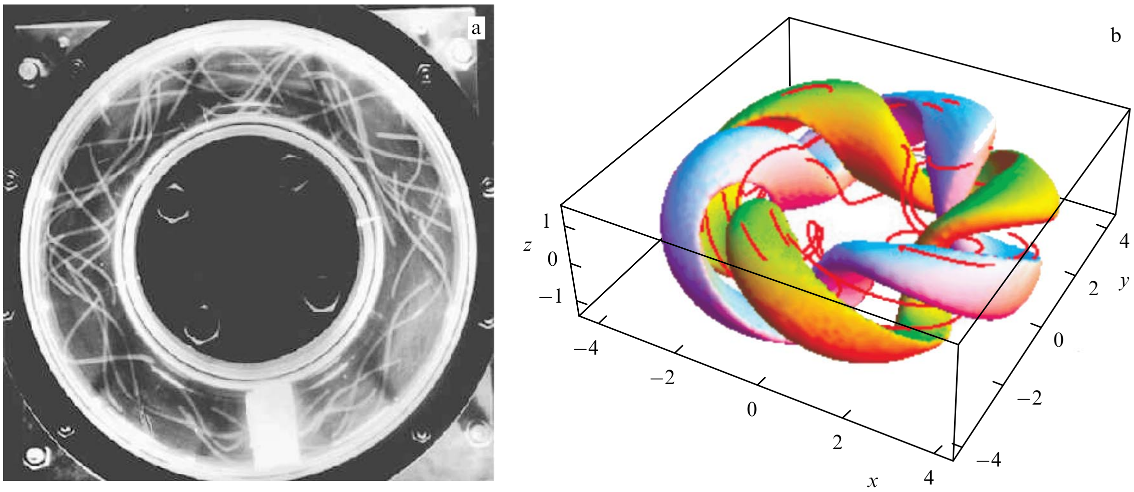

Figure 6. (Color online). Screw-like water flow in a rotating toroidal channel after rotation, visualized by suspended spherical particles. The light body inside the channel is the diverter twisting the flow. (a) Photograph borrowed from Ref. [67]. (b) Magnetic field generated by the screw flow in the torus (calculated in Ref. [73]).

The problem of studying screw dynamos in toroidal geometry has some important peculiarities. On the one hand, the channel wall thickness and conductivity need to be taken into consideration. What is more, the channel must be rugged but not very heavy (otherwise, it will be difficult to stop). Calculations of the generation threshold for various materials and wall thicknesses showed that the critical number in a toroidal screw flow is always higher than in a cylindrical channel placed in a practically unrestricted conducting medium. However, for real materials and a constructively acceptable geometry, the critical value of the magnetic Reynolds number may be within the range of 30 [69].

On the other hand, passage to a toroidal geometry has an advantage as well, lying in the fact that the convective nature of the resulting instability is no longer a problem, because disturbances develop within a closed flow but do not move off with it. Numerical investigations showed that a magnetic field has time to form in a nonstationary closed screw flow before it degenerates to a critical level [74].

One more important feature of dynamos is that increasing a curvature of the channel (in the 'thick' torus limit corresponding to small Γ) may generate a global mode giving a magnetic field that does not decay on the torus axis (i.e., in the axial hole) [73, 75].

The idea to produce an intense pulsed flow of a liquid metal in a rapidly rotating channel by means of its abrupt deceleration proved very fruitful for laboratory studies of MHD flows with moderate and high magnetic Reynolds numbers, even though it has not thus far been translated into the dynamo effect. Investigations into screw flow hydrodynamics on the water prototypes operated by the Laboratory of Physical Hydrodynamics at the Institute of Continuous Media Mechanics (Perm) were continued on three liquid-metal facilities with toroidal channels, which were designed and fabricated in the same Laboratory: (1) a 1-litre laminated fabric channel filled with gallium; (2) a 115-l bronze channel filled with sodium, and (3) a 25-l titanium channel filled with sodium. Screw flows with a low magnetic Reynolds number ($\mathrm{Rm} < 0.3$), obtained in the small (gallium-filled) channel (1), were utilized to study the effects of magnetic field transfer by the screw flow [70]. In addition, priority measurements of turbulent transport coefficients were conducted [71, 76]. They will be considered below in Section 5.

The installation with the large bronze channel (2) was specially designed and constructed for the dynamo experiment. The channel was rotated at a rate of 40 rps to generate a pulsewise flow with a magnetic Reynolds number on the order of 40, which was expected to ensure the dynamo regime [74]. Major difficulties were associated with the fabrication of the toroidal channel itself from chromium bronze ensuring the necessary combination of strength and electrical conductivity under strict limitations on the torus mass. One of the big homogeneous ingots of the alloy (half torus) cracked, to the extent of losing pressurization. The defect was corrected after time-consuming investigations and the studies were carried out in the subcritical regime [77]. However, the strength properties of the reconstructed channel do not yet allow achieving the critical regime.

The titanium channel (3), having very low electrical conductivity compared with sodium, was designed to study sodium flows with moderate magnetic Reynolds numbers ($1 < \mathrm{Rm} < \mathrm{Rm}_{\mathrm{cr}}$). It was used to measure turbulent transport coefficients for $\mathrm{Rm} > 1$ [78, 79] (see below Section 5).

According to the classification presented in Fig. 4, screw dynamos are quasilaminar and cannot serve as prototypes of astrophysical dynamos. Another approach to the laboratory realization of dynamos is based on the use of mean-field dynamos, which are of direct interest for the simulation of natural dynamos. The key problem in this context is to create the 'correct' turbulence capable of generating a magnetic field. A screw-like flow combined with other factors can induce turbulent EMF in accordance with formula (10), and cause a variety of dynamo effects [80].

As shown in Section 2, the effective EMF caused by the action of turbulent pulsations involves a wide spectrum of physical mechanisms. Historically, the first attempts to explain the appearance of the pseudoscalar factor in the expression for a turbulent EMF were based on helicity considerations (Parker). The relevance of this idea was confirmed by an experiment conducted at the Institute of Physics, Riga [81]. The experimental setup, known as an 'alpha-box', consisted of two entangled copper channels through which liquid sodium was pumped (Fig. 7). The

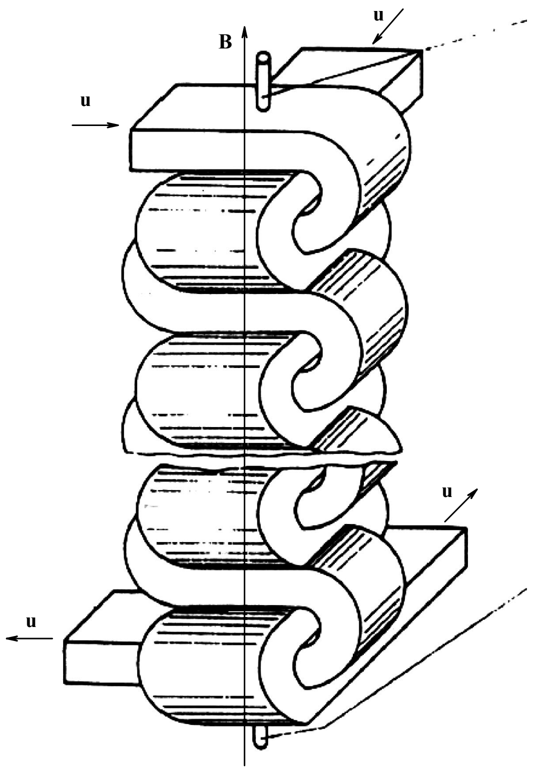

Figure 7. Diagram of experimental setup known as the 'alpha-box' [81].

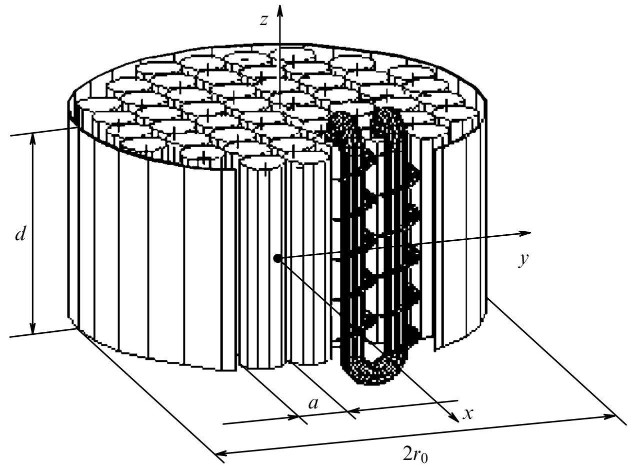

Figure 8. Schematic of the Karlsruhe experimental facility [50].

imposed magnetic field is subjected to crossed sodium fluxes properly in accordance with Parker's scheme (to stretch out a mesh and then rotate it), resulting in the emergence of EMF along the imposed magnetic field.

To recall, the alpha-effect in the mean-field dynamo theory implies the turbulent (random) structure of the small-scale velocity field maintaining this effect. The alpha-box and all other laboratory prototypes of mean-field dynamos create flows of a regular small-scale structure providing the alpha-effect. From this standpoint, it is more appropriate to call laboratory turbulent mean-field dynamo models two-scale dynamos. Examples of flows whose small-scale (but regular) structure ensures the generation of a macro-scale flow have been discussed in literature since the mid-1960s (see, e.g., Refs [82–85]). They provided the basis for the Karlsruhe experiment. The possibility of producing a dynamo effect in a double-period flow was confirmed by Roberts [84].

The Karlsruhe module comprises an assembly of 52 'spin generators', each in the form of two co-axial cylinders (Fig. 8). The 'direct' sodium flow passes through the inner cylinder, and the screw flow formed by special guiding devices flows through the outer one. The characteristic sizes of this assembly (total diameter and spin generator length) measure 1 m.

Magnetic field generation in such a flow was considered first in simple models [86], and thereafter studied in more detail in terms of the mean-field theory [87] and by direct numerical simulation [88]. The comprehensive theory of mean-field dynamos with reference to the Karlsruhe facility is presented in Refs [89, 90].

The Karlsruhe experiment was conducted as follows. The sodium flow rate in the direct (central) tubes was kept constant (115 $m^3 h^{-1}$), and in the screw channels increased from 95 to 107 $m^3 h^{-1}$, giving rise to slow growth of the magnetic field. The growth process up to the onset of saturation took the rather long time of 90 s. The resulting field had the shape of a dipole oriented perpendicular to the axis of the block of spin generators [50, 91].

The most widespread type of natural dynamo is the so-called $\alpha - \Omega$-dynamo operated by combining differential rotation (omega) and small-scale turbulence (alpha). An attempt to realize in laboratory conditions this type of dynamo was undertaken at the Los Alamos Technological Institute (New Mexico Tech), USA. In this device, the alpha-effect was reached by injecting twisted sodium jets into the sodium flow between rotating co-axial cylinders with a radius of 0.3 and 0.6 m, respectively (Couette flow maintaining differential rotation) [92, 93]. Such a scheme was expected to produce the alpha-effect in a free liquid sodium flow, unlike the Karlsruhe experiment, where it was achieved in a system of tubes. However, the attempt failed.

Attractive models of natural dynamos are those in which the field-generating flow of a conducting fluid is produced in a closed (preferably spherical) volume. In such a geometry, the so-called T2S2 flows with two toroidal and two poloidal components appear to be of special interest. A T2S2 flow is possible to create with the help of two counter-rotating propellers set on the axis of a sphere. It is this flow that was thoroughly investigated by the group of C B Forest at the University of Wisconsin (Madison, USA). After running experiments with a water prototype and relevant calculations, the researchers designed a sodium-filled stainless steel sphere 0.5 m in diameter containing two propellers, each driven by a 75-kW engine [94]. A number of experiments were conducted with this device, including the observation of the effects of transformation of the imposed axial magnetic field [95] with the separation of contributions from the mean velocity field and turbulent pulsations [96]. Also, bursts of generation that did not, however, cause sustained induction of the magnetic field were recorded [97]. In a word, the generation regime was never reached in this experiment, despite the favorable forecast based on numerical calculations.

A kinematically similar scheme was taken as a basis for the French experimental dynamo program using von Kármán flow arising in a cylindrical cavity between two rotating disks [98]. Counter-rotating disks (in fact, impellers of different shapes) twisted the fluid and created centrifugal flows near the cylinder blind ends, thus generating just the same T2S2 flow. Notice that the French program attracted an especially large number of participants from Lyon, Paris, Orsay, Nice, and Grenoble. The sodium-based device was designed and constructed in the Cadarache Atomic Energy Centre. The experiment was thoroughly prepared and preceded by numerous studies of von Kármán flows using different types and rotation rates of impellers [99]. The effects of magnetic field induction in subcritical regimes were analyzed on the gallium prototype of the experimental dynamo setup [100, 101]. The theory predicted generation of a magnetic field for the sodium-filled device in the form of a magnetic dipole oriented normally to the cylinder axis [102]. However, the dynamo effect was not observed (as in the Madison facility), either in the first variant of the experiment [103] or in the second one, conducted after substantial modification of the device with a view to increasing the effective magnetic Reynolds number and stabilizing the conducting sodium flow by guiding elements [104].

Still, the French researchers managed to obtain the sought-after dynamo effect in von Kármán sodium flow [51] by introducing ferromagnetic impellers to shield the region behind the rotating disks, where flows increased the generation threshold [105]. The true role of these ferromagnetic elements thus far remains a matter of discussion, although they were found to enhance the mechanisms of the dynamo cycle (thus decreasing the generation threshold) that were initially disregarded altogether. This resulted in a qualitatively different type of dynamo; namely, an axial dipole (the field oriented along the cylinder axis) was generated in the system instead of the expected transverse dipole. One explanation may be that the von Kármán flow performs only half the work of the dynamo, while the second half is done by the α-effect arising on ordered vortices in the vicinity of the ferromagnetic elements [106]. According to an alternative viewpoint, variation of magnetic permeability intruded by the embedded ferromagnetic elements may be a missing link in the dynamo cycle [107].

Disk rotation in the Cadarache setup occurred independently, which made it possible to substantially modify the flow structure in the cavity (from quasiaxial rotation between co-directionally rotating disks via single-vortex T1S1 flow, generated when one of the disks does not rotate, to symmetric T2S2 in the limit of two disks counter-rotating with a similar speed, as shown schematically in Fig. 9). The diversity of hydrodynamic regimes accounts for a wide spectrum of various dynamo regimes, such as a stationary dynamo, oscillatory regimes, dynamo bursts, and chaotic inversions of the magnetic field [52].

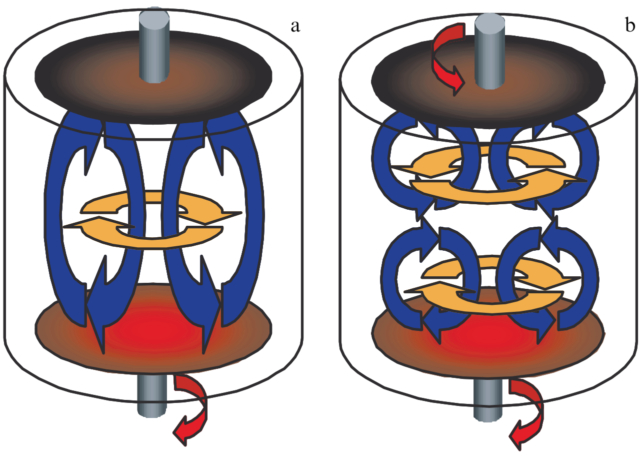

Figure 9. (Color online). Schematic representation of T1S1 (a) and T2S2 (b) flows used in the French dynamo experiment [51].

The dynamo problem most relevant to the present-day needs of humankind is that of a geodynamo realized by virtue of convective flows of liquid matter containing large amounts of iron in Earth's outer core. In this case, a source of energy for the dynamo can be a heat flux from the inner core, suggestive of its convective nature. It cannot be ruled out either that the flow is maintained by gravitational differentiation of Earth's matter. Laboratory-scale generation of convective flows of the desired intensity is impracticable without direct mechanical excitation of the flow, the more so since convection in a rotating spherical layer under the influence of the central gravity field is needed. A most important feature of intracore flows is a dynamic balance between Coriolis forces and Lorentz forces (magnetostrophic regime) amid the absolute predominance of Coriolis forces over viscosity forces (the Ekman number is estimated as 10^{-15}). All attempts to simulate intracore flows in lab experiments are based on the use of the spherical Couette flow that makes it possible to control differential rotation in the spherical layer. Estimated characteristics of the model of the spherical Couette flow in liquid sodium that can potentially reach the dynamo regime suggest the necessity of applying some 700 kW to a sphere around 1 m in diameter [108], which is a nontrivial task. The DTS (standing for Derviche Tourneur Sodium) setup with the outer sphere 0.4 m in diameter was designed in Grenoble to operate in a deliberately subcritical regime; it was used to study the structure of emerging flows under the effect of an imposed magnetic field and in its absence [109]. In addition, magnetoinertial waves and magnetic convection were investigated [110, 111]. Since the DTS setup is equipped with a variety of measuring instruments, the MHD flow generated in this installation could be characterized better than any of its analogs in other experiments [112].

A huge experimental facility for studying the spherical Couette flow of liquid sodium has recently been commissioned at the University of Maryland, USA. It was designed in the hope of reaching the dynamo regime (an outer sphere diameter of 3 m to contain over 3 metric tons of sodium). The first studies were aimed to elucidate the structure of the emerging flows in the sphere [113]. Thus far, regimes with the magnetic Reynolds number $\mathrm{Rm} = \Delta\Omega\Delta r^2/\eta < 500$ (where $\Delta\Omega$ is the difference between the angular rotational velocities of the spheres, and Δr is the difference between their radii) have been attained that amount to almost half of the theoretical maximum throughput of this setup (provided it is equipped with an additional cooling system); no dynamo effect has yet been documented [114].

5. Experimental verification of individual dynamo elements

The 15-year dramatic history of dynamo experiments reveals that only the qualitative aspect of this phenomenon is actually understood, despite advances in the dynamo theory. Its basic principles have been formulated, making it clear what flows can or cannot bring about the dynamo effect; also, viable models of stellar and planetary dynamos have been proposed. However, in laboratory experiments where the generation threshold is hardly reached, researchers require results of exact calculations taking into account all characteristics of a given system. As evidenced by the foregoing, predictions based on all available information were fulfilled only in two cases where the desired flows were forcibly brought into consideration. It may even be argued that the results of negative dynamo experiments are as useful as those of positive ones. Overall, the dynamo question still poses many problems that cannot be resolved by simple realizations of the dynamo process, so that experiments are needed that would promote understanding the mechanisms governing dynamo systems.

In this context, experimental studies of individual effects playing an important role in turbulent dynamo processes are of a special value. The simplest effect of the turbulent flow of a conducting medium on the magnetic field is the enhancement of effective field diffusion, which is analogous to enhanced effective momentum diffusion in a turbulent flow. Momentum diffusion is closely related to turbulent viscosity, whereas turbulent field diffusion can be introduced by analogy in the case of a magnetic field. In the expansion of turbulent electromotive force (9), turbulent diffusion is described by tensor $\boldsymbol{\beta}$ and is frequently referred to as the β-effect. Under conditions of isotropic turbulence, this effect can be interpreted as a large-scale decrease in mean (effective) electrical conductivity of the turbulent medium.

Although this effect is seemingly self-evident, it is extremely complicated to record small variations of conductivity in a turbulent flow; reliable direct measurements of effective electrical conductivity of a turbulent conducting fluid flow have been unavailable until recently. The first attempt to measure it in a container with vigorously mixed fluid sodium was described in Ref. [115], but the results seemed doubtful as regards both the extent of variations and the estimated measurement error.

Direct measurements of effective conductivity in a turbulent metal flow were made in experiments with nonstationary flows in toroidal channels at the Institute of Continuous Media Mechanics (Perm). The first studies were carried out using a low-temperature gallium alloy flowing in a toroidal laminated fabric channel [76]. The maximum Reynolds numbers $\mathrm{Re} \approx 3 \times 10^5$ corresponded to the magnetic Reynolds number $\mathrm{Rm} \approx 0.3$. Metal conductivity in the channel was estimated from the phase shift of forced harmonic oscillations in a series oscillatory circuit with an inductor in the form of a toroidal coil wound around the channel. The maximum difference between the effective conductivity of the turbulent medium and the ohmic conductivity of the metal was roughly 1%.

Clearly, experimental studies of the beta-effect at significantly higher magnetic Reynolds numbers are of greater interest. Moderate magnetic Reynolds numbers ($1 < \mathrm{Rm} < \mathrm{Rm}_{\mathrm{cr}}$) became possible to reach in a much more stiff titanium channel of greater geometric sizes using liquid sodium — an essentially lighter and better conducting metal than gallium. The maximum Reynolds number in an experimental device using liquid sodium was $\mathrm{Re} \approx 3 \times 10^6$, which corresponds to the magnetic Reynolds number $\mathrm{Rm} \approx 30$. The maximum deviation of the diffusion coefficient from the reference (laminar) value was about 50% [78]. The first direct measurements of turbulent viscosity and turbulent magnetic field diffusion are reported in Ref. [79].

It was revealed that the behavior of the turbulent magnetic field diffusion coefficient is determined by the $\mathrm{Rm}_{\mathrm{rms}}$ values evaluated from a root-mean-square value of turbulent velocity pulsations. For $\mathrm{Rm}_{\mathrm{rms}} < 1$, the turbulent diffusion coefficient grows together with the Reynolds number according to a square law, whereas for $\mathrm{Rm}_{\mathrm{rms}} > 1$ the turbulent magnetic field diffusion appears to be proportional to turbulent viscosity [79].

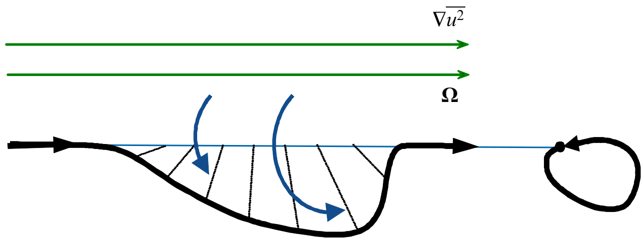

Figure 10. Schematic of the experimentally discovered α-effect.

Notice that experimental verification of the turbulent alpha-effect is much more difficult than that of the beta-effect. Its simplest manifestation is believed to be related to helical turbulence. However, the very problem of the production of homogeneous helical turbulence is far from being solved, and the sole laboratory evidence of the turbulent alpha-effect has been received for the same conducting fluid flow in the toroidal channel where small-scale helicity can make no contribution. Short-term generation of a magnetic field of necessary symmetry turned out well in the toroidal channel during the formation of the screw flow behind diverters. In this case, the alpha-effect was due to the joint action of the turbulent pulsation gradient and large-scale vorticity [71]. The nature of the alpha-effect thus obtained is significantly different from the nature of the classical alpha-effect (Fig. 10) associated with the contribution from density gradient and angular velocity. Nevertheless, the community of dynamo experts unanimously considers the above results as the first experimental evidence of the α-effect and emphasizes that these studies are substantially different from the earlier research of the Riga group reproducing this effect in a system of twisted tubes [81].

It is worth mentioning one more experiment conducted at the Institute of Continuous Media Mechanics in the context of astrophysical dynamo inquiries. The interstellar medium where the galactic dynamo operates is highly inhomogeneous (see, e.g., Ref. [16]). Its parameters of importance for dynamo effects may be by orders of magnitude different depending on concrete phases. This fact needs to be considered when deriving mean-field dynamo equations, which implies the necessity of developing a relevant theory. Remarkably, at least one such parameter (the magnetic constant μ) was measured in a mixture of two substances with markedly different μ; as a result, useful analytical approximations were proposed for the dependence of effective magnetic permeability on the mixture composition [70].

6. Laboratory experiments in the context of astrophysical dynamos

Because industrial units with very high magnetic Reynolds numbers are still a long way in the future, astrophysics and geophysics remain the main fields of dynamo experiments. The questions are what knowledge astrophysicists expect to derive from these observations, to what extent current experiments contribute to the implementation of their plans, and what the prospects are for the further development of these studies.

The dynamo theory has for a long time remained unsupported by experimental findings. For this reason, many more theoretical models than collections of experimental data are currently available. That is why experiments must first confirm that the present theories consider model situations that are actually interesting in the astrophysical context.

First and foremost, experiments conducted in Riga and Karlsruhe demonstrated the very possibility of self-excitation of magnetic fields in a flowing conducting fluid, i.e., dynamos. Certainly, specialists in astrophysical dynamos and geomagnetism did not question such a possibility for a long time, but many of those working in many related fields had no idea of this phenomenon. The fact is that an adequate description of magnetic field self-excitation is virtually different from the stereotype approach to electromagnetism based on different physical problems. Specifically, all researchers developing dynamo models, unlike experts concerned with related disciplines, find it more convenient to consider the magnetic field rather than electric current traversing the liquid medium. E Parker, one of the pioneers of dynamo research, devoted a special book to this collision [117].

Another aspect of the dynamo theory, clarified by the results of experimental studies, is the variety of flows producing dynamos, many of which were subjected to theoretical analysis, in the first place by representatives of the English school of magnetic hydrodynamics (see, for instance, paper [118]). At the same time, there are not as many dynamos of interest for astrophysics and geophysics as that. As a rule, the mean large-scale velocity in astrophysical dynamos has to do with differential rotation that cannot by itself results in self-excitation of a magnetic field. One differential rotation may cause transient enhancement of the magnetic field that decays thereafter as a result of a scale decrease and dissipation, even if initially weak.

The effect of differential rotation must be supplemented by another factor if an astrophysical dynamo is to be realized. As a rule, such a factor is the mirror asymmetry of small-scale turbulence or convection, even if other types of mirror asymmetry may be involved, e.g., the mirror asymmetry of a small-scale magnetic field. The importance of mirror asymmetry for the operation of dynamos was first emphasized by Steenbeck, Krause, and Rädler [20], who proposed parametrization of the influence of this asymmetry in the form of the famous α-effect. The very asymmetry can arise, for example, under the influence of the Coriolis force acting on emerging and submerging vortices in stratified convection, even if other mechanisms are equally possible. Practically all pragmatically useful models of astrophysical dynamos have been constructed in the framework of the paradigm of the joint action of differential rotation (the so-called Ω-effect) and the α-effect, or, as a limiting case, of the α-effect alone. However, the concept of the α-effect is difficult to illustrate by examples from other branches of physics for the sole reason that this mirror asymmetry plays no special role there.

On the other hand, the α-effect emerges in the formulation of the equation of the mean (large-scale) magnetic field, which is of primary interest in the astrophysical context. This effect does not manifest itself beyond the framework of a detailed description of a magnetic field. Such a situation is not unusual in physics. For example, the notion of temperature in the molecular-kinetic theory of gases arises from their averaged description, whereas a detailed description of gases in terms of individual molecules does not require an explicit introduction of this quantity. Certainly, it is difficult to demonstratively avoid using temperature or related notions when dealing with a macroscopic gas volume containing huge numbers of molecules. However, the number of convection (or turbulence) cells in astrophysical dynamos is far less than the Avogadro number; therefore, modern computers allow us not to work directly at the α-effect. Another difficulty is that the peculiar forms of the α-effect, once postulated by the German school and known as the Krause formula, by no means exhaust the diversity of possibilities. By analogy, the simplest formulas for the calculation of, say, heat capacity do not exhaust all the physical effects in this field one can think of. Specialists engaged in the dynamo theory have gone through and continue to go through much trouble to show how to avoid using the α-effect in the description of concrete dynamos and how utterly inadequate the elementary Krause formula can be. No matter how interesting these studies might be, they are not aimed at clarifying the astrophysical aspects of the problem.

Historically, parametrization of the α-effect in the models of Earth's magnetic field generation (the so-called geodynamo problem) was employed much more rarely than in astrophysical problems, thus introducing additional difficulties. Conversely, direct numerical simulation of magnetic hydrodynamic equations is extensively used as giving simultaneously the flow pattern inside Earth's liquid outer core and the magnetic field. Of course, there were good reasons for such development of the theory. To begin with, we know very little about flows in the liquid core of Earth. However, it is difficult even to check if a concrete flow identified by direct numerical simulation has a mirror-asymmetric component and to calculate the respective α; as a rule, this problem cannot be resolved by interpreting the results.

Similar discussions are certainly conceivable in other domains of condensed matter physics, but the utility of a notion, e.g., electrical resistance, is usually confirmed by direct measurement of this parameter rather than by general theoretical reasoning. The specific character of the dynamo problem arises here from the fact that observational astronomy had not until recently possessed methods for observing the α-effect and determining other large-scale transport coefficients inherent in dynamo research (see Section 2). Additionally, the α-effect is expressed through the correlation between velocity and vortex; therefore, all three velocity components must be known to calculate it. In astronomy, however, the velocity is usually estimated from the Doppler effect, which gives only the line-of-sight component. Only in the last few years has solar astronomy found alternative ways to study the mirror asymmetry of MHD flows in the Sun's active regions. In the present context, one such way is taking account of the so-called Joy's law, i.e., the significant deflection of the line connecting the centers of sunspots in bipolar groups from the latitudinal direction. Such a deflection has long been known [119], but the amount of data necessary for reliable calculation of the α-effect or its analogs [121] from observations of sunspots proved sufficient only recently [120]. On the whole, these observational data confirm basic provisions of standard astrophysical dynamo concepts that, however, do not use up all possibilities.

This explains why direct laboratory measurements of the α-effect and the coefficient of turbulent (convective) diffusion of the magnetic field (as much discussed in the literature as the α-effect) have for several decades been regarded as an attractive but unreal hope. To recall, the historic dynamo experiments started in Riga and developed up to laboratory realization of dynamos were initiated by the proposal of M Steenbeck to verify the possibility of achieving the α-effect under laboratory conditions.

Today, both the α-effect and turbulent magnetic field diffusion coefficients are measured in laboratory dynamo experiments (see Section 5), and estimates of the respective transport coefficients fit fairly well into the current concepts of the dynamo theory. It should be emphasized that these pioneering measurements of turbulent transport coefficients in liquid metal flows do not finally resolve the problem but only open up the opportunity for its experimental exploration. For instance, mechanisms underlying mirror asymmetry in experiment and astrophysics are altogether different. For example, compressibility of matter is an important characteristic of astrophysical flows, whereas liquid metals are virtually incompressible. A similar problem is encountered in observations of the α-effect. Nevertheless, recent positive developments have begun to prevail over the seemingly hopeless situation with which dynamo investigations were faced in the preceding period. Among other things, the utilization of plasma flows instead of liquid metals appears to be a promising way to overcome the aforementioned compressibility problem.

Another sore spot in astrophysical discussions concerning details of dynamo models is estimating the limiting strength of the magnetic field created by the dynamo mechanism. Naturally, such an estimation implies some balance relations. The simplest idea of this kind is based on the energy conservation law; specifically, the energy that instability can convert to magnetic energy must not exceed the total energy stored in the system. In many concrete astronomical problems, even such very rough estimates lead to sensible conclusions allowing us to characterize the magnetic fields of many celestial bodies from equipartition of the magnetic energy and the kinetic energy due to movements. The intricate question is what movements should be considered. As a rule, the kinetic energy of the general rotation of a celestial body is incomparably higher than its magnetic energy; therefore, estimation of equipartition usually concerns the energy of random movements, and its separation from the total energy is not a trivial problem. By way of example, the density along the radius of the Sun's convective zone is known to vary essentially, and estimates of the convection rate are rather indeterminate.

It became clear in the 1990s that estimating energy equipartition does not help to understand the physics of the phenomenon. Results of pioneering research were published by Vainshtein, Cattaneo, Gruzinov, and Diamond [122–124], but the true current understanding of the problem came after decades of ardent debates, which it would be superfluous to discuss here. The core of the subject is that the store of kinetic energy (first and foremost, general rotation energy) is always more than sufficient to generate a desired magnetic field, but a dynamo may cease to operate if a weak chain in the magnetic field self-excitation process (α-effect) is suppressed.

The α coefficient has a velocity dimension, with the general rotation velocity of a typical spiral galaxy being circa 250 km s^{-1}, root-mean square velocity of random movements ca. 10 km s^{-1}, and the typical estimate of the α-effect ca. 1 km s^{-1}. It is natural to think that magnetic forces first and foremost suppress small α. On the other hand, symmetry considerations compel the researcher to doubt that energy balance needs considering in the estimation of this suppression: α is a pseudoscalar, while the energy is a scalar [125]. Indeed, it turns out that the energy balance is a less important limiting factor in estimating α-effect suppression than magnetic helicity (another specific conserved quantity of magnetic hydrodynamics [126]), defined as $\int \mathbf{A}\cdot\mathbf{B}\,\mathrm{d}^3x$, where $\mathbf{B}$ is the magnetic field, and $\mathbf{A}$ is the magnetic potential.

Because the magnetic potential is defined up to gauge, the law of conservation of magnetic helicity is nonlocal; this means that magnetic helicity density $\mathbf{A}\cdot\mathbf{B}$ at a given point can always be turned into zero by the proper choice of gauge. Nonlocal laws are certainly more difficult to work with than the local energy conservation law, but the dynamo community learnt how to do it (see review [127]). At this point, ideological cross-talk between the dynamo theory and quantum mechanics arises once again. As was shown in the famous Aharonov–Bohm experiment, the magnetic potential also plays a much more autonomous role in quantum mechanics than in classical domains of physics. It may be argued that here, too, quantum mechanics helps to better understand its classical counterpart [128].

For the purpose of this review, it is important that the direct application of the reasoning behind the magnetic helicity balance to the estimate of the dynamo-generated magnetic field yields a significantly smaller magnitude than the equipartitioned field. In this case, the ratio between the strengths of the limiting magnetic field to the equipartitioned field depends on the magnetic Reynolds number which may be enormously high in astrophysical problems. The situation is resolved due to the fact that magnetic helicity is not only conserved during generation of the magnetic field but also transferred by different mechanisms. That is why, the strength of the magnetic field proves to be similar to that of the equipartitioned field, as shown by a detailed analysis of concrete models (see, for instance, review [127]). Notice that the necessity of considering magnetic helicity balance and transfer was first postulated in Ref. [129], written when one of the authors (N I Kleeorin) was the last postgraduate student of Ya B Zeldovich.

Naturally, the contradictory results of the discussion concerning suppression of helicity need experimental and observational confirmation, with astronomical observation being a priority at this junction. Years-long targeted efforts of solar astronomers from different countries successfully contributed to their learning of the measurement (in the framework of certain approximations inevitable in the interpretation of astronomical observations) of magnetic field current helicity density (i.e., the quantity $\mathbf{B}\cdot\mathbf{j}$, where $\mathbf{j}$ is electrical current) in the Sun's active regions. The results were then used to calculate magnetic helicity. The method was proposed by the German astronomer N Seehafer [130], who worked in cooperation with the group that coined the notion of the α-effect. Chinese astronomers led by H Zhang greatly contributed to the application of this method to monitor by the common procedure the evolution of magnetic helicity in the Sun's active regions during the last two solar cycles [131]. It should be emphasized that Russian Foundation for Basic Research (RFBR) also greatly contributed to the treatment of the results of the Chinese astronomers by enabling access of Russian researchers to observational data as soon as they became available. This work brought together researchers from China, Russia, the UK, Israel, the USA, and Japan into the international collaboration group. Their joint efforts made it possible to reconstruct the latitudinal and temporal evolution of current (and magnetic) helicity during two solar cycles [132] or, in astronomical language, to create a butterfly diagram of current helicity in two cycles. A detailed description of these results is beyond the scope of the present review. Notice only that the form of the distribution thus obtained agrees, at least in general, with the predictions of the standard solar dynamo theory [133, 134].

It is noteworthy that researchers concerned with solar dynamo simulation did not care to theoretically describe the latitudinal and temporal distribution of current helicity until it was substantiated based on direct observations. The first attempt to explain the observational data was made in Ref. [135] in the framework of the simplest solar dynamo model. It turned out that the distribution patterns can be predicted based on symmetry considerations and other elementary ideas. Certainly, a more detailed comparison of observational and theoretical data is the most challenging problem.

Dynamo experiments also provide, of course, materials for the study of nonlinear saturation of dynamos. But here, we are actually at the commencement of the work, and methods for comparison of these data with theoretical expectations ensuing from the analysis of astrophysical models remain to be developed. It appears to be a promising avenue of future research.

Further insight into natural dynamo models is aimed at relating them to a concrete flow structure in celestial bodies. The situation appears to be manageable in the rough approximation, because celestial bodies having fairly well explored large-scale magnetic fields are actually few, viz. the Sun and stars with similar properties, Earth and some planets with their satellites, and spiral galaxies. The governing parameters of dynamos in the Sun and galaxies do not markedly exceed their critical values, and the suppression of dynamo's operation may be regarded as due to the influence of the growing magnetic field on the α-effect. Under these conditions, a large-scale monotonically growing and thereafter stabilized magnetic field appears in galactic disks where the angular velocity gradient is parallel to the disk plane. The magnetic configuration in the form of a magnetic field wave traveling over the shell that periodically (or quasiperiodically) emerges in the relatively thin convective shell of the Sun for the angular velocity gradient transversal to the shell boundaries gives rise to a solar cycle. A more intricate situation takes place in Earth's outer core, where the general rotation rate measured in natural units is extremely high. Indeed, it is reasonable to match the Sun's rotational period (circa 1 month) with the solar cycle period (11 years), and Earth's rotational period (a day) with the time of significant changes in the general magnetic field (not less than thousands of years). Explorations show that magnetic forces under these conditions can substantially rearrange large-scale flows in Earth's outer core. Anyway, the geomagnetic field in the first approximation is constant within relatively short time periods and, near Earth's surface, resembles a dipole magnetic field roughly parallel to the rotation axis. However, paleomagnetic data [136] indicate that the direction of the dipole underwent multiple variations (hundreds of times) during geological history and very rapidly (in geological terms) changed orientation. Inversions of magnetic dipole direction appear to have occurred in a random sequence and were far from periodic.

The first two regimes of natural dynamo operation could be expected to be attributable to the appearance of complex roots with a positive real part in dispersion relations. The emergence of regimes with chaotic inversions of the magnetic dipole is universally recognized in geological science (see, e.g., Ref. [137]) and confirmed by direct numerical simulation in the geodynamo [138], but it awaits experimental verification.

Characteristically, all three types of temporal behavior of magnetic configurations created by the dynamo mechanism were observed in the Cadarache VKS experiment (see Section 4). Of course, this experiment does not directly reproduce the dynamo regime in any of the aforementioned celestial bodies, but experimental confirmation of the possibility, in principle, of the regime reminiscent of the geomagnetic field behavior in geological history is a fundamental scientific discovery.

We may ascertain that even the early dynamo experiments clarified a number of difficult problems related to dynamo exploration by astrophysical methods. Current projects of further experiments give hope for the development of laboratory-scale dynamos in the near future, which are based on the joint action of the differential rotation and the α-effect.

7. Advances in computational experiments

There is one more nontrivial aspect of dynamo experiments. Today, it is hardly possible to surprise anyone by applying computational physics methods (numerical simulation) to solve physical problems, but computer experiments hold a special place in dynamo research. It is not just the amount of computed data in the total body of information about dynamos that matters — quantum chemistry, for example, comprises as many different numerical results as that. However, there is no doubt that standard textbooks on quantum mechanics provide a useful introduction to the subject of interest. Suffice it to mention *Quantum Mechanics* by L D Landau and E M Lifshitz as an excellent example where numerical results do not predominate over a variety of simple models describing quantum-mechanical systems, and the exactly or approximately solved problems serve as a basis for the interpretation of numerical data obtained in quantum chemistry.

A formal analogy with quantum mechanics is very useful for work with dynamo models (see, e.g., Ref. [139]), but numerical methods occupy a particular place in studying the dynamo effect. Historically, the dynamo theory developed in parallel with computational physics, and the complexity of dynamo problems increased approximately in parallel with the growing potential of computational physics. Due to this, physicists did not have to devise at first exactly or approximately solvable model problems and to resort to numerical methods after all other possibilities were exhausted. The culminating stage of these parallel developments fell in the 1990s, when direct numerical methods of geodynamo research reached the level at which mean field or other simple models became redundant, and dynamo processes could be described by complete equations of magnetic hydrodynamics allowing physically plausible solutions and reasonably reproducing observable phenomena [140]. This created for the time being an illusion that dynamo research could only be based on numerical experiments and astronomical observations, while traditional methods of theoretical physics were put aside. In this country, physicists proved especially vulnerable to the changes in the scientific atmosphere, because at that time they neither possess the necessary technical capabilities for solving numerical problems of this level nor were they properly trained in such a work. The situation has radically improved over the past time: Russia has seen the advent of supercomputers and the appearance of highly skilled experts in computational physics (of special importance in this context was the work of M Yu Reshetnyak on direct numerical simulation of geodynamos; see, e.g., Ref. [141]).

Simultaneously, researchers came to a better understanding of the potential of direct numerical simulation. It became clear that real parameters of cosmic plasma will be possible to reach only in the distant future, if at all. Developing realistic models of dynamos in concrete celestial bodies by these direct methods, even if necessary computer resources and software packages were available, requires a great variety of initial and other additional conditions to be involved, while relevant observational data are either scarce or refer to essentially different quantities. Immediate demonstration of the adequacy of simulated results usually requires an unbelievably huge number of calculations. In other words, a numerical experiment has as many disadvantages as a laboratory one and they can only be compensated by methods of theoretical physics.

The value of theoretical methods currently employed to study different aspects of dynamos varies. The best ones are those applied to simulate galactic dynamos [142]. Some of them, relatively simple and adequate for the available observational data, are suitable for describing galactic magnetic fields and based on the at least potentially observational data. In particular, they allow avoiding the description of the structural details of a magnetic field across the galactic disk. The respective coordinate is usually denoted by z, whence the term 'no z-model' [143, 144] for such models. The use of direct numerical simulation of galactic dynamos is restricted to relatively rare problems for which detailed characteristics are actually justified.

Simplified dynamo models, e.g., Parker's dynamo, are also extensively used in stellar dynamo simulation, and the wealth of information about the Sun justifies wide application of direct numerical simulation without compromising mean-field models (see, for instance, Ref. [145]). In contrast, it is rather difficult to find readily accessible simple methods for studying geodynamo models (see, e.g., Ref. [146]), and there is little data on the structure of Earth's liquid outer core.

Although the 'heroic epoch' of direct numerical simulation of dynamos is possibly gone, the role of numerical techniques remains paramount (see, for instance, Refs [147, 148]).

In this review, we confined ourselves to considering only two aspects of the application of computational physics methods to dynamo research. First, the direct application of numerical simulation yields a number of useless facts, besides the really interesting data. For example, we would like to have information about the Sun's large-scale magnetic field and averaged data on the solar small-scale fields, because concrete realization of a random small-scale field inevitable with the application of direct methods is, all the same, unpredictable. It is somehow possible to avoid obtaining unwanted information using methods adapted to describing random components of the solution by means of cascade models (see Section 7.2). On the other hand, even complete solutions of magnetic hydrodynamics equations may prove useless without knowing how to measure the quantities of interest (e.g., the α-effect). The application of naïve methods encounters many difficulties [149] that can be obviated by developing the special so-called test field method.

7.1 Test field method

In the foregoing, we comprehensively discussed theoretical and experimental results of the determination of transport coefficients and conditions for the manifestation of mean-field dynamo effects. It turns out that theoretical approaches are especially efficacious for small Reynolds numbers, while the real measurement of turbulent EMF is more practical in 'pure' experiment and depends on the potential of measuring techniques tapped in liquid metals. Reproduction of consistent results is really a challenge. Direct numerical methods may be of help in this situation. The so-called test field method has recently been formulated and verified with respect to a large class of problems [45, 46]. It does not exclude limitations on Reynolds numbers, but they are 2–3 orders of magnitude lower than in theoretical calculations of smoothed values. At the same time, the method opens up unrestricted possibilities for staging computational experiments and analyses of their results.

In the test field method, calculated data are split apart into two flows (main and test). In the main flow, a complete set of magnetic hydrodynamics equations is solved. In the test calculation, only magnetic induction equation (3) is solved, in which the velocity field is taken from the calculation of the main flow, while a mean magnetic field is chosen depending on the objective of the numerical experiment (finding one or the other of components of tensors $\mathcal{A}$ and $\mathcal{B}$ in expression (8)). The solution obtained in the test flow allows us to calculate $\mathcal{E}$ defined by Eqn (5) and thereby to distinguish the value of any coefficient in parametrization (10). To recall, the mean magnetic field is imposed as if from the outside, i.e., it is originally specified. A fundamental requirement in this method is the independence of the results from the choice of a concrete test field. The method was initially designed for the kinematic regime in which the magnetic field grows exponentially but remains weak enough to have no effect on the velocity field. It turned out, however, that the method works just as well in the nonlinear regime, thus making it possible to verify the theoretical models of quenching turbulent effects. The test field method was successively applied to determine turbulent EMF under conditions of homogeneous isotropic turbulence and taking into account helicity [150], shear [51], and both effects [152]. The contribution of cross helicity to mean field generation in the presence of background MHD turbulence is described in Ref. [153].

7.2 Combined models

The development of computer technologies made it possible to improve spatial and temporal resolution of direct numerical simulation methods to the point enabling them to reproduce the most essential properties of the dynamo process and, thereby, to promote solutions to theoretical problems. However, the Reynolds numbers involved remain too far from the values characteristic of astrophysical objects. It should be noted that an order of magnitude rise in the resolving power leads to an increase in the node count by three orders of magnitude and a decrease in the time step by two orders of magnitude (with reference to diffusion time). As a result, the computational labor intensity enhances by a factor of 10^5. We are far from making forecasts as regards the computer performance, but a known empirical law stating that the computer throughput increases roughly by an order of magnitude every 10 years suggests that such calculations will be feasible only in 50 years. Therefore, direct numerical methods will be most likely applied in the immediate future for the study of complicated physical models and more detailed analysis of numerical results.

Notice that a transition to higher Reynolds numbers is associated not only with upgrading computer resources but also with the radical improvement in computational algorithms and methods for interpreting calculated results. This means that simulation of the multiscale dynamo process does not allow reproducing the turbulent flow in its entirety and requires that only a relatively small number of variables describing the small-scale field behavior be chosen from a great number of degrees of freedom formally needed to describe small-scale fields. The idea to combine mean-field dynamo models with dynamic models of small-scale turbulence provided a basis for mixed dynamo models; the first such model was considered in paper [154].

A mixed or multiscale model makes up a combination of a large-scale magnetic field dynamo model in the paradigm of the mean-field theory and a dynamic small-mode, small-scale dynamo model. There are many candidates for the role of the large-scale model. The simplest one is the α^2-dynamo model, written out in the form of a system of two ordinary differential equations for the amplitudes of the toroidal and poloidal components of a large-scale magnetic field [154, 155]. An advanced model reproducing the dynamics of the large-scale field profile is exemplified by the dynamo model in the thin galactic disk, described by one-dimensional partial differential equations [156, 157]. Moreover, large-scale magnetic fields can be described by quasi-two-dimensional models, e.g., the aforementioned 'no z-models', where taking account of the small-scale field also leads to realistic reproduction of the magnetic field [158]. Importantly, all these models only include large-scale variables, while the influence of the small-scale field is expressed through turbulent EMF. Cascade models of MHD turbulence constitute most suitable and convenient ones for the description and calculation of small-scale turbulent field dynamics.

The mathematical apparatus of cascade (shell) models of turbulence was created and developed beginning in the 1970s with the active participation of the domestic researchers belonging to Kolmogorov–Obukhov school. Cascade equations substitute complex partial differential equations by a set of low-order ordinary differential equations relatively simple and suitable for numerical investigation even at very high Reynolds numbers, with a few dozen variables characterizing pulsations of velocity and magnetic fields on different (up to dissipative) scales. Cascade equations make it possible to keep track of the spectral density dynamics of the pulsation energy of both fields and to calculate also spectral fluxes and other statistical characteristics affecting large-scale fields. The construction of cascade equations requires strict fulfillment of all conservation laws that hold true for complete equations. Generalization of shell models over the case of magnetic hydrodynamics [159–161] revealed their high efficacy in the elucidation of the spectral properties of MHD turbulence. At present, it is possible to adequately formalize the derivation of cascade equations, categorize models by their physical parameters, and substantiate the choice of the model for the solution of a concrete problem. A systematic presentation of these materials with a description of the most important results of MHD turbulence studies can be found in review [162].

The key feature of a combined model is the relations connecting large- and small-scale field models. Expressions for transport coefficients in the mean-field theory (see Section 2.3) have the form of integrals of correlation characteristics over the spectrum; they are naturally rewritten through the sum over cascade variables. The formation of a closed system of equations requires that the influence of large-scale variables be introduced into cascade equations. The development of a concrete conjugation depends on the specific features of the problem of interest, but some of the accompanying issues of a general character are, as a rule, resolved in the context of conservation laws for the system as a whole [157]. The resulting combined model makes it possible to study the nonlinear dynamics of magnetic fields in a very wide range of parameters. By way of example, Fig. 11 shows the result obtained with the use of a combined model of α^2-dynamo efficacy at different values of the magnetic Prandtl number (over a range of 10 orders of magnitude!). The construction of combined dynamo models appears to be a promising area of further research in which outstanding theoretical ideas can be realized by means of breakthrough computational methods.

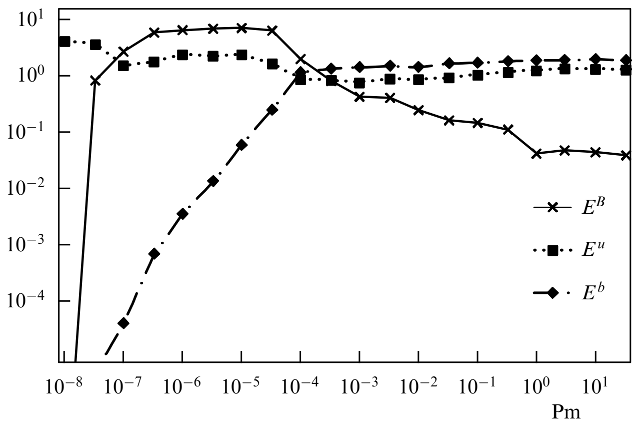

Figure 11. Energy E^B of a large-scale field, small-scale kinetic energy E^u, and magnetic energy E^b plotted versus magnetic Prandtl number Pm at $\mathrm{Re} = 10^6$ in a combined α^2-dynamo model (taken from Ref. [155]).

8. Conclusion

The decades-long history of research on magnetic field generation in a flowing conducting fluid expounded in the present review fairly well illustrates the progress in this area of physics. The very term 'dynamo theory', still used by convention, is no longer quite adequate, because it is not broad enough to encompass the new concept that puts laboratory and computational experiments on an equal footing with the theoretical constituent. We wished to demonstrate how the original theoretical ideas were supported by the results of experimental and numerical studies obtained by those who now formulate subjects of further investigations.

Russian science went through trying times in the decades covered by this review. We believe that it coped successfully with all the challenges as far as dynamo studies are concerned. The present generation of researchers was able not only to continue traditions set up by the generation of Zeldovich and his co-workers but also to open up new prospects for the search for meeting points between dynamo science and industry. The attached list of references reflects the great interest of the international physical community in the dynamo problem and universally recognized contribution of

Russian specialists to this research area. It would be certainly naïve to anticipate an unclouded future, but a certain work has been done in anticipation and there are now themes deserving thorough considerations.

Acknowledgments
This study was supported by RFBR projects Nos 11-01-00423-a, 12-02-00170-a, joint RFBR and Perm Krai Administration Projects (Nos 11-01-96031-a, MIG-245), and grant MD-520.2013.2 of the President, Russian Federation for the Support of Young Russian Scientists–Doctors of Sciences.

References

1. Zel'dovich Ya B, Ruzmaikin A A, Sokoloff D D *Magnetic Fields in Astrophysics* (New York: Gordon and Breach, 1983); Translated into Russian: *Magnitnye Polya v Astrofizike* (Moscow–Izhevsk: Inst. Komp'yut. Issled., RKhD, 2006)
2. Baliunas S L et al. *Astrophys. J.* **438** 269 (1995)
3. Fermi E *Phys. Rev.* **75** 1169 (1949)
4. Chandrasekhar S, Fermi E *Astrophys. J.* **118** 113 (1953)
5. Beck R et al. *Annu. Rev. Astron. Astrophys.* **34** 155 (1996)
6. Larmor J (Sir), in *Report of the 87th Meeting of the British Association for the Advancement of Science, Bournemouth, September 9 – 13, 1919* (London: John Murray, 1920) p. 159; http://www.biodiversitylibrary.org/item/96028
7. Vainshtein S I, Zel'dovich Ya B *Sov. Phys. Usp.* **15** 159 (1972); *Usp. Fiz. Nauk* **106** 431 (1972)
8. Zel'dovich Ya B *Sov. Phys. Usp.* **30** 353 (1987); *Usp. Fiz. Nauk* **152** 3 (1987)
9. Zeldovich Ya B, Ruzmaikin A A *Sov. Phys. Usp.* **30** 494 (1987)]; *Usp. Fiz. Nauk* **152** 263 (1987)
10. Ruzmaikin A A *Sov. Phys. Usp.* **29** 1069 (1986); *Usp. Fiz. Nauk* **150** 469 (1986)
11. Molchanov S A, Ruzmaikin A A, Sokolov D D *Sov. Phys. Usp.* **28** 307 (1985); *Usp. Fiz. Nauk* **145** 593 (1985)
12. Severnyi A B *Sov. Phys. Usp.* **9** 1 (1966); *Usp. Fiz. Nauk* **88** 3 (1966)
13. Childress S, Gilbert A D *Stretch, Twist, Fold: the Fast Dynamo* (Berlin: Springer, 1995)
14. Alfvén H *Rev. Geophys.* **15** 271 (1977)
15. Ponomarenko Yu B *J. Appl. Mech. Tech. Phys.* **14** 775 (1973); *Zh. Prikl. Mekh. Tekh. Fiz.* (6) 47 (1973)
16. Zeldovich Ya B, Ruzmaikin A A, Sokoloff D D *The Almighty Chance* (World Scientific Lecture Notes in Physics, Vol. 20) (Singapore: World Scientific, 1990) p. 316
17. Kazantsev A P *Sov. Phys. JETP* **26** 1031 (1968); *Zh. Eksp. Teor. Fiz.* **53** 1806 (1967)
18. Kraichnan R H, Nagarajan S *Phys. Fluids* **10** 859 (1967)
19. Parker E N *Astrophys. J.* **122** 293 (1955)
20. Steenbeck M, Krause F, Rädler K-H *Z. Naturforsch. A* **21** 369 (1966)
21. Rädler K-H, Brandenburg A *Phys. Rev. E* **77** 026405 (2008)
22. Zel'dovich Ya B *Sov. Phys. JETP* **4** 460 (1957); *Zh. Eksp. Teor. Fiz.* **31** 154 (1956)
23. Zel'dovich Ya B et al. *J. Fluid Mech.* **144** 1 (1984)
24. Arnol'd V I et al. *Sov. Phys. Dokl.* **27** 814 (1982); *Dokl. Akad. Nauk SSSR* **266** 1357 (1982)
25. Zel'dovich Ya B *Zh. Eksp. Teor. Fiz.* **7** 1466 (1937)
26. Kogan V R, Kolokolov I V, Lebedev V V *J. Phys. A Math. Theor.* **43** 182001 (2010)
27. Velikhov E P *Sov. Phys. JETP* **9** 995 (1959); *Zh. Eksp. Teor. Fiz.* **36** 1399 (1959)
28. Velikhov E P *JETP Lett.* **82** 696 (2005); *Pis'ma Zh. Eksp. Teor. Fiz.* **82** 785 (2005)
29. Kichatinov L L *Phys. Usp.* **48** 449 (2005); *Usp. Fiz. Nauk* **175** 475 (2005)
30. Krause F, Rädler K-H *Mean-Field Magnetohydrodynamics and Dynamo Theory* (Oxford: Pergamon Press, 1980)
31. Rädler K-H, Kleeorin N, Rogachevskii I *Geophys. Astrophys. Fluid Dyn.* **97** 249 (2003)
32. Rädler K-H, Stepanov R *Phys. Rev. E* **73** 056311 (2006)
33. Rädler K-H *Monatsber. Deutsch. Akad. Wissenschaft. Berlin* **11** 194 (1969)
34. Moffatt H K, Proctor M R E *Geophys. Astrophys. Fluid Dyn.* **21** 265 (1982)
35. Tobias S M, Cattaneo F *Nature* **497** 463 (2013)
36. Weiss N O *Proc. R. Soc. Lond. A* **293** 310 (1966)
37. Stepanov R A *Astron. Rep.* **43** 205 (1999); *Astron. Zh.* **76** 243 (1999)
38. Brandenburg A, Urpin V *Astron. Astrophys.* **332** L41 (1998)
39. Rädler K-H, Brandenburg A *Astron. Nachr.* **331** 14 (2010)
40. Yoshizawa A *Phys. Fluids B* **2** 1589 (1990)
41. Yoshizawa A *Publ. Astron. Soc. Jpn.* **45** 129 (1993)
42. Sur S, Brandenburg A *Mon. Not. R. Astron. Soc.* **399** 273 (2009)
43. Yokoi N *Geophys. Astrophys. Fluid Dyn.* **107** 114 (2013)
44. Krause F, Rädler K H, in *Ergebnisse der Plasmaphysik und der Gaselektronik* (Eds R Rompe, M Steenbeck) Vol. 2 (Berlin: Akademie-Verlag, 1971) p. 1
45. Schrinner M et al. *Astron. Nachr.* **326** 245 (2005)
46. Schrinner M et al. *Geophys. Astrophys. Fluid Dyn.* **101** 81 (2007)
47. Rädler K-H, Rheinhardt M *Geophys. Astrophys. Fluid Dyn.* **101** 117 (2007)
48. Gailitis A et al. *Phys. Rev. Lett.* **84** 4365 (2000)
49. Gailitis A et al. *Phys. Rev. Lett.* **86** 3024 (2001)
50. Stieglitz R, Müller U *Phys. Fluids* **13** 561 (2001)
51. Monchaux R et al. *Phys. Rev. Lett.* **98** 044502 (2007)
52. Ravelet F et al. *Phys. Rev. Lett.* **101** 074502 (2008)
53. Fuchs H, Rädler K-H, Rheinhardt M *Astron. Nachr.* **320** 129 (1999)
54. Bullard E (Sir) *Math. Proc. Cambr. Philos. Soc.* **51** 744 (1955)
55. Bourgoin M et al. *New J. Phys.* **8** 329 (2006)
56. Herzenberg A *Philos. Trans. R. Soc. London A* **250** 543 (1958)
57. Brandenburg A, Moss D, Soward A M *Proc. R. Soc. Lond. A* **454** 1283 (1998)
58. Lowes F J, Wilkinson I *Nature* **198** 1158 (1963)
59. Ruzmaikin A, Sokoloff D, Shukurov A *J. Fluid Mech.* **197** 39 (1988)
60. Gilbert A D *Geophys. Astrophys. Fluid Dyn.* **44** 241 (1988)
61. Gailitis A, Freiberg Ya *Magnetohydrodynamics* **12** 127 (1976); *Magn. Gidrodinamika* **12** (2) 3 (1976)
62. Solov'ev A A *Dokl. Akad. Nauk SSSR* **282** 44 (1985)
63. Stepanov R A, Frick P G "Vintovoe MGD-dinamo v real'nykh potokakh v trubakh" ("Screw MHD dynamo in real lows in tubes"), in *Gidrodinamika. Sb. Nauchnykh Trudov PGU* (Hydrodynamics. Collection of Scientific Papers of Perm State University) Iss. 12 (Perm': Permskii Gos. Univ., 1999) p. 240
64. Stefani F, Gerbeth G, Gailitis A, in *Transfer Phenomena in Magnetohydrodynamic and Electroconducting Flows* (Fluid Mechanics and Its Applications, Vol. 51, Eds A Alemany, Ph Marty, J P Thibault) (Dordrecht: Kluwer Acad. Publ., 1999) p. 31
65. Lupyan E A, Shukurov A *Magnetohydrodynamics* **28** 234 (1992); *Magn. Gidrodinamika* **28** (3) 29 (1992)
66. Gailitis A K et al. *Magnetohydrodynamics* **23** 349 (1987); *Magn. Gidrodinamika* **23** (4) 3 (1987)
67. Denisov S A et al. *Dokl. Phys.* **44** 231 (1999); *Dokl. Ross. Akad. Nauk* **365** 478 (1999)
68. Denisov S A et al. *Fluid Dynamics* **36** 734 (2001); *Izv. Ross. Akad. Nauk. Mekh. Zhidk. Gaza* (5) 73 (2001)
69. Frick P et al. *Magnetohydrodynamics* **38** 143 (2002)
70. Noskov V et al. *Eur. Phys. J. B* **41** 561 (2004)
71. Stepanov R et al. *Phys. Rev. E* **73** 046310 (2006)
72. Frick P et al. *Magnetohydrodynamics* **48** 31 (2012)
73. Stepanov R A, Chupin A V, Frick P G *Vychisl. Mekh. Sploshnykh Sred* **1** (1) 109 (2008)
74. Dobler W, Frick P, Stepanov R *Phys. Rev. E* **67** 056309 (2003)
75. Chupin A, Frick P, Stepanov R *Astron. Nachr.* **332** 11 (2011)
76. Denisov S A et al. *JETP Lett.* **88** 167 (2008); *Pis'ma Zh. Eksp. Teor. Fiz.* **88** 198 (2008)
77. Khalilov R, Kolesnichenko I, Stepanov R *Magnetohydrodynamics* **49** 73 (2013)
78. Frick P et al. *Phys. Rev. Lett.* **105** 184502 (2010)
79. Noskov V et al. *Phys. Rev. E* **85** 016303 (2012)
80. Rädler K-H, Stepanov R *Geophys. Astrophys. Fluid Dyn.* **100** 379 (2006)
81. Steenbeck M et al. *Monatsber. Deutsch. Akad. Wissenschaft. Berlin* **9** 714 (1967)

82. Steenbeck M, Krauze F *Magnetohydrodynamics* **3** 8 (1967); *Magn. Gidrodinamika* **3** (3) 19 (1967)
83. Gailitis A *Magnetohydrodynamics* **3** 23 (1967); *Magn. Gidrodinamika* **3** (3) 45 (1967)
84. Roberts G O *Philos. Trans. R. Soc. London A* **271** 411 (1972)
85. Busse F H *Geophys. J. Int.* **42** 437 (1975)
86. Busse F et al. *Magnetohydrodynamics* **32** 235 (1996); *Magn. Gidrodinamika* **32** 259 (1996)
87. Rädler K-H et al. *Studia Geophys. Geodaetica* **42** 224 (1998)
88. Tilgner A *Phys. Lett. A* **226** 75 (1997)
89. Rädler K-H et al. *Magnetohydrodynamics* **38** 41 (2002)
90. Rädler K-H et al. *Magnetohydrodynamics* **38** 73 (2002)
91. Stieglitz R, Müller U *Magnetohydrodynamics* **38** 27 (2002)
92. Colgate S A et al. *Magnetohydrodynamics* **38** 129 (2002)
93. Colgate S A *Astron. Nachr.* **327** 456 (2006)
94. Forest C B et al. *Magnetohydrodynamics* **38** 107 (2002)
95. Spence E J et al. *Phys. Rev. Lett.* **96** 055002 (2006)
96. Spence E J et al. *Phys. Rev. Lett.* **98** 164503 (2007)
97. Nornberg M D et al. *Phys. Rev. Lett.* **97** 044503 (2006)
98. Marié L et al. *Magnetohydrodynamics* **38** 163 (2002)
99. Marié L et al. *Eur. Phys. J. B* **33** 469 (2003)
100. Odier P, Pinton J-F, Fauve S *Phys. Rev. E* **58** 7397 (1998)
101. Bourgoin M et al. *Magnetohydrodynamics* **40** 3 (2004)
102. Ravelet F et al. *Phys. Fluids* **17** 117104 (2005)
103. Pétrélis F et al. *Phys. Rev. Lett.* **90** 174501 (2003)
104. Volk R et al. *Phys. Rev. Lett.* **97** 074501 (2006)
105. Stefani F et al. *Eur. J. Mech. B* **25** 894 (2006)
106. Giesecke A, Stefani F, Burguete J *Phys. Rev. E* **86** 066303 (2012)
107. Gallet B, Pétrélis F, Fauve S *J. Fluid Mech.* **727** 161 (2013)
108. Cardin P et al. *Magnetohydrodynamics* **38** 177 (2002)
109. Nataf H-C et al. *Geophys. Astrophys. Fluid Dyn.* **100** 281 (2006)
110. Gillet N et al. *J. Fluid Mech.* **580** 83 (2007)
111. Schmitt D et al. *J. Fluid Mech.* **604** 175 (2008)
112. Nataf H-C *Comptes Rendus Physique* **14** 248 (2013)
113. Rieutord M et al. *Phys. Rev. E* **86** 026304 (2012)
114. Lathrop D, in *549. Wilhelm und Else Heraeus-Seminar "Liquid Metal MHD", 15th–18th October, 2013, Physikzentrum Bad Honnef, Germany*
115. Reighard A B, Brown M R *Phys. Rev. Lett.* **86** 2794 (2001)
116. Ruzmaikin A A, Sokoloff D D, Shukurov A M *Magnitnye Polya Galaktik* (Galactic Magnetic Fields) (Moscow: Nauka, 1988)
117. Parker E N *Conversations on Electric and Magnetic Fields in the Cosmos* (Princeton: Princeton Univ. Press, 2007); Translated into Russian: *Besedy ob Elektricheskikh i Magnitnykh Polyakh v Kosmose* (Moscow–Izhevsk: Inst. Komp'yut. Issl., RKhD, 2010)
118. Hughes D W, Proctor M R E, Cattaneo F *Mon. Not. R. Astron. Soc.* **414** L45 (2011)
119. Hale G E et al. *Astrophys. J.* **49** 153 (1919)
120. Stenflo J O, Kosovichev A G *Astrophys. J.* **745** 129 (2012)
121. Tlatov A et al. *Mon. Not. R. Astron. Soc.* **432** 2975 (2013)
122. Cattaneo F, Vainshtein S I *Astrophys. J. Lett.* **376** L21 (1991)
123. Vainshtein S I, Cattaneo F *Astrophys. J.* **393** 165 (1992)
124. Gruzinov A V, Diamond P H *Phys. Rev. Lett.* **72** 1651 (1994)
125. Kleeorin N et al. *Astron. Astrophys.* **361** L5 (2000)
126. Moffatt H K *Magnetic Field Generation in Electrically Conducting Fluids* (Cambridge: Cambridge Univ. Press, 1978); Translated into Russian *Vozbuzhdenie Magnitnogo Polya v Provodyashchei Srede* (Moscow: Mir, 1980)
127. Brandenburg A, Subramanian K *Phys. Rep.* **417** 1 (2005)
128. Paradoksov P *Sov. Phys. Usp.* **9** 618 (1967); *Usp. Fiz. Nauk* **89** 707 (1966)
129. Kleeorin N I, Ruzmaikin A A *Astron. Nachr.* **305** 265 (1984)
130. Seehafer N *Solar Phys.* **125** 219 (1990)
131. Bao S, Zhang H *Astrophys. J. Lett.* **496** L43 (1998)
132. Zhang H et al. *Mon. Not. R. Astron. Soc. Lett.* **402** L30 (2010)
133. Zhang H et al. *Astrophys. J.* **751** 47 (2012)
134. Pipin V V et al. *Astrophys. J.* **768** 46 (2013)
135. Xu H et al. *Astron. Rep.* **53** 160 (2009); *Astron. Zh.* **86** 182 (2009)
136. Gradstein F M, Ogg J G, Smith A G (Eds) *A Geologic Time Scale 2004* (Cambridge: Cambridge Univ. Press, 2005)
137. Pecherskii D M, Sokoloff D D (Comp.) "Paleomagnitnyi Slovar" ("Paleomagnetic Dictionary") (2011), http://paleomag.ifz.ru/books.html
138. Olson P L et al. *Phys. Earth Planet. Inter.* **180** 66 (2010)
139. Galitsky V M, Sokoloff D D *Acta Astron. Geophys. Univ. Comenianae* **19** 1 (1997)
140. Glatzmaier G A et al. *Nature* **401** 885 (1999)
141. Hejda P, Reshetnyak M *Phys. Earth Planet. Inter.* **177** 152 (2009)
142. Ruzmaikin A A, Shukurov A M, Sokoloff D D *Magnetic Fields of Galaxies* (Astrophysics and Space Science Library, Vol. 133) (Dordrecht: Kluwer Acad. Publ., 1988)
143. Subramanian K, Mestel L *Mon. Not. R. Astron. Soc.* **265** 649 (1993)
144. Moss D *Mon. Not. R. Astron. Soc.* **275** 191 (1995)
145. Pipin V V, in *Solar and Astrophysical Dynamos and Magnetic Activity, Proc. of the International Astronomical Union* (IAU Symp., Vol. 294, Eds A G Kosovichev, E de Gouveia Dal Pino, Y Yan) (Cambridge: Cambridge Univ. Press, 2013) p. 375
146. Sobko G S et al. *Geomagn. Aeron.* **52** 254 (2012); *Geomagn. Aeronomiya* **52** 271 (2012)
147. Brandenburg A *Space Sci. Rev.* **144** 87 (2009)
148. Brandenburg A, Sokoloff D, Subramanian K *Space Sci. Rev.* **169** 123 (2012)
149. Brandenburg A, Sokoloff D *Geophys. Astrophys. Fluid Dyn.* **96** 319 (2002)
150. Sur S, Brandenburg A, Subramanian K *Mon. Not. R. Astron. Soc.* **385** L15 (2008)
151. Brandenburg A et al. *Astrophys. J.* **676** 740 (2008)
152. Mitra D et al. *Astron. Astrophys.* **495** 1 (2009)
153. Rheinhardt M, Brandenburg A *Astron. Astrophys.* **520** A28 (2010)
154. Sokoloff D D, Frick P G *Astron. Rep.* **47** 511 (2003); *Astron. Zh.* **80** 556 (2003)
155. Frick P, Stepanov R, Sokoloff D *Phys. Rev. E* **74** 066310 (2006)
156. Stepanov R, Frick P, Sokoloff D *Astron. Nachr.* **327** 481 (2006)
157. Stepanov R A, Frick P G, Sokoloff D D *Vychisl. Mekh. Sploshnykh Sred* **1** (4) 97 (2008)
158. Moss D et al. *Astron. Astrophys.* **537** A68 (2012)
159. Frick P G *Magnetohydrodynamics* **20** 262 (1984); *Magn. Gidrodinamika* **20** (3) 48 (1984)
160. Frick P, Sokoloff D *Phys. Rev. E* **57** 4155 (1998)
161. Mizeva I A, Stepanov R A, Frick P G *Dokl. Phys.* **54** 93 (2009); *Dokl. Ross. Akad. Nauk* **54** 93 (2009)
162. Plunian F, Stepanov R, Frick P *Phys. Rep.* **523** 1 (2013)