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Critical charge in a superstrong magnetic field

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Abstract. **The phenomenon of a critical charge in a superstrong magnetic field is discussed taking into account the screening of the Coulomb potential and the finite size of the nucleus.**

1. Introduction

The problem of a critical nuclear charge was considered for the first time by I Ya Pomeranchuk and Ya A Smorodinsky [1]. They discovered that it is possible to remove the singularity of the solution to the Dirac equation for an electron moving in a Coulomb field. This singularity emerges at the nucleus charge $Z = 137$, when the ground state energy reaches $\varepsilon_0 = 0$. If the finite size of the nucleus is taken into account, the solution of the Dirac equation also exists at larger Z, and the ground energy level drops until it reaches a negative lower continuum, $\varepsilon_0 = -m_e$, where m_e is the electron mass. In Ref. [1], the value of the nucleus charge at which this happens was called critical. According to this paper, $Z_{cr} = 175-200$, depending on the nucleus radius.

A physical picture of the phenomenon which takes place at $Z = Z_{cr} = 172$ was established about 20 years later in Refs [2–7]. When the charge of a hydrogenlike ion reaches the critical value, two e^+e^- pairs are produced from a vacuum. The electrons occupy the ground atomic state, while the positrons are emitted to infinity.

When the results reported in Ref. [1] were obtained, Pomeranchuk exclaimed in delight: "It would be great to collide two uranium nuclei" (see memoirs by Ya A Smorodinsky [8]).

A natural question arises as to whether is it possible to achieve criticality at smaller Z, which correspond to nuclei existing in nature. The answer turned out yes: in external magnetic fields[1] $B > B_0 \equiv m_e^2/e$, even ions with moderate Z are critical [9].

When B grows further, the Coulomb potential of the nucleus becomes screened [10, 11] due to the radiative corrections. And we are going to study how this screening modifies the dependence of Z_{cr} on the magnetic field strength.

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2. Screening of the Coulomb potential in $d = 1$ and $d = 3$

There is a similarity between the radiative corrections to the Coulomb potential in three space dimensions ($d = 3$) in a strong external magnetic field, and in one space dimension ($d = 1$). That is why we are starting from a simpler problem: the Coulomb potential in $d = 1$ [12].

Let us consider $1 + 1$ dimensional QED with massive charged fermions. The electrical potential of the pointlike charge with the account of polarization effects (Fig. 1) takes the form

$$\Phi(k) = -\frac{4\pi g}{k^2 + \Pi(k^2)}\,, \qquad (1)$$

[1] We use the Gaussian units: $e^2 = \alpha = 1/137.0359\ldots$.

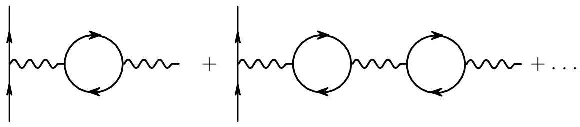

Figure 1. Modification of the Coulomb potential due to 'dressing' of the photon propagator.

where $\Pi(k^2)$ is the one-loop expression for the photon polarization operator:

$$\Pi(k^2) = 4g^2 \left[\frac{1}{\sqrt{t(1+t)}} \ln\left(\sqrt{1+t} + \sqrt{t}\,\right) - 1 \right] \equiv -4g^2 P(t)\,, \tag{2}$$

and $t \equiv -k^2/4m^2$, with g having the dimension of mass.

In the coordinate representation for $k = (0, k_\parallel)$, we obtain

$$\Phi(z) = 4\pi g \int_{-\infty}^{\infty} \frac{\exp(\mathrm{i}k_\parallel z)\, \mathrm{d}k_\parallel/(2\pi)}{k_\parallel^2 + 4g^2 P\left[k_\parallel^2/(4m^2)\right]}\,. \tag{3}$$

With the help of the interpolation formula

$$\bar{P}(t) = \frac{2t}{3+2t}\,, \tag{4}$$

the accuracy of which is better than 10% for $0 < t < \infty$, one obtains

$$\Phi = 4\pi g \int_{-\infty}^{\infty} \frac{\exp(\mathrm{i}k_\parallel z)\, \mathrm{d}k_\parallel/(2\pi)}{k_\parallel^2 + 4g^2\left[k_\parallel^2/(2m^2)\right]/\left[3 + k_\parallel^2/(2m^2)\right]}$$
$$= \frac{4\pi g}{1 + 2g^2/(3m^2)} \left[-\frac{1}{2}\,|z| + \frac{g^2/(3m^2)}{\sqrt{6m^2+4g^2}} \times \exp\left(-\sqrt{6m^2+4g^2}\,|z|\right) \right]. \tag{5}$$

In the case of heavy fermions ($m \gg g$), the potential is given by the tree-level expression; the corrections are suppressed as g^2/m^2.

In the case of light fermions ($m \ll g$), one finds

$$\Phi(z)\big|_{m \ll g} = \begin{cases} \pi \exp\left(-2g|z|\right), & z \ll \dfrac{1}{g} \ln \dfrac{g}{m}\,, \\ -2\pi g\, \dfrac{3m^2}{2g^2}\,|z|\,, & z \gg \dfrac{1}{g} \ln \dfrac{g}{m}\,. \end{cases} \tag{6}$$

The massless case ($m = 0$) corresponds to the Schwinger model: a photon acquires a mass due to the photon polarization operator with massless fermions. Light fermions provide the continuous transition from $m > g$ to $m = 0$.

To get an expression for the Coulomb potential when $d = 3$ in a strong external magnetic field, we need an expression for the polarization operator. It is greatly simplified for $B \gg B_0 \equiv m_\mathrm{e}^2/e$. The following result was obtained in Refs [10, 11]:

$$\Phi(k) = \frac{4\pi e}{k_\parallel^2 + k_\perp^2 + (2e^3B/\pi)\exp\left[-k_\perp^2/(2eB)\right] P\left(k_\parallel^2/(4m_\mathrm{e}^2)\right)}\,, \tag{7}$$

where P is the same as in $d = 1$. A natural question now arises of whether are the two-loop terms enhanced as $(e^3B)^2$. According to Ref. [14], the two-loop corrections are very small, and the physical reason for their smallness is the nullification of the higher loops in $d = 1$ QED with massless fermions (see, e.g., Ref. [15]).

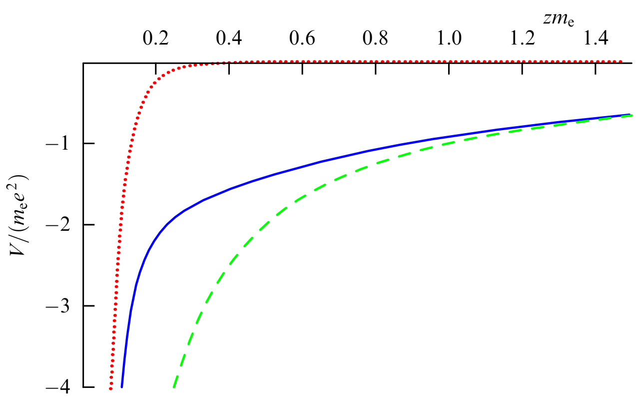

Figure 2. Screened Coulomb potential along the magnetic field ($\rho = 0$) at $B = 5 \times 10^4 B_0$. The dashed (green) line corresponds to the Coulomb potential; solid (blue) line represents the screened potential, and dotted (red) line illustrates the asymptotic behavior of the modified potential at small distances.

In the coordinate representation when $\rho = 0$ (where ρ is the coordinate in the direction transverse to a magnetic field), we obtain

$$\Phi(z) = 4\pi e \int \frac{\exp(\mathrm{i}k_\parallel z)\, \mathrm{d}k_\parallel\, \mathrm{d}^2k_\perp}{(2\pi)^3}$$
$$\times \left[k_\parallel^2 + k_\perp^2 + \frac{2e^3B}{\pi} \exp\left(-\frac{k_\perp^2}{2eB}\right) \frac{k_\parallel^2/(2m_\mathrm{e}^2)}{3 + k_\parallel^2/(2m_\mathrm{e}^2)} \right]^{-1}$$
$$= \frac{e}{|z|}\left[1 - \exp\left(-\sqrt{6m_\mathrm{e}^2}\,|z|\right) + \exp\left(-\sqrt{\frac{2}{\pi}\,e^3B + 6m_\mathrm{e}^2}\,|z|\right)\right]. \tag{8}$$

For $B \ll 3\pi m_\mathrm{e}^2/e^3$, the potential is of a Coulomb type up to the small power suppressed terms:

$$\Phi(z)\big|_{e^3B \ll m_\mathrm{e}^2} = \frac{e}{|z|}\left[1 + O\left(\frac{e^3B}{m_\mathrm{e}^2}\right)\right], \tag{9}$$

in full accordance with the $d = 1$ case, where g^2 plays the role of e^3B.

In the opposite case of superstrong magnetic fields $B \gg 3\pi m_\mathrm{e}^2/e^3$, we arrive at

$$\Phi(z) = \begin{cases} \dfrac{e}{|z|} \exp\left(-|z|\sqrt{\dfrac{2e^3B}{\pi}}\right), & \dfrac{1}{\sqrt{2e^3B/\pi}} \ln\sqrt{\dfrac{e^3B}{3\pi m_\mathrm{e}^2}} > |z| > \dfrac{1}{\sqrt{eB}}\,, \\ \dfrac{e}{|z|}\left[1 - \exp\left(-|z|\sqrt{6m_\mathrm{e}^2}\right)\right], & \dfrac{1}{m_\mathrm{e}} > |z| > \dfrac{1}{\sqrt{2e^3B/\pi}} \ln\sqrt{\dfrac{e^3B}{3\pi m_\mathrm{e}^2}}\,, \\ \dfrac{e}{|z|}\,, & |z| > \dfrac{1}{m_\mathrm{e}}\,. \end{cases} \tag{10}$$

The potential energy of the electron in the modified potential $V(z) = -e\Phi(z)$ is shown in Fig. 2.

3. Energy levels of the electron in the modified potential

3.1 Nonrelativistic approach

The following equation governs the energies of the even states of the hydrogen atom in a strong magnetic field, when taking into account the screening of the Coulomb potential [13]:

$$\ln \frac{B}{m_e^2 e^3 + e^6 B/(3\pi)} = \lambda + 2\ln\lambda + 2\psi\left(1 - \frac{1}{\lambda}\right) + \ln 2 + 4\gamma + \psi\big(1 + |m|\big)\,, \qquad (11)$$

where γ is the Euler constant, $\psi(x)$ is the logarithmic derivative of the gamma-function, $m = 0, -1, -2, \ldots$ is the projection of the electron angular momentum onto the direction of the magnetic field, and the binding energy is defined by λ:

$$E \equiv -\frac{m_e e^4}{2}\,\lambda^2\,. \qquad (12)$$

The analogous equation without screening was derived in Refs [16, 17] (see also review [18]).

In the limit $B \gg 3\pi m_e^2/e^3$, for the ground state energy we get

$$\lambda_{gr} \to 11.2\,, \qquad E_{gr} \to -1.7\ \text{keV}\,. \qquad (13)$$

Freezing of the ground state in the limit $B \to \infty$ was discovered by Shabad and Usov [10, 11].

3.2 Relativistic approach

Without taking screening of the Coulomb potential into account, the problem was solved in the framework of the Dirac equation in Ref. [9]. Let us follow this paper.

The bispinor of the electron on the lowest Landau level looks like the following:

$$\psi_e = \begin{pmatrix} \varphi_e \\ \chi_e \end{pmatrix}, \qquad \varphi_e = \begin{pmatrix} 0 \\ g(z)\exp\left(-\dfrac{\rho^2}{4a_H^2}\right) \end{pmatrix},$$
$$\chi_e = \begin{pmatrix} 0 \\ \mathrm{i} f(z)\exp\left(-\dfrac{\rho^2}{4a_H^2}\right) \end{pmatrix}, \qquad (14)$$

and the Dirac equation

$$\big[\boldsymbol{\alpha}(\mathbf{p} - e\mathbf{A}) + V + \beta m_e\big]\,\psi_e = \varepsilon\psi_e \qquad (15)$$

takes the following form:

$$\begin{cases} g_z - (\varepsilon + m_e - \bar{V})\,f = 0\,, \\ f_z + (\varepsilon - m_e - \bar{V})\,g = 0\,, \end{cases} \qquad (16)$$

where $g_z \equiv \mathrm{d}g/\mathrm{d}z$, and $f_z \equiv \mathrm{d}f/\mathrm{d}z$. System of equations (16) describes electron motion in the effective potential $\bar{V}(z)$ (averaged over fast transverse motion):

$$\bar{V}(z) = -\frac{Ze^2}{a_H^2}\int_0^\infty \frac{\exp\left[-\rho^2/(2a_H^2)\right]}{\sqrt{\rho^2 + z^2}}\,\rho\,\mathrm{d}\rho\,. \qquad (17)$$

At the distances $z \gg a_H$, Eqn (17) is essentially simplified, so that $\bar{V} \approx -Ze^2/|z|$. The solution to system (16) for $\bar{V} = -Ze^2/|z|$ is well known and it is the linear combination of Whittaker functions (see Ref. [9] for details).

The solution at small distances was found in Ref. [9] in the limit of $\bar{V}(z) \gg 2m_e$, i.e., for $|z| \ll Ze^2/(2m_e)$. Therefore, there is a matching region in the nonscreened case: $a_H \ll |z| \ll Ze^2/2m_e$, as soon as the condition $B \gg B_0/(Ze^2)^2$ is satisfied.

Matching the long-distance and short-distance solutions yields the equation for the energy levels of the electron in the Coulomb field of the nucleus with the charge Z and in the external magnetic field B, which was given in Ref. [9]. This equation allows us to find the magnetic field B_{cr} at which the ions with the charge Z become critical, i.e., the ground state energy reaches a lower continuum, $\varepsilon_0 = -m_e$:

$$\frac{B_{cr}}{B_0} = 2(Ze^2)^2\exp\left[-\gamma + \frac{\pi - 2\arg\Gamma(1 + 2\mathrm{i}Ze^2)}{Ze^2}\right], \qquad (18)$$

where $\Gamma(\ldots)$ is the gamma-function. According to the last formula, uranium becomes critical at $B \approx 10^2 B_0$, and in stronger magnetic fields even ions with smaller Z are critical.

To take screening into account, instead of expression (17) one should use the following formula for $\bar{V}$:

$$\bar{V}(z) = -\frac{Ze^2}{a_H^2}\left[1 - \exp\left(-\sqrt{6m_e^2}\,|z|\right) + \exp\left(-\sqrt{\frac{2}{\pi}\,e^3 B + 6m_e^2}\,|z|\right)\right]\int_0^\infty \frac{\exp\left(-\rho^2/2a_H^2\right)}{\sqrt{\rho^2 + z^2}}\,\rho\,\mathrm{d}\rho. \qquad (19)$$

We see that the screened Coulomb potential follows its asymptotic behavior, $\bar{V} = -Ze^2/|z|$, only at large distances, $|z| \gg 1/m_e$. Thus, it is impossible to match two solutions, as was done in Ref. [9], since the solution at small distances is valid only for $|z| < Ze^2/m_e$. That is why the analytical equation for the ground energy level in the screened potential has not been derived yet.

For this reason, we have solved the problem numerically. Following Popov [6], we reduced the Dirac equation to the effective Schrödinger equation

$$\frac{\mathrm{d}^2\chi}{\mathrm{d}z^2} + 2m_e(E - U)\chi = 0\,, \qquad (20)$$
$$E = \frac{\varepsilon^2 - m_e^2}{2m_e}\,,$$
$$U = \frac{\varepsilon}{m_e}\,\bar{V} - \frac{1}{2m_e}\,\bar{V}^2 + \frac{\bar{V}''}{4m_e(\varepsilon + m_e - \bar{V})} + \frac{3/8(\bar{V}')^2}{m_e(\varepsilon + m_e - \bar{V})^2}\,.$$

For $B \ll B_0$, relativistic corrections are small, and the binding energy $E \approx \varepsilon - m_e$ is defined by the nonrelativistic equation. However, for $B \gg B_0$ relativistic corrections grow as powers of B/B_0 and correction terms have different signs, considerably complicating the numerical calculations.

We have found that the relativistic corrections for a hydrogen atom are small even in very strong magnetic fields, and the value of the freezing energy barely changes. We have also considered ions with larger Z and revealed the freezing effect in the relativistic domain. For example, the ground energy level for ions with $Z = 40$ freezes at $\varepsilon_0 \approx -m_e/2$.

The freezing of the ground energy level is crucial for the phenomenon of the critical nucleus charge. We have found that the ions with $Z < 50$ never become critical and calculated the values of the critical magnetic field B_{cr} for the ions with larger Z. These results are given in Fig. 3. The ions with $Z \lesssim 55$ achieve criticality in such a strong magnetic field that a_H becomes smaller than the size of the nucleus. Thus, the finiteness of the nucleus radius should be taken into consideration.

Without screening of the Coulomb potential, the magnetic field B appears in formula (17) only through Landau

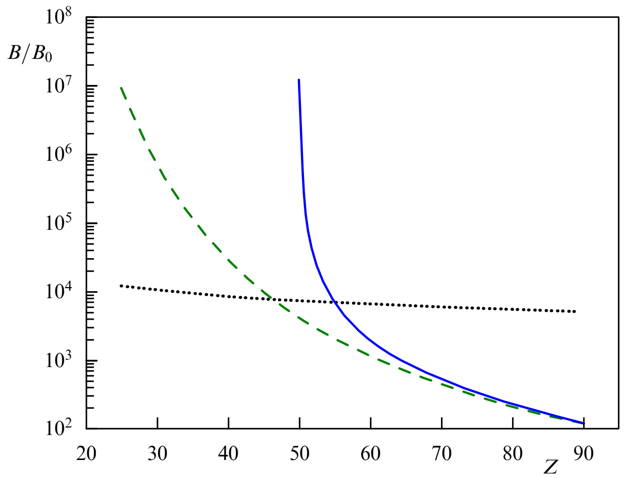

Figure 3. Critical magnetic field expressed in units of B_0. The dashed (green) line is the fit by formula (18) originally obtained in Ref. [9]; the solid (blue) line corresponds to numerical results with the account of screening according to Ref. [19]. The dotted (black) line corresponds to the magnetic field in which Landau radius a_H becomes smaller than the size of the nucleus.

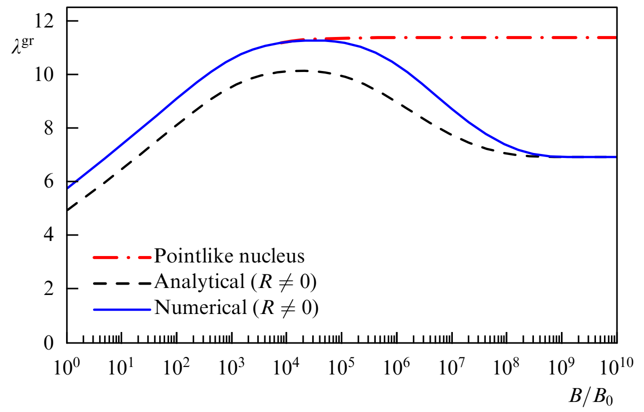

Figure 4. Dependence of λ^{gr} on the magnetic field. The dashed-dotted (red) line corresponds to the pointlike nucleus; the dashed (green) line is the fit by the analytical formula (22) for $h(|z|) = 1$; the solid (blue) line corresponds to the numerical solution at $h(|z|) = 1$.

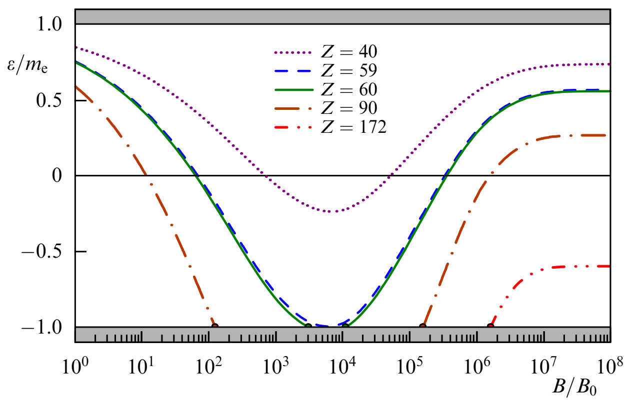

Figure 5. Dependence of the ground state energy on the magnetic field for $Z = 40, 59, 60, 90, 172$. The correspondence between charge Z and the line style (color) is shown in the legend to the figure.

radius $a_H \equiv 1/\sqrt{eB}$. When a_H becomes smaller than the nucleus size R, one should substitute R for a_H. This means that the spectrum we are looking for coincides with the one in the magnetic field $B = 1/(eR^2)$, which corresponds to $a_H = R$.

However, the magnetic field B appears directly in the expression for the screened potential (19). As a result, special consideration is needed in the case of screening.

4. Finite nucleus size and the ground energy level

The three-dimensional formula for the screened potential has not been derived yet, and the distribution of the electric charge inside a nucleus in such a strong magnetic fields is not known. Thus, one cannot find the analytical formula for the electrical potential of a nucleus having a finite size. However, we have found the approximate expression for the potential along the magnetic field (see Ref. [20] for details). In the case of protons ($Z = 1$), it looks like this:

$$\Phi(z) = \begin{cases} \dfrac{e}{|z|}\left[1 - \exp\left(-|z|\sqrt{6m_e^2}\right) + h(R)\exp\left(-\mu|z|\right)\right], & |z| \geqslant R, \\ \dfrac{e}{R}\left[1 - \exp\left(-R\sqrt{6m_e^2}\right) + h(|z|)\exp\left(-\mu R\right)\right], & |z| < R, \end{cases} \tag{21}$$

where $h(|z|)$ is determined by the charge distribution inside the proton, $\mu \equiv \sqrt{6m_e^2 + (2e^3B/\pi)}$, and $R = 0.877$ fm is the proton charge radius.

Formula (21) allows us to derive an approximate nonrelativistic formula for the hydrogen energy levels analogous to Eqn (11):

$$\ln\frac{a_B}{\sqrt{R^2 + a_H^2}} - E_1\left(\sqrt{R^2 + a_H^2}\sqrt{6m_e^2}\right) + h(R)E_1\left(\mu\sqrt{R^2 + a_H^2}\right) = \frac{\lambda}{2} + \ln\lambda + \psi\left(1 - \frac{1}{\lambda}\right) + 2\gamma + \ln 2, \tag{22}$$

where $a_B \equiv 1/(m_e\alpha)$ is the Bohr radius; λ defines the electron binding energy, $E \equiv -(m_e e^4/2)\lambda^2$, and

$$E_1(x) \equiv \int_x^\infty \frac{\exp(-t)}{t}\,dt\,. \tag{23}$$

According to formula (22), the value of λ in the limit of $B \to \infty$ equals 6.9 instead of $\lambda = 11.2$, which was obtained for the pointlike proton. The dependence of λ^{gr} (corresponding to the ground energy level) on the magnetic field at $h(|z|) = 1$ is shown in Fig. 4.[2] It is evident that the ground energy level goes up (and the binding energy diminishes) until it reaches the final freezing energy. This effect is even more pronounced for heavier ions (Fig. 5). Due to the rise in the ground state energy, the ions with $Z = 60-210$ stop being critical in a strong enough magnetic field. Even the ion with $Z = 172$ becomes noncritical for $B/B_0 \gtrsim 2 \times 10^6$, while it is critical in the absence of the magnetic field. At $Z \approx 210$, the final freezing energy reaches a lower continuum, and the nuclei with $Z > 210$ *are critical regardless of the magnetic field strength*. In Fig. 6, the dependence of the critical nucleus charge on the magnetic field B is given.

[2] The function $h(|z|) = 1$ was chosen for simplicity, and it was verified that other distributions (like that for the homogeneously charged sphere) lead to quite close results (see Ref. [20] for details).

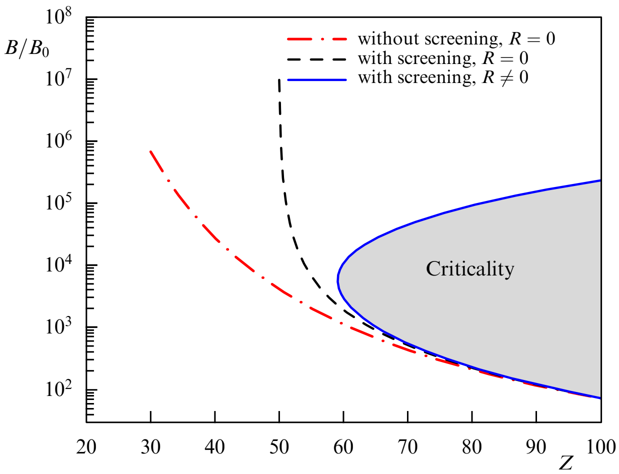

Figure 6. Values of the critical magnetic field. The dashed-dotted (red) line corresponds to formula (18); the dashed (green) line represents the numerical results for the screened potential of the pointlike charge; the solid (blue) line corresponds to the numerical results with account for both the screening and the finite nucleus size.

5. Conclusion

The influence of the Coulomb potential screening and the finite nucleus size on the energy levels of hydrogenlike ions has been studied. Both screening and the finite nucleus size push the ground energy level up. Screening starts at $B \sim m_e^2/e^3 \approx 6 \times 10^{15}$ G and leads to the freezing of the ground state energy. The finite nucleus radius R comes into play at $B \sim 1/(eR^2) = 10^{17} - 10^{18}$ G, and the ground energy level rises until it reaches the final freezing energy in the magnetic field of $B \sim 1/(e^3 R^2) = 10^{19} - 10^{20}$ G.

The dependence of the ground state energy on the magnetic field was calculated analytically using a nonrelativistic approach, and numerically by solving the Dirac equation. Our main result comprises the calculation of the critical nucleus charge in the magnetic field, whose outcome is shown in Fig. 6.

The effects discussed manifest themselves only in superstrong magnetic fields which have not been found in Nature yet. However, considering such an asymptotic behavior is in the spirit of I Ya Pomeranchuk's approach. According to Ya A Smorodinsky [8], looking for the asymptotic behavior of different quantities was Pomeranchuk's approach to various physical problems (extremely low temperatures or extremely high energies).

Acknowledgments
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