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Gravitational four-fermion interaction in the early Universe

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Contents

Abstract. **If torsion exists, it generates gravitational four-fermion interaction (GFFI), essential on the Planck scale. We analyze the influence of this interaction on the Friedmann–Lemaitre–Robertson–Walker cosmology. An explicit analytic solution is derived for the problem where both the energy–momentum tensor generated by GFFI and the common ultra-relativistic energy–momentum tensor are included. We demonstrate that gravitational four-fermion interaction does not result in a Big Bounce.**

1. Introduction

According to the common belief, the present-day expansion of the Universe is the result of the Big Bang. The idea is popular that this expansion had been preceded by compression with a subsequent Big Bounce. We here analyze the assumption that the Big Bounce is due to the gravitational four-fermion interaction.

The observation that in the presence of (nonpropagating) torsion, the interaction of fermions with gravity results in the four-fermion interaction of axial currents, goes back at least to [1, 2].

The most general form of the gravitational four-fermion interaction is as follows:

$$S_{\mathrm{ff}} = \frac{3\pi G\gamma^2}{2(\gamma^2+1)} \int \mathrm{d}^4x\sqrt{-g}\,\eta_{IJ}\left[\left(1-\beta^2+\frac{2\beta}{\gamma}\right)A^I A^J - 2\alpha\left(\beta-\frac{1}{\gamma}\right)V^I A^J - \alpha^2 V^I V^J\right]. \quad (1)$$

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Here and below, G is the Newton gravitational constant, g is the determinant of the metric tensor, A^I and V^I are the total axial and vector neutral currents,

$$A^I = \sum_a A^I_a = \sum_a \bar{\psi}_a \gamma^5 \gamma^I \psi_a\,,$$
$$V^I = \sum_a V^I_a = \sum_a \bar{\psi}_a \gamma^I \psi_a\,, \quad (2)$$

where the sums over a in (2) extend over all species of elementary fermions with spin 1/2, and α, β, and γ are numerical parameters of the problem. The values of α and β are unknown. For to the so-called Barbero–Immirzi parameter γ, we assume the value $\gamma = 0.274$ [3]. In fact, the exact numerical values of these parameters are insignificant in our problem.

The AA contribution to expression (1) corresponds (up to a factor) to the action derived long ago in [1, 2]. This contribution was subsequently obtained in the limit $\beta \to 0$, $\gamma \to \infty$ in [4]. The present form of the AA interaction, given in (1), was derived in [5, 6]. The VV and VA terms in (1) were respectively derived in [7] and [6, 7].

Simple dimensional arguments demonstrate that interaction (1), being proportional to the Newton constant G and to the particle number density squared, n^2, could be significant and comparable to the common interactions only at very high densities, i.e., on the Planck scale.

A quite extensive list of references of papers where the gravitational four-fermion interaction is discussed in connection with cosmology can be found in [6, 8, 9].

2. Energy–momentum tensor

The energy–momentum tensor (EMT) $T^{\mathrm{ff}}_{\mu\nu}$ generated by action (1) is

$$T^{\mathrm{ff}}_{\mu\nu} = -\frac{3\pi}{2}\,G\,\frac{\gamma^2}{\gamma^2+1}\,g_{\mu\nu}\eta_{IJ}\left[\left(1-\beta^2+\frac{2\beta}{\gamma}\right)A^I A^J - 2\alpha\left(\beta-\frac{1}{\gamma}\right)V^I A^J - \alpha^2 V^I V^J\right]. \quad (3)$$

The nonvanishing components of expression (3), written in a locally inertial frame, are the energy density $T^{\mathrm{ff}}_{00} = \rho_{\mathrm{ff}}$ and the pressure $T_{11}^{\rm ff} = T_{22}^{\rm ff} = T_{33}^{\rm ff} = p_{\rm ff}$ (they are marked by ff here and below to indicate their origin from the four-fermion interaction; for the correspondence among ρ, p, and EMT components, see [10], § 35).

We analyze the expressions for $\rho_{\rm ff}$ and $p_{\rm ff}$ in our case of the interaction of two ultrarelativistic fermions (labeled a and b) in their locally inertial center-of-mass system. We here follow the argument in [11]. The axial and vector currents of fermion a are

$$A_a^I = \frac{1}{4E^2}\,\phi_a^\dagger\{E\,\boldsymbol{\sigma}_a(\mathbf{p}'+\mathbf{p}),\ \ (E^2-\mathbf{p}'\mathbf{p})\,\boldsymbol{\sigma}_a+\mathbf{p}'(\boldsymbol{\sigma}_a\mathbf{p}) + \mathbf{p}(\boldsymbol{\sigma}_a\mathbf{p}') - \mathrm{i}\mathbf{p}'\times\mathbf{p}\}\,\phi_a = \frac{1}{4}\,\phi_a^\dagger\{\boldsymbol{\sigma}_a(\mathbf{n}'+\mathbf{n}),\,(1-\mathbf{n}'\mathbf{n}) \times\,\boldsymbol{\sigma}_a+\mathbf{n}'(\boldsymbol{\sigma}_a\,\mathbf{n})+\mathbf{n}\,(\boldsymbol{\sigma}_a\mathbf{n}')-\mathrm{i}\mathbf{n}'\times\mathbf{n}\}\,\phi_a\,, \tag{4}$$

$$V_a^I = \frac{1}{4E^2}\,\phi_a^\dagger\{E^2+\mathbf{p}'\mathbf{p}+\mathrm{i}\boldsymbol{\sigma}_a[\mathbf{p}'\times\mathbf{p}]\,,\,E\left(\mathbf{p}'+\mathbf{p}-\mathrm{i}\boldsymbol{\sigma}_a \times(\mathbf{p}'-\mathbf{p})\right)\}\,\phi_a = \frac{1}{4}\,\phi_a^\dagger\{1+\mathbf{n}'\mathbf{n}+\mathrm{i}\boldsymbol{\sigma}_a[\mathbf{n}'\times\mathbf{n}]\,,\,\mathbf{n}'+\mathbf{n} - \mathrm{i}\boldsymbol{\sigma}_a\times(\mathbf{n}'-\mathbf{n})\}\,\phi_a\,, \tag{5}$$

where E is the energy of fermion a, ϕ_a is a two-component spinor, and $\mathbf{n}$ and $\mathbf{n}'$ are the unit vectors of its initial and final momenta $\mathbf{p}$ and $\mathbf{p}'$; under the discussed extreme conditions, all fermion masses can be neglected. In the center-of-mass system, the axial and vector currents of fermion b are obtained from these expressions by changing the signs: $\mathbf{n}\to-\mathbf{n}$, $\mathbf{n}'\to-\mathbf{n}'$. Then, after averaging over the directions of $\mathbf{n}$ and $\mathbf{n}'$, we arrive at the following semiclassical expressions for the nonvanishing components of the energy–momentum tensor, i.e., for the energy density $\rho_{\rm ff}$ and pressure $p_{\rm ff}$:

$$\rho_{\rm ff} = T_{00} = -\frac{\pi}{48}\,G\,\frac{\gamma^2}{\gamma^2+1}\sum_{a,\,b} n_a n_b\left[\left(1-\beta^2+\frac{2\beta}{\gamma}\right) \times\left(3-11\langle\boldsymbol{\sigma}_a\boldsymbol{\sigma}_b\rangle\right)-\alpha^2\left(60-28\langle\boldsymbol{\sigma}_a\boldsymbol{\sigma}_b\rangle\right)\right] = -\frac{\pi}{48}\,G\,\frac{\gamma^2}{\gamma^2+1}\,n^2\left[\left(1-\beta^2+\frac{2\beta}{\gamma}\right) \times(3-11\zeta)-\alpha^2(60-28\zeta)\right], \tag{6}$$

$$p_{\rm ff} = T_{11} = T_{22} = T_{33} = \frac{\pi}{48}\,G\frac{\gamma^2}{\gamma^2+1}\sum_{a,\,b} n_a\,n_b \times\left[\left(1-\beta^2+\frac{2\beta}{\gamma}\right)\left(3-11\langle\boldsymbol{\sigma}_a\boldsymbol{\sigma}_b\rangle\right)-\alpha^2\left(60-28\langle\boldsymbol{\sigma}_a\boldsymbol{\sigma}_b\rangle\right)\right] = \frac{\pi}{48}\,G\frac{\gamma^2}{\gamma^2+1}\,n^2\left[\left(1-\beta^2+\frac{2\beta}{\gamma}\right) \times(3-11\zeta)-\alpha^2(60-28\zeta)\right]. \tag{7}$$

Here and below, n_a and n_b are the number densities of the corresponding species of fermions and antifermions, $n=\sum_a n_a$ is the total density of fermions and antifermions, the summation $\sum_{a,b}$ extends over all species of fermions and antifermions, and $\zeta=\langle\boldsymbol{\sigma}_a\boldsymbol{\sigma}_b\rangle$ is the average value of the product of corresponding $\boldsymbol{\sigma}$ matrices, presumably universal for any $a\neq b$. Because the number of fermion and antifermion species is large, we can neglect the exchange and annihilation contributions for numerical reasons, and also neglect the fact that if $\boldsymbol{\sigma}_a$ and $\boldsymbol{\sigma}_b$ refer to the same particle, then $\langle\boldsymbol{\sigma}_a\boldsymbol{\sigma}_b\rangle=3$. It is only natural that after the performed averaging over all momenta orientations, the P-odd contributions of VA to $\rho_{\rm ff}$ and $p_{\rm ff}$ vanish.

Thus, the equation of state (EOS) becomes

$$\rho_{\rm ff} = -p_{\rm ff} = -\frac{\pi}{48}\,G\,\frac{\gamma^2}{\gamma^2+1}\,n^2 \times\left[\left(1-\beta^2+\frac{2\beta}{\gamma}\right)(3-11\zeta)-\alpha^2(60-28\zeta)\right]. \tag{8}$$

Four-fermion energy density (8) can then be conveniently rewritten as

$$\rho_{\rm ff} = \varepsilon G n^2\,,\quad \varepsilon = -\frac{\pi}{48}\,\frac{\gamma^2}{\gamma^2+1} \times\left[\left(1-\beta^2+\frac{2\beta}{\gamma}\right)(3-11\zeta)-\alpha^2(60-28\zeta)\right]. \tag{9}$$

The parameter $\zeta=\langle\boldsymbol{\sigma}_a\boldsymbol{\sigma}_b\rangle$ for $a\neq b$, just by its physical meaning, can in principle range the interval from 0 (which corresponds to complete thermal incoherence or to antiferromagnetic ordering) to 1 (which corresponds to complete ferromagnetic ordering). Correspondingly, ε ranges from

$$\varepsilon = -\frac{\pi}{16}\,\frac{\gamma^2}{\gamma^2+1}\left(1-\beta^2+\frac{2\beta}{\gamma}-20\alpha^2\right)\ \text{at}\ \zeta=0 \tag{10}$$

to

$$\varepsilon = \frac{\pi}{6}\,\frac{\gamma^2}{\gamma^2+1}\left(1-\beta^2+\frac{2\beta}{\gamma}+4\alpha^2\right)\ \text{at}\ \zeta=1\,. \tag{11}$$

The absolute numerical value of the parameter ε is inessential for the analysis below. Its sign, however, is crucial for the physical implications, and depends on α, β, and ζ. As regards ζ, at the discussed extreme conditions of high densities and high temperatures, this correlation function is most probably negligibly small.

We next discuss the contributions of common matter to the energy density and pressure. For extreme densities, where gravitational four-fermion interaction is significant, this matter is certainly ultrarelativistic, and its contribution to the energy density can be written, for simple dimensional reasons, as

$$\rho = \nu n^{4/3}\,, \tag{12}$$

where ν is a numerical factor. One power of $n^{1/3}$ here is an estimate of the energy per particle. Another factor n in this expression is the total density of ultrarelativistic particles and antiparticles, fermions and bosons, contributing to (12). Because bosons also contribute to the total energy density, this factor should exceed the fermion density n entering the above four-fermion expressions. This difference, however, is absorbed in (12) by the factor ν. As was the case with $\rho_{\rm ff}$, it is natural to assume that ρ is also independent of the spin correlations.

We now consider the energy–momentum tensor of the common ultrarelativistic matter in our problem. Because the problem is isotropic, the mixed components of the energy–momentum tensor must vanish:

$$T_{0m} = T_{m0} = 0\,,\quad m=1,2,3\,.$$

Then the spatial components of the energy–momentum tensor can be diagonalized, and again due to the isotropy, we arrive at

$$T_{11} = T_{22} = T_{33} .$$

Finally, the trace of the energy–momentum tensor of this ultrarelativistic matter should vanish, $T^{\mu}_{\mu} = 0$. Hence, the discussed energy–momentum tensor can be written as

$$T^{\mu}_{\nu} = \rho \, \mathrm{diag}\left(1, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}\right),$$

$$\text{or} \quad T_{\mu\nu} = \rho \, \mathrm{diag}\left(1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right). \tag{13}$$

Here and below, ρ is the energy density of the common ultrarelativistic matter, and its pressure is $p = \rho/3$.

With $\rho_{\mathrm{ff}} \sim Gn^2$, close to the Planck scale, gravitational four-fermion interaction is quite comparable to $\rho \sim n^{4/3}$, and therefore both contributions should be included on this scale. Unfortunately, in our previous papers on the subject, the contribution of the common ultrarelativistic matter was not taken into account.

3. Friedmann–Lemaitre–Robertson–Walker equations

We assume that even on a scale close to the Planck one, the Universe is homogeneous and isotropic, and can therefore be described by the Friedmann–Lemaitre–Robertson–Walker (FLRW) metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left[\mathrm{d}r^2 + f(r)\,(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right], \tag{14}$$

where $f(r)$ depends on the topology of the Universe as a whole:

$$f(r) = r^2, \quad \sin^2 r, \quad \sinh^2 r$$

respectively for the spatial flat, closed, and open Universe.

Now, the total energy density and total pressure are

$$\rho_{\mathrm{tot}} = \rho_{\mathrm{ff}} + \rho, \quad p_{\mathrm{tot}} = -\rho_{\mathrm{ff}} + \frac{1}{3}\rho .$$

The fact that ρ_{ff} and ρ enter the expression for the total pressure with opposite signs can be traced back to the difference between algebraic structures of the tensors T^{ff} and T. The first is proportional to $\delta^{\mu}_{\nu} = \mathrm{diag}\,(1, 1, 1, 1)$ in the mixed components, and the second is proportional to $\mathrm{diag}\,(1, -1/3, -1/3, -1/3)$.

Thus, the Einstein equations for FLRW metric (14) are

$$\left(\frac{\dot a}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\,\rho_{\mathrm{tot}} = \frac{8\pi G}{3}\,(\rho_{\mathrm{ff}} + \rho), \tag{15}$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}\,(\rho_{\mathrm{tot}} + 3p_{\mathrm{tot}}) = \frac{8\pi G}{3}\,(\rho_{\mathrm{ff}} - \rho). \tag{16}$$

The parameter k in Eqn (15) is respectively equal to 0, 1, and -1 for the spatial flat, closed, and open Universe. These equations result in the covariant conservation law for the total energy–momentum tensor:

$$\dot\rho_{\mathrm{tot}} + 3\,\frac{\dot a}{a}\,(\rho_{\mathrm{tot}} + p_{\mathrm{tot}}) = \dot\rho_{\mathrm{ff}} + \dot\rho + 4\,\frac{\dot a}{a}\,\rho = 0 . \tag{17}$$

We note that in the absence of the four-fermion interaction, i.e., for $\rho_{\mathrm{ff}} = 0$, this equation reduces to the well-known equation for standard ultrarelativistic matter: $\dot\rho + 4(\dot a/a)\,\rho = 0$.

On the other hand, without standard matter, i.e., for $\rho = 0$, Eqn (17) degenerates into $\dot\rho_{\mathrm{ff}} = 0$. This is quite natural, since energy–momentum tensor (3), generated by the four-fermion interaction, can be conserved by itself only if $\rho_{\mathrm{ff}} = \mathrm{const}$ [12].

In fact, observational data strongly favor the idea that our Universe is spatially flat, i.e., $k = 0$. Then Eqn (15) is simplified to

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\,(\rho_{\mathrm{ff}} + \rho). \tag{18}$$

Obviously, if the gravitational four-fermion interaction exists, our equations (15)–(18) are as firmly established as the common FLRW equations in the absence of gravitational four-fermion interaction.

4. Solutions and conclusions

We now turn to the solution of the FLRW equations. With the substitution

$$a(t) = a_0 \exp f(t), \tag{19}$$

Eqns (16) and (18) become

$$\frac{8\pi G}{3}\,(\rho_{\mathrm{ff}} + \rho) = \dot f^2, \tag{20}$$

$$\frac{8\pi G}{3}\,\rho = -\frac{1}{2}\,\ddot f. \tag{21}$$

Differentiating Eqn (20) with respect to t and combining the result with (21), we arrive at the solution

$$f = -\frac{3}{4\nu}\,\varepsilon G n^{2/3} - \frac{1}{3}\ln n, \tag{22}$$

with the numerical factor ν introduced in (12). A comment on the ratio $\varepsilon G/\nu$ in this expression is pertinent. It can be easily demonstrated that in the absence of the four-fermion interaction, the relation $f = -(1/3)\ln n$ implies the law $a(t) = \sqrt{t}$. Therefore, it is only natural that the relative weight of the four-fermion interaction enters formula (22) via the ratio $\varepsilon G/\nu$.

Thus, we obtain

$$a(t) = a_0 \exp f(t) \sim n^{-1/3} \exp\left(-\frac{3}{4\nu}\,\varepsilon G n^{2/3}\right). \tag{23}$$

We introduce the dimensionless ratio $\xi(t)$ of the four-fermion energy density ρ_{ff} and the energy density ρ of ultrarelativistic matter:

$$\xi(t) = \frac{\rho_{\mathrm{ff}}}{\rho} = \frac{\varepsilon G}{\nu}\,n^{2/3}. \tag{24}$$

Then

$$a(t) \sim \frac{1}{\sqrt{\xi(t)}}\exp\left(-\frac{3}{4}\,\xi(t)\right). \tag{25}$$

Combining Eqns (20) and (22), we arrive at

$$\dot\xi = \mp\frac{4}{3}\sqrt{\frac{8\pi G}{3}\,\frac{\nu^{3/2}}{\varepsilon G}}\;\xi^2\,\frac{\sqrt{\xi + 1}}{\xi + 2/3}, \tag{26}$$

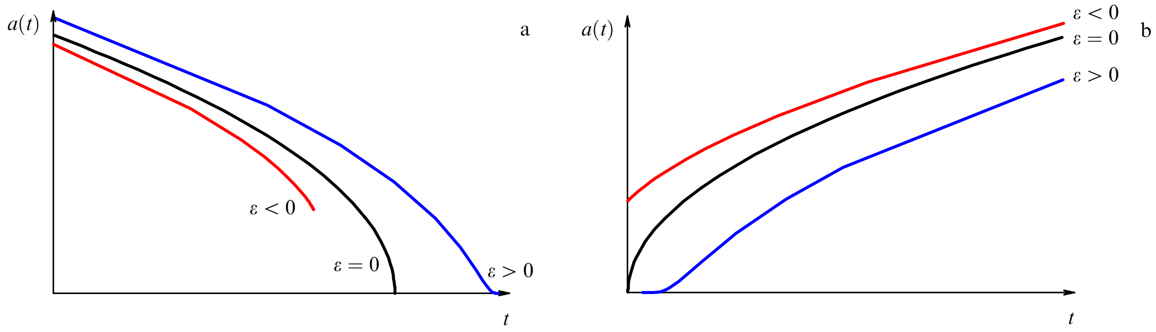

Figure. (Color online). Time dependence of the scale factor.

which yields relations between ξ and t:

$$\ln\left(\frac{\sqrt{\xi(t)}}{1+\sqrt{1+\xi(t)}}\right)-\frac{1}{2}\,\frac{\sqrt{1+\xi(t)}}{\xi(t)}$$
$$=\mp\sqrt{\frac{8\pi G}{3}}\,\frac{v^{3/2}}{\varepsilon G}\,t+\text{const}\quad\text{for}\quad\varepsilon>0\,, \tag{27}$$

$$-\ln\left(\frac{\sqrt{|\xi(t)|}}{1+\sqrt{1-|\xi(t)|}}\right)-\frac{1}{2}\,\frac{\sqrt{1-|\xi(t)|}}{|\xi(t)|}$$
$$=\mp\sqrt{\frac{8\pi G}{3}}\,\frac{v^{3/2}}{|\varepsilon|\,G}\,t+\text{const}\quad\text{for}\quad\varepsilon<0\,. \tag{28}$$

The constants in the right-hand sides of (27) and (28) are fixed by initial conditions. As regards the signs in formulas (26)–(28), $-$ and $+$ therein respectively refer to expansion and compression.

The physical implications of formula (23) for positive and negative values of ε are quite different.

For positive ε, both factors in (23), $n^{-1/3}$ and $\exp\left[-3/(4v)\,\varepsilon G n^{2/3}\right]$, and of course their product $a(t)$, shrink to zero as the density n increases. To analyze the compression, we rewrite Eqns (16) and (18) as

$$\dot a=-\sqrt{\frac{8\pi G}{3}}\,a\sqrt{\rho_{\text{ff}}+\rho}\,, \tag{29}$$

$$\ddot a=\frac{8\pi G}{3}\,a(\rho_{\text{ff}}-\rho)\,. \tag{30}$$

At the initial moment, when $\rho_{\text{ff}}\ll\rho$, both $\dot a$ and $\ddot a$ are negative; therefore, the Universe shrinks with acceleration. Then, at $\rho_{\text{ff}}=\rho$, the acceleration $\ddot a$ changes sign, while $\dot a$ remains negative; therefore, the compression of the Universe decelerates. According to relations (23) and (27), it takes finite time for a to shrink to zero. Due to the exponential factor in (23), $\dot a$ and $\ddot a$ also vanish at the same moment (the curve $\varepsilon>0$ in Fig. a). Therefore, repulsive gravitational four-fermion interaction does not stop the collapse, but only reduces its rate. The asymptotic behavior of $a(t)$ is

$$a(t)\sim(t_1-t)\,\exp\left(-\frac{9\varepsilon^2 G}{128\pi v^3}\,\frac{1}{(t_1-t)^2}\right), \tag{31}$$

where t_1 is the moment of the collapse for $\varepsilon>0$.

For negative ε, the situation is different. Here, the right-hand side of (25),

$$a(t)\sim\frac{1}{\sqrt{|\xi(t)|}}\,\exp\left(\frac{3}{4}|\xi(t)|\right),$$

reaches its minimum value at $|\xi_{\text{m}}|=2/3$, i.e., $a(t)$ cannot decrease further. It follows from (18), however, that the compression rate $\dot a$ at this point does not vanish and remains finite (the curve $\varepsilon<0$ in Fig. a). In a sense, the situation here resembles that in the standard cosmology with ultrarelativistic particles: there, $a(t)\sim\sqrt{t_0-t}\to 0$ as $t\to t_0$ (t_0 is the moment of the collapse in this case), although $\dot a$ does not vanish, but tends to infinity at this point (the curve $\varepsilon=0$ in Fig. a). In the standard cosmology, we do not expect that this compression to the origin is followed by expansion. Therefore, in the present case, with $\varepsilon<0$, it looks natural to also assume that the compression does not change to expansion.

Thus, contrary to possible naïve expectations [11], the gravitational four-fermion interaction does not result in the Big Bounce.

We note in conclusion that it is difficult (if not impossible) to imagine a realistic possibility of detecting any effect of the gravitational four-fermion interaction.

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