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Cyclotron autoresonance — 50 years since its discovery

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Contents

Abstract. **Commemorating the 50th anniversary of the discovery of cyclotron autoresonance, this paper briefly reviews how the experimental and theoretical aspects of this field of research have been developed over time.**

1. Introduction

Fifty years ago, A A Kolomenskii, A N Lebedev [1] (see also Ref. [2]), and independently V Ya Davydovskii [3] discovered *cyclotron autoresonance* effect, which is a remarkable phenomenon in physics. The gist of this phenomenon resides in that the initial condition of the cyclotron autoresonance of a particle with an electromagnetic wave propagating along a constant magnetic field persists 'by itself' during all the time of particle motion, i.e., is the integral of motion. Cyclotron autoresonance is based on the fact that, in the relativistic motion of a charged particle in the field of a plane electromagnetic wave propagating along a constant magnetic field (z-axis), an exact integral exists, which was first found by I A Gilinskii [4]:

$$N\gamma - P_z = I = \text{const} . \qquad (1)$$

Here, γ is a relativistic factor, P_z is a dimensionless projection of particle momentum onto the direction of the magnetic field (in mc units), $N = kc/\omega \equiv \beta_{\text{ph}}^{-1}$ is the refractive index, and $\beta_{\text{ph}} = v_{\text{ph}}/c$ is a dimensionless phase velocity of the wave.

The condition of the particle–wave cyclotron resonance is written down as

$$\omega - kv_z = \frac{\omega_{\text{c0}}}{\gamma} \equiv \omega_{\text{c}} ,$$

or in the dimensionless form as

$$\gamma - NP_z = \Omega_0 , \qquad (2)$$

where $\Omega_0 = eB_0/(mc\omega) \equiv \omega_{\text{c0}}/\omega$ is the ratio of the classical gyrofrequency to the wave frequency.

In a constant external magnetic field, the dimensionless gyrofrequency $\Omega_0 = \text{const}$. The cyclotron resonance condition (2) holds true all the time during particle motion — that is, it coincides with the integral of motion (1) in the case of a 'vacuum wave' only, when

$$N = 1 , \qquad I = \Omega_0 . \qquad (3)$$

This mode of motion of charged particles is precisely the *cyclotron autoresonance* [1, 2]. The parameter Ω_0 varies over a wide range, namely, in the case of a light wave for electrons one has $\Omega_0 \sim 10^{-5}$, and in the centimeter wave range Ω_0 may be greater than unity.

The physical mechanism of cyclotron autoresonance consists of the following: the exact resonance condition (2) which holds true at the initial instant of time can be violated by the Doppler frequency shift and a relativistic change in the cyclotron frequency. For $N > 1$, the Doppler shift prevails, while for $N < 1$, the relativistic change in the cyclotron frequency dominates. It is only for $N = 1$ that these two competing effects are mutually compensated. A deeper reason for the appearance of autoresonance in the case of $N = 1$ only is as follows [5]. For a fixed wave-field frequency, the absorption (or emission) of a photon by the charged particle is possible if the charge absorption (or emission) spectrum in the magnetic field is equidistant. Based on the laws of conservation of energy and momentum projection onto the magnetic field direction upon photon absorption (or emission) and the charged particle passing to a new quantum state without a spin change, one can show [5] that an equidistant spectrum can only exist at $N = 1$.

Autoresonance is impossible if at least one of conditions (3) is violated, i.e., when $N = 1$, but $I \neq \Omega_0$, or when $N \neq 1$. In any of these cases, the frequency is mismatched and beats appear leading to energy oscillations [1, 2, 6, 7]. The qualitative picture of energy oscillations involves the follow-

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ing. Let us suppose that at the initial moment the exact resonance condition (2) is met and the particle velocity (with a positive sign) is parallel to the electric field strength. In this case, the particle energy increases. As a result of resonance detuning, the angle the particle velocity makes with the field strength changes and can become obtuse. Then the particle loses energy until the angle again becomes acute, and the particle energy again begins increasing. This leads to periodicity of energy variations.

The term *autoresonance* was introduced, obviously for the first time, in 1937 by Andronov, Vitt, and Khaikin [8]. Considering self-oscillating systems, the authors of monograph [8] defined autoresonance as resonance "*under the action of force induced by the motion of the system itself*." It turned out that autoresonance provides maximum efficiency of functioning for different engines and devices [9]. The *cyclotron autoresonance* discovered in studies [1–3] has a different physical nature. Further on, the term *autoresonance* has been used in very different situations. In paper [10], one of the methods of ion acceleration was referred to as autoresonance. To accelerate ions by this method, traveling charge-density waves are used, which are excited in a relativistic electron beam propagating in a waveguide along a strong longitudinal magnetic field. In some papers [11, 12], autoresonance is identified with Veksler's [13] (see also Refs [14–21]) and McMillan's [22] *self-phasing principle*, playing an important role in charged-particle accelerator physics [23]. In this context, autoresonance phenomena are investigated under various conditions, namely, charged particle acceleration [24–26], the generation and nonlinear interaction of waves [27–29], the excitation of Rydberg atoms [30], molecular dissociation [31], and many others [32].

The *cyclotron autoresonance* discovered in works [1–3] constitutes a particular phenomenon. It is specific for the fact that this is a purely relativistic effect for a charged particle motion, whereas under other conditions autoresonance is considered in a weakly relativistic and even nonrelativistic approximation [27, 33]. Moreover, cyclotron autoresonance is only possible in a traveling electromagnetic wave in which the Doppler frequency shift is significant, whereas the authors of Refs [11, 27] examine oscillations in the absence of Doppler shift.

2. Discovery of the cyclotron autoresonance phenomenon

The paper by A A Kolomenskii and A N Lebedev, which was the first to report the possibility of autoresonance particle motion in a plane vacuum wave traveling along a constant magnetic field, was published in *Sov. Phys. Dokl.* [1] in 1962. Almost simultaneously, the paper by V Ya Davydovskii [3] of analogous content came out in journal *Sov. Phys. JETP*. Therefore, the honor of the discovery of cyclotron autoresonance effect is rightfully given to the three authors. We shall present a brief review of their original papers.

A little later, Kolomenskii and Lebedev [2] considered the motion of a charged particle in an isotropic refracting medium in the field of a plane electromagnetic wave propagating along a constant magnetic field B_0. The wave field vectors satisfy the conditions $\mathbf{B} = \beta_{\text{ph}}^{-1}[\mathbf{nE}]$, $\mathbf{nE} = \mathbf{nB} = 0$, where $\mathbf{n}$ is a unit vector in the wave vector direction: $\mathbf{k} = k\mathbf{n}$. The electric field E of the wave is considered to vary by the law $\exp[\mathrm{i}\omega(t - \boldsymbol{\rho}\mathbf{n}/\omega)]$, where $\boldsymbol{\rho} = k\mathbf{r}$ is a dimensionless radius vector of the particle. The particle motion is described by the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\gamma\,\frac{\mathrm{d}\boldsymbol{\rho}}{\mathrm{d}t}\right) = \frac{e}{m}\left\{\left(1 - \frac{\mathrm{d}\boldsymbol{\rho}}{\mathrm{d}t}\,\frac{\mathbf{n}}{\omega}\right)k\mathbf{E} + \frac{\mathbf{n}}{c\beta_{\text{ph}}\left[(\mathrm{d}\boldsymbol{\rho}/\mathrm{d}t)\,\mathbf{E}\right]} + \frac{B_0}{c}\left[\frac{\mathrm{d}\boldsymbol{\rho}}{\mathrm{d}t}\,\mathbf{n}\right]\right\}.$$

This equation predicts the integral coincident with expression (1):

$$\gamma\left(1 - \beta_{\text{ph}}^2\,\frac{\mathrm{d}\boldsymbol{\rho}}{\mathrm{d}t}\,\frac{\mathbf{n}}{\omega}\right) = \text{const} = \gamma_0(1 - \beta_{\text{ph}}\,\beta_{\mathbf{n}0})\,,$$

where $\beta_{\mathbf{n}0}$ is the projection of particle velocity onto the wave propagation direction at the initial instant of time. As an independent variable, instead of time we introduce the particle phase relative to the wave, $\psi = \omega t - \boldsymbol{\rho}\mathbf{n} + \psi_0$, and thus reducing the equation of motion to the form

$$\frac{\mathrm{d}(M\,\mathrm{d}\boldsymbol{\rho}/\mathrm{d}\psi)}{\mathrm{d}\psi} = \boldsymbol{\eta} + \mathbf{n}\left(\frac{\mathrm{d}\boldsymbol{\rho}}{\mathrm{d}\psi}\,\boldsymbol{\eta}\right) + \frac{\Omega}{\omega}\left[\frac{\mathrm{d}\boldsymbol{\rho}}{\mathrm{d}\psi}\,\mathbf{n}\right]. \tag{4}$$

Here, the following notation was introduced:

$$\Omega = \frac{\omega_{\text{c}0}}{\gamma_0(1 - \beta_{\text{ph}}\,\beta_{\mathbf{n}0})}\,, \qquad \boldsymbol{\eta} = \frac{\Omega\mathbf{E}}{\omega B_0\beta_{\text{ph}}} = \boldsymbol{\eta}_0\sin\psi\,,$$

$$M = \left[1 + (1 - \beta_{\text{ph}}^2)\,\frac{\mathrm{d}\boldsymbol{\rho}}{\mathrm{d}\psi}\,\mathbf{n}\right]^{-1}.$$

Then, we consider equation (4) in particular cases $\beta_{\text{ph}} = 1$ and $\beta_{\text{ph}} \neq 1$. In the case of a vacuum wave linearly polarized along the y-axis ($\beta_{\text{ph}} = 1$) and propagating along the constant magnetic field (z-axis), the particle motion is described as

$$\begin{aligned} &\frac{\mathrm{d}^2 y}{\mathrm{d}\psi^2} + \left(\frac{\Omega}{\omega}\right)^2 y = \eta_0\sin\psi - \frac{\Omega}{\omega}\,\frac{\mathrm{d}x_0}{\mathrm{d}\psi}\,,\\ &\frac{\mathrm{d}z}{\mathrm{d}\psi} - \frac{\mathrm{d}z_0}{\mathrm{d}\psi} = \frac{1}{2}\left[\left(\frac{\mathrm{d}y}{\mathrm{d}\psi}\right)^2 + \left(\frac{\Omega}{\omega}\right)^2 y^2\right] - \frac{1}{2}\left(\frac{\mathrm{d}y_0}{\mathrm{d}\psi}\right)^2 - \frac{\Omega}{\omega}\,\frac{\mathrm{d}x}{\mathrm{d}\psi}\,y\,. \end{aligned} \tag{5}$$

From these equations follow that under the condition $\Omega = \omega$ coincident with formula (2), resonance or, more precisely, *autoresonance* exists, because the indicated condition also coincides with the integral of motion (1). At resonance, the amplitude of oscillations increases in the wave polarization direction, and the velocity $\mathrm{d}z/\mathrm{d}\psi$ in the direction along the guiding magnetic field also increases. The asymptotic $\tau = \eta_0(\psi - \psi_0)$ dependences (for $\psi \gg 1$) of dimensionless energy γ, 'acceleration length' $L = \eta_0 z$, and trajectory radius $R = (x^2 + y^2)^{1/2}$ of a charged particle were obtained:

$$\begin{aligned} &\gamma \approx \gamma_0 + \gamma_0(1 - \beta_{\mathbf{n}0})\\ &\quad\times\left\{\frac{\tau^2}{8} + \frac{\tau}{2\left[(\mathrm{d}y_0/\mathrm{d}\psi)\sin\psi_0 - (\mathrm{d}x_0/\mathrm{d}\psi)\cos\psi_0\right]}\right\},\\ &L \approx \frac{\tau^3}{24} + \frac{\tau^2}{4}\left(\frac{\mathrm{d}y_0}{\mathrm{d}\psi}\sin\psi_0 - \frac{\mathrm{d}x_0}{\mathrm{d}\psi}\cos\psi_0\right) + \tau\,\frac{\beta_{\mathbf{n}0}}{1 - \beta_{\mathbf{n}0}}\,,\\ &R \approx \frac{\tau}{2}\,. \end{aligned} \tag{6}$$

This implies that the trajectory of an accelerating resonance particle is a helix with a growing radius and pitch. The initial conditions of a particle inlet into a plane wave, permissible in the autoresonance regime, were also considered. In particular, the parameter Ω_0 in formula (2) must meet the condition

$$\Omega_0^2 > \frac{1-\beta_{\mathbf{n}0}}{1+\beta_{\mathbf{n}0}}\,.$$

This condition defines the angle of a particle inlet into the wave (the angle between the initial momentum vector and the wave propagation direction). For $\Omega_0 > 1$, the particle can be let in at any arbitrary angle up to $\pi/2$. For $\Omega_0 \leqslant 1$, the inlet angle is restricted: $-\Omega_0 < \alpha < \Omega_0$.

According to equations (4), in a fast or slow electromagnetic wave ($\beta_{\mathrm{ph}} \neq 1$), "*resonance cannot be maintained automatically all the time during motion.... Thus, in a slow or fast plane wave, no resonant motion would be possible*" [2]. The global analysis of a charged particle motion was carried out by the phase trajectory method.

"In addition to a fundamental aspect of this issue, the considered 'autoresonance' mechanism is significant for the fact that, in particular, it can play a certain role in space processes leading to charged particle acceleration by radio waves and by luminous fluxes in cosmic fields. This effect can find application in the case of particle acceleration by high-power light beams or amplification of radio waves of different bands, etc." [2].

In his brief paper [3], Davydovskii considered the possibility of "charged-particle resonant acceleration by electromagnetic waves in a constant magnetic field" on the basis of the integral of motion derived by him in the form similar to Eqn (1). This integral implies that "cyclotron resonance can hold out rather long, which can cause a considerable increase in the particle energy" [3]. Thus, the "mechanism providing natural self-phasing" was discovered [3].

Ionized gas was considered under the conditions that the mean free path of charged particles greatly exceed the radius of curvature of their trajectories in the magnetic field. The equation of particle motion

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\varepsilon}{c^2}\,\mathbf{v}\right) = e\mathbf{E} + \frac{e}{c}\left[\mathbf{v}(\mathbf{H}+\mathbf{h})\right] \tag{7}$$

is considered input. Here, ε is the particle energy, $\mathbf{H}$ is a constant homogeneous magnetic field, and $\mathbf{E}$, $\mathbf{h}$ are field vectors of a plane electromagnetic wave. The energy variation equation

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = e\mathbf{E}\mathbf{v} \tag{8}$$

is also considered. For a wave running along the external magnetic field direction, equations (7) and (8) can yield the relation

$$\frac{1}{\omega'}\,\frac{\mathrm{d}\omega'}{\mathrm{d}t} = -\frac{1}{\varepsilon}\,\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\,, \tag{9}$$

where $\omega' = \omega - \mathbf{k}\mathbf{v}$ is the "difference in the wave frequency relative to the particle" [3]. This implies the exact integral of the equations of motion, $\varepsilon(1 - v_H/c) = \mathrm{const}$, where $v_H = \mathbf{k}\mathbf{v}\,c/\omega$. This integral coincides with integral (1). Considering further the cyclotron frequency variation rate $\omega_H = eHc/\varepsilon$, one can obtain the relation

$$\omega' = C_1\omega_H = C_2\varepsilon^{-1}\,. \tag{10}$$

This means that "*the particle motion is automatically synchronized with the wave*" [3]. In particular, at $C_1 = 1$, the cyclotron resonance condition (2) holds true. "*For such particles, the acceleration time can be on the order of the mean free time*" [3].

Thus, the mechanism of cyclotron autoresonance was discovered in Refs [1, 3]. To keep to historical truth, we should note that the authors of paper [34] were very close to the discovery of autoresonance particle motion, but this did not happen.

After the discovery of cyclotron autoresonance effect [1, 3], a thorough analysis of the relativistic motion of a charged particle in a plane transverse electromagnetic wave propagating along a constant magnetic field (z-axis) was carried out in Ref. [6]. An exact equation was deduced, which only contained the particle energy $\mathcal{H} = mc^2\gamma$:

$$\left(\frac{\mathrm{d}\mathcal{H}}{\mathrm{d}t}\right)^2 + V(\mathcal{H}) = 0\,. \tag{11}$$

Here, we have the 'potential' $V(\mathcal{H}) = (1/\mathcal{H}^2)(\alpha_4\mathcal{H}^4 + \alpha_3\mathcal{H}^3 + \alpha_2\mathcal{H}^2 + \alpha_1\mathcal{H} + \alpha_0)$, where α_i are constants determined by the initial conditions. The one-dimensional 'potential' $V(\mathcal{H})$ contains powers of $\mathcal{H}$ and, therefore, equation (11) generally admits a solution in the form of elliptic integrals. The motion of a charged particle with the initial condition of exact cyclotron resonance in the field of a circularly polarized wave was considered. The authors of Ref. [6] called the particle motion *synchronous* (and not autoresonant) at $N = 1$, and *oscillatory* for $N \neq 1$. In the synchronous regime, equation (11) is solved elementarily. The solution defines time as a function of energy. To find the energy as a function of time, one should solve a cubic equation. It was shown that as $t \to \infty$, the energy increases with time by the law

$$\mathcal{H} \approx a\,\mathcal{H}_0(\omega t)^{2/3}\,, \tag{12}$$

where a is a certain constant. A similar estimate follows from formulas (6).

Solutions in the *oscillatory* regime were also found and examined in paper [6]. In general, the solution is a combination of elliptic functions and certain elementary functions. However, an analysis of the solution is rather complicated. Hence, problems are most frequently solved using numerical methods. Notice that equation (11) can be arrived at by a simpler way [7] than it was in Ref. [6].

It was pointed out in paper [2] that cyclotron autoresonance can find application for both charged particle acceleration and electromagnetic wave generation. The problems of electromagnetic wave generation using different gyrodevices, including cyclotron autoresonance masers (CARMs), have continually been discussed in literature [35–37].

In what follows, we shall analyze the results of research covering particle acceleration in the regime of cyclotron autoresonance.

3. Further studies of cyclotron autoresonance

In the first experiments confirming the existence of the autoresonance mechanism of charged particle acceleration, waves propagating inside waveguides were utilized [38–40]. Simple electromagnetic waves were excited, namely, the wave H_{11} of a circular waveguide, and the wave H_{01} (close to it in structure) of a rectangular waveguide [38]. However, the phase velocity of the wave in the waveguide exceeds the speed of light. Hence, the conditions of cyclotron autoresonance are certainly not met in the case of a constant guiding magnetic field, and therefore continuous particle acceleration is impossible. To demonstrate the possibility of a charge particle acceleration, the effective acceleration length was defined as $z_a = \lambda\beta_{ph}/[2(\beta_{ph} - 1)]$. For the wave H_{11}, we have $\beta_{ph} = 2$ (the acceleration length is $z_a = 3.6$ cm). If the length of the uniform part of the solenoid is more than 3.6 cm, the particle energy must periodically change on the waveguide length. This effect has been observed when the inner waveguide wall is covered with a thin luminophor layer. For an optimum magnetic field strength, luminous rings appeared on the waveguide walls. The distance between the rings was ≈ 6 to 7 cm — that is, $\approx 2z_a$. The kinetic energy of the accelerated electrons appeared to be ≈ 700 keV for the electric field strength of 3 to 5 kV cm^{-1}. This energy greatly exceeds the energy which the electron could gain under the given conditions in the case of ordinary cyclotron acceleration. A similar experiment was realized in Ref. [39], where a wave with $\beta_{ph} = 1.14$ was used.

Different properties of autoresonance particle motion have been examined in many papers. The authors of Ref. [40] calculated the particle motion in linearly and circularly polarized TE-11 modes of a circular waveguide, and in the TE-111 resonator mode in a homogeneous magnetic field. The results of calculations showed that in the electromagnetic field inside a waveguide or a resonator the motion of the particle and the energy it gained differ substantially from those of a plane wave in ideal conditions. However, the experiments designed around circular resonators in which TE-111 mode waves were excited demonstrated that in real conditions, too, a rather efficient energy gain by the particle is possible. This results in the occurrence of a ring of accelerated particles. A thorough analysis of the formation of an accelerated relativistic electron ring considered as a driver for a gyrotron was carried out in work [41]. To maintain the synchronism between a particle and a standing wave in a resonator, a stationary magnetic field was applied, which increased linearly along the resonator axis.

The autoresonance mechanism of interaction between electrons and TE and TM waves in waveguides was considered in paper [42]. The conditions of particle motion in the synchronous regime were found in Ref. [43] for a plane wave propagating at an angle to the magnetic field vector. The particular mechanism of autoresonance stochastic acceleration of charged particles was discussed in papers [44, 45].

Radiation loss is an important issue in particle acceleration [46, 47]. It was shown that in the case of a circularly polarized electromagnetic wave (at $N = 1$) propagating along the magnetic field the maximum permissible energy gained by the particle before resonance detuning is determined from the relation

$$\gamma_{max} \approx \frac{\varepsilon}{\Omega}\sqrt{\frac{3}{\mu}\,\Omega_0}\,. \tag{13}$$

Here, the parameter $\mu = 2e^2\omega/(3mc^3) \equiv 4\pi r_0/(3\lambda) \ll 1$, where $r_0 = e^2/(mc^2)$ is the classical electron radius. The maximum permissible energy at autoresonance does not depend on the energy value with which particle acceleration begins. The estimates showed that, in principle, the reaction of radiation sets up a limit for the autoresonant acceleration mechanism, although this limit may be fairly high. This limit can, however, become almost infinite if a longitudinal electrostatic field $E_z(Z)$ (where $Z = kz$) is applied [48].

It should be noted, incidentally, that the expression for the radiative friction force is still a subject of discussion [49–51].

As mentioned above, the initial condition of cyclotron resonance cannot persist 'by itself' during all the time of particle motion in the case of slow ($N > 1$) or fast ($N < 1$) plane electromagnetic waves propagating along a constant magnetic field. Then a 'forced' maintenance of particle–wave phase synchronism is possible through a change in the wave phase velocity, an appropriate profiling of the guiding magnetic field, switching on a quasistationary electric field, and so forth.

Particle phase synchronization with a plane electromagnetic wave is generally impossible when the *phase velocity of the wave* (the *refractive index*) is changed arbitrarily. If, however, the refractive index changes in a certain way in the wave propagation direction, it is then possible to seek maintenance of the initial condition of cyclotron resonance during all the time of a particle motion:

$$\gamma_0 - N_0 P_{z0} = \gamma - N(Z) P_z = \Omega\,. \tag{14}$$

Thus, the main goal is to find the dependence $N(Z)$ under condition (14). Different versions of this problem were considered in Refs [52–56].

The first indications of the possibility of maintaining a synchronism between a particle and a nonvacuum wave through a *special profiling of the external magnetic field* were presented in paper [57]. This issue was later discussed in a number of papers [25, 58–62]. The magnetic field profile must generally be concordant with the condition of cyclotron resonance retention during all the time of particle motion. However, a synchronizing magnetic field as a solution of a certain equation is rather complicated and can hardly be realized in practice [61]. It is therefore natural to consider simple magnetic field profiles, for instance, a linear dependence of the form

$$\Omega_0(z) = \Omega_{00}\left[1 + \alpha(Z - Z_0)\right]. \tag{15}$$

In this case, synchronism cannot be conserved all the time during particle motion. At the same time, a distance exists at which particle motion has a synchronous character and the particle gains substantial energy. One can determine the optimum value of the magnetic field gradient at which the charged particle acquires maximum energy:

$$\alpha_{opt} = A\varepsilon^2\,\frac{(1 - N^2)^2}{N\Omega_{00}}\,, \tag{16}$$

where the parameter $A = 5 \times 10^2$ was found from the analysis of numerical results.

For a successive determination of the profile of the magnetic field providing maintenance of the particle–wave phase matching, the transverse components of the field and the drift velocity of the particle should be explicitly taken into account. A simplified version of this problem was examined in Ref. [62]. On the basis of the numerical solution, scalings during cyclotron laser acceleration of electrons were found: the maximum acceleration length kz_m, when dependent on the magnitude of the accelerating field, changes by the law

ε^{-1}, and when dependent on phase velocity it changes by the law $(\beta_{\mathrm{ph}}-1)^{-\nu}$, where $\nu = 0.6$. The variation of an accelerated particle energy is defined by the relationship

$$\frac{\gamma_{\mathrm{f}}-\gamma_{\mathrm{i}}}{\gamma_{\mathrm{i}}-1} \approx (\beta_{\mathrm{ph}}-1)^{-\mu}\,,$$

where $\mu \approx 0.5$, and γ_{i}, γ_{f} are the initial and final particle energies, respectively.

The problem of maintaining particle–wave synchronism *with the aid of an electrostatic field oriented in the direction of a guiding magnetic field* was discussed in papers [63–65]. The necessary profile of the synchronizing electrostatic field, $\varepsilon_0 f(Z) = -\varepsilon_0\,\mathrm{d}U/\mathrm{d}Z$, was found in an implicit form in Ref. [64]. By requiring that the initial condition of resonance be met during all the time of particle motion, one can select the electrostatic field so that it can compensate for the phase shift occurring for $N \neq 1$. In general, this field has a rather complicated form and can hardly be realized in experiment. It is hence natural to consider first the simple case of an electric field with potential $U(z) = 1 + \alpha Z$.

This case was very thoroughly investigated in paper [63]. Particles turned out to behave differently in fast and slow waves; namely, with increasing potential for $\varepsilon_0 > 0$, the particle accelerates in a slow wave, and slows down in a fast wave.

A constant electric field cannot provide a lasting maintenance of particle–wave phase matching. It is possible to estimate the maximum energy gained by the particle before resonance breakdown:

$$\gamma_{\max} \approx \varepsilon_0\alpha N|q|s_{\max} + |q|\left[\varepsilon^2(1-N^2) + (\varepsilon_0\alpha)^2\right]\frac{s_{\max}^2}{2}\,, \qquad (17)$$

where

$$s_{\max} \approx \left\{\frac{6}{\Omega\left[\varepsilon^2(1-N^2)+(\varepsilon_0\alpha)^2\right]}\right\}^{1/3}. \qquad (18)$$

So long as the field is low, it affects weakly the character of energy variation, i.e., the energy oscillates and increases on average. A value of the field gradient $\varepsilon_0\alpha_{\mathrm{opt}}$ exists for which the energy heightens greatly before the resonance breakdown. As soon as this value is exceeded, the resonance breaks down earlier, and the particle does not have enough time to gain sufficient energy. The optimum value of the electrostatic field gradient can be estimated as

$$\varepsilon_0\alpha_{\mathrm{opt}} = \frac{A}{N}\left[\frac{\varepsilon^2(1-N^2)}{\sqrt{N}}\right]^{2/3}, \qquad (19)$$

where the empirical constant $A = 1.2$ was found in the numerical experiment [63].

Under the influence of an *electrostatic field crossed with a constant magnetic field*, the character of wave–particle resonance interaction is affected appreciably, which leads to a number of physical effects. In particular, the mechanism of Cherenkov absorption of a transverse electromagnetic wave propagating along a constant magnetic field appears to be possible. Based on the result of its action, this mechanism can be comparable with the mechanism of cyclotron absorption [66]. A peculiar autoresonance of a nonrelativistic charged particle in the field of a plasma wave propagating at an angle to the external magnetic field was also found [33]. This effect is entirely due to a weakly inhomogeneous electric field crossed with the magnetic field.

The possibility of maintaining the synchronous motion of a relativistic charged particle in a nonvacuum wave was considered in papers [67–69] with allowance made for electric drift. By passing to a co-moving frame of reference which executes a motion with the electric drift velocity, the conditions were found under which cyclotron autoresonance occurs in this frame. Under such conditions — depending substantially on the type of wave and its polarization — the particle energy can increase monotonically. In the laboratory frame of reference, the energy gained by the particle on average turns out to exceed the energy in the co-moving frame.

Autoresonance and the possibility of its maintenance were discussed above for particle motion in a vacuum in the single-particle approximation. The one-particle approximation is valid if the energy density $E_0^2/(4\pi)$ of the accelerating field exceeds appreciably the energy density of the beam of accelerated particles, $n_{\mathrm{b}}mc^2\gamma$, where n_{b} is the number of particles in the beam. When this condition is violated, the back action of the beam accelerated onto the accelerating field and the space charge effects become substantial.

Clearly, when a particle beam is moving in a medium, such as plasma, in particular, the autoresonance regime of acceleration cannot be retained for a long time. Papers [70, 71] were devoted to the study of this problem. The authors investigated the possibility of autoresonance acceleration of a sufficiently dense electron beam in the field of a plane circularly polarized wave propagating along a constant magnetic field in homogeneous electron–ion plasma. In a self-consistent problem, solutions have been sought depending on time only. The results of computations showed that autoresonance acceleration of electrons coincides with that for a one-particle picture at the initial stage only. Then, the particle beam–wave phase matching is violated, which leads to a periodic failure of the acceleration process. As a result, the electron energy varies periodically, the beam energy density being then comparable to the energy density of the accelerating field.

Coulomb collisions upon autoresonance acceleration of an electron beam were also taken into consideration [71]. The cyclotron autoresonance condition was assumed to be met for only a small part of the beam particles.

The authors of Refs [72, 73] developed the self-consistent theory of interaction between a modulated relativistic electron beam and a plane electromagnetic wave in autoresonance conditions. A class of periodic solutions was found for which the phase velocity of the wave is constant and equals the speed of light, also in the presence of a beam.

The mechanisms of autoresonance and stepwise trapping of particles were investigated in Refs [74, 75] in the case of two circularly polarized electromagnetic waves propagating in a dispersive medium in opposite directions to the constant magnetic field vector. It was shown that under these conditions drive current can be generated. The autoresonance mechanism of acceleration taking into account the beats of two electromagnetic waves was also analyzed [76]. The autoresonance particle acceleration effects in astrophysical conditions were discussed in papers [77, 78].

4. Microwave autoresonance acceleration

In paper [40] it was first shown that a particle which was initially at rest in a plane wave with amplitude E at a distance z in the autoresonance regime gains energy (in units of the rest energy) $\gamma_c = [(3/\sqrt{2})AZ]^{2/3}$, where $A = eE/(m\omega c)$ is a dimensionless wave amplitude, and $Z = z\omega/c$ is a dimensionless acceleration length. In a field of the same magnitude in a linear accelerator, the particle acquires energy $\gamma_L = AZ$. This implies that the linear accelerator is more efficient in particle acceleration up to high energies, whereas the autoresonance accelerator is more efficient for $AZ < 4.5$. Therefore, the microwave autoresonance accelerator can serve as a source of rather low-energy electrons needed for different purposes.

The character of the autoresonance acceleration of particles in a running wave differs appreciably from that in a standing wave. In a standing wave in a resonator, the energy of an accelerated electron beam is mainly stored in the transverse component of velocity. Therefore, an autoresonance microwave accelerator built around a resonator is an effective means of obtaining relativistic rotating electron beams which can be used, in particular, as a source of coherent radiation [79, 80]. In the scheme of an autoresonance microwave accelerator on a traveling wave (waveguide), a greater part of the beam energy is stored in the axial component of the velocity.

The first experiments showing acceleration of electrons with an energy of 10 keV to 150 keV at a distance of 1.5 m in an autoresonance microwave accelerator were described in paper [81]. In a circular waveguide, a TE_{11} mode wave was excited. The autoresonance acceleration regime was maintained by a weak profiling of the guiding magnetic field along the waveguide central axis. The limiting value of the synchronizing magnetic field was determined — for field strengths under this limiting value the particles were trapped into the accelerating phase which turned out to be equal to π.

The experiments performed showed that under the considered conditions electrons can, in principle, be accelerated up to high energies. However, realization of this mechanism requires exceedingly high values of the magnetic field and high-frequency (HF) power, and the radiation loss must be negligibly low. The author of Ref. [82] analyzed the stability and the terminal beam current effects in the accelerator [81]. It was experimentally demonstrated that by choosing the optimal magnetic field profiling, one can heighten the energy of accelerated electrons up to 200 keV, the expected value being 250 keV. The possibility of acceleration with the TE_{21} mode was also considered. Note that the global analysis of charged-particle motion stability and the calculation of stable equilibrium phases are presented in paper [60]. It should also be mentioned that the authors of Refs [81, 82] do not refer to the pioneering works [1–3]. This is, unfortunately, typical of many subsequent publications.

The results of experimental research of resonance interactions between electrons and an H_{10} wave of a rectangular waveguide are presented in Ref. [83]. To maintain the particle–wave synchronism, a guiding magnetic field increasing along the axis was invoked. It was found that in the acceleration regime under consideration, mutually exclusive requirements must be met: on the one hand, to heighten the electron energy, the strength of an electric field of the accelerating wave should be increased, which, on the other hand, violates the regime of stable acceleration. Hence, the optimal wave parameters and injection conditions should be chosen for autoresonance acceleration in a waveguide. In the experiments [83], at an injection energy of nearly 120 keV, hollow electron beams with an energy of 800 to 850 keV and accelerated current of nearly 0.15 A in a pulse of about 0.8 μs and a repetition rate of 50 Hz were produced.

The series of studies [84–89] was devoted to further theoretical and experimental research on the electron cyclotron autoresonance accelerator CARA, in which phase matching between particles and an accelerating wave is maintained by profiling the guiding magnetic field and increasing the waveguide radius. It is pointed out that the beam particles are rigidly trapped into the accelerating phase and turn out to be insensitive to deviations from the condition of exact autoresonance. CARA is applied as a compact injector of low-energy electron beams for a high-gradient accelerator or is used for radiation sources. The quality of such beams determined by particle spread over the axial velocity and the energy limit of the accelerated beam were discussed in Refs [84, 85]. CARA was shown to have an upper energy limit related to a decrease in the axial velocity upon heightening of the guiding magnetic field:

$$\gamma_{\max} = \gamma_0 + \left(\frac{\gamma_0^2 - 1}{1 - N_f^2}\right)^{1/2}.$$

Here, γ_0 is a relativistic factor of injected particles, and N_f is the refractive index at the end of acceleration process. This limit can be exceeded if a multistage CARA is used, which at each subsequent stage should work at a higher gyrofrequency harmonic [86]. The calculations showed that for the injection of a beam with an energy of 250 keV and a current of 15 A into a three-stage CARA with an HF power of 75 MW and a frequency of 11.424 GHz, the energy acquired by the beam at the output is higher than 5 MeV. For the injection of an electron beam with an energy of 250 keV into a single-stage CARA, the accelerated beam energy does not exceed 1.7 MeV. From the calculations and experiments carried out in Refs [87, 88] it follows that the cyclotron autoresonance acceleration of electrons in a waveguide with the TE_{11} mode field proceeds with very high efficiency, namely, more than 90% of the waveguide HF field power is transferred to the electron beam accelerated. It was also shown that autoresonance acceleration may result in generating coherent radiation at the seventh laser harmonic [89].

5. Autoresonance acceleration by laser radiation

Different methods of laser-driven acceleration, which are conditionally divided into vacuum [90–92] and plasma [93–97] methods, are presently under investigation. It has been demonstrated that by using sufficiently compact devices these methods are able, under certain conditions, to ensure a high electron acceleration rate. Compared to plasma methods, vacuum methods of laser acceleration have some advantages, namely, the absence of plasma instabilities, of fairly rigid restrictions on plasma homogeneities, of the effects of interaction of accelerated electrons with plasma particles, etc. Among the many schemes of vacuum laser acceleration, an effective one is that of cyclotron autoresonance. The physical mechanism underlying autoresonance laser acceleration is, in principle, the same as that in the scheme of microwave acceleration. Some distinctions, however, exist:

— laser radiation is much more intensive than microwaves, which provides a higher acceleration rate;

— the laser radiation source is external relative to the acceleration region; this increases the efficiency of energy transfer from wave to particle and eliminates the difficulties due to breakdown, etc.;

— laser radiation propagates as beams, which leads to a specific wave–particle interaction;

— the parameter Ω_0 is very small in resonance conditions (2) when lasing. Therefore, the autoresonance acceleration can involve electrons that already possess considerable energy exceeding their rest energy. Rather high Ω_0 values can be reached with the help of superhigh magnetic fields. Such fields can be generated by high-power ultrashort laser pulses interacting with solid targets [96].

The first studies of autoresonance laser acceleration were carried out in the approximation of a plane accelerating wave [97–99]. However, such an approximation is rather rough for laser radiation, and so the optimistic results thus obtained can only be of methodical interest. In many cases, laser radiation is rather well described in a quasioptical approximation in the form of a Gaussian beam [100–103]. A paraxial region is typically considered, in which the characteristic diffraction angle is defined as

$$\theta = \frac{\lambda}{\pi a} = \frac{a}{z_{\mathrm{R}}} \ll 1 \, .$$

Here, λ is the radiation wavelength, a is the beam waist (its radius in the focal plane), $z_{\mathrm{R}} = ka^2/2$ is the Rayleigh length, and $k = 2\pi/\lambda = \omega/c$ is the wave number. The Gaussian monochromatic radiation is described in the quasioptical approximation by a parabolic equation which, in the case of axisymmetric beams propagating along the z-axis, has a general solution in the form of the linear superposition of modes [100]:

$$A(r,z) = \exp(-\zeta) \sum_{m=0}^{\infty} \frac{A_m(0)}{(1+\mathrm{i}D)^{m+1}} \, L_m(\zeta) \, .$$

Here, $\zeta \equiv r^2/[a^2(1+\mathrm{i}D)]$, $r = \sqrt{x^2+y^2}$ is the distance from the beam axis (z-axis), $D = 2z/(ka^2)$ is a dimensionless diffraction length, and $L_m(\zeta)$ are Chebyshev–Laguerre polynomials of order m. The number m must not be too large lest it goes out of the applicability coverage of the parabolic equation. In general, TE_{lm} type multimode laser radiation is described by Hermitian polynomials [102]. The distinctive feature of Gaussian electromagnetic radiation is the fact that it contains all the field vector components, although not all their terms have the same order in the parameter $1/(ka)$.

The amplitude of the electric field strength in a focused laser beam can be estimated by the formula [93]: $E_0 = 3 \times 10^{-9}\sqrt{I_{\mathrm{r}}}$ [TV m^{-1}], where the radiation intensity I_{r} is given in W cm^{-2} units. Thus, for $I_{\mathrm{r}} > 10^{18}$ W cm^{-2}, colossal electric fields are possible, which exceed interatomic ones and are many orders of magnitude higher than the fields explored in traditional charged-particle accelerators.

In the single-particle approximation, the intensity of accelerating laser radiation is characterized by the dimensionless parameter $g = eE_0/(m\omega c)$, which stands for the ratio of the amplitude of electron oscillatory velocity to the speed of light. The parameter g is determined by the peak intensity I_{r} [W cm^{-2}] and the radiation wavelength λ [μm]: $g = 0.85 \times 10^{-9}\lambda\sqrt{I}$. For a sufficiently low radiation intensity, we have $g \ll 1$. For high-power pulsed laser radiation, the g value can be higher. For example, the parameter $g \approx 0.85$ for $I_{\mathrm{r}} \approx 10^{18}$ W cm^{-2}, and $\lambda \approx 1$ μm.

The problem of particle acceleration by laser radiation of the principal mode, including pulsed radiation, is the most well-studied. In this case, the pulsed character of radiation is usually taken into account by simple multiplication of the Gaussian beam expression by a certain impulse function. Such a procedure is, however, not quite correct for very short pulses (strongly focused radiation) [103, 104].

The condition of cyclotron resonance of a particle with Gaussian radiation, as distinct from the condition of resonance in a plane wave, is not the integral of motion. This means that the cyclotron resonance in a Gaussian beam (GB) does not survive automatically upon particle motion. For resonance maintenance, it is therefore necessary that, e.g., the guiding magnetic field be varied. Meeting the cyclotron resonance condition at the initial instant of time leads to a restriction on the injected electron energy:

$$\gamma \geqslant \frac{\Omega_0^2 + 1}{2\Omega_0} \, .$$

In the case of laser radiation, we have $\Omega_0 \ll 1$. Consequently, the electrons accelerated in the autoresonance regime must already possess considerable energy. For fairly wide GBs, the initial energy of accelerated electrons cannot be below 25 MeV for a CO_2 laser, while for a Nd-glass laser the injection energy must exceed 300 MeV.

The first estimations of the autoresonance mechanism of electron acceleration by Gaussian lowest-mode laser radiation were obtained in Ref. [105]. It was shown that in a 10^{13}-W cm^{-2} CO_2 laser field with a 0.5-cm spot in a magnetic field of 100 kG, electrons can be accelerated from an energy of 25 MeV to 500 MeV at a distance of about two Rayleigh lengths (nearly 15 m) with an appropriate profiling of the guiding magnetic field. The authors of Ref. [105], however, neglected the dependences of the amplitude and phase of the Gaussian beam on the transverse coordinates, which appeared to be inadmissible in consideration of the acceleration of even a very narrow electron beam.

It was later shown [106, 107] that in the field of a GB lowest-mode, electrons can undergo high-rate acceleration at a small accelerating distance (of about 1 m). The acceleration rate in this interval can be increased by an appropriate profiling of the guiding magnetic field. As the particle accelerates, it moves helically in the transverse plane xy until it reaches the limiting cycle (at a distance of about 100 cm). The limiting cycle radius is smaller than the GB waist, and so the accelerated particle does not leave the accelerating field. After reaching the limiting cycle, the accelerated electrons form a tubular beam of constant radius, the same as in autoresonance acceleration by microwaves in a cavity.

The formation of a tubular electron beam accelerated by Gaussian lowest-mode laser radiation was considered in detail in Refs [108, 109]. It was shown that, irrespective of the phase and position of injection, all the particles are monotonically accelerated at different rates and each particle has its limiting cycle at a distance on the order of the double Rayleigh length, the cycle center depending substantially on the particle position at the moment of injection. For a width of the injected bunch smaller than 1/4 of the Gaussian beam waist, the accelerated bunch takes the shape of a ring. The mean ring radius does not exceed half of GB waist. The length of the accelerated ring is determined by the length of an injected electron bunch. For a wider injected bunch, no ring is formed, and the accelerated electrons are nonuniformly distributed over the bunch cross section. Thus, the shape of an accelerated electron bunch is mostly determined by the injection parameters.

Autoresonance acceleration of electrons by Gaussian radiation of a *first-mode* CO_2 laser was also examined [110]. As distinct from the principal-mode GB radiation with a maximum on the radiation propagation axis, first-mode radiation has a minimum on such an axis [101]. The study of energy variation of electrons injected on the GB axis in the plane of its waist for different resonance phases on the interval from 0 to 2π has shown that the synchronous acceleration regime involves all the electrons, irrespective of their initial phases. On the interval on the order of the Rayleigh length, all the particles acquire approximately the same energy of about 275 MeV, i.e., the particle energy increases more than 10-fold. On the accelerating interval on the order of the Rayleigh length, the deviation of frequency mismatch from zero turns out to be insignificant, which ensures the synchronous acceleration regime. A transverse declination of particles from the symmetry axis on the accelerating interval is small and does not reach even half of the GB width. The particle being accelerated then moves helically in the transverse plane, the same as in the GB lowest-mode, until it reaches the limiting cycle (at a distance of nearly 120 cm).

It turned out that the particle acceleration rate in the GB first-mode field can be higher than in the GB lowest-mode field. Even for 'unfavorable' phases, electron acceleration in the GB first-mode field takes place with higher efficiency than for the GB lowest-mode, and at a distance on the order of the Rayleigh length the particles gain almost the same energy as in the case of a vacuum plane wave. This means that the choice of lowest-mode accelerating laser radiation is not optimal.

The electron acceleration rate can be appreciably enhanced if combined laser radiation is used composed of GB lowest- and first-mode [110]. The greater part of injected particles for very different phase relations acquires noticeable energy before resonance breakdown. It is only a small part of particles with unfavorable phases that falls out of the acceleration regime. The energy gain by particles appears to be the largest when first-mode radiation only slightly 'spoils' the lowest-mode field. Acceleration by combined radiation appears to be more efficient than acceleration by lowest or first-mode radiation separately. But the acceleration interval shortens to half the Rayleigh length.

Thus, in a field of powerful Gaussian laser radiation, the effective acceleration of relativistic electrons is possible in the high-rate cyclotron autoresonance regime at distances on the order of 1 m.

Similar conclusions can be found in Refs [111–114]. The important studies [112–114] were aimed at the design of the laser cyclotron autoresonance accelerator LACARA (laser-driven CARA). Autoresonance acceleration by a principal-mode CO_2 laser with a power of nearly 1 TW and a wavelength of 10.6 μm was considered (GB waist of 1 mm, and a Rayleigh length of nearly 60 cm.) To maintain autoresonance conditions, it was suggested that the magnetic field vary at a distance of approximately 1 m. A 50-MeV electron bunch was injected. The calculations show that under autoresonance laser acceleration, electron bunches can be formed with a duration of about 3 fs and a period of 35 fs [112]. This effect makes it possible to employ the accelerator considered as an injector for another types of accelerators [113]. The first experimental results which were obtained on the LACARA accelerator in 2008 demonstrated agreement with the calculation data [114].

6. Conclusion

The cyclotron autoresonance mechanism discovered by Kolomenskii, Lebedev, and Davydovskii 50 years ago was based on idealized conditions of charged particle injection at exact cyclotron resonance in the field of a plane electromagnetic wave propagating along the direction of a constant magnetic field with the speed of light in a vacuum. Violation of these conditions leads to periodic variation of energy gained by the particle instead of its monotonic growing.

Further investigations revealed that the particle motion in the autoresonance regime can proceed in real conditions upon a corresponding change in the parameters of the accelerating electric field, the guiding magnetic field, etc. The conditions of autoresonance acceleration by microwaves and laser radiation appeared to differ substantially. Microwave autoresonance acceleration is effective for obtaining comparatively low-energy particles, whereas laser acceleration is possible for electrons that already possess high energy. This follows from resonance condition (2).

For autoresonance laser acceleration of low-energy electrons, superhigh magnetic fields are necessary. Upon microwave autoresonance acceleration, a traveling wave (in a waveguide) and a standing wave (in a cavity) gain energy differently. In a cavity, the accelerated electron energy is mainly stored in the transverse velocity component, and in a waveguide in the axial one, the HF-field power being transferred to an electron beam undergoing acceleration with high efficiency. The autoresonance laser acceleration may cause the occurrence of high-energy electron rings and electron bunches. The properties of the electron beams produced in autoresonance acceleration allow their use as drivers for gyrotrons, for further acceleration in injectors, and so forth.

A theoretical analysis of autoresonance motion is normally made in the single-particle approximation, which is valid if the accelerating field energy density greatly exceeds the energy density of the accelerated particle beam. If this requirement is not met, the back action of the accelerated beam on the accelerating field and the space charge effects become significant.

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