# Excitation of cyclic Sommerfeld waves and Wood's anomalies in plane wave scattering from a dielectric cylinder at oblique incidence 

A D Pryamikov, A S Biriukov

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#### Abstract

An analysis is presented of the scattering of a plane electromagnetic wave in the case of oblique incidence from a circular dielectric cylinder. Under certain conditions, this scattering process involves the excitation of cyclic surface Sommerfeld waves (SWs) capable of propagating over large distances along the cylinder. It is shown that the interaction of SWs of low azimuthal order with the cylinder continuous (radiation) modes gives rise to cyclic Sommerfeld resonances (SRs), analogous to well-known Wood's anomalies in a plane wave scattering from one-dimensional metallic diffraction gratings. Conditions necessary for the effective excitation of SWs and SRs are established and the SW and SR contributions to mode formation in microstructured optical fibers are discussed.


## 1. Introduction

This paper was motivated by the recent advent of a number of promising optical fibers for fiber optics, in which guiding of light is obtained not through total internal reflection but through a different mechanism, which is not completely understood, in our opinion. The case in point is one of the types of the so-called microstructured optical fibers (MOFs) with a central core whose optical density is lower than that of a cladding, in particular, MOFs with a hollow core (filled with atmospheric air).

It is well known that guiding of a light beam with a complete compensation of its diffraction divergence is provided by refraction from the layers of a medium surrounding the region of light propagation. Guided radiation in fibers is typically achieved due to reflection of radiation from the

[^0]core-cladding interface, followed by the constructive interference of scattered waves in the core. In particular, in standard telecommunication fibers, this mechanism is caused by the effect of total internal reflection from the cladding, whereas in waveguides where total internal reflection plays no role, other effects are involved. These effects are determined by the cladding structure. For example, for hollow fibers with a uniform cladding, this is the usual Fresnel reflection, for fibers with a microstructured cladding, this is the Bragg reflection (when the microstructure is ordered with the characteristic period of variation of the permittivity comparable to the wavelength of light) or reflections of other types (for example, Mie scattering from small-scale inhomogeneities of the cladding).

As regards optical losses in waveguides, it is well known that they include material and waveguide losses. We note that material losses are always present to some extent, whereas waveguide losses play a key role in fibers based not on total internal reflection but on other effects. Losses in such fibers are determined by imperfections of their inhomogeneous cladding and the transfer of the guided radiation energy out of the core to the cladding. In other words, the cladding structure in such fibers determines the level of their optical losses. The authors of many theoretical and experimental papers noted that resonance scattering from elements of the cladding structure plays a role in the formation of modes in such fibers.

In this paper, we analyze the nature and types of this resonance scattering and its influence on the level of optical losses in MOFs.

There exists a rather wide scope of problems whose solution requires an analysis of light scattering from a dielectric rod (or a capillary). One of them is an interesting and urgent physical problem of establishing the proper light guiding mechanism in a number of MOFs promising for applications in fiber optics.

We recall that the use of the term MOF has gradually extended to any fibers whose claddings have optical inhomogeneities that extend along the fiber length and are arranged according to some symmetry in the fiber cross section. The transverse size of these nonoverlapping inhomogeneities is
typically comparable with the guided radiation wavelength, while their permittivity can be either lower or higher than the permittivity of the ambient matrix. A rather detailed review of MOFs is presented, for example, in [1].

Microstructured optical fibers discussed here include, in particular, all-glass optical fibers with the cladding whose cross section is typically a finite two-dimensional photonic crystal with the hexagonal symmetry of the arrangement of cylindrical inclusions with the refractive index higher than that of the surrounding silica matrix. We call such a microstructure an all-solid microstructure. The core is formed by removing one or several most closely spaced cylinders from the photonic crystal, thereby producing a defect in the crystal structure. It is assumed that the principal property of all-solid microstructured fibers is the presence of the so-called photonic bandgaps (PBGs) in their transmission spectra, where radiation at a given wavelength is efficiently reflected from the periodic structure of the photonic crystal and light in this case can propagate only along crystal defects (see, e.g., review [2] for the details). It is the presence of PBGs in photonic crystals and light guiding different from the one due to the effect of total internal reflection that stimulated studies of MOFs of this type.

At the same time, it was shown in [3] that radiation in allsolid microstructured fibers is efficiently localized not only for an ordered arrangement of rods in the cladding in the form of a photonic crystal but also for their random arrangement. In other words, the presence of PBGs, determined by the periodicity of the spatial distribution of the permittivity of the medium, is not necessary for light localization in the core of an all-solid fiber. As a rule, the transverse size $d$ of inhomogeneities in the cladding exceeds the guided light wavelength $\lambda(d>\lambda)$. In this case, the frequencies corresponding to the boundaries of PBGs in photonic bandgap fibers and the frequencies corresponding to the minimum transmission of fibers with a disordered arrangement of rods in the cladding (without PBGs) coincide. These frequencies also coincide with the cutoff frequencies of the eigenmodes of dielectric rods in the cladding [ 3,4$]$, which can be treated in corresponding spectral ranges as individual fibers in which light is guided through total internal reflection. (Cutoff frequencies in fiber optics are frequencies at which a fiber mode ceases to be guided and becomes a radiation mode.)

The possibility of light guiding in the absence of PBGs means that the localization of light in all-solid fibers is governed by a different mechanism, which, however, forms transmission spectra similar to the spectra of PBG fibers. But what is the nature of this mechanism? This question was discussed, for example, in [3, 5-8]. In particular, in the study of the transmission spectra of all-solid fibers with different cladding geometries [3], an invariable parameter of their structure, along with the refractive index of glasses used in the fibers, was only the diameter of rods in the cladding. Therefore, it is natural to conclude, as was done in [3], that the similarity of the transmission spectra of all types of fibers studied in [3] is determined to a great extent by the optical properties of an individual glass cylinder in the cladding (more exactly, by its interaction with a fiber mode, i.e., scattering).

All-solid microstructured fibers attract interest because they can be efficiently used as dispersion compensators in allfiber laser systems and as fibers with a large cross section of the mode field, which can be used for developing high-power fiber lasers and amplifiers.

Fibers with a hollow core (containing atmospheric air) belong to the same type of MOFs. They do not differ principally from all-solid microstructured fibers; however, because the nonlinearity of air is at least three orders of magnitude lower than that of silica, hollow fibers are quite promising for fiber optics, which we will discuss elsewhere.

The scattering of a plane electromagnetic wave incident at a grazing angle on an infinitely long circular dielectric cylinder (it is in this way that the interaction of the mode with each inhomogeneity in the cladding of an all-solid microstructured fiber can be treated in good approximation) was considered by many authors. This problem was first solved in [9]. Later, this solution was used in [10] to determine the continuous-spectrum modes (radiation modes) of a dielectric cylinder. The authors of [11] numerically calculated the scattering and extinction coefficients for a plane wave incident at a grazing angle on a cylinder. It was shown that in the case of grazing incidence, the extinction coefficients have much higher- $Q$ resonances compared to those in the case of a normally incident plane wave. We consider the cases of scattering of a plane wave from a dielectric cylinder when cyclic Sommerfeld waves (SWs) and cyclic Sommerfeld resonances (SRs) of different orders can be excited.

Indeed, modes in all-solid fibers are formed due to interference of fields in the core scattered from all inhomogeneities (rods) in the cladding. But according to numerical calculations performed by a number of authors, many characteristic features of scattering of a plane wave from a rod are manifested in the transmission spectra of all-solid microstructured fibers. It is for this reason that we first analyze the problem in the presence of only one rod in the cladding. A more complex problem of formation of modes in real all-solid microstructured fibers containing many rods in the cladding will be the subject of our next publications.

It is known [9] that the components of a scattered field inside a cylinder, as the components of the incident-wave field, can be expanded into infinite Fourier-Bessel series. All terms of the expansion have the dependence $\exp ( \pm \mathrm{i} m \varphi)$, where $\varphi$ is the azimuthal angle in a cylindrical coordinate system associated with a particular cylinder and $m$ is an integer. We show in what follows that certain relations between the expansion amplitudes of the same order for waves in opposite azimuthal directions lead to the excitation of SWs on the cylinder surface.

The excitation of SWs and corresponding SRs appearing upon scattering of a plane wave from a dielectric or a metal cylinder with a nanometer diameter was first analyzed in [12]. It was found that surface waves can be excited that propagate over long distances along the rod due to interaction between continuous-spectrum modes of the rod and its leaky modes of low azimuthal orders. Sommerfeld resonances occurring in this interaction have a much higher $Q$ factor compared to that for known plasmon resonances excited on the surface of metal nanorods.

In Section 2, we find conditions for the excitation of cyclic SWs and cyclic SRs in the case of grazing incidence of a plane wave on a dielectric cylinder and show that these effects are closely related to a system of resonances formed in plane wave scattering from a one-dimensional metallic diffraction grating.

A system of resonances on a diffraction grating was first observed by Robert Wood in 1902 [13]. He discovered the existence of narrow bright and dark bands in the emission spectrum scattered from a metallic diffraction grating
irradiated by a source with a slowly changing spectral intensity. These anomalies in spectra were later called after him. It was also found that the appearance of such bands in the scattered radiation spectrum strongly depends on the polarization of the incident radiation and the shape and depth of grooves of the diffraction grating.

The first theoretical explanation of Wood's anomalies was proposed by Rayleigh in 1907 [14, 15]. Using the Huygens principle, Rayleigh has shown [14] that each element of a diffraction grating is a source of a scattered spherical wave. Rayleigh assumed the appearance of diffracted waves grazing along the diffraction grating surface. This means that a part of the scattered wave propagates along the grating surface and arrives at a neighboring groove in phase with the incident wave and the waves scattered by other grooves. The interference of these waves gives rise to Wood's anomalies. In [15], scattered electromagnetic fields were expanded in series containing only outgoing waves. In this interpretation, scattered fields have singularities at the wavelengths for which one of the above-mentioned diffraction orders appears at grazing angles. The wavelengths (called Rayleigh wavelengths) at which these singularities appear correspond to Wood's anomalies.

One of the main disadvantages of the Rayleigh theory is that it cannot describe the shape (or the spectral dependence) of Wood's anomalies. Fano [16] introduced the method of successive approximations for the description of interference effects related to Wood's anomalies and explained these anomalies in terms of surface waves excited on the surface of a metallic diffraction grating. These surface waves can be treated as quasistationary surface waves theoretically obtained by Sommerfeld in solving the problem of radiation of a dipole near a conducting plane [17]. It was shown in [16] that the problem of the propagation of radiation along a onedimensional diffraction grating is directly related to the Sommerfeld problem.

A similar approach to the explanation of the nature of Wood's anomalies was proposed in [18]. Unlike the wellknown methods based on the consideration of multiple scattering [19], the theory of Wood's anomalies developed in [18] is based on the effect of excitation of surface waves on a diffraction grating. The authors of [18] showed that two types of anomalies exist: Rayleigh anomalies characterized by the appearance and disappearance of new spectral orders, and resonance anomalies corresponding to resonances of complex surface waves on a diffraction grating. It was found that anomalies of these two types can be observed both separately and simultaneously. In Sections 2 and 3, based on [18], we compare resonance phenomena appearing upon the incidence of a plane wave at a grazing angle on a dielectric cylinder and a one-dimensional metallic diffraction grating.

Fano resonances appearing upon scattering of a plane wave incident at a grazing angle on a dielectric cylinder were first described by the authors of [8], who showed that such resonances are observed in the spectral dependences of the imaginary part of the effective refractive index of a mode of an all-solid fiber. According to [8], such a behavior of the loss spectrum is possible for small ratios $d / \Lambda$, where $d$ is the diameter of cylinders in the cladding and $\Lambda$ is the distance between them. In this case, it is assumed that the interaction between cylinders in the cladding is weak and the loss spectrum of the fiber is determined by the scattering properties of an individual cylinder according to the ARROW (antiresonant reflective optical waveguides) model [4, 20].

## 2. Excitation of cyclic Sommerfeld waves on the surface of a dielectric cylinder. Analogies with plane wave scattering from a one-dimensional diffraction grating

Because the solution of the problem of plane wave scattering from a dielectric cylinder is well known, we discuss its main aspects only briefly.

We consider a plane electromagnetic wave with the electric and magnetic vectors

$$
\begin{equation*}
\mathbf{E}=\mathbf{E}_{0} \exp [\mathrm{i}(\omega t+\mathbf{k r})], \quad \mathbf{H}=\mathbf{H}_{0} \exp [\mathrm{i}(\omega t+\mathbf{k r})] \tag{1}
\end{equation*}
$$

propagating in a medium with the refractive index $n$. The absolute value of the wave vector in (1) is $|\mathbf{k}|=n k_{0}$, where $k_{0}=\omega / c$ is the wave number in the vacuum, and $\omega$ and $c$ are the circular frequency and the speed of light. The wave is incident on an infinitely long homogeneous cylinder with radius $a$ made of a dielectric with the refractive index $n_{1}$.

We introduce a coaxial cylindrical coordinate system ( $\rho, \varphi, z$ ) and assume for definiteness, following [9], that the vector $\mathbf{k}$ makes a small angle $\theta$ with the negative direction of the $z$ axis. Then the exponentials in (1) can be written as

$$
\begin{equation*}
\exp [\mathrm{i}(\omega t+\mathbf{k r})]=\exp [\mathrm{i}(\omega t-\beta z)] \exp \left[\mathrm{i}\left(k_{\perp} \rho \cos \varphi\right)\right] \tag{2}
\end{equation*}
$$

where $k_{\perp}=\left(k^{2}-\beta^{2}\right)^{1 / 2}=k_{0} n \sin \theta$ is the transverse component of the wave vector and $\beta=k_{0} n \cos \theta$ is its longitudinal component (propagation constant). For definiteness, we measure the azimuthal angle $\varphi$ with respect to the $(\mathbf{z}, \mathbf{k})$ plane.

For simplicity in what follows, we omit the factor $\exp [\mathrm{i}(\omega t-\beta z)]$, which is the same for all components of the electromagnetic field, and expand the azimuthal dependence in (2) in a Fourier-Bessel series in cylindrical harmonics. For this, we use the known relation [21] following from Sommerfeld integral representations for cylindrical functions,

$$
\begin{gather*}
\exp \left[\mathrm{i}\left(k_{\perp} \rho \cos \varphi\right)\right]=\sum_{m=-\infty}^{+\infty} \mathrm{i}^{m} J_{m}\left(k_{\perp} \rho\right) \exp (\mathrm{i} m \varphi) \\
\quad=\sum_{m=-\infty}^{+\infty} \mathrm{i}^{m} J_{m}\left(k_{\perp} \rho\right) \exp (-\mathrm{i} m \varphi) \tag{3}
\end{gather*}
$$

Then the longitudinal components of the vectors $\mathbf{H}$ and $\mathbf{E}$ of the incident wave take the form

$$
\begin{align*}
& H_{z}^{\mathrm{i}}=C_{1} \sum_{m=-\infty}^{\infty} \mathrm{i}^{m} J_{m}(q) \exp (-\mathrm{i} m \varphi)  \tag{4}\\
& E_{z}^{\mathrm{i}}=C_{2} \sum_{m=-\infty}^{\infty} \mathrm{i}^{m} J_{m}(q) \exp (-\mathrm{i} m \varphi)
\end{align*}
$$

where $\quad q=k_{\perp} \rho=\rho k_{0} n \sin \theta, \quad C_{1}=H_{0} \sin \theta \cos \delta, \quad C_{2}=$ $E_{0} \sin \theta \sin \delta$, and $\delta$ is the rotation angle of the field components $\mathbf{H}$ and $\mathbf{E}$ with respect to the wave vector $\mathbf{k}$, measured from the $(\mathbf{z}, \mathbf{k})$ plane selected above. The angle $\delta$ characterizes the polarization of incident radiation. For example, $\delta=0$ corresponds to the TE (transverse electric) polarization, when the vector $\mathbf{H}$ lies in the plane of incidence and $C_{2}=0$, while $\delta=\pi / 2$ corresponds to another limit case of the TM (transverse magnetic) polarization with $C_{1}=0$.

We seek the solution of the wave equation for the longitudinal components of the scattered wave, for the
assumed time dependence of the field $\sim \exp (\mathrm{i} \omega t)$, in a form similar to (4):

$$
\begin{align*}
& E_{z}^{\mathrm{s}}=\sum_{m=-\infty}^{\infty} b_{m}^{\mathrm{s}} H_{m}^{(2)}(q) \exp (-\mathrm{i} m \varphi),  \tag{5}\\
& H_{z}^{\mathrm{s}}=\sum_{m=-\infty}^{\infty} a_{m}^{\mathrm{s}} H_{m}^{(2)}(q) \exp (-\mathrm{i} m \varphi),
\end{align*}
$$

where $H_{m}^{(2)}(q)$ are Hankel function of the second kind; inside the cylinder, the solution is sought in the form

$$
\begin{align*}
& E_{z}=\sum_{m=-\infty}^{\infty} b_{m} J_{m}\left(q_{1}\right) \exp (-\mathrm{i} m \varphi),  \tag{6}\\
& H_{z}=\sum_{m=-\infty}^{\infty} a_{m} J_{m}\left(q_{1}\right) \exp (-\mathrm{i} m \varphi) .
\end{align*}
$$

Here, $a_{m}^{\mathrm{s}}, b_{m}^{\mathrm{s}}, a_{m}$, and $b_{m}$ are the so far unknown amplitudes of harmonics and

$$
q_{1}=k_{1 \perp} \rho=\rho \sqrt{\left(n_{1} k_{0}\right)^{2}-\beta^{2}}=\rho k_{0} \sqrt{n_{1}^{2}-n^{2} \cos ^{2} \theta} .
$$

The azimuthal components of the fields can be found from the known relations

$$
\begin{align*}
& E_{\varphi}=-\mathrm{i} \frac{\rho^{2}}{q^{2}}\left(\frac{\beta}{\rho} \frac{\partial E_{z}}{\partial \varphi}-k_{0} \frac{\partial H_{z}}{\partial \rho}\right),  \tag{7}\\
& H_{\varphi}=-\mathrm{i} \frac{\rho^{2}}{q^{2}}\left(\frac{\beta}{\rho} \frac{\partial H_{z}}{\partial \varphi}+\varepsilon k_{0} \frac{\partial E_{z}}{\partial \rho}\right),
\end{align*}
$$

following from Maxwell's equations. All the variables in (7) are related to a corresponding medium (the rod or its environment); $\varepsilon=n^{2}$ is the permittivity. The magnetic susceptibility for all media is assumed equal to unity.

Taking Eqns (4)-(7) into account, we can write the azimuthal components of the incident and scattered fields and of the fields inside the cylinder as
$H_{\varphi}^{\mathrm{i}}=\mathrm{i} \frac{\rho}{q} k_{0}\left[\sum_{m=-\infty}^{\infty}\left(\mathrm{i} \frac{\beta m}{q k_{0}} C_{1} J_{m}(q)-\varepsilon C_{2} J_{m}^{\prime}(q)\right) \mathrm{i}^{m} \exp (-\mathrm{i} m \varphi)\right]$,
$E_{\varphi}^{\mathrm{i}}=\mathrm{i} \frac{\rho}{q} k_{0}\left[\sum_{m=-\infty}^{\infty}\left(\mathrm{i} \frac{\beta m}{q k_{0}} C_{2} J_{m}(q)+C_{1} J_{m}^{\prime}(q)\right) \mathrm{i}^{m} \exp (-\mathrm{i} m \varphi)\right]$,
$H_{\varphi}^{\mathrm{s}}=\mathrm{i} \frac{\rho}{q} k_{0}\left[\sum_{m=-\infty}^{\infty}\left(\mathrm{i} \frac{\beta m}{q k_{0}} a_{m}^{\mathrm{s}} H_{m}^{(2)}(q)-\varepsilon b_{m}^{\mathrm{s}} H_{m}^{(2)^{\prime}}(q)\right) \exp (-\mathrm{i} m \varphi)\right]$,
$E_{\varphi}^{\mathrm{s}}=\mathrm{i} \frac{\rho}{q} k_{0}\left[\sum_{m=-\infty}^{\infty}\left(\mathrm{i} \frac{\beta m}{q k_{0}} b_{m}^{\mathrm{s}} H_{m}^{(2)}(q)+a_{m}^{\mathrm{s}} H_{m}^{(2)^{\prime}}(q)\right) \exp (-\mathrm{i} m \varphi)\right]$,
$H_{\varphi}=\mathrm{i} \frac{\rho}{q_{1}} k_{0}\left[\sum_{m=-\infty}^{\infty}\left(\mathrm{i} \frac{\beta m}{q_{1} k_{0}} a_{m} J_{m}\left(q_{1}\right)-\varepsilon_{1} b_{m} J_{m}^{\prime}\left(q_{1}\right)\right) \exp (-\mathrm{i} m \varphi)\right]$,
$E_{\varphi}=\mathrm{i} \frac{\rho}{q_{1}} k_{0}\left[\sum_{m=-\infty}^{\infty}\left(\mathrm{i} \frac{\beta m}{q_{1} k_{0}} b_{m} J_{m}\left(q_{1}\right)+a_{m} J_{m}^{\prime}\left(q_{1}\right)\right) \exp (-\mathrm{i} m \varphi)\right]$.

Here and hereafter, the prime at cylindrical functions denotes differentiation with respect to the argument.

As a result, boundary conditions requiring the continuity of the tangential components $\left(H_{z}, E_{z}, H_{\varphi}, E_{\varphi}\right)$ of the fields on the cylinder surface $(\rho=a)$ lead to a system of linear
inhomogeneous algebraic equations for the unknown amplitudes $a_{m}^{\mathrm{s}}, b_{m}^{\mathrm{s}}, a_{m}$, and $b_{m}$ :

$$
\begin{align*}
& a_{m}^{\mathrm{s}} f+b_{m}^{\mathrm{s}} \mathrm{i} h=X_{m}\left(C_{1} g+C_{2} \mathrm{i} h\right),  \tag{9}\\
& a_{m}^{\mathrm{s}} h+b_{m}^{\mathrm{s}} \mathrm{i} \varepsilon f_{1}=X_{m}\left(C_{1} h+C_{2} \mathrm{i} \varepsilon g_{1}\right),
\end{align*}
$$

where
$a_{m}=\left(a_{m}^{\mathrm{s}}-C_{1} X_{m}\right) \frac{H_{m}^{(2)}(q)}{J_{m}\left(q_{1}\right)}, \quad b_{m}=\left(b_{m}^{\mathrm{s}}-C_{2} X_{m}\right) \frac{H_{m}^{(2)}(q)}{J_{m}\left(q_{1}\right)}$,
and $q$ and $q_{1}$ are defined by expressions (4) and (6). Below, $q$ and $q_{1}$ denote the same quantities, but for the fixed radial component $\rho=a$. For brevity, we introduce the notation

$$
\begin{aligned}
& X_{m}=-\mathrm{i}^{m} \frac{J_{m}(q)}{H_{m}^{(2)}(q)}, \quad h=n m \cos \theta\left(\frac{1}{q^{2}}-\frac{1}{q_{1}^{2}}\right), \\
& g=\frac{J_{m}^{\prime}(q)}{q J_{m}(q)}-\frac{J_{m}^{\prime}\left(q_{1}\right)}{q_{1} J_{m}\left(q_{1}\right)}, \quad g_{1}=\frac{J_{m}^{\prime}(q)}{q J_{m}(q)}-\frac{\varepsilon_{1}}{\varepsilon} \frac{J_{m}^{\prime}\left(q_{1}\right)}{q_{1} J_{m}\left(q_{1}\right)}, \\
& f=\frac{H_{m}^{(2)^{\prime}}(q)}{q H_{m}^{(2)}(q)}-\frac{J_{m}^{\prime}\left(q_{1}\right)}{q_{1} J_{m}\left(q_{1}\right)}, \quad f_{1}=\frac{H_{m}^{(2)^{\prime}}(q)}{q H_{m}^{(2)}(q)}-\frac{\varepsilon_{1}}{\varepsilon} \frac{J_{m}^{\prime}\left(q_{1}\right)}{q_{1} J_{m}\left(q_{1}\right)} .
\end{aligned}
$$

The solution of (9) is

$$
\begin{align*}
& b_{m}^{\mathrm{s}}=X_{m}\left[C_{2}+\left(C_{2} \mathrm{i} f-\frac{C_{1} h}{\varepsilon}\right) Z_{m}\right], \\
& a_{m}^{\mathrm{s}}=X_{m}\left[C_{1}+\left(C_{1} \mathrm{i} f_{1}+C_{2} h\right) Z_{m}\right],  \tag{10}\\
& b_{m}=X_{m} Z_{m}\left(C_{2} \mathrm{i} f-\frac{C_{1} h}{\varepsilon}\right) \frac{H_{m}^{(2)}(q)}{J_{m}\left(q_{1}\right)}, \\
& a_{m}=X_{m} Z_{m}\left(C_{1} \mathrm{i} f_{1}+C_{2} h\right) \frac{H_{m}^{(2)}(q)}{J_{m}\left(q_{1}\right)},
\end{align*}
$$

where the notation

$$
\begin{align*}
Z_{m}= & \frac{2}{\pi q^{2} J_{m}(q) H_{m}^{(2)}(q) D_{m}},  \tag{11}\\
D_{m}= & \left(\frac{H_{m}^{(2)^{\prime}}(q)}{q H_{m}^{(2)}(q)}-\frac{\varepsilon_{1}}{\varepsilon} \frac{J_{m}^{\prime}\left(q_{1}\right)}{q_{1} J_{m}\left(q_{1}\right)}\right)\left(\frac{H_{m}^{(2)^{\prime}}(q)}{q H_{m}^{(2)}(q)}-\frac{J_{m}^{\prime}\left(q_{1}\right)}{q_{1} J_{m}\left(q_{1}\right)}\right) \\
& -\left[m \cos \theta\left(\frac{1}{q^{2}}-\frac{1}{q_{1}^{2}}\right)\right]^{2}
\end{align*}
$$

is introduced.
We note that equating expression (11) for $D_{m}$ to zero, we obtain the well-known dispersion equation for the propagation constants of modes in a dielectric cylinder (when the radiation source is located inside it). Therefore, the presence of $D_{m}$ in the denominators of all amplitudes (10) in field expansions gives rise to singularities of these amplitudes near the cutoff wavelengths.

From (10), using the properties of cylindrical functions (see, e.g., [22]), we can easily find relations between the amplitudes of similar harmonics entering expansions (5), (6), and (8) with different signs before $m$ :

$$
\begin{array}{ll}
a_{-m}^{\mathrm{s}}=(-1)^{m} a_{m}^{\mathrm{s}} A, & a_{-m}=(-1)^{m} a_{m} \bar{A},  \tag{12}\\
b_{-m}^{\mathrm{s}}=(-1)^{m} b_{m}^{\mathrm{s}} B, & b_{-m}=(-1)^{m} b_{m} \bar{B},
\end{array}
$$

where
$A=\frac{C_{1}+\left(C_{1} \mathrm{i} f_{1}-C_{2} h\right) Z_{m}}{C_{1}+\left(C_{1} \mathrm{i} f_{1}+C_{2} h\right) Z_{m}}, \quad B=\frac{C_{2}+\left(C_{2} \mathrm{i} f+C_{1} h / \varepsilon\right) Z_{m}}{C_{2}+\left(C_{2} \mathrm{i} f-C_{1} h / \varepsilon\right) Z_{m}}$,
$\bar{A}=\frac{C_{1} \mathrm{i} f_{1}-C_{2} h}{C_{1} \mathrm{i} f_{1}+C_{2} h}, \quad \bar{B}=\frac{C_{2} \mathrm{i} f+C_{1} h / \varepsilon}{C_{2} \mathrm{i} f-C_{1} h / \varepsilon}$.
The substitution of these relations, for example, in expansion (5) for the longitudinal magnetic component of the scattered field gives

$$
\begin{align*}
H_{z}^{\mathrm{s}} & =a_{0}^{\mathrm{s}} H_{0}^{(2)}(q)+\sum_{m=1}^{\infty} a_{m}^{\mathrm{s}} H_{m}^{(2)}(q) \\
& \times[(A+1) \cos (m \varphi)+\mathrm{i}(A-1) \sin (m \varphi)] . \tag{13}
\end{align*}
$$

This shows that other components of the scattered field and the field inside the cylinder are, like (13), superpositions of standing azimuthal harmonics. In other words, the surface harmonics $\exp (\mathrm{i} m \varphi)$ and $\exp (-\mathrm{i} m \varphi)$, having equal projections of the azimuthal 'momentum' $m$ with opposite signs, describe a standing wave (a cyclic Sommerfeld wave).

Solution (10) is noticeably simplified when either a TEpolarized or a TM-polarized wave is incident on the cylinder.

In the case of the TE polarization $\left(C_{2}=0\right)$, solution (10) becomes

$$
\begin{array}{ll}
b_{m}^{\mathrm{s}}=-X_{m} Z_{m} \frac{h}{\varepsilon}, & a_{m}^{\mathrm{s}}=X_{m}\left(Z_{m} \mathrm{i} f_{1}+1\right), \\
b_{m}=-X_{m} Z_{m} \frac{h}{\varepsilon} \frac{H_{m}^{(2)}(q)}{J_{m}\left(q_{1}\right)}, & a_{m}=X_{m} Z_{m} \mathrm{i} f_{1} \frac{H_{m}^{(2)}(q)}{J_{m}\left(q_{1}\right)}, \tag{14}
\end{array}
$$

where

$$
\begin{equation*}
X_{m}=-C_{1} i^{m} \frac{J_{m}(q)}{H_{m}^{(2)}(q)}=-H_{0} \sin \theta \mathrm{i}^{m} \frac{J_{m}(q)}{H_{m}^{(2)}(q)} . \tag{15}
\end{equation*}
$$

Relations (12) are also simplified ( $A=\bar{A}=1, B=\bar{B}=-1$ ):

$$
\begin{array}{ll}
a_{-m}^{\mathrm{s}}=(-1)^{m} a_{m}^{\mathrm{s}}, & a_{-m}=(-1)^{m} a_{m}  \tag{16}\\
b_{-m}^{\mathrm{s}}=(-1)^{m+1} b_{m}^{\mathrm{s}}, & b_{-m}=(-1)^{m+1} b_{m}
\end{array}
$$

and the expression for the longitudinal magnetic component of the scattered field reduces to the form

$$
H_{z}^{\mathrm{s}}=a_{0}^{\mathrm{s}} H_{0}^{(2)}(q)+2 \sum_{m=1}^{\infty} a_{m}^{\mathrm{s}} H_{m}^{(2)}(q) \cos (m \varphi) .
$$

As a whole, all the components of the field turn out to be even functions of $m$, representing a superposition of standing azimuthal harmonics.

In the case of a TM-polarized incident plane wave ( $C_{1}=0$ ), solution (10) takes the form

$$
\begin{aligned}
& b_{m}^{\mathrm{s}}=X_{m} C_{2}\left(Z_{m} \mathrm{i} f+1\right), \quad a_{m}^{\mathrm{s}}=X_{m} C_{2} h Z_{m}, \\
& b_{m}=X_{m} C_{2} Z_{m} \mathrm{i} f \frac{H_{m}^{(2)}(q)}{J_{m}\left(q_{1}\right)}, \quad a_{m}=X_{m} C_{2} h Z_{m} \frac{H_{m}^{(2)}(q)}{J_{m}\left(q_{1}\right)},
\end{aligned}
$$

where $C_{2}=E_{0} \sin \theta=\left(H_{0} / n\right) \sin \theta$ and the known relation between numerical values of the electric and magnetic field strengths in a plane wave is taken into account [23]. The rest of the notation is the same. Relations between the expansion amplitudes of the field components with different signs of $m$
for the TM polarization have the form

$$
\begin{array}{ll}
a_{m}^{\mathrm{s}}=(-1)^{m+1} a_{-m}^{\mathrm{s}}, & a_{m}=(-1)^{m+1} a_{-m}, \\
b_{m}^{\mathrm{s}}=(-1)^{m} b_{-m}^{\mathrm{s}}, & b_{m}=(-1)^{m} b_{-m} .
\end{array}
$$

In this case, all components of the field are also even functions of $m$ and represent superpositions of standing azimuthal harmonics.

An arbitrary polarization must be considered as the case intermediate between those with TE and TM polarizations, but this requires using the more complex general solution in (10). For simplicity in what follows, we restrict ourselves to the analysis of scattering of a TE-polarized wave from a cylinder.

We now use the obtained results to find quantities characterizing the efficiency of plane wave scattering from a cylinder.

The ability of a set of dielectric cylinders located in a fiber cladding to localize light in the fiber core can be largely characterized, for example, by the Umov-Poynting vector of the radiation scattered by a separate cylinder. The efficiency of scattering of a plane wave from a cylinder is best demonstrated by the average value (for the field oscillation period) of the radial component of the Umov-Poynting vector

$$
\begin{equation*}
S_{\mathrm{r}}^{\mathrm{s}}=\frac{c}{8 \pi} \operatorname{Re}\left[\mathbf{E}^{\mathrm{s}} \mathbf{H}^{\mathrm{s} *}\right]_{\mathrm{r}}=\frac{c}{8 \pi} \operatorname{Re}\left(E_{\varphi}^{\mathrm{s}} H_{z}^{\mathrm{s} *}-E_{z}^{\mathrm{s}} H_{\varphi}^{\mathrm{s} *}\right) . \tag{17}
\end{equation*}
$$

Based on physical considerations, we are interested only in the values of $S_{\mathrm{r}}^{\mathrm{s}}$ in the far-field zone (for $\rho \rightarrow \infty$ ). Modes are formed in the fiber core due to the constructive interference of waves backscattered by all the cylinders in the cladding.

We calculate (17) using expressions (5) and (8) for the scattered components of the field and the asymptotic forms of the Hunkel functions $H_{m}^{(2)}(q)$ and their derivatives for large values of $q$ (see [22]), omitting terms decreasing faster than $\sim q^{-\alpha}(\alpha>1 / 2)$ :

$$
\begin{aligned}
H_{m}^{(2)}(q) & \approx \sqrt{\frac{2}{\pi q}} \exp \left[-\mathrm{i}\left(q-\frac{\pi m}{2}-\frac{\pi}{4}\right)\right] \\
& =\sqrt{\frac{2}{\pi q}} \mathrm{i}^{m} \exp \left[-\mathrm{i}\left(q-\frac{\pi}{4}\right)\right] \\
H_{m}^{(2)^{\prime}}(q) & =-H_{m+1}^{(2)}(q)+\frac{m}{q} H_{m}^{(2)}(q) \\
& \cong \sqrt{\frac{2}{\pi q}}\left(\frac{m}{q}-\exp \frac{\mathrm{i} \pi}{2}\right) \exp \left[-\mathrm{i}\left(q-\frac{\pi m}{2}-\frac{\pi}{4}\right)\right] \approx-\mathrm{i} H_{m}^{(2)}(q) .
\end{aligned}
$$

Then the field components in (17) for a TE-polarized incident wave become

$$
\begin{aligned}
& H_{z}^{\mathrm{s}} \cong \sqrt{\frac{2}{\pi q}} \exp \left[-\mathrm{i}\left(q-\frac{\pi}{4}\right)\right]\left(a_{0}^{\mathrm{s}}+2 \sum_{m=1}^{\infty} a_{m}^{\mathrm{s}} \mathrm{i}^{m} \cos (m \varphi)\right), \\
& E_{z}^{\mathrm{s}} \cong-2 \mathrm{i} \sqrt{\frac{2}{\pi q}} \exp \left[-\mathrm{i}\left(q-\frac{\pi}{4}\right)\right] \sum_{m=1}^{\infty} b_{m}^{\mathrm{s}} \mathrm{i}^{m} \sin (m \varphi),
\end{aligned}
$$

$$
H_{\varphi}^{\mathrm{s}} \cong 2 \mathrm{i} \frac{\rho}{q} k_{0} \varepsilon \sqrt{\frac{2}{\pi q}} \exp \left[-\mathrm{i}\left(q-\frac{\pi}{4}\right)\right] \sum_{m=1}^{\infty} b_{m}^{\mathrm{s}} \mathrm{i}^{m} \sin (m \varphi),
$$

$$
E_{\varphi}^{\mathrm{s}} \cong \frac{\rho}{q} k_{0} \sqrt{\frac{2}{\pi q}} \exp \left[-\mathrm{i}\left(q-\frac{\pi}{4}\right)\right]\left(a_{0}^{\mathrm{s}}+2 \sum_{m=1}^{\infty} a_{m}^{\mathrm{s}} \mathrm{i}^{m} \cos (m \varphi)\right) .
$$

Substituting (18) into (17), we obtain

$$
\begin{align*}
S_{\mathrm{r}}^{\mathrm{s}}(\rho, \varphi)= & \frac{\rho k_{0} c}{\pi^{2} q^{2}}\left[\left|\frac{a_{0}^{\mathrm{s}}}{2}+\sum_{m=1}^{\infty} a_{m}^{\mathrm{s}} \mathrm{i}^{m} \cos (m \varphi)\right|^{2}\right. \\
& \left.+\varepsilon\left|\sum_{m=1}^{\infty} b_{m}^{\mathrm{s}} \mathrm{i}^{m} \sin (m \varphi)\right|^{2}\right] \tag{19}
\end{align*}
$$

Because $q$ is proportional to the distance $\rho$ from the rod, relation (19) means that the scattered radiation power per unit surface area decreases proportionally to $\rho$. But the area itself of the surface moving away as $\rho$ increases as $2 \pi \rho$ (per rod unit length along $z$ ). As a result, the power $P_{\mathrm{s}}$ scattered by the rod unit length turns out to be independent of $\rho$ and is determined only by the azimuthal angle $\varphi$ :

$$
\begin{align*}
P_{\mathrm{s}}(\varphi)= & \frac{c \lambda}{(\pi n \sin \theta)^{2}}\left[\left|\frac{a_{0}^{\mathrm{s}}}{2}+\sum_{m=1}^{\infty} a_{m}^{\mathrm{s}} \mathrm{i}^{m} \cos (m \varphi)\right|^{2}\right. \\
& \left.+\varepsilon\left|\sum_{m=1}^{\infty} b_{m}^{\mathrm{s}} \mathrm{i}^{m} \sin (m \varphi)\right|^{2}\right] . \tag{20}
\end{align*}
$$

Using the known definition of the electromagnetic energy density in a plane wave [23] and multiplying this density by the phase velocity component directed normally to the cylinder surface, we find the incident radiation intensity $I_{i}$. As a result, the differential scattering cross section defined as $\sigma_{\mathrm{d}}=P_{\mathrm{s}} / I_{\mathrm{i}}$ takes the form

$$
\begin{align*}
\sigma_{\mathrm{d}}(\varphi)= & \frac{8 \lambda}{\pi n H_{0}^{2} \sin ^{3} \theta}\left[\left|\frac{a_{0}^{\mathrm{s}}}{2}+\sum_{m=1}^{\infty} a_{m}^{\mathrm{s}} \mathrm{i}^{m} \cos (m \varphi)\right|^{2}\right. \\
& \left.+\varepsilon\left|\sum_{m=1}^{\infty} b_{m}^{\mathrm{s}} \mathrm{i}^{m} \sin (m \varphi)\right|^{2}\right] . \tag{21}
\end{align*}
$$

Because all expansion coefficients in (14) are proportional to the product $H_{0} \sin \theta$, the factor in front of the brackets in (21) is actually independent of $H_{0}$ and depends on the angle of incidence of the wave as $1 / \sin \theta$.

In particular, for backscattered radiation $(\varphi=0)$, it follows from (21) that

$$
\begin{equation*}
\sigma_{\mathrm{d}}(0)=\frac{8 \lambda}{\pi n H_{0}^{2} \sin ^{3} \theta}\left|\frac{a_{0}^{\mathrm{s}}}{2}+\sum_{m=1}^{\infty} a_{m}^{\mathrm{s}} \mathrm{i}^{m}\right|^{2} . \tag{22}
\end{equation*}
$$

For a TM-polarized incident wave, we obtain

$$
\begin{align*}
\sigma_{\mathrm{d}}(\varphi) & =\frac{8 \lambda}{\pi n H_{0}^{2} \sin ^{3} \theta}\left[\varepsilon\left|\frac{b_{0}^{\mathrm{s}}}{2}+\sum_{m=1}^{\infty} b_{m}^{\mathrm{s}} \mathrm{i}^{m} \cos (m \varphi)\right|^{2}\right. \\
& \left.+\left|\sum_{m=1}^{\infty} a_{m}^{\mathrm{s}} \mathrm{i}^{m} \sin (m \varphi)\right|^{2}\right],  \tag{23}\\
\sigma_{\mathrm{d}}(0) & =\frac{8 \lambda}{\pi n H_{0}^{2} \sin ^{3} \theta}\left|\frac{b_{0}^{\mathrm{s}}}{2}+\sum_{m=1}^{\infty} b_{m}^{\mathrm{s}} \mathrm{i}^{m}\right|^{2} . \tag{24}
\end{align*}
$$

We now discuss analogies between scattering of a plane wave by a one-dimensional metallic diffraction grating and excitation of a cyclic Sommerfeld wave on a dielectric cylinder surface. We analyze scattering of a plane wave by a diffraction grating based on papers [16, 18].

We consider a diffraction grating of infinite sizes along $z$ and $y$ directions and the modulation period $d$ of the refractive index along the $z$ axis. According to [18], the periodic
dependence of the refractive index of such a diffraction grating can be represented by the surface impedance

$$
\begin{equation*}
Z^{\mathrm{s}}(z)=\sum_{n=-\infty}^{+\infty} Z_{n}^{\mathrm{s}} \exp \left(\mathrm{i} \frac{2 \pi n}{d} z\right) . \tag{25}
\end{equation*}
$$

It is assumed that a TM-polarized plane wave (whose magnetic field vector is directed along the $y$ axis) is incident on the diffraction grating from the vacuum. The angle of incidence with respect to the normal to the grating is $\theta$, and the vector $H$ can be written in the form

$$
\begin{equation*}
H_{y}^{\mathrm{i}}(x, z)=H_{0} \exp (\mathrm{i} p x) \exp (-\mathrm{i} \beta z), \tag{26}
\end{equation*}
$$

where $\beta=k \cos \theta, p=\left(k^{2}-\beta^{2}\right)^{1 / 2}$, and $k=\omega / c$ is the wave number in free space.

In this case, the scattered wave field can be written as
$H_{y}^{\mathrm{s}}(x, z)=\sum_{n=-\infty}^{+\infty} A_{n}(\beta) \exp \left(\mathrm{i} p_{n} x\right) \exp \left(-\mathrm{i} \frac{2 \pi n z}{d}\right) \exp (-\mathrm{i} \beta z)$,
where $x \geqslant 0$ and $p_{n}=\left[k^{2}-(\beta+2 \pi n / d)^{2}\right]^{1 / 2}$.
Using periodic boundary conditions for impedance (25) in the $x=0$ plane, we obtain an infinite system of inhomogeneous linear algebraic equations for the amplitudes $A_{n}(\beta)$ of scattered fields [18]. As a result, each of the amplitudes of the $n$ th-order harmonic is found from the relation

$$
\begin{equation*}
A_{n}=\frac{\Delta_{n}}{\Delta}, \tag{28}
\end{equation*}
$$

where $\Delta$ is the determinant of the corresponding system of linear homogeneous equations and $\Delta_{n}$ is the determinant obtained by replacing the $n$th column of $\Delta$ with a vector column containing the right-hand part of the system of linear inhomogeneous equations, associated with the amplitude of the incident plane wave.

According to $[16,18]$, the infinite system of equations obtained in this way determines two types of resonances forming the structure of Wood's anomalies. The first type is related to the existence of propagating eigenmodes (leaky modes) of the diffraction grating with propagation constants that can be found by solving the dispersion equation $\Delta=0$. These propagation constants are complex and the corresponding resonances have a Lorentzian spectral profile. The second type of resonances is related to a rapid change in the amplitudes of diffraction orders corresponding to the appearance or disappearance of one of them in a narrow spectral interval.

For example, we consider the $(n-1)$ th diffraction order with the amplitude $A_{n-1}(\beta)$ and assume that the determinant $\Delta_{n-1}$ is zero for some wavelength. If the determinant $\Delta$ determining a leaky mode of the diffraction grating also vanishes in the immediate vicinity, then expression (28) is given by a product of two factors one of which has a complex pole and the other has a first-order zero in a narrow spectral region. As a result, $A_{n-1}(\beta)$ has a minimum and a maximum located close to each other. This means that the spectral dependence of the resonance has an asymmetric shape of the Fano resonance type. In other words, anomalies of this type are determined by the coexistence of resonance transmission and resonance reflection in a narrow spectral range, which corresponds to the interaction of discrete-spectrum modes associated with the diffraction grating with the continuum of propagating radiation modes [24].

In [16], the diffraction of a plane wave from a diffraction grating was considered as the momentum transfer from the grating to the incident wave. In this case, a pair of waves with tangential components of the wave vector (the wave momentum) $k_{\mathrm{pt}}=k_{0} \pm 2 \pi n / \Lambda$ excited by the incident plane wave on the grating surface can be treated as induced oscillations. If the phase-matching condition of the coincidence of the $k_{\mathrm{pt}}$ for individual diffraction orders and the real part of the propagation constant of a surface quasistationary wave is satisfied, these induced oscillations can have the same intensity as the quasistationary wave itself. Surface quasistationary waves of this type, excited on the diffraction grating surface, have the same nature as surface waves excited on a metal surface by an oscillating dipole located above it (Sommerfeld waves). It is these waves that, in Fano's opinion, cause Wood's anomalies.

We can therefore state that scattering of a plane TEpolarized wave from a cylinder corresponds to scattering of a plane TM wave by a one-dimensional diffraction grating if the cylinder surface is treated as a diffraction quasigrating with the period $2 \pi$ in the azimuthal direction. The processes of scattering by two physically different objects are similar because the momentum transfer to the incident wave from the diffraction grating in the wave propagation direction in one case and the momentum transfer to a polarized wave incident at a grazing angle on a cylinder due to the curvature and cyclic periodicity of the cylindrical surface in the other case give rise to both polarization states in the scattered wave field. Thus, scattering of a plane wave by a dielectric cylinder is physically similar to scattering of a plane wave by a diffraction grating, and therefore resonance phenomena similar to the known Wood anomalies should also be observed upon scattering by a cylinder.

We illustrate these assertions with specific examples.

## 3. Results of calculations

We consider a TE-polarized plane wave incident at the angle $\theta=1^{\circ}$ from a medium with the refractive index $n=1.45$ on a cylinder that has the cross-sectional radius $a=1.2 \mu \mathrm{~m}$ and is made of a dielectric with the refractive index $n_{1}=1.48$. Such a small difference between refractive indices is typical for allsolid fibers. Spectral dependences of the amplitudes of cylindrical harmonics (radiation modes) of the first five orders (up to $m=4$ ) of the scattered magnetic field are presented in Fig. 1. Spectral dependences of the amplitudes of higher-order harmonics ( $m>4$ ) also have a pronounced resonance character. The dependences presented in Fig. 1 are similar to those obtained in [18] for light scattering by a diffraction grating.

To analyze the resonance behavior of the amplitudes of scattered cylindrical harmonics in detail, we consider the spectral dependence of the amplitude $a_{0}^{\mathrm{s}}(\lambda)$ of the zeroth harmonic. We can see from Fig. 1 that this dependence contains both Fano resonances and Lorentzian resonances in the long-wavelength spectral region, which also resembles the structure of resonances in the scattering of a plane wave by a diffraction grating [18]. Using expression (14), which for $a_{0}^{\mathrm{s}}(\lambda)$ takes the form

$$
\begin{equation*}
a_{0}^{\mathrm{s}}=X_{0}\left(\frac{1}{q} \frac{J_{0}^{\prime}(q)}{J_{0}(q)}-\frac{1}{q_{1}} \frac{J_{0}^{\prime}\left(q_{1}\right)}{J_{0}\left(q_{1}\right)}\right)\left(\frac{1}{q} \frac{H_{0}^{(2)^{\prime}}(q)}{H_{0}^{(2)}(q)}-\frac{1}{q_{1}} \frac{J_{0}^{\prime}\left(q_{1}\right)}{J_{0}\left(q_{1}\right)}\right)^{-1}, \tag{29}
\end{equation*}
$$



Figure 1. Spectral dependences of the absolute values of amplitudes $a_{m}^{\mathrm{s}}(\lambda)$ of harmonics of the scattered magnetic field normalized to $H_{0}$ : (a) $m=0$ (solid curve), $m=1$ (dashed curve), and $m=2$ (dotted curve); (b) $m=3$ (solid curve) and $m=4$ (dashed curve).
we can analyze the dependence $a_{0}^{\mathrm{s}}(\lambda)$ from the standpoint of the interaction of the discrete spectrum modes of a cylinder with propagating continuous-spectrum modes in this cylinder [25]. For this, we consider the spectral dependences of the numerator and denominator in (29) separately, which themselves exhibit the resonance behavior. These dependences, without the factor $X_{0}$, are shown in Fig. 2. We can see that the resonances $a_{0}^{\mathrm{s}}(\lambda)$ are mainly determined by resonances in the numerator of (29). To analyze the spectral dependence $a_{0}^{\mathrm{s}}(\lambda)$ in more detail, it is necessary to study both terms in the numerator, $A_{0}^{\mathrm{s}}(\lambda)=J_{0}^{\prime}\left(q_{1}\right) /\left(J_{0}\left(q_{1}\right) q_{1}\right)$ and $B_{0}^{s}(\lambda)=J_{0}^{\prime}(q) /\left(J_{0}(q) q\right)$, separately. Their spectral dependences are presented in Fig. 3.

The first of the terms is related to the presence of intrinsic transverse resonances in the cylinder (its discrete modes corresponding to $m=0$ ). In the spectral range under study ( $\lambda \leqslant 1.2 \mu \mathrm{~m}$ ), the argument $q_{1}$ is large, and we can use the asymptotic approximation for cylindrical functions. The dependence $A_{0}^{\mathrm{s}}(\lambda)$ calculated in this approximation (see Fig. 3) is well described by the function

$$
\begin{equation*}
A_{0}^{\mathrm{s}}(\lambda) \approx-\frac{\tan \left(q_{1}-\pi / 4\right)}{q_{1}} \tag{30}
\end{equation*}
$$

This function is alternating and has discontinuities at points corresponding to the cutoff wavelengths, which are deter-


Figure 2. Spectral dependences of absolute values of the numerator $N_{0}^{\mathrm{s}}(\lambda)$ of expression (29) (dashed curve), its denominator $D_{0}^{\mathrm{s}}(\lambda)$ (dotted curve) and the ratio $N_{0}^{\mathrm{s}}(\lambda) / D_{0}^{\mathrm{s}}(\lambda)$ (solid curve) (logarithmic scale).
mined for $m=0$ [26] from the characteristic equation $J_{0}\left(q_{1}\right)=0$; in the case under consideration, the cutoff wavelengths are $0.936,0.417,0.270 \mu \mathrm{~m}$, etc.

The second term, $B_{0}^{\mathrm{s}}(\lambda)$, in the numerator in (29) is related to the continuum of propagating modes of the continuous spectrum, and, as can be seen from Fig. 3, has the nonresonance character in the relevant spectral range [the function $J_{0}(q)$ has no roots] and is a slowly varying function.

Obviously, the numerator in (29) vanishes at the intersection points of the dependences $A_{0}^{\mathrm{s}}(\lambda)$ and $B_{0}^{\mathrm{s}}(\lambda)$. This occurs mainly near the cutoff wavelengths [except for the longest cutoff wavelength $\lambda=0.936 \mu \mathrm{~m}$, above which the asymptotic approximation for $A_{0}^{\mathrm{s}}(\lambda)$ is no longer valid and the intersection with $B_{0}^{\mathrm{s}}(\lambda)$ is absent]. The existence of a zero and a pole of the numerator located close to each other leads to the appearance of characteristic asymmetric Fano resonances in the spectral dependences of its absolute value. However, for the longest cutoff wavelength, the standard symmetric Lorentzian resonance appears (see Fig. 2).

A similar analysis can also be performed for the amplitudes of higher-order harmonics ( $m \geqslant 1$ ). However, it is more complicated because a simple expression like (29) cannot be obtained for these harmonics.

The presence of resonances in the spectral dependences of amplitudes of azimuthal harmonics of the field scattered by a cylinder causes a similar behavior of the spectral dependence of the differential backscattering cross section $\sigma_{\mathrm{d}}(0)$ (Fig. 4). It follows that this cross section is mainly determined by the azimuthal harmonic with $m=1$ (see Fig. 1). The spectral dependence of the amplitude of this harmonic is comparatively weak and has minima at $\lambda \approx 0.9,0.55,0.3 \mu \mathrm{~m}$, etc. The contribution of other harmonics (mainly with $m=0$ and 2 ) is noticeable only in some narrow spectral ranges (for example, at $\lambda=0.40$ and $0.26 \mu \mathrm{~m}$ ). This is explained by the alternating behavior of terms in the sum in (22). Indeed, we can see from (22) and the definition of $a_{m}^{\mathrm{s}}$ in (14) that each term in this sum contains the factor $(-1)^{m+1}$, which determines the nonmonotonic


Figure 3. Spectral dependences of the absolute value $N_{0}^{\mathrm{s}}(\lambda)=$ $B_{0}^{\mathrm{s}}(\lambda)-A_{0}^{\mathrm{s}}(\lambda)$ (solid curve) and the terms $A_{0}^{\mathrm{s}}(\lambda)$ (dashed curve) and $B_{0}^{\mathrm{s}}(\lambda)$ (dotted curve) (linear scale).


Figure 4. The differential backscattering cross section $\sigma_{d}(0)[\mu \mathrm{m}]$ for a grazing angle of incidence $\theta=1^{\circ}$ of a plane wave (logarithmic scale).
behavior of $\sigma_{\mathrm{d}}(0)$ in these spectral regions. This also leads to an inhomogeneous spectral dependence of $\sigma_{\mathrm{d}}(0)$ in the region away from the resonances: the cross section in the first region $(\lambda \geqslant 1 \mu \mathrm{~m})$ is approximately an order of magnitude higher than in the second region $(0.6 \leqslant \lambda \leqslant 0.9 \mu \mathrm{~m})$; in the second region, on the contrary, it is an order of magnitude lower than in the third region $(0.40 \leqslant \lambda \leqslant 0.55 \mu \mathrm{~m})$, and so on.

Differential scattering cross sections $\sigma_{\mathrm{d}}(\varphi)$ have similar spectral dependences for other, nonzero scattering angles.

We note that the differential cross section of scattering strictly backward or at some other angle is small and contains a comparatively small amount of physical information because, obviously, a mode in a fiber is formed by the interference of waves scattered by cylinders in the cladding not to a specific angle but to some range of predominantly backscattering angles $(|\varphi| \leqslant \pi / 2)$. Here, this cross section is presented only to demonstrate that harmonics of different azimuthal orders are involved in scattering.

Integrating (21) with respect to the azimuthal angle (from $-\varphi$ to $+\varphi$ ) with a solid angle $2 \varphi \leqslant \pi$, we find the integrated


Figure 5. Integrated cross section $\sigma_{\mathrm{s}}[\mu \mathrm{m}]$ for solid scattering angles $2 \varphi=\pi / 4$ (solid curve), $2 \varphi=\pi / 3$ (dashed curve), and $2 \varphi=\pi / 2$ (dotted curve) (logarithmic scale).
scattering cross section

$$
\begin{align*}
\sigma_{\mathrm{s}}(\varphi) & =\frac{8 \lambda}{\pi n H_{0}^{2} \sin ^{3} \theta}\left\{\varphi\left(\frac{\left|a_{0}^{\mathrm{s}}\right|^{2}}{2}+\sum_{m=1}^{\infty}\left|a_{m}^{\mathrm{s}}\right|^{2}+\varepsilon \sum_{m=1}^{\infty}\left|b_{m}^{\mathrm{s}}\right|^{2}\right)\right. \\
& +\sum_{m=1}^{\infty}\left(\left|a_{m}^{\mathrm{s}}\right|^{2}-\varepsilon\left|b_{m}^{\mathrm{s}}\right|^{2}\right) \frac{\sin (2 m \varphi)}{2 m} \\
& +\sum_{m=1}^{\infty} \mathrm{i}^{m}\left[a_{0}^{\mathrm{s} *} a_{m}^{\mathrm{s}}+a_{0}^{\mathrm{s}} a_{m}^{\mathrm{s} *}(-1)^{m}\right] \frac{\sin (m \varphi)}{m} \\
& +\sum_{n=1}^{\infty} \sum_{m=1, m \neq n}^{\infty}\left[\left(a_{n}^{\mathrm{s}} a_{m}^{\mathrm{s} *}+\varepsilon b_{n}^{\mathrm{s}} b_{m}^{\mathrm{s} *}\right) \frac{\sin [(m-n) \varphi]}{m-n}\right. \\
& \left.\left.+\left(a_{n}^{\mathrm{s}} a_{m}^{\mathrm{s} *}-\varepsilon b_{n}^{\mathrm{s}} b_{m}^{\mathrm{s} *}\right) \frac{\sin [(m+n) \varphi]}{m+n}\right] \mathrm{i}^{n-m}\right\} \tag{31}
\end{align*}
$$

The spectral dependences of $\sigma_{\mathrm{s}}$ for three scattering solid angles are presented in Fig. 5. We can see that all the calculated curves are similar and correlate well with the differential backscattering cross section (see Fig. 4). They not only have resonance properties but also exhibit inhomogeneous behavior in alternating spectral regions separated by singularities. We also note that the integrated cross section is high, its value for $2 \varphi \geqslant \pi / 3$ becoming comparable with the transverse size of the cylinder (with the radius $1.2 \mu \mathrm{~m}$ ) and even exceeding it. This suggests that backscattering (to the core) is very efficient, which should provide favorable conditions for the formation of a mode in a fiber with the cladding containing such cylinders. The results of our calculations and calculations of other authors confirm this conclusion.

## 4. Conclusions

Our analysis has shown that scattering of a plane electromagnetic wave by a dielectric rod occurs via excitation of standing cyclic Sommerfeld waves on the rod surface. At the cutoff wavelengths of the rod eigenmodes, this process has a resonance character caused by the interference of the corresponding leaky quasidiscrete modes and the scattered radiation continuum.

Obviously, part of the incident wave energy is spent to excite cyclic SWs, which, along with material losses, deter-
mines total optical losses in all-solid microstructured fibers. Losses for the cyclic SW excitation are determined by the density of eigenstates of the rod. These losses can be reduced by replacing a solid rod by a capillary with the same diameter.

Calculations have shown that the smaller the wall thickness of the capillary is, the lower the density of its eigenstates and the lower the energy spent for a cyclic SW excitation. Indeed, the higher density of the eigenstates of a solid cylinder corresponds to the higher power flux in it compared to the power flux in capillary walls. This in turn produces higher material losses in the rod compared to those in a capillary of the same size. This is confirmed by many calculations performed for MOFs with the cladding consisting of a row of cylinders or capillaries.

The replacement of rods in all-solid microstructured fibers by capillaries was first considered in [27].

However, while scattering from solid rods is accompanied by the excitation of SWs mainly in low azimuthal orders, in the case of capillaries, on the contrary, modes with large azimuthal numbers dominate (resembling whispering-gallery modes). As mentioned, we will analyze hollow MOFs with a capillary cladding elsewhere.

We have shown in this paper that the resonances that we found, of both Fano and Lorentzian type, are analogs of wellknown Wood's anomalies appearing in the scattering of a plane wave by a metallic diffraction grating. This analogy is more descriptive if the rod is treated as a quasigrating with a period that is determined, unlike that of a planar diffraction grating, by the curvilinear azimuthal coordinate and is equal to $2 \pi$.

We note that we considered the problem of scattering by a rod from the standpoint of the applicability of our results to all-solid MOFs with the cladding containing cylindrical inhomogeneities with a circular cross section. We have shown that already for the solid scattering angle $2 \varphi \approx \pi / 3$, the scattering cross section becomes comparable with the transverse size of the cylinder or even exceeds it, which provides the efficiency required for the formation of a fiber mode.

The analysis performed above cannot be applied in the case of inhomogeneities of other shapes (rectangular, triangular, etc.) because the use of cylindrical coordinates in our analysis was important. It seems that only in this case (and possibly also for inhomogeneities with elliptical sections) it is possible to excite cyclic standing surface waves, which mainly determine the scattering efficiency.

In our first papers [28,29] devoted to the study of hollow MOFs, we assumed that their main advantage over the existing analogs was the negative curvature of the corecladding interface. However, we currently see that although the negative curvature is a necessary condition, it is not sufficient for obtaining low losses in these fibers. It is important to have the possibility to excite standing azimuthal Sommerfeld waves for inhomogeneities in the cladding. We note that this condition is not satisfied, for example, in the case of inhomogeneities in claddings of hollow fibers studied in [30, 31].

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[^0]:    A D Pryamikov, A S Biriukov Fiber Optics Research Center, Russian Academy of Sciences,
    ul. Vavilova 38, 119333 Moscow, Russian Federation
    Tel. +7 (499) 50381 93. Fax +7 (499) 1358139
    E-mail: pryamikov@fo.gpi.ru, biriukov@fo.gpi.ru
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