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METHODOLOGICAL NOTES

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On the physical interpretation of Thomson scattering in a plasma

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Contents

Abstract. The scattering of a wave by an individual particle is due to the fact that the particle oscillates in the field of the incident wave and these oscillations radiate the scattered wave. It is usually believed that scattering in a plasma, even though the cross section in it is on the order of the Thomson scattering cross section in a vacuum, takes place by plasma density fluctuations, which also involve ions, so that the total scattered radiation is not the sum of Thomson scattering by individual electrons. Although the scattering formulas widely used in processing observations are correct, their interpretation often is not. This note proves rigorously that scattering in a plasma is the sum of the scattering from the electrons and ions, with the total momentum difference between the incident and scattered waves being distributed among the electrons and ions, and that it is only based on this interpretation that we can obtain the conservation laws for waves and particles in the plasma. General physical, astrophysical, and other implications of the correct interpretation of scattering processes for radiation frequencies much larger than the plasma frequency are discussed.

1. Introduction. Fundamentals of the physics and research history of scattering processes

The necessity of changing terminology in the studies of plasma scattering of electromagnetic and other waves became clear comparatively long ago [1]. However, so far a clear understanding that this terminological change is necessary for a more accurate description of the physics of scattering processes has often been absent. In the excellent and one of the most often cited monographs on the scattering of electromagnetic waves in plasmas, John Sheffield [2] states already in the Introduction, where the contents of the monograph chapters is presented, that "because the scattered intensity is inversely proportional to a charge mass (more exactly, to the charge mass squared, V N Ts), we should conclude at once that scattering is mainly produced by electrons only." This statement, which was adopted a long time ago, is incorrect, however. Indeed, it does not follow at all that if the expression for the scattered intensity contains only the electron mass (in this case, the mass squared in the denominator), scattering cannot be produced by plasma ions as well. It was shown in Ref. [2] that correct expressions for the scattered intensity do contain the distribution function of plasma ions, which is proportional to the ion concentration (see expression (6.3.4) in monograph [2]). Then, in this book the distinction is drawn between 'incoherent' scattering, for which the wavelength $2\pi/|\mathbf{k}_{\mathrm{inc}} - \mathbf{k}_{\mathrm{sc}}|$ is much smaller than the Debye screening radius, and 'coherent' scattering, for which the relation between these quantities becomes inverse (here, $2\pi/|\mathbf{k}_{\mathrm{inc}}|$ is the incident radiation wavelength, and $2\pi/|\mathbf{k}_{\mathrm{sc}}|$ is the scattered radiation wavelength). In the limit of 'incoherent' scattering, the scattered radiation intensity is the sum of radiation intensities scattered by free electrons in a vacuum. In this case, it is often asserted that scattering in a medium (plasma) should differ from that by separate particles because a homogeneous medium as a whole should not scatter radiation, and scattering can only be produced by fluctuations in the medium.

Wave scattering by plasma fluctuations was first considered within the framework of the consistent theory of plasma fluctuations [3–5] (see monograph [2] and references cited therein). Indeed, the scattered intensity contained two terms [2]: one was proportional to the electron distribution

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function, while the other term, proportional to the ion distribution function, gave the so-called incoherent scattering in the limit indicated above. The mechanism of coherent scattering when terms proportional to the ion distribution function appear and scattering cross sections change in the term proportional to the electron distribution function is explained in the following way [2]. The ions have polarization electron 'clouds', which are also involved in scattering, and density fluctuations affect the polarization electron clouds of ions, which makes scattering by plasma fluctuations different from the sum of scatterings by individual electrons. The statement that the electron clouds of plasma particles are involved in wave scattering is correct. But why can we not say that a part of the scattered intensity proportional to the ion distribution occurs namely by ions? The reason is that ions are heavy and therefore cannot have a large oscillation amplitude in the field of the incident wave. However, the radiation of the scattered wave, as stated, is caused by the electrons of the polarization cloud of ions, and because electron oscillations are inversely proportional to their mass, the total scattered intensity is only determined by the electron mass. These are standard words accompanying the explanation of 'coherent' scattering.

The correctness or inaccuracy of such a picture can be most simply verified based on the energy and momentum conservation laws during scattering (see Section 2). And the statement that the energy and momentum are imparted to the electrons of polarization clouds does not stand up to such a verification, as has been shown already in book [1]. Clearly, the total momentum and total energy of scattered radiation are not equal to the total momentum and total energy of incident radiation. The question is where the energy and momentum are imparted to? Of course, they are imparted to scattering particles. But are they completely imparted to electrons or only partially, or maybe they are imparted almost completely to ions? The result of calculations shows that the energy and momentum can be transferred in certain cases mainly to ions. As pointed out, the scattered radiation intensity contains two terms, one of them being proportional to the electron distribution function, while the other is proportional to the ion distribution function. We can consider both these terms separately, calculate the reverse action of wave scattering processes described by these terms on the electron and ion distributions (see in Section 2 the expressions of the standard theory, containing both these terms, and the calculation of scattered intensity described by the second term), and confirm that scattering occurs both from plasma electrons and ions. It turns out that the second term describes the energy and momentum transfer to ions, while the first term describes the transfer to electrons.

Thus, a correct interpretation is as follows: wave scattering occurs both from plasma electrons and ions, and the resulting scattered intensity *is simply a sum of radiation intensities scattered by plasma electrons and ions.* Note that scattered intensity from ions is on the same order of magnitude as scattered intensity from electrons (namely, the scattering cross section is on the order of the Thomson scattering cross section in vacuum). This follows from the fluctuation theory. So-called incoherent scattering corresponds to the case in which wave scattering by ions is weak. Thus, electron clouds constitute simply an intermediate link that is not involved in the energy and momentum exchange between incident waves and particles.

The fact that radiation of heavy particles can be determined by surrounding polarization cloud, which depends at high frequencies only on the electron mass, is well known due to the Vavilov–Cherenkov radiation phenomenon, and therefore it is not surprising that a heavy ion in the incident wave field can emit waves with the intensity determined by the electron mass. It was found in Ref. [1] that in calculating such radiation, nonlinear responses should be used, which depend on two fields: in the given case, on the incident wave field and the ion field unperturbed by the incoming wave field. These nonlinear responses, which contain only the electron mass, give rise to appearing the above-mentioned second term in the scattered radiation field (scattering by ions) and changing and considerably reducing the first term (scattering by electrons). In studies [6] (performed after investigations [3–5]), wave scattering caused by nonlinearity was discovered for the first time, which is possible for arbitrarily heavy particles, in particular, for plasma ions. This scattering was called nonlinear. Although (as for usual scattering due to charge oscillations) the scattered wave field is in the given case proportional to the incident wave field (i.e., is linear in the field), scattering of the new type can be calculated from nonlinear responses of a plasma, and therefore it was called 'nonlinear scattering'. This term is widely used in Ref. [1].

Later on, a more general consideration of wave scattering processes caused by the permittivity modulation with the incident wave field revealed the *transition scattering* effect [7–9]. Transition scattering in plasmas proved to be identical to the nonlinear scattering considered earlier. This process in plasmas was called 'transition scattering' [7–9]. The last term not only is more correct from the physical point of view, but also reflects more deeply the essence of the process and allows one to distinguish spontaneous scattering from stimulated scattering which can be proportional to higher orders in the fields of the incident and scattered waves (i.e., to the waves nonlinear in the fields). Transition scattering was calculated for individual particles in plasmas. But is it contained in formulas for wave scattering produced by fluctuations? These formulas do contain transition scattering. Both types of scattering can have the same order of magnitude and interfere with each other, so that scattering from electrons in plasmas can be much weaker than that in a vacuum and that from ions. Transition scattering appears due to a change in scattering by plasma electrons compared to scattering in a vacuum (the first term in the standard scattering theory). We will show here that expressions for scattering by fluctuations include both *usual scattering*, caused by oscillations of particles in the incident wave field, and *transition scattering*, as well as *interference between them.*

This paper is devoted only to *purely classical effects*, the quantum scattering probabilities being introduced phenomenologically and the results obtained being discussed only in the classical limit. In this sense, this paper does not overlap with papers [10, 11] in which the role of quantum corrections to scattering and the character of scattering in the presence of electron beams are discussed in detail. The quantum theory of transition scattering was considered in Ref. [12]. In the classical limit, parametric instabilities discussed in papers [10, 11] take into account properly polarization clouds, and, as was shown in lectures [13], also describe induced transition scattering in a certain limit. Therefore, we will present here only a classical consideration, putting more emphasis on those cases where wave scattering by ions dominates.

As is seen from the brief history of studying the scattering of waves in plasmas presented above, it has been finally found that the interference between transition and usual scatterings due to oscillations of particles in the wave field plays a considerable role, and also the important role of ions in wave scattering has been confirmed, which was pointed out long ago in papers published in the late 1960s–early 1970s. Therefore, it is surprising to see the persistence of prejudices (which are still reflected in the literature, especially astrophysical), assuming that high-frequency waves are scattered only by electrons. In plasma physics, it is now generally accepted that the induced and spontaneous scattering of longitudinal waves and electromagnetic waves mainly by ions plays an important role. To emphasize the illusiveness of the assumption that high-frequency waves are scattered only by plasma electrons and to show the necessity of analysis of scattering by ions, we present in Section 2 a simplified calculation procedure for obtaining the energy and momentum conservation laws between waves and plasma particles in scattering processes for arbitrary nonequilibrium distributions of particles and high-frequency waves (such calculations have not been performed before). A comparison of the results obtained taking into account usual and transition scatterings with the standard results on scattering by density fluctuations in a thermal plasma, which are used for processing experimental data on the scattering of electromagnetic waves, showed that they completely coincide.

2. Methodical notes

2.1 Scattering of electromagnetic waves by plasma density fluctuations

First of all, we write out the result of the fluctuation theory using notations that are closer to those in book [1]. Thus, ω and $\mathbf{k}$ in monograph [2] denote the differences between the frequencies and wave vectors of incident and scattered waves, each of them being considered as a monochromatic high-frequency electromagnetic wave, while plasma particles are assumed nonrelativistic, whereas in book [1] the scattering of any waves with a broad set of frequencies and wave numbers was considered. To compare the results presented in Refs [1] and [2], it is expedient to use standard notations ω and $\mathbf{k}$ for the frequency and wave vector, respectively, for the convenience of representation in calculations of the Fourier transforms of any quantities. For example, the expansion of the field $\mathbf{E}$ into Fourier harmonics in space and time is written in the form

$$\mathbf{E}(\mathbf{r}, t) = \int \mathbf{E}_{\mathbf{k},\omega} \exp(\mathrm{i}\mathbf{kr} - \mathrm{i}\omega t)\, \mathrm{d}\mathbf{k}\, \mathrm{d}\omega\,. \tag{1}$$

By following Ref. [2], we denote the frequency and wave vector of the incident wave by ω_{inc} and $\mathbf{k}_{\text{inc}}$, and the frequency and wave vector of the scattered wave by ω_{sc} and $\mathbf{k}_{\text{sc}}$. Then, unlike the notation in monograph [2] and in accordance with the notation used in book [1], we denote the difference between the frequencies and wave vectors of the incident and scattered waves by $\omega_- = \omega_{\text{inc}} - \omega_{\text{sc}}$ and $\mathbf{k}_- = \mathbf{k}_{\text{inc}} - \mathbf{k}_{\text{sc}}$, $k_- = |\mathbf{k}_-|$ (in Ref. [2], the corresponding quantities are simply denoted by $\mathbf{k}$ and ω). For arbitrary waves in a plasma in the case considered in Ref. [1] the frequencies ω_{inc} and ω_{sc} for each branch of a wave are functions of $\mathbf{k}_{\text{inc}}$ and $\mathbf{k}_{\text{sc}}$, and for high-frequency electromagnetic waves considered in Ref. [2], $\omega_{\text{inc}} = |\mathbf{k}_{\text{inc}}|c$ and $\omega_{\text{sc}} = |\mathbf{k}_{\text{sc}}|c$ (where c is the speed of light). We denote the incident wave field by $\mathbf{E}_{\text{inc}}$, and the scattered wave field by $\mathbf{E}_{\text{sc}}$. The subscripts 'inc' and 'sc', which refer to the incident and scattered waves, respectively, will be placed in front of lower indices i and j for vector components. Quantities corresponding to electrons and ions will be indicated by superscripts e and i, respectively. All plasma particles are assumed nonrelativistic, $v^{\mathrm{e,i}}/c \ll 1$, and wave frequencies are much lower than $m_\mathrm{e}c^2$, which corresponds to the definition of Thomson scattering in monograph [2]. Then, by omitting coefficients depending on polarization, we obtain, in accordance with expression (6.3.4) in Ref. [2], the intensity ratio of the scattered and incident waves, determined by the factor $S(\mathbf{k}_-, \omega_-)$:

$$S(\mathbf{k}_-, \omega_-) = \frac{2\pi}{|k_-|}\left|1 - \frac{G^\mathrm{e}_-}{\epsilon_-}\right|^2 f^\mathrm{e}_{0,\parallel}\left(\frac{\omega_-}{k_-}\right) + \frac{2\pi Z^\mathrm{i}}{|k_-|}\left|\frac{G^\mathrm{e}_-}{\epsilon_-}\right|^2 f^\mathrm{i}_{0,\parallel}\left(\frac{\omega_-}{k_-}\right), \tag{2}$$

where $f^\mathrm{e}_{0,\parallel}(v_\parallel)$ and $f^\mathrm{i}_{0,\parallel}(v_\parallel)$ are one-dimensional electron and ion distribution functions in the direction of the vector $\mathbf{k}_-$, and the minus in the subscript ϵ_- and susceptibility G^e_- corresponds to the index $\mathbf{k}_-, \omega_-$. The longitudinal permittivity ϵ_- is expressed in terms of the electron and ion susceptibilities $G^\mathrm{e}(\mathbf{k}, \omega)$ and $G^\mathrm{i}(\mathbf{k}, \omega)$, respectively (according to the notation in Ref. [2], we omit the minus in the subscript), as

$$\epsilon(\mathbf{k}, \omega) = 1 + G^\mathrm{e}(\mathbf{k}, \omega) + G^\mathrm{i}(\mathbf{k}, \omega)\,, \tag{3}$$

where the electron and ion susceptibilities in a plasma are defined as

$$G^\mathrm{e}(\mathbf{k}, \omega) = \int \mathrm{d}\mathbf{v}\, \frac{4\pi e^2}{k^2(\omega - \mathbf{kv} + \mathrm{i}0)}\left(\mathbf{k}\,\frac{\partial \Phi^\mathrm{e}}{\partial \mathbf{p}}\right), \tag{4}$$

$$G^\mathrm{i}(\mathbf{k}, \omega) = \int \mathrm{d}\mathbf{v}\, \frac{4\pi (Z^\mathrm{i})^2 e^2}{k^2(\omega - \mathbf{kv} + \mathrm{i}0)}\left(\mathbf{k}\,\frac{\partial \Phi^\mathrm{i}}{\partial \mathbf{p}}\right), \tag{5}$$

with Φ^e and Φ^i, as pointed out in Ref. [2], being the electron and ion distribution functions averaged over fluctuations. According to Ref. [2], particle distributions in formulas (4) and (5) depend on the particle velocity $\mathbf{v}$ and are normalized to $\mathrm{d}\mathbf{v}$, whereas in Ref. [1] they are normalized to $\mathrm{d}\mathbf{p}/(2\pi)^3$, where $\mathbf{p}$ is the particle momentum. The assumption [2] that all the ions have the same charge Z^i makes a comparison of the results in Refs [1] and [2] easier. Obviously, $\mathbf{v}$ in formulas (4) and (5) is the electron and ion velocity, respectively. In Ref. [2], the unperturbed electron and ion concentrations n^e_0 and n^i_0, respectively, are found from the corresponding distribution functions $\Phi^\mathrm{i} = n^\mathrm{e}_0 f^\mathrm{e}_0$ and $\Phi^\mathrm{e} = n^\mathrm{i}_0 f^\mathrm{i}_0$. The zero subscript indicates that these distributions set in before the action of incident radiation. In the more general case considered in book [1], it is pointed out that, if a slow change in particle distributions caused by the same scattering process is taken into account, these particle distributions should be introduced in formulas (4) and (5). In monograph [2], the quasineutrality condition $n^\mathrm{i}_0 = n^\mathrm{e}_0/Z^\mathrm{i}$ is used to express all the quantities in terms of the electron concentration, which masks, to a certain degree, the involvement of ions in scattering, creating the illusion that wave scattering is produced only by electrons. By the way, the total scattered intensity in Ref. [2] is deliberately written proportional to the electron concentration. If the quasineutrality condition is applied, the ion polarizability (5) will contain the first degree of Z^{i}, as in expression (6.2.18) from Ref. [2] (notice that the second line of expression (6.2.18) contains a misprint because the unperturbed electron concentration should be employed). In monograph [2], the notation is used that can lead readers into error. Thus, $f_{\mathrm{e}0}(\omega/k)$ and $f_{\mathrm{i}0}(\omega/k)$ are one-dimensional velocity distribution functions of electrons and ions along the vector $\mathbf{k}$, normalized to $(2\pi T_{\mathrm{e}}/m_{\mathrm{e}})^{1/2}$ and $(2\pi T_{\mathrm{i}}/m_{\mathrm{i}})^{1/2}$, respectively (see expressions (3.3.5), (6.3.4), etc. in Ref. [2]), whereas for the inverse order of subscripts, they are three-dimensional distribution functions [for example, $f_{0,\mathrm{e}}$ in the polarizability is a three-dimensional distribution function normalized to $(2\pi T_{\mathrm{e}}/m_{\mathrm{e}})^{3/2}$]. Therefore, here we changed somewhat the notation in Ref. [2] to avoid misunderstandings. Note also that the quasineutrality condition for the ground state taking into account only electrons and ions cannot be fulfilled if a system has other charged particles (for example, dust particles). It is convenient to introduce the common factor n_0^{e} in the scattered intensity in Ref. [2] into expression (2) and represent it in the form

$$n_0^{\mathrm{e}} S(\mathbf{k}_-, \omega_-) = 2\pi \left| 1 - \frac{G_-^{\mathrm{e}}}{\epsilon_-} \right|^2 \int \Phi_0^{\mathrm{e}}(v^{\mathrm{e}}) \delta(\omega_- - \mathbf{k}_- \mathbf{v}^{\mathrm{e}})\, \mathrm{d}\mathbf{v}^{\mathrm{e}} + 2\pi (Z^{\mathrm{i}})^2 \left| \frac{G_-^{\mathrm{e}}}{\epsilon_-} \right|^2 \int \Phi_0^{\mathrm{i}}(v^{\mathrm{i}}) \delta(\omega_- - \mathbf{k}_- \mathbf{v}^{\mathrm{i}})\, \mathrm{d}\mathbf{v}^{\mathrm{i}} , \quad (6)$$

where Φ_0^{e} and Φ_0^{i} are the total initial electron and ion distribution functions normalized to particle velocities and containing their concentrations. Expression (6) implicitly shows that the first term on the right-hand side is proportional to the total electron distribution function, while the second term is proportional to the total ion distribution function integrated over velocities, taking into account the energy and momentum conservation laws in elementary acts of wave scattering by electrons and ions, respectively:

$$\delta(\omega_- - \mathbf{k}_- \mathbf{v}^{\mathrm{e}}) = \delta(\omega_{\mathrm{sc}} - \omega_{\mathrm{inc}} - (\mathbf{k}_{\mathrm{sc}} - \mathbf{k}_{\mathrm{inc}}) \mathbf{v}^{\mathrm{e}}) , \quad (7)$$

$$\delta(\omega_- - \mathbf{k}_- \mathbf{v}^{\mathrm{i}}) = \delta(\omega_{\mathrm{sc}} - \omega_{\mathrm{inc}} - (\mathbf{k}_{\mathrm{sc}} - \mathbf{k}_{\mathrm{inc}}) \mathbf{v}^{\mathrm{i}}) . \quad (8)$$

The first term on the right-hand side of formula (6), describing scattering by electrons, takes into account the interference of usual scattering due to oscillations of electrons in the incident wave field and transition scattering by the electron polarization cloud caused by the lack of plasma electrons in the electron cloud of a scattering electron. This lack appears due to their repulsion from the latter (amplitudes having opposite signs in the given case are summed rather than intensities).

By denoting the ion energies before and after scattering by $\varepsilon_{\mathbf{p}}^{\mathrm{i}}$ and $\varepsilon_{\mathbf{p}'}^{\mathrm{i}}$, respectively, we write out the momentum and energy conservation laws in an elementary act of scattering by an ion in the form

$$\mathbf{p}' = \mathbf{p} + \hbar \mathbf{k}_{\mathrm{inc}} - \hbar \mathbf{k}_{\mathrm{sc}} , \quad (9)$$

$$\varepsilon_{\mathbf{p}} + \hbar \omega_{\mathrm{inc}} = \varepsilon_{\mathbf{p} + \hbar \mathbf{k}_{\mathrm{inc}} - \hbar \mathbf{k}_{\mathrm{sc}}} + \hbar \omega_{\mathrm{sc}} . \quad (10)$$

By using the expansion in momenta of electromagnetic waves in the classical limit, we obtain relation (8) from expressions (9) and (10).

All this suggests that the second term on the right-hand side of formula (6) describes wave scattering by ions. This can also be verified by considering transition scattering from a probe ion and calculating a change in the ion distribution function during scattering of electromagnetic waves.

2.2 Transition scattering by probe nonrelativistic ions

The method of *probe particles* is quite efficient in plasma physics because any plasma particle, being indistinguishable from others, can be treated as a probe particle, and the description of processes for a separate probe particle should correspond to that for any plasma particle. Therefore, let us consider the transition scattering of electromagnetic waves by a single probe nonrelativistic ion with the charge $Z^{(q)}e$ and compare the result with the result of the theory of scattering by fluctuations.

The qualitative description of transition scattering and its theory are presented in detail in Refs [1, 7–9]. Here, we will consider only a general scheme for calculating the scattering of high-frequency electromagnetic waves by nonrelativistic ions. The amplitude of an incident wave is small, and therefore it gives rise to only weak ion velocity oscillations that can be neglected, as opposed to the thermal ion velocities. The Fourier components of the field $\mathbf{E}^{(q)}$ created by a nonrelativistic *probe ion* with the charge $Z^{(q)}e$ are described by the expression

$$\mathbf{E}_{\mathbf{k},\omega}^{(q)} = \frac{Z^{(q)} e \mathbf{k}}{2\pi^2 k^2 \epsilon_{\mathbf{k},\omega}} \delta(\omega - \mathbf{k} \mathbf{v}^{(q)}) . \quad (11)$$

This expression for the field follows from the Poisson equation for an ion uniformly moving in a plasma with velocity $\mathbf{v}^{(q)}$. Here, $\epsilon_{\mathbf{k},\omega}$ is the longitudinal permittivity defined by expressions (3)–(6). For nonrelativistic velocities of ions, only the longitudinal component of the particle field should be taken into account (in Fourier components, the component directed along $\mathbf{k}$). According to Refs [1, 7–9], transition scattering is determined by the nonlinear current $\mathbf{j}^{\,\mathrm{nl},2}$ of the second order in the field, which is a plasma response simultaneously to the incident wave field $\mathbf{E}_i$ and ion field (11). The ith component of this current is given by the standard expression

$$j_i^{\mathrm{nl},2}(\mathbf{k}, \omega) = 2 \int S_{i,j,l} E_{\mathrm{inc},j}(\mathbf{k}_1, \omega_1) E_l^{(q)}(\mathbf{k}_2, \omega_2) \mathrm{d}_{1,2} , \quad (12)$$

$$\mathrm{d}_{1,2} = \mathrm{d}\mathbf{k}_1\, \mathrm{d}\mathbf{k}_2\, \mathrm{d}\omega_1\, \mathrm{d}\omega_2 \delta(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2) \delta(\omega - \omega_1 - \omega_2) . \quad (13)$$

Relationship (12) can be written symbolically as

$$j_i^{\mathrm{nl},2} = 2 \int \hat{S} E_{\mathrm{inc}} E^{(q)} \mathrm{d}_{1,2} . \quad (14)$$

The nonlinear plasma response tensor entering into formulas (12) and (14) and symmetrized over j, $\mathbf{k}_1$, ω_1 and l, $\mathbf{k}_2$, ω_2 can easily be found using the perturbation theory in the fields (see Refs [1, 12]). The coefficient 2 on the right-hand side of formula (12) appears due to this symmetrization and the presence of two terms in Eqn (12), when the ion field $E^{(q)}$ is present in formula (12) as cofactors either in the first or second place. Expression (12) can be further easily simplified: first, it should be taken into account that the incident wave field is a high-frequency electromagnetic field; second, the main contribution is introduced by perturbations of nonrelativistic electrons, and third, the denominators of responses containing frequency differences greatly exceed denominators containing only frequencies themselves [these are standard assumptions (see Refs [1, 10])]. Then, the tensor S will be proportional to G_{-}^{e}, and the probe ion field gives ϵ_{-} in the denominator, so that nonlinear current (14) will be proportional to $G_{-}^{\mathrm{e}}/\epsilon_{-}$. To calculate the radiation intensity Q for waves scattered by the probe ion, it is sufficient to calculate the work of the scattered wave done by current (14) appearing due to the presence of the probe ion, which is described by the combined action of the incident wave field and the probe ion field:

$$Q = -\int \mathbf{j}^{\,\mathrm{nl},\,2}\mathbf{E}_{\mathrm{sc}}\,\mathrm{d}\mathbf{r}\,. \tag{15}$$

The field $\mathbf{E}_{\mathrm{sc}}$ excited by current (14) can be found from Maxwell's equation with current (14) on the right-hand side. After moving to Fourier components in formula (15), the current squared appears in the numerator of the integrand in Eqn (15) during the division of the current by the Maxwell operator to find $\mathbf{E}_{\mathrm{sc}}$, and expression (15) becomes proportional to the modulus $G_{-}^{\mathrm{e}}/\epsilon_{-}$ squared, while a contribution to the denominator of formula (14) will be introduced only by the imaginary part $\mathrm{Im}\,\{1/(\omega_{\mathrm{sc}}^2 - k_{\mathrm{sc}}^2c^2)\}$ of the Maxwell operator, because the frequency of the high-frequency scattered wave is $k_{\mathrm{sc}}c$. Thus, we briefly described the scheme of calculations which finally give the expression for the factor $S^{(q)}$ relevant to the probe ion, which is similar to expression (2) written above for the factor S:

$$S^{(q)}(\mathbf{k}_{-},\omega_{-}) = 2\pi(Z^{(q)})^2\left|\frac{G_{-}^{\mathrm{e}}}{\epsilon_{-}}\right|^2\delta(\omega_{-} - \mathbf{k}_{-}\mathbf{v}^{(q)})\,. \tag{16}$$

We will use the fact that *each plasma ion can be treated as a probe ion*. To evaluate radiation scattered by all ions, it is sufficient to replace the superscript (q) by the superscript i and integrate the result over the ion distribution. We obtain exactly the second term of relation (6), which independently proves that wave scattering occurs from plasma ions.

2.3 Balance of particles and photons in scattering processes

The incident wave can be either monochromatic or a set of waves with random phases, and in the latter case the characteristics of a packet of randomly scattered waves can be conveniently described using the concept of photons. In this case, as pointed out in books [1, 12], it is expedient to introduce occupation numbers both for particles and waves (to normalize distribution functions $\Phi_{\mathbf{p}}^{\mathrm{e,i}}$ to $\mathrm{d}\mathbf{p}/(2\pi)^3$) and to determine the numbers $N_{\mathbf{k}}^{\mathrm{inc,sc}}$ of photons from the expression for the radiation energy density W:

$$W_{\mathrm{inc,sc}} = \int \omega_{\mathrm{inc,sc}}N_{\mathbf{k},\mathrm{inc,sc}}\,\frac{\mathrm{d}\mathbf{k}}{(2\pi)^3}\,. \tag{17}$$

Here, integration for incident radiation is performed over the wave vectors of incident radiation, and for scattered radiation it is over the wave vectors of scattered radiation. Photon numbers $N_{\mathbf{k},\,\mathrm{inc,sc}}$ are directly expressed in terms of classical correlation functions of radiation fields [13], and $\omega_{\mathrm{inc,sc}}$ are classical radiation frequencies (although the quantum analogs are obvious, the photon energy contains $\hbar$, while the number of photons in the classical limit contains $1/\hbar$, and $\hbar$ is cancelled). Sometimes it is convenient to utilize units in which $\hbar = 1$. Strictly speaking, formula (17) defines the number of photons in the classical description, but it turns out that the quantum analogy used in calculations of fluctuation effects always correctly describes the corresponding quantities entering classical conservation laws. Thus, the classical photon momentum $\mathbf{P}_{\mathrm{inc,sc}}$, calculated from known classical expressions for the momentum of a packet of random waves with the correlation function entering into the definition of $N_{\mathbf{k},\,\mathrm{inc,sc}}$, is always expressed in the form

$$\mathbf{P}_{\mathrm{inc,sc}} = \int \mathbf{k}N_{\mathbf{k},\,\mathrm{inc,sc}}\,\frac{\mathrm{d}\mathbf{k}}{(2\pi)^3}\,, \tag{18}$$

which corresponds to the momentum $\hbar\mathbf{k}$ of a single photon. Therefore, to obtain correct expressions following from the fluctuation theory, we can take advantage of simple quantum balance conditions taking into account Einstein rules for induced processes. To this end, we can introduce the scattering probability $w_{\mathbf{p}}(\mathbf{k}_{\mathrm{inc}},\mathbf{k}_{\mathrm{sc}})$ per unit time by a particle with the momentum $\mathbf{p}$ with the absorption of a photon with the momentum $\hbar\mathbf{k}_{\mathrm{inc}}$ and emission of a photon with the momentum $\hbar\mathbf{k}_{\mathrm{sc}}$ [normalized to elementary phase volumes $\mathrm{d}\mathbf{k}_{\mathrm{sc}}/(2\pi)^3$ and $\mathrm{d}\mathbf{k}_{\mathrm{inc}}/(2\pi)^3$]. Expression (15) for the scattered power for the probe ion can serve for defining the probability of scattering by electrons and ions:

$$Q_{\mathbf{p},\,\mathrm{sc}}^{\mathrm{i,e}} = \int \omega_{\mathrm{sc}}w_{\mathbf{p}}^{\mathrm{i,e}}(\mathbf{k}_{\mathrm{inc}},\mathbf{k}_{\mathrm{sc}})N_{\mathbf{k},\,\mathrm{inc}}\,\frac{\mathrm{d}\mathbf{k}_{\mathrm{sc}}}{(2\pi)^3}\,\frac{\mathrm{d}\mathbf{k}_{\mathrm{inc}}}{(2\pi)^3}\,. \tag{19}$$

Upon introducing scattering probability (19), we can calculate a change in the number of photons using simple balance equations by multiplying the probability for emitted waves by $N_{\mathbf{k}} + 1$, and for absorbed waves by $N_{\mathbf{k}}$, and also multiplying by the number of particles with the specified momentum and integrating over all phase volumes of waves and particles [1] (which corresponds to the simplest Einstein method):

$$\frac{\mathrm{d}N_{\mathbf{k}',\,\mathrm{sc}}}{\mathrm{d}t} = \int w_{\mathbf{p}}(\mathbf{k},\mathbf{k}')\big[(N_{\mathbf{k}',\,\mathrm{sc}}+1)N_{\mathbf{k},\,\mathrm{inc}}\Phi_{\mathbf{p}} - N_{\mathbf{k}',\,\mathrm{sc}}(N_{\mathbf{k},\,\mathrm{inc}}+1)\Phi_{\mathbf{p}+\hbar\mathbf{k}-\hbar\mathbf{k}'}\big]\,\frac{\mathrm{d}\mathbf{p}\,\mathrm{d}\mathbf{k}}{(2\pi)^6}\,. \tag{20}$$

Here, we introduced for simplicity the notation $\mathbf{k}_{\mathrm{inc}} = \mathbf{k}$ and $\mathbf{k}_{\mathrm{sc}} = \mathbf{k}'$, and omitted summation over electrons and ions. The number of photons in equation (20) has the usual quantum meaning, but above we defined the classical number of photons in terms of the photon correlation function which corresponds to $\hbar N_{\mathbf{k}} \to N_{\mathbf{k}}$. Multiplying (20) by $\hbar$ and expanding in a series in the ratio of the photon momentum to the particle momentum, we arrive at a completely classical relationship containing only the classical number of photons, defined in terms of their correlation function (the classical number of photons is denoted by $N_{\mathbf{k}}$, as above):

$$\frac{\mathrm{d}N_{\mathbf{k}',\,\mathrm{sc}}}{\mathrm{d}t} = \int w_{\mathbf{p}}(\mathbf{k},\mathbf{k}')\left\{N_{\mathbf{k}',\,\mathrm{sc}}N_{\mathbf{k},\,\mathrm{inc}}\left[(\mathbf{k}'-\mathbf{k})\,\frac{\partial\Phi_{\mathbf{p}}}{\partial\mathbf{p}}\right] + N_{\mathbf{k},\,\mathrm{inc}}\Phi_{\mathbf{p}} - N_{\mathbf{k}',\,\mathrm{sc}}\Phi_{\mathbf{p}}\right\}\frac{\mathrm{d}\mathbf{p}\,\mathrm{d}\mathbf{k}}{(2\pi)^6}\,. \tag{21}$$

Obviously, expression (21) takes into account the following effects, which were neglected in monograph [2] and in the above discussion, namely that: (1) the distribution function of particles can slowly change under the effect of the scattering itself (without this, it is impossible to verify the conservation laws, in particular, that the energy and momentum are transferred during scattering not only to plasma electrons but also to plasma ions); (2) the intensity of scattered waves is small, and therefore we can neglect on the right-hand side of equation (21) all the terms proportional to $N_{\mathbf{k}',\,\mathrm{sc}}$, i.e., the so-called radiation extinction and induced scattering, and (3) a change in the spatio–temporal distribution of photons is sufficiently slow. But in the limit when $N_{\mathbf{k}',\,\mathrm{sc}} \to 0$ on the right-hand side of equation (21), we can also verify the energy and momentum transfer to ions in the scattering process, taking into account simultaneously a change in the ion distribution (i.e., not assuming that the ion distribution function is constant and equal to its value before the scattering act). Of course, conservation laws are valid when all the terms on the right-hand side of equation (21) are taken into account, but we will focus our attention only on wave scattering by ions, described by the second term on the right-hand side of equation (21), because the presence of this scattering is sometimes in doubt. Then, Eqn (21) is reduced to the relationship

$$\frac{\mathrm{d}N_{\mathbf{k}',\,\mathrm{sc}}}{\mathrm{d}t} = \int w^{\mathrm{i}}_{\mathbf{p}}(\mathbf{k},\mathbf{k}')N_{\mathbf{k},\,\mathrm{inc}}\Phi^{\mathrm{i}}_{\mathbf{p}}\,\frac{\mathrm{d}\mathbf{p}\,\mathrm{d}\mathbf{k}}{(2\pi)^6}\,. \tag{22}$$

We will follow the same procedure when determining a change in the number of incident photons by considering the limit $N^{\mathrm{sc}}_{\mathbf{k}'} \to 0$ on the right-hand side of above formula:

$$\frac{\mathrm{d}N_{\mathbf{k},\,\mathrm{inc}}}{\mathrm{d}t} = -\int w^{\mathrm{i}}_{\mathbf{p}}(\mathbf{k},\mathbf{k}')N_{\mathbf{k},\,\mathrm{inc}}\Phi^{\mathrm{i}}_{\mathbf{p}}\,\frac{\mathrm{d}\mathbf{p}\,\mathrm{d}\mathbf{k}'}{(2\pi)^6}\,. \tag{23}$$

Hence it follows that the total change in the photon energy is given by

$$\begin{aligned}\frac{\mathrm{d}W_{\mathrm{sc}}}{\mathrm{d}t} + \frac{\mathrm{d}W_{\mathrm{inc}}}{\mathrm{d}t} &= \int \omega_{\mathrm{sc}}\,\frac{\mathrm{d}N_{\mathbf{k}',\,\mathrm{sc}}}{\mathrm{d}t}\,\frac{\mathrm{d}\mathbf{k}'}{(2\pi)^3} \\ &\quad - \int \omega_{\mathrm{inc}}\,\frac{\mathrm{d}N_{\mathbf{k},\,\mathrm{inc}}}{\mathrm{d}t}\,\frac{\mathrm{d}\mathbf{k}}{(2\pi)^3} \\ &= \int (\omega_{\mathrm{sc}} - \omega_{\mathrm{inc}})w^{\mathrm{i}}_{\mathbf{p}}(\mathbf{k},\mathbf{k}')N_{\mathbf{k},\,\mathrm{inc}}\Phi^{\mathrm{i}}_{\mathbf{p}}\,\frac{\mathrm{d}\mathbf{p}\,\mathrm{d}\mathbf{k}\,\mathrm{d}\mathbf{k}'}{(2\pi)^9}\,.\end{aligned} \tag{24}$$

We will perform similar calculations for the ion distribution function $\Phi^{\mathrm{i}}_{\mathbf{p}}$ by writing down for a change in this function per unit time the quantum balance equations governing the transition of an ion with the probability $w^{\mathrm{i}}_{\mathbf{p}}(\mathbf{k},\mathbf{k}')$ from its state with the momentum $\mathbf{p}$ to the state with the momentum $\mathbf{p} + \hbar\mathbf{k} - \hbar\mathbf{k}'$ and back, and for the transition of the ion with the probability $w^{\mathrm{i}}_{\mathbf{p}-\hbar\mathbf{k}+\hbar\mathbf{k}'}(\mathbf{k},\mathbf{k}')$ from the state with the momentum $\mathbf{p} - \hbar\mathbf{k} + \hbar\mathbf{k}'$ to the state with the momentum $\mathbf{p}$ and back, taking into consideration spontaneous and induced processes, and will pass to the classical limit, introducing, as above, classical numbers of photons via their correlation function and using the limit $N^{\mathrm{sc}}_{\mathbf{k}'} \to 0$. In the upshot, we arrive at

$$\frac{\mathrm{d}\Phi^{\mathrm{i}}_{\mathbf{p}}}{\mathrm{d}t} = \frac{\partial}{\partial\mathbf{p}}\int (\mathbf{k}' - \mathbf{k})N_{\mathbf{k},\,\mathrm{inc}}w^{\mathrm{i}}_{\mathbf{p}}(\mathbf{k},\mathbf{k}')\Phi^{\mathrm{i}}_{\mathbf{p}}\,\frac{\mathrm{d}\mathbf{k}\,\mathrm{d}\mathbf{k}'}{(2\pi)^6}\,. \tag{25}$$

This equation will be used for determining a change in the ion energy E^{i} per unit time:

$$\begin{aligned}\frac{\mathrm{d}E^{\mathrm{i}}}{\mathrm{d}t} &= \int \varepsilon^{\mathrm{i}}_{\mathbf{p}}\,\frac{\mathrm{d}\Phi^{\mathrm{i}}_{\mathbf{p}}}{\mathrm{d}t}\,\frac{\mathrm{d}\mathbf{p}}{(2\pi)^3} \\ &= \int (\omega_{\mathrm{inc}} - \omega_{\mathrm{sc}})N_{\mathbf{k},\,\mathrm{inc}}w^{\mathrm{i}}_{\mathbf{p}}(\mathbf{k},\mathbf{k}')\Phi^{\mathrm{i}}_{\mathbf{p}}\,\frac{\mathrm{d}\mathbf{p}\,\mathrm{d}\mathbf{k}\,\mathrm{d}\mathbf{k}'}{(2\pi)^9}\,,\end{aligned} \tag{26}$$

where $\varepsilon^{\mathrm{i}}_{\mathbf{p}}$ is the ion energy, and $\mathbf{v}^{\mathrm{i}} = \mathrm{d}\varepsilon^{\mathrm{i}}_{\mathbf{p}}/\mathrm{d}\mathbf{p}$; here, after integration by parts over momenta, we used the energy conservation law in the elementary scattering act: $\omega_{\mathrm{inc}} - \omega_{\mathrm{sc}} = (\mathbf{k} - \mathbf{k}')\mathbf{v}^{\mathrm{i}}$. By comparing Eqns (24) and (26), we see that energy is indeed imparted to ions during scattering:

$$\frac{\mathrm{d}W_{\mathrm{sc}}}{\mathrm{d}t} + \frac{\mathrm{d}W_{\mathrm{inc}}}{\mathrm{d}t} + \frac{\mathrm{d}E^{\mathrm{i}}}{\mathrm{d}t} = 0\,. \tag{27}$$

Taking into account expression (18) for the photon momentum, we arrive at the conclusion that the momentum is also transferred to ions during scattering:

$$\frac{\mathrm{d}\mathbf{P}_{\mathrm{sc}}}{\mathrm{d}t} + \frac{\mathrm{d}\mathbf{P}_{\mathrm{inc}}}{\mathrm{d}t} + \frac{\mathrm{d}\mathbf{P}^{\mathrm{i}}}{\mathrm{d}t} = 0\,, \qquad \mathbf{P}^{\mathrm{i}} = \int \mathbf{p}\Phi^{\mathrm{i}}_{\mathbf{p}}\,\frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}\,. \tag{28}$$

The conservation laws are fulfilled taking into consideration all the other terms which were omitted for the simplicity of analysis, and taking into account wave scattering by electrons. This *completely proves that scattering in plasmas occurs from both electrons and ions*. Unfortunately, such detailed calculations have not been performed earlier, and statements that wave scattering by plasma ions plays a significant role have been ignored in a number of cases.

2.4 Derivation of classical balance equations taking into account plasma fluctuations and randomness of incident waves

The question can arise: why do we need to resort to quantum considerations for a classical system? Of course, all classical results can be directly obtained only from classical, not quantum, considerations. However, quantum considerations serve like guiding ones and are quite fine and, in addition, quantum calculations give in the limit exactly the same results as classical ones. Final equations for the balance of photons and particles presented in Section 2.3 are completely classical in nature (the same equations can be obtained using only classical calculations) and contain all the terms proportional both to $N_{\mathbf{k},\,\mathrm{inc}}$ and $N_{\mathbf{k}',\,\mathrm{sc}}$ and their squares [1]. In this case, consideration must be given to plasma fluctuations in more detail than in Ref. [2], and to the fact that incident and scattered waves in the approach presented in Sections 2.1–2.3 are wave packets with random phases and correlation functions determined by $N_{\mathbf{k}}$. The latter restriction is not necessary, however, because the packets of incident and scattered waves can be of any kind, in particular, monochromatic, but then the results obtained somewhat differ from those presented in Section 2.3 in the classical limit.

We dwell here on the main features of such calculations for random wave packets. Then, nonlinear plasma responses of both the second and the third order in the field should be used, as was demonstrated in Section 2.2 for accounting for second-order nonlinear responses in calculations of transition scattering by ions. A change in the ion distribution under the action of electromagnetic waves scattered by ions requires the inclusion of third-order nonlinear responses. We will retain the notation E_{inc} and E_{sc} for random incident and scattered waves, respectively (not using the operator δ in front of them, but utilizing δ for the random part of the particle distribution function $f^{\,\mathrm{e,i}}$):

$$f^{\,\mathrm{e,i}}_{\mathbf{p}} = \Phi^{\mathrm{e,i}}_{\mathbf{p}} + \delta f^{\,\mathrm{e,i}}_{\mathbf{p}}\,, \qquad \langle \delta f^{\,\mathrm{e,i}}_{\mathbf{p}} \rangle = 0\,, \tag{29}$$

where $\Phi^{\mathrm{e,i}}_{\mathbf{p}}$ is the distribution function averaged over both plasma fluctuations and an ensemble of random incident and scattered waves. At present, plasma fluctuations can be accounted for much more simply than was done in monograph [2] by applying a cumbersome apparatus [14] or the method of correlation functions [13]. It is sufficient to write down the equation for the one-particle distribution function (the distribution function integrated over all particles except for the specified one), to average it, and to subtract the averaged equation from the original one:

$$\frac{\mathrm{d}}{\mathrm{d}t} f_{\mathbf{p}}^{\mathrm{e,i}} \equiv \left(\frac{\partial}{\partial t} + \mathbf{v}\,\frac{\partial}{\partial \mathbf{r}}\right) f_{\mathbf{p}}^{\mathrm{e,i}} = -\frac{\partial}{\partial \mathbf{p}}\,\mathbf{F} f_{\mathbf{p}}^{\mathrm{e,i}}\,, \tag{30}$$

where $\mathbf{F}$ is the random force of scattered and incident waves and plasma fluctuations, and therefore

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\Phi_{\mathbf{p}}^{\mathrm{e,i}} = -\frac{\partial}{\partial \mathbf{p}}\,\langle \mathbf{F}\,\delta f_{\mathbf{p}}^{\mathrm{e,i}}\rangle\,, \tag{31}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\delta f_{\mathbf{p}}^{\mathrm{e,i}} = -\frac{\partial}{\partial \mathbf{p}}\left(\mathbf{F}\,\delta f_{\mathbf{p}}^{\mathrm{e,i}} - \langle \mathbf{F}\,\delta f_{\mathbf{p}}^{\mathrm{e,i}}\rangle\right). \tag{32}$$

The force $\mathbf{F}$ for scattered and incident waves reduces to the Lorentz force:

$$\mathbf{F}_{\mathbf{k},\omega} = e^{\mathrm{e,i}}\left(\mathbf{E}_{\mathbf{k},\omega} + \frac{\mathbf{v}\times\mathbf{H}_{\mathbf{k},\omega}}{c}\right)$$
$$= e^{\mathrm{e,i}}\left[\mathbf{E}_{\mathbf{k},\omega}\left(1 - \frac{\mathbf{kv}}{\omega}\right) + \frac{\mathbf{k}(\mathbf{v}\mathbf{E}_{\mathbf{k},\omega})}{\omega}\right],$$

and because of the condition $v \ll c$, this force slightly differs from the electric force $e^{\mathrm{e,i}}\mathbf{E}$. The force caused by plasma fluctuations slightly differs (also because of the condition $v \ll c$) from the electrostatic force of natural 'zero' fluctuations of noninteracting particles:

$$\delta\mathbf{E}^{0}_{\mathbf{k},\omega} = \frac{4\pi\mathbf{k}}{\epsilon_{\mathbf{k},\omega}k^2}\int\left[e^{\mathrm{e}}\delta f^{\mathrm{e},0}_{\mathbf{p},\mathbf{k},\omega} + e^{\mathrm{i}}\delta f^{\mathrm{i},0}_{\mathbf{p},\mathbf{k},\omega}\right]\frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}\,, \tag{33}$$

where fluctuations of nonintersecting particles correspond to the fact that the average of particle fluctuations squared in the given volume is equal to the average number of particles in this volume (see a set of lectures [13]):

$$\left\langle \delta f^{\mathrm{e,i},0}_{\mathbf{p},\mathbf{k},\omega}\,\delta f^{\mathrm{e,i},0}_{\mathbf{p}',\mathbf{k}',\omega'}\right\rangle$$
$$= \Phi_{\mathbf{p}}^{\mathrm{e,i}}\,\delta(\mathbf{k}+\mathbf{k}')\,\delta(\mathbf{p}-\mathbf{p}')\,\delta(\omega+\omega')\,\delta(\omega-\mathbf{kv})\,. \tag{34}$$

Relations (34) are exact, and the expansion in $\mathbf{E}_{\mathrm{sc,inc}}$, $\delta\mathbf{E}^0$, δf^0, with allowance made for second- and third-order nonlinearities over the field and δf^0 followed by averaging, enables one to obtain a complete system of equations (24)–(28) for scattering of photons by plasma particles. By the way, if only 'zero' fluctuations are taken into account, the calculation of the Balescu collision integral (see book [13]) is performed in one line. To obtain equation (25), it is sufficient to use equation (31) for ions by substituting into its right-hand side $\delta f^{\mathrm{i}} = \delta f^{\mathrm{i},0}$ and the cubic quantity $E = \delta E^{\mathrm{sc},3}$ containing a term quadratic over the incident wave field and linear over the plasma fluctuation field. Then, if we denote the inverse Maxwell operator for cubic current by M_{sc}^{-1} and symbolically write out the current cubic in field in the form

$$j^{\mathrm{nl},3} = \int \hat{\Sigma} E_{\mathrm{inc},1} E_{(0),2} E_{\mathrm{inc},3}\,\mathrm{d}_{1,2,3}\,,$$
$$\mathrm{d}_{1,2,3} = \mathrm{d}\mathbf{k}_1\,\mathrm{d}\mathbf{k}_2\,\mathrm{d}\mathbf{k}_3\,\mathrm{d}\omega_1\,\mathrm{d}\omega_2\,\mathrm{d}\omega_3\,\delta(\omega-\omega_1-\omega_2-\omega_3)$$
$$\times\,\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3) \tag{35}$$

(see book [1]), the averaging over zero fluctuations will give, according to (34), the expression $\propto (1/\epsilon_-)\delta(\omega_- - \mathbf{k}_-\mathbf{v})\Phi^{\mathrm{i}}_{\mathbf{p}}$, whereas averaging over the incident wave field in the cubic nonlinear response obtained as a result of the iterations of quadratic responses via an intermediate (virtual) longitudinal wave (see Ref. [1]) will give the quantity $\propto |G^{\mathrm{e}}|^2 N_{\mathbf{k}}/\epsilon^*_-$. The presence of the operator M_{sc}^{-1} leads to the equality $\omega = \omega_{\mathrm{sc}}$, while averaging over the frequencies of incident waves leads to the equality $\omega = \omega_{\mathrm{inc}}$, which gives $\omega_- = \omega_{\mathrm{sc}} - \omega_{\mathrm{inc}}$. As a result, we obtain exactly relation (25). This is the general scheme for calculating the scattering effect on the ion distribution from the theory of fluctuations. In addition, the operator $\mathrm{d}/\mathrm{d}t$ on the left-hand side of equation (31) obtained in this way corresponds to the identity on the left-hand side of equation (30), i.e., $\mathrm{d}/\mathrm{d}t = (\partial/\partial t) + \mathbf{v}^{\mathrm{i}}\,\partial/\partial\mathbf{r}$. Equations for $(\mathrm{d}/\mathrm{d}t)N_{\mathrm{sc}}$ and $(\mathrm{d}/\mathrm{d}t)N_{\mathrm{inc}}$ are derived similarly. Notice that the operator $\mathrm{d}/\mathrm{d}t$ is obtained containing the group velocity of photons:

$$\frac{\mathrm{d}}{\mathrm{d}t_{\mathrm{sc,inc}}} = \frac{\partial}{\partial t} + \mathbf{v}_{\mathrm{sc,inc,gr}}\,\frac{\partial}{\partial \mathbf{r}}\,, \tag{36}$$

where $\mathbf{v}_{\mathrm{sc,inc,gr}}$ are group velocities of incident and scattered photons:

$$\mathbf{v}_{\mathrm{sc,inc,gr}} = \frac{\mathrm{d}\omega_{\mathrm{sc,inc}}}{\mathrm{d}\mathbf{k}}\,. \tag{37}$$

Thus, in the presence of spatial derivatives or, in other words, the translation effects of photons and particles, conservation laws in the energy and momentum exchange between particles and photons during scattering are fulfilled after integration over the entire space of the possible motion of particles and photons.

2.5 Discussion

It may seem surprising that the photon energy is proportional to frequency $\omega_{\mathbf{k}}$, while the momentum is proportional to $\mathbf{k}$, i.e., it appears that some quantum relationships are revealed already in the classical description. At the same time, it is clear that Maxwell's equations already contain this, at least for high-frequency electromagnetic waves. Indeed, writing the momentum of a packet of random transverse electromagnetic waves, expanding electric and magnetic fields in the Poynting vector over wave numbers, and determining the number of waves as the correlator of electric fields divided by frequency with the normalization to $\mathrm{d}\mathbf{k}/(2\pi)^3$, we obtain relations (17) and (18). But it turns out that for scattering of waves of a different type in plasmas, for example, waves in which the perturbation of the magnetic field is absent and the Poynting vector is zero, relations like (17), (18) are retained, the momentum of waves being determined by the momentum of plasma particles involved in the creation of the waves according to their dispersion in plasma.

It should be noted that the presence of plasma wave scattering by ions is confirmed experimentally.

For induced scattering of waves in plasmas, quadratic in the wave intensity, scattering by ions is a dominating process, and the proof that energy is transferred just to ions during induced scattering is rather simple, and it therefore seemed trivial that the scattered intensity in processes linear in the wave intensity should also be described by the sum of scattered intensities from electrons and ions. However, a direct proof of the fact that scattering is also caused by ions or even schemes for calculating the energy and momentum transfer to ions and electrons during scattering were absent in the literature. This paper eliminates this deficiency. We present the proof for electromagnetic waves with frequencies greatly exceeding plasma frequencies, when at first glance the plasma role can be small. It is for this reason that we focused our attention on this case, although the statement itself about the role of ions in scattering is general for any waves in plasmas. One should bear in mind that the scattered intensity, as is evident from the results of calculations presented in Sections 2.1–2.4, is determined by the frequency difference of incident and scattered waves, not only by the frequency of the waves themselves.

A detailed enough explanation of the fact that wave scattering in plasmas is caused by both electrons and ions, which was presented above, is necessary because it is still erroneously stated sometimes that scattering is caused only by electrons. An example is a referee report on a recent paper in an astrophysical journal concerning the role of scattering by ions: "Possibly, the authors are indeed right, but none of the editors and reviewers can understand how ions can scatter radiation." This demonstrates the erroneous dogma that has taken root, possibly not only among astrophysics but also among experimentalists utilizing laser radiation in plasmas for measuring its parameters, in particular, the electron and ion temperatures (judging by some publications). If we keep in mind the correct answer, it is easy to make estimates and introduce different corrections to electron and ion distributions, because many additional processes changing the ion distribution differ from those that change the electron distributions.

3. Comparison of radiation powers backscattered by electrons and ions

Curiously, scattering by ions always dominates in the backscattered radiation power. Therefore, backscattering can be applied to detecting the ion component (see Section 4). In many applications, the electron and ion concentrations are not determined by the quasineutrality condition, or ions of different kinds or with different degrees of ionization are present. Scattering corresponds to the sum of scatterings by particles of each kind. Here, we will give the example of scattering by singly ionized ions of the same kind with concentration n^{i} and by electrons with concentration n^{e}, having temperatures T^{i} and T^{e}, respectively, and compare the intensity of backscattering by electrons and ions. According to formula (6), we will normalize the scattered intensity so that the expression for scattering by ions would have the simplest form. For this purpose, we will multiply the left-hand side of Eqn (6) by $v_{T\mathrm{i}}/\sqrt{2\pi}$, where $v_{T\mathrm{i}} = \sqrt{T^{\mathrm{i}}/m^{\mathrm{i}}}$ is the thermal ion velocity, and denote in this way the normalized left-hand side of Eqn (6) by $\tilde{S}_-$:

$$\tilde{S}_- = n^{\mathrm{i}}\left|\frac{G_-^{\mathrm{e}}}{\epsilon_-}\right|^2 \exp\left(-\frac{\omega_-^2}{2k_-^2 v_{T\mathrm{i}}^2}\right) + n^{\mathrm{e}}\sqrt{\frac{m^{\mathrm{e}}T^{\mathrm{i}}}{m^{\mathrm{i}}T^{\mathrm{e}}}}\left|1-\frac{G_-^{\mathrm{e}}}{\epsilon_-}\right|^2 \exp\left(-\frac{\omega_-^2}{2k_-^2 v_{T\mathrm{i}}^2}\,\frac{m^{\mathrm{e}}T^{\mathrm{i}}}{m^{\mathrm{i}}T^{\mathrm{e}}}\right). \quad (38)$$

The first term on the right-hand side of the last expression describes scattering by ions, and the second one describes scattering by electrons. For backscattering, when $k_-^2 \approx 4\omega_{\mathrm{inc}}^2/c^2$, it is convenient to introduce the Doppler ion width ξ of the scattered signal:

$$\xi^2 = \frac{(\omega_{\mathrm{sc}} - \omega_{\mathrm{inc}})^2 c^2}{4\omega_{\mathrm{inc}}^2 v_{T\mathrm{i}}^2}\,, \qquad v_{T\mathrm{i}} = \sqrt{\frac{T^{\mathrm{i}}}{m^{\mathrm{i}}}}\,. \quad (39)$$

Then, exponentials in scattering by electrons and ions will be written out as

$$\exp\left(-\frac{\xi^2}{2}\right),\ \exp\left(-\frac{\xi^2}{2}\,\frac{m^{\mathrm{e}}T^{\mathrm{i}}}{m^{\mathrm{i}}T^{\mathrm{e}}}\right),$$

respectively. Let us consider scattering within the Doppler ion width, when the electron component is virtually equal to unity for

$$\xi^2 \ll \frac{2m^{\mathrm{i}}T^{\mathrm{e}}}{m^{\mathrm{e}}T^{\mathrm{i}}}\,. \quad (40)$$

Because the ion mass is large, this integral embraces all the ion line from $\xi^2 \ll 1$ to $\xi^2 \gg 1$. Consider wavelengths that are much greater than the Debye radius of both electrons and ions. For $\xi^2 \ll 1$, we have

$$G_-^{\mathrm{i}} \approx \frac{c^2}{4\omega_{\mathrm{inc}}^2 \lambda_{\mathrm{Di}}^2} \gg 1\,, \qquad G_-^{\mathrm{e}} \approx \frac{c^2}{4\omega_{\mathrm{inc}}^2 \lambda_{\mathrm{Di}}^2}\,\frac{n^{\mathrm{e}}T^{\mathrm{i}}}{n^{\mathrm{i}}T^{\mathrm{e}}} \gg 1\,,$$
$$\lambda_{\mathrm{Di}}^2 = \frac{T^{\mathrm{i}}}{4\pi n^{\mathrm{i}} e^2}\,. \quad (41)$$

Then, pre-exponential factors in formula (38), defining scattering by electrons and ions, can be written in the form

$$\left|\frac{G_-^{\mathrm{e}}}{\epsilon_-}\right|^2 \approx \left(\frac{T^{\mathrm{i}}n^{\mathrm{e}}}{T^{\mathrm{i}}n^{\mathrm{e}} + T^{\mathrm{e}}n^{\mathrm{i}}}\right)^2,$$
$$\left|1-\frac{G_-^{\mathrm{e}}}{\epsilon_-}\right|^2 \approx \left(\frac{T^{\mathrm{e}}n^{\mathrm{i}}}{T^{\mathrm{i}}n^{\mathrm{e}} + T^{\mathrm{e}}n^{\mathrm{i}}}\right)^2, \quad (42)$$

and, therefore, scattering by ions dominates in the total scattered intensity by electrons and ions:

$$\tilde{S}_- = \left(\frac{T^{\mathrm{i}}n^{\mathrm{e}}}{T^{\mathrm{i}}n^{\mathrm{e}} + T^{\mathrm{e}}n^{\mathrm{i}}}\right)^2 + \sqrt{\frac{m^{\mathrm{e}}T^{\mathrm{i}}}{m^{\mathrm{i}}T^{\mathrm{e}}}}\left(\frac{T^{\mathrm{e}}n^{\mathrm{i}}}{T^{\mathrm{i}}n^{\mathrm{e}} + T^{\mathrm{e}}n^{\mathrm{i}}}\right)^2. \quad (43)$$

In the opposite limit of $\xi \gg 1$, scattering by ions decreases exponentially, $\propto \exp(-\xi^2/2)$, but scattering by electrons also decreases due to the presence of the factor $|1 - G_-/\epsilon_-^2|$ varying as $\propto 1/\xi^4$. For $\xi^2 \gg 1$, we have

$$\tilde{S}_- = \exp\left(-\frac{\xi^2}{2}\right) + \sqrt{\frac{m^{\mathrm{e}}T^{\mathrm{i}}}{m^{\mathrm{i}}T^{\mathrm{e}}}}\left(\frac{T^{\mathrm{e}}n^{\mathrm{i}}}{T^{\mathrm{i}}n^{\mathrm{e}}}\right)^2 \frac{1}{\xi^4}\,, \quad (44)$$

i.e., *scattering by both ions and electrons has the characteristic width of Doppler scattering by ions* and sharply decreases for $\xi^2 \gg 1$. Because of the presence of the factor $\sqrt{m^{\mathrm{e}}/m^{\mathrm{i}}}$ in the expression for scattering by electrons, *backscattering by electrons can be neglected without any noticeable sacrifice of precision*. Then, we can estimate scattering by ions within the ion Doppler width by the scattered intensity at the Doppler line center, namely for $\xi^2 \ll 1$:

$$\tilde{S}_- \approx \left(\frac{T^{\mathrm{i}}n^{\mathrm{e}}}{T^{\mathrm{i}}n^{\mathrm{e}} + T^{\mathrm{e}}n^{\mathrm{i}}}\right)^2. \quad (45)$$

This expression is convenient for qualitative estimates.

4. Examples of estimates for applications

As an example, we will demonstrate how the physical concepts described above facilitate making estimates in new active experiments on the scattering of radio waves in dusty clouds in the lower ionosphere [15]. Active experiments involve the heating of electrons in the region of dusty clouds by high-power radio-frequency pulses at frequencies different from that of the probe radar radiation. Enhanced scattering from the regions of ionospheric dusty clouds in the absence of heating was discovered long ago, and many attempts have been made to interpret it, in particular, by using transition scattering effect in the dust in 1990 [16]. Since then, the vigorous development of dusty plasma physics, in particular, laboratory experiments and experiments on the International Space Station (ISS), followed by the publication of numerous reviews and monographs (see, for example, Refs [17–20]) and references cited therein) devoted to dusty plasma resulted in a deeper understanding of processes in this new state of matter in modern physics. What is important is the general conclusion that, in the presence of many dust particles forming dusty clouds or, so-called dusty structures, a noticeable accumulation of ions [21] and possibly electrons appears, for which such structures become potential wells. The nature of this phenomenon is rather simple. To maintain the charge of dusty clouds, collective plasma flows should exist in the entire region of clouds, which collect electrons and ions even from regions with dimensions noticeably exceeding the cloud sizes. However, the specific features, formation, and dynamics of dusty clouds in the ionosphere and the scattering of radio waves have not been completely analyzed so far, and additional investigations are required. Because the collective effect of ion and electron accumulation in ionospheric dusty clouds have not been studied in detail, we can only assume that ionospheric clouds are not, in this case, the exception vis-à-vis a number of other structures observed in experiments.

Assuming that such a concentration effect of ions and electrons exists, we will show how, by using estimated scattered intensity (45), to conduct simplest estimations of radio wave scattering for explaining both observations of enhanced scattering in the absence of electron heating and the results of active experiments in the presence of this heating. This estimate is presented only to illustrate the possible application of the physical picture described above. It is certainly clear that transition scattering is independent of the mass of a scattering particle and is determined by the electron mass, but depends on the square of the charge which can coherently take part in scattering. Therefore, ions, dust particles, and coherent dusty structures can be candidates for an important role in the explanation of backscattering observed in experiments. We will begin with a description of observations and then assess all three possibilities.

4.1 Radar measurements of scattering from dusty clouds in the lower ionosphere

We shall touch briefly on observations. The anomalous scattering of a radar signal from regions with the minimal temperature (approximately 90 km from Earth's surface) was first discovered in Earth's polar regions around 1996. This altitude corresponding to the upper atmosphere or lower ionosphere is often called the mesosphere region. The minimal temperature (below $-100\,^{\circ}\mathrm{C}$) in this region usually appears in summer [22]. Because of this, the effect in which scattering enhanced by almost two orders of magnitude was called the polar mesospheric summer echo (PMSE) [23]. We shall discuss whether the PMSE effect can be explained by the wave scattering from ions at increased concentrations, and scattering by coherent electrons in dusty structures [24]. Radar scatterings were performed to determine the distribution of electron parameters in the ionosphere at the EISCAT (European International Incoherent Scatter Scientific Association) facility at Tromsø University by using, in principle, the same technique as at facilities of controllable fusion and applying the expression for wave scattering by fluctuations [2]. A backscattering signal was measured, and in this sense the scattered signal can be called a radar signal. It turned out that the scattered intensity during certain summer periods increased by two orders of magnitude, and the altitude at which scattering appeared (about 90 km, as pointed out above) was determined from the signal delay. Because the Doppler shift of the scattered signal is small, scattering should be related to heavy particles or large coherent dusty clouds with small thermal velocities and heavy masses. It was established that scattering evidently correlated with the appearance of dusty clouds. Dusty clouds, which were observed by astronauts in the first space flights, were then associated with noctilucient clouds [22, 23]. Such clouds were observed beginning from the mid-20th century, and several monographs on their morphology were published in the geophysical literature. It is assumed that there are several dust sources in the mesosphere: meteorites, volcanic activity, and industrial pollution. It was pointed out that such clouds were not observed before the beginning of the technological revolution boom, and systematic observations have been performed only since the mid-20th century, which gives no way of unambiguously answering the question of whether the abundance of these clouds is the result of the pollution of the environment by industrial waste, but this is quite possible because, at present, PMSEs are appearing more often and the scattered intensity is increasing with time, although slowly. Therefore, a PMSE study can have an ecological direction. The minimum temperature at an altitude of 90 km is observed in summer, and icy shells grow around dust particles, making them larger and heterogeneous, i.e., consisting of ice with inclusions of solid particles or solid particles with icy shells.

Recently, active experiments were performed in which electrons in the region of dusty clouds were heated by additionally using an intense ground radio-frequency radiation source [15]. The heating source was periodically switched off and on. It was rightly pointed out in paper [15] that such active experiments can help us to understand the PMSE mechanisms. The most illustrative results are those obtained by switching on and off the heating source for 20 s, followed by its switching off completely for 120 s. When the heating source was switched on, a considerable decrease in the PMSE scattered intensity was observed, whereas after switching off for a certain time, the PMSE scattered intensity exceeded its value before switching on the source (the so-called overshot effect).

This scattering mechanism attracts attention in connection with the results of recent rocket measurements of the size of dust particles in the ionosphere, which showed that the size of dust particles is probably smaller than the size required for explaining the PMSE effect as transition scattering from dust particles [16], although this result is not unambiguous. Direct rocket measurements of the size distribution of dust particles in noctilucient clouds confirmed only the presence of dust particles, but not their size distribution (because of the crushing of particles in devices); however, observers are inclined to believe that dust particles in dusty clouds are small enough. For this reason, we shall not consider here the mechanism of transition scattering by dust proposed in paper [16], and we restrict the discussion to the possibility of explaining observations upon wave scattering by ions and coherent structures.

4.2 Estimates of scattering by ions for the polar mesospheric summer echo effect

The necessary condition for wave scattering from ions is the requirement that the wavelength of the incident wave exceed the Debye radius of a scatterer. This condition is wittingly fulfilled, because the Debye radius measures approximately 1 cm, while the wavelength of the incident radiation in modern experiments varies from a few centimeters to 3 m. Under conditions of the lower ionosphere, ions are singly ionized: $Z_{\rm i} \approx 1$. To gain a rough idea of the charge of dust particles, we should bear in mind that considerations based on the floating potential lead to the conclusion that the charge $Z_{\rm d}$ of dust particles is proportional to their size and the electron temperature in the surrounding plasma, and $Z_{\rm d}$ for micron-sized dust particles in laboratory plasma at the electron temperature of ~ 2 eV is equal to $\sim 10^4$. At temperatures in the region of noctilucient clouds (where the identical electron and ion temperatures are two orders of magnitude lower), this gives $Z_{\rm d} \approx 100$ for micron-sized particles, and $Z_{\rm d} \sim 2-5$ for smaller particles. The presence of dust changes the quasineutrality condition, which now looks like $Z_{\rm d}\, n_{\rm d} = n - n_{\rm e}$, and which, due to possible variations in the dust charge, requires the consideration of different relationships between the electron and ion number densities according to formula (45).

At $T_{\rm i} = T_{\rm e}$, the scattered intensity by ions, according to Eqn (45), is proportional to

$$n^{\rm i}\left(\frac{n^{\rm e}}{n^{\rm e}+n^{\rm i}}\right)^2 . \qquad (46)$$

The simplest PMSE mechanism manifesting itself in the absence of electron heating is the enhancement of scattering when ions and electrons are concentrated in the regions of dusty clouds. This possibility is suggested by the presence of numerous structures and compact dusty clouds discovered in experiments with a dusty plasma aboard the ISS. The physics of structuring processes studied in Ref. [25] and numerical simulations [21] suggest an important role of plasma flows in dusty clouds. Assuming that the PMSE also takes place in mesospheric dusty clouds, the enhancement of scattering in the PMSE can be explained by an increase in the ion and electron densities at the center of the clouds (the necessary condition for this — that the wavelength of the incident radiation considerably exceed the Debye radius — is fulfilled in the entire wavelength range of incident waves).

4.3 Estimates for electron heating experiments in dusty clouds

Active experiments on electron heating in the region of mesospheric dusty clouds have shown that the scattered intensity decreases during heating. This also follows from formula (45) because, for $T_{\rm e} n_{\rm i} > T_{\rm i} n_{\rm e}$, the scattered intensity is inversely proportional to $T_{\rm e}^2$, which also takes place for the total increase in the electron and ion concentrations. We can also propose a simple explanation of the recently discovered overshot effects, when scattering from the region of dusty clouds after switching off heating turns out to be greater than that before heating. The charge of dust particles is determined by the electron temperature, which before heating was equal to the rather low temperature of ions and neutral components in this region. As electrons are heated, the charge of dust particles and plasma fluxes noticeably increase. As a result, the ion concentration in dusty clouds increases, which leads to a certain increase in the ion and electron densities at the center of the clouds. Indeed, ion fluxes absorbed in dusty clouds and providing the charge of dust particles are proportional to the charge of dust particles and considerably increase proportionally to $T_{\rm e}$. But in this case, global plasma fluxes directed to the center of structures also increase to compensate for the increasing absorption of ions and electrons by dust particles (these fluxes are controlled only by ions). Electron fluxes to individual dust particles also increase, but proportionally to $\sqrt{T_{\rm e}}$. Therefore, the dimensionless charge of dust particles determined by the floating potential, namely $Z_{\rm d}e^2/aT_{\rm e}$, decreases but insignificantly, because it logarithmically depends on the fluxes to dust particles, and therefore the relationship $Z_{\rm d} \propto T_{\rm e}$ is approximately fulfilled, i.e., the charge considerably increases with electron heating. But how can the decrease in scattered intensity during heating be explained? According to Eqn (46), the decrease in the scattered intensity as $1/T_{\rm e}^2$ with increasing $T_{\rm e}$ is greater than the increase in the ion concentration caused by the establishment of a new balance of global fluxes. Then, wave scattering should weaken upon heating, which is observed in experiments. When the heating source is switched off, the ion concentration increased during heating begins to 'work' because the factor $1/T_{\rm e}^2$ disappears. The heating overshot effect can be related to the time required for the diffusion of the excess of ions from the region inside the dusty clouds. This model weakly depends on the dust particle size, but satisfies the necessary requirement of observations indicating that the described effects should be unambiguously related to a dusty component.

The use of the wave scattering effect by ions is hindered by the observed small width of the scattered signal, which is much smaller than $v_{T\rm i}/c$ even for those low temperatures that are inherent in the PMSE region. Thus, the above-presented estimate can be considered as preliminary (or a 'trial' for the correct formulation of the problem). We emphasize that, by using Eqn (46), such estimates can be very simply obtained.

4.4 Estimates of scattering by coherent dusty structures

Now we can proceed further with our estimations, using the fact that scattering is undoubtedly related to dust and rather long waves (longer than 3 m) are scattered, i.e., scattering objects should be very heavy. These particles can only be coherent electrons, for example, in dusty clouds with a size on the order of the wavelength, which, as ions, will scatter radiation proportionally to the square of the number of electrons in the coherent structure. The correctness of this interpretation can be confirmed only by a detailed study of structuring processes in a dusty plasma, which was started in Ref. [24]. Notice that it is necessary to take into account thermophoretic forces under conditions of inhomogeneous gas temperatures in the regions of temperature minimum, where the PMSE appears (which were neglected in Ref. [24] for the linear stage of the structuring instability). At the nonlinear stage, frictional and gravitational forces and the growth in the size of dust particles should also be taken into account. Our estimates suggest the reality of such a scenario, although the detailed study and verification of its implications should be, of course, performed in the future. The first experiments [27] on probing the dusty component distribution in PMSE regions revealed that the dust distributions are strongly inhomogeneous, and dusty clouds on the order of a few meters in size are formed, which can be the first evidence of the structuring and formation of coherent dusty structures in PMSE regions. Further, more detailed, three-dimensional measurements can facilitate the construction of an adequate theory of wave scattering by charged particles. The estimates made above considerably restrict the possibilities of using models different from the one described. Of course, if coherent dusty structures do have a size close to that obtained in today's preliminary experiments, wave scattering (as for ions) will be determined by the square of the number of coherent electrons, which is quite sufficient in such structures for the qualitative explanation of observations.

It is possible that wave scattering by heavy particles and structures can be used for analyzing parameters in experiments.

(1) *Dusty structures in laboratory and space experiments*. Various structures have been observed in experiments with dusty plasma on the ISS: compact dusty clouds, dusty voids (regions in which dust particles are completely absent), and dusty vortices. They have been detected with the help of laser illumination and only according to the dusty component distribution. Of current interest is the analysis of ion distributions: the ion concentration at the center of structures, the ion concentration continuity on rather sharp surfaces of voids, and small ion perturbations in vortex structures. To date, the properties of dusty structures have been able to be qualitatively determined only from the distribution of dust particles, and they have been in satisfactory agreement with the results of numerical simulations of equilibrium structures, predicting the distributions of ions and charges of dust particles in these structures. The scattering of submillimeter radiation by ions could give the information required for a more complete understanding of the equilibrium state of such structures, the nature of the predicted confinement of ions in compact structures, and some other their characteristics. But the incident radiation intensity required for detecting the scattered signal is quite high, and this technique could probably be used only in structures formed in laboratory dusty plasma, which were recently observed under Earth conditions.

(2) *Wall plasma sheets*.The ion diagnostics is also needed in various technological and industrial facilities in wall plasma sheets, especially in regions where they are formed (presheets). The ion diagnostics can also find applications, in particular, in plasma chemistry and plasma processing of various surfaces. The use of wave scattering by ions can possibly give additional information.

5. Conclusions

The scattering of electromagnetic waves in usual (dustless) plasmas by particle fluctuations, with allowance made for the interference between usual and transition scatterings, is rigorously equal to the sum of scatterings by plasma electrons and ions. This statement concerns any nonequilibrium distribution of particles and waves only if collective modes, scattering from which may have the nature of induced Raman scattering, are not excited in the plasma due to instability. As shown in book [13], spontaneous Raman scattering and Mandelstam–Brillouin scattering correspond to resonance transition scattering. Polarization effects also play a great role in plasma and other processes. It has long been known [9, 28] that collisions between particles described by the so-called Balescu collision integral [13] correspond to collisions of particles having dynamically screened polarization clouds, while bremsstrahlung substantially changes due to the so-called polarization bremsstrahlung interfering with usual bremsstrahlung, as in the case of scattering processes [9, 28]. Polarization bremsstrahlung, which also appears in the case of heavy particles, can be determined by the mass of electrons in their polarization clouds. It is quite natural that scattering, collisions, and bremsstrahlung in plasma physics depend on the local permittivity, i.e., on the distribution of all plasma particles, and therefore are collective processes.

Proof remark

The scattering of electromagnetic waves by structures is independent of the sign of the coherent charge of the structure: the excess or lack of electrons. A simple calculation of a weak perturbation of the structure field by the incident wave field using the nonlinear current from Section 2 gives the scattering cross section for the structure, which is equal to the Thomson cross section for scattering by free electrons multiplied by the coherence factor

$$\left(\int n_{\mathrm{e}} \exp(\mathrm{i}\mathbf{k}_{-}\mathbf{r})\, \mathrm{d}\mathbf{r}\right)^{2}$$

(integration is performed over the structure volume). For wavelengths greatly exceeding the structure size, the coherence factor is equal to the square of the total coherent charge of the structure. The necessary condition of the smallness of the incident wave field compared to the field providing coherence is fulfilled for all examples discussed above.

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