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Dynamics of frequency-modulated wave packets in optical guides with complex-valued material parameters

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Abstract. Features of wave packet dynamics in linear and nonlinear optical fibers are studied allowing for complex-valued dispersion parameters. It is shown that the presence of imaginary components in dispersion parameters affects considerably the compression behavior of a wave packet in single-element and cascaded optical fibers; causes a shift in carrier frequency, and creates frequency-modulated wave packets with a superluminal envelope maximum.

1. Introduction

The dynamics of optical wave packets (WPs) propagating in an amplifying medium has been attracting attention recently both in fundamental and applied studies [1–9]. However, despite a variety of methods applied for studying these systems, the models used neglect, as a rule, the complex nature of dispersion parameters playing an important role in the dynamics of optical WPs propagating in active optical fibers [10–13]. Meanwhile, amplifying fibers are described by the complex refractive index. Guided modes in such fibers have a complex propagation constant and, therefore, a complex group velocity and dispersion parameters. The authors of papers [14, 15] pointed out significant differences in the behavior of WPs in optical fibers with complex dispersion parameters from that in 'classical' active fibers, in which amplification in the corresponding dynamic equation is taken into account only by introducing a linear term resulting in an exponential increase in the amplitude [16, 17]. Thus, the degree of the linear compression of optical radiation is determined to a large extent by the relation between the imaginary and real components of the group velocity dispersion (GVD) parameter and the initial frequency modulation (FM) rate. In this case, compression is possible even without the initial FM of the input WP, unlike fibers with the real GVD parameter.

The compression of a WP in an active optical fiber, as a rule, is accompanied by a shift of its carrier frequency, caused by the presence of the imaginary component of the first-order dispersion parameter. The carrier frequency shift, in turn, can result in WP distortion and the deformation of the signal envelope, thereby violating the compression regime and producing other dynamic effects in the propagation of WPs in such fibers [18–20].

The pulsed radiation dynamics in optical fibers with material parameters inhomogeneously distributed over the fiber length has attracted great recent attention [21–24]. Such an interest is explained by the fact that optical fibers with modulated GVD, nonlinearity, or amplification find wide applications as highly efficient systems for controling laser radiation. In this paper, we study the features of the linear and nonlinear WP dynamics in active and inhomogeneous (in length) optical fibers with a complex propagation constant and, consequently, dispersion parameters and discuss the possibility of the temporal compression of WPs and achieving the superluminal velocity of the WP envelope. This possibility is associated with the presence of the imaginary components of the optical fiber material parameters.

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2. Linear dynamics of a wave packet in an active optical fiber

2.1 Temporal compression

Let us consider the optical radiation dynamics in an inhomogeneous (in length) optical fiber with a complex refractive index and, therefore, the complex propagation constant $\beta = \beta' - i\beta''$. In this case, the field of a WP propagating in a fiber can be written out in the form

$$\mathbf{E}(t, r, z) = \frac{1}{2}\,\mathbf{e}R(r) \times \left[B(t, z)\,\exp\left[i\left(\omega_0 t - \int_0^z \beta'(\xi)\,d\xi\right)\right] + \text{c.c.}\right], \quad (1)$$

where $\mathbf{e}$ is the unit polarization vector of the light field, the function $R(r)$ describes the radial field distribution in the fiber, ω_0 is the carrier frequency of the coupled WP, and c.c. stands for the complex conjugate. Taking into account first- and second-order dispersion effects, we obtain in the linear approximation the equation [17]

$$\frac{\partial B}{\partial z} + k(z)\,\frac{\partial B}{\partial t} - i\,\frac{D(z)}{2}\,\frac{\partial^2 B}{\partial t^2} = -\beta''(z)\,B \quad (2)$$

for the WP temporal envelope, where complex dispersion parameters are introduced: the first-order parameter $k = k' - ik'' = (\partial\beta/\partial\omega)_0$, and the second-order parameter $D = D' - iD'' = (\partial^2\beta/\partial\omega^2)_0$, where the values of the derivatives are taken at the WP carrier frequency ω_0. In the case of the real propagation constant, the first dispersion parameter k determines the group velocity, and the second dispersion parameter D determines the GVD of the light wave propagating in the fiber.

Taking into account the complex propagation constant, the slowly varying complex amplitude $B(t, z)$ can be conveniently written out in the form

$$B(t, z) = A(t, z)\,\exp\left(-\int_0^z \beta''(\xi)\,d\xi\right), \quad (3)$$

where $\beta'' > 0$ for an absorbing fiber, and $\beta'' < 0$ for an amplifying fiber. By substituting expression (3) into equation (2), we obtain the equation

$$\frac{\partial A}{\partial z} - ik''(z)\,\frac{\partial A}{\partial \tau} - i\,\frac{D(z)}{2}\,\frac{\partial^2 A}{\partial \tau^2} = 0 \quad (4)$$

for the WP temporal envelope, where $\tau = t - \int_0^z k'(\xi)\,d\xi$ is the time taken in a moving coordinate system.

Let us consider a frequency-modulated Gaussian WP with quadratically changing phase, which is coupled to the input of an optical fiber ($z = 0$):

$$A(\tau, 0) = A_0 \exp\left[-\frac{(1 + i\alpha_0\tau_0^2)\,\tau^2}{2\tau_0^2}\right], \quad (5)$$

where the parameter α_0 characterizes the input FM rate, and τ_0 is the initial WP duration. The solution of Eqn (4) for the initial excitation conditions of fiber (5) in a moving coordinate system can be written out in the form

$$A(\tau, z) = \rho(\tau, z)\exp\left[i\varphi(\tau, z)\right], \quad (6)$$

where the amplitude of the WP temporal envelope and its phase were introduced:

$$\rho(\tau, z) = A_0\sqrt{\frac{\tau_0}{\tau_p}}\exp\left[\frac{(1 + S^2)\,K''^2 z^2 - \tau_s^2}{2\tau_p^2}\right],$$
$$2\varphi(\tau, z) = \frac{S\tau_s^2 - 2\tau_s K''(1 + S^2)\,z + K''^2 S(1 + S^2)\,z^2}{\tau_p^2} + \arctan\left(\frac{S - \alpha_0\tau_0^2}{1 + \alpha_0\tau_0^2 S}\right). \quad (7)$$

Here, we also introduced the refined running time $\tau_s = \tau - SK''z$, the duration of WP propagation through the fiber

$$\tau_p(z) = \tau_0^2\sqrt{\frac{(1 - \chi_1 z)^2 + \chi_2^2 z^2}{\tau_0^2 + D''(1 + \alpha_0^2\tau_0^4)\,z}}, \quad (8)$$

and the parameters

$$S = \frac{(\alpha_0^2\tau_0^2 + \tau_0^{-2})\,D'z - \alpha_0\tau_0^2}{1 + (\alpha_0^2\tau_0^2 + \tau_0^{-2})\,D''z},$$

$$\chi_1 = \alpha_0 D' - D''\tau_0^{-2}, \quad \chi_2 = \alpha_0 D'' + D'\tau_0^{-2},$$

$$D'(z) = z^{-1}\int_0^z d'(\xi)\,d\xi, \quad D''(z) = z^{-1}\int_0^z d''(\xi)\,d\xi,$$

$$K''(z) = z^{-1}\int_0^z k''(\xi)\,d\xi$$

depending on the coordinate z.

An analysis of the equations obtained shows that, depending on the relation among parameters τ_0, α_0, D', and D'', the WP spreads or undergoes compression and acquires additional FM during its propagation. The compression condition for the input WP, $(\partial\tau_p/\partial z) < 0$, is determined by the inequality

$$(\alpha_0^2\tau_0^4 - 1)\,D_0'' + 2\alpha_0\tau_0^2 D_0' > 0, \quad (9)$$

in the case of complex dispersion parameters and taking expression (8) into account, where D_0' and D_0'' are the values of the appropriate parameters at the fiber input. It follows from inequality (9) that under consideration of the real propagation constant β, i.e., for the real GVD parameter ($D'' = 0$), compression takes place only for $\alpha_0 \neq 0$ in the region of parameters where $\alpha_0 D' > 0$. Then, we obtain the well-known expression

$$\tau_p(z) = \tau_0\sqrt{(1 - \alpha_0 D'z)^2 + \left(\frac{D'z}{\tau_0^2}\right)^2} \quad (10)$$

for the WP duration. In another limiting case, $D' = 0$ and $D'' \neq 0$, the WP duration takes the following form

$$\tau_p(z) = \tau_0^2\sqrt{\frac{(1 + D''z/\tau_0^2)^2 + (\alpha_0 D''z)^2}{\tau_0^2 + D''(1 + \alpha_0^2\tau_0^4)\,z}}, \quad (11)$$

which means that the WP compression is realized for $D_0'' < 0$, even in the absence of the initial FM. In this conditions, $\alpha_0 = 0$ and the WP duration is expressed as $\tau_p(z) = (\tau_0^2 + D''z)^{1/2}$.

Figure 1 shows the dependence of the WP duration on the distance propagated in the fiber for $D'' < 0$ and different

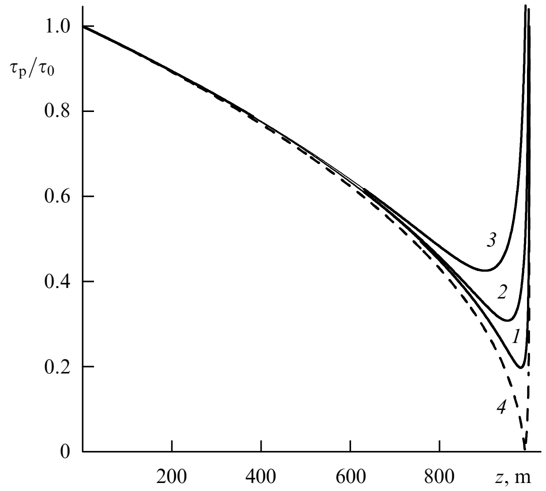

Figure 1. Dependence of the WP duration on the traversed distance for $D'' < 0$, $|D''/D'| = 10, 20, 50$, $\alpha_0 = 10^{20}$ s^{-2} (curves *1–3*); dashed curve *4* corresponds to $|D''/D'| = 10$ and $\alpha_0 = 10^{21}$ s^{-2}.

values of the parameter $\eta = D''/D'$. The initial WP duration is $\tau_0 = 10^{-11}$ s, the FM rate is $\alpha_0 = 10^{20}$ s^{-2}, and the value of $D'' = -10^{-25}$ s^2 m^{-1} is fixed (only the value of $D' > 0$ changes). As the parameter $|\eta|$ increases, the WP compression degree also increases. The dashed curve fits the values of $\alpha_0 = 10^{21}$ s^{-2} and $|\eta| = 10$. In this case, the compression degree increases to an order of magnitude compared to the case where $\alpha_0 = 10^{20}$ s^{-2}.

2.2 Spectral description of the wave-packet dynamics

An analysis of the WP dynamics in an amplifying medium with complex dispersion parameters should take into account effects related to the possible carrier-frequency shift [18–20]. We will more to the spectral representation of the WP temporal envelope

$$\tilde{A}(\omega, z) = \frac{1}{2\pi} \int A(t, z) \exp(-\mathrm{i}\omega t)\, \mathrm{d}t \,. \tag{12}$$

The spectral representation of the temporal envelope for a Gaussian frequency-modulated pulse takes the form

$$\tilde{A}(\omega, z) = A_0 \left(\frac{\tau_0}{2\pi \Delta\omega_s} \right)^{1/2} \times \exp\left(-\frac{\left(\omega_s(z) - \omega\right)^2}{2\Delta\omega_s^2} + \frac{\Omega^2}{2\Delta\omega_s^2} + \mathrm{i}\varphi(\omega) \right), \tag{13}$$

where $\omega_s(z)$ is the coordinate-dependent effective carrier frequency, and

$$\Omega(z) = \omega_s(z) - \omega_0 = -\Delta\omega_s^2(z) \int_0^z \frac{\partial \beta''(\xi)}{\partial \omega}\, \mathrm{d}\xi \tag{14}$$

is the carrier-frequency shift. Then, the coordinate-dependent spectral width of the WP is given by the relation $\Delta\omega_s = [\tau_p^{-2} + \alpha^2(z)\,\tau_p^2]^{1/2}$. The sign of the carrier-frequency shift is determined by the sign of $k'' = \partial\beta''/\partial\omega$, which, in turn, depends on the amplification line shape and the position of the carrier frequency with respect to the resonance frequency. It follows from the relationships presented above that, during the propagation of a WP in an optical fiber, its effective carrier frequency shifts and the spectral width changes. For $D'' < 0$, the spectral broadening of the WP occurs in the path of $z \leqslant L$. The mechanism of this broadening is related to the specific features of the radiation propagation in media with a strong dispersion of the gain. Because of the fast FM, spectral components at the pulse edges escape from the maximum gain region, so that their amplification proves to be much smaller than that for components located near the maximum of the WP envelope. Therefore, considerable amplification occurs only near the maximum of the WP envelope, where the instantaneous frequency shift $\Omega(t) = -\partial\varphi(\omega)/\partial t$ does not considerably exceed the gain linewidth.

Because the carrier frequency ω_0 of a WP coupled to an optical fiber may not coincide with the gain line center ω_r in the region $D'' < 0$, the sign of the carrier frequency shift for a gain line depends on the sign of the difference $\omega_0 - \omega_r$. If $\omega_0 > \omega_r$, then $\Omega > 0$, while for $\omega_0 < \omega_r$, the opposite situation takes place, i.e., $\Omega < 0$. In this case, the carrier frequency in an amplifying medium is pulled into the maximum gain region. The occurrence of the shift of the carrier frequency considerably restricts the possibilities of the compression mechanism under study, because the carrier frequency at large enough values of the parameter k'' can leave a region with the negative imaginary component of the GVD parameter.

Consider next the dependence of the imaginary components of dispersion parameters on the carrier-frequency detuning from the resonance frequency and the linewidth by the example of a Lorentzian gain line. The gain increment for radiation intensity well below the saturation intensity in the case under study can be described by the formula [25]

$$\gamma(\omega) = -2\beta''(\omega) = \frac{\rho N \Delta\omega^2}{\delta\omega^2 + \Delta\omega^2} \,, \tag{15}$$

where $\delta\omega = \omega_0(z) - \omega_r$ is the detuning from the induced transition frequency ω_r, ρ is the induced transition cross section, N is the concentration of active particles in the absence of lasing, and $\Delta\omega$ is the gain linewidth. In this case, the imaginary components of the first- and second-order dispersion parameters are defined by the expressions

$$k'' = \frac{\rho N \,\delta\omega\, \Delta\omega^2}{(\delta\omega^2 + \Delta\omega^2)^2} \,, \quad D'' = \frac{\rho N \Delta\omega^2 (\Delta\omega^2 - 3\delta\omega^2)}{(\delta\omega^2 + \Delta\omega^2)^3} \,. \tag{16}$$

To obtain the compression of a WP coupled to a fiber at $\alpha_0 = 0$, according to inequality (9), the condition $D'' < 0$ must be fulfilled, where the frequency ω is set equal to the WP carrier frequency ω_0. It follows from formulas (16) that this condition can be fulfilled if the carrier frequency is chosen so that the detuning from the resonance frequency equals $|\delta\omega| > \Delta\omega/\sqrt{3}$. Because the carrier frequency is pulled to the gain line center, where $k'' \approx 0$ and $D'' > 0$, the realization of compression mechanism related to the phase modulation caused by the imaginary component of the GVD parameter in the model of a single Lorentzian line proves to be quite problematic.

It is necessary to note that, for $k'' \neq 0$, the carrier frequency always shifts. For a Lorentzian gain line, the carrier frequency shift is determined by the equation

$$\Omega_s(z) = -\Delta\omega_s^2(z) \int_0^z \rho N(\xi)\, \frac{\left(\omega_s(\xi) - \omega_r\right) \Delta\omega^2}{\left(\Delta\omega^2 + \left(\omega_s(\xi) - \omega_r\right)^2\right)^2}\, \mathrm{d}\xi \,. \tag{17}$$

In general, having regard to the pump degeneracy, the parameter N determining the presence of active centers almost always depends on a coordinate (for example, due to the pump degeneracy effect).

The problem of the carrier-frequency shift can be solved, in particular, by using media with the complex profile of the gain increment possessing a local extremum in the frequency range with $D'' < 0$. It can be formed if the medium is approximated by a sum of two Lorentz oscillators. In this case, the gain is defined by the expression

$$\gamma(\omega) = -2\beta''(\omega) = N \sum_{i=1,2} \frac{\rho_i f_i \Delta\omega_i^2}{(\omega - \omega_{ri})^2 + \Delta\omega_i^2} , \qquad (18)$$

where the quantity $f_i = N_i/N$ determines the contribution from the corresponding group of oscillators to the total gain curve and $f_1 + f_2 = 1$. Here, it is always possible to select the carrier frequency $\omega = \omega_s$ so that, on the one hand, $k'' = 0$ and, on the other hand, the condition $D'' < 0$ is fulfilled.

Figure 2 plots the frequency dependences of dispersion parameters k'' and D'' for a single Lorentzian line (Fig. 2a) and a complex gain line (Fig. 2b) determined by relations (16) and (18). Numerical calculations were performed using parameters of widely distributed erbium-doped silica fibers: $\rho = 2\times10^{-24}$ m^2, $N = 5\times10^{21}$ m^{-3}, $\omega_r = 1.8\times10^{15}$ s^{-1}, $\Delta\omega = 0.5\times10^{12}$ s^{-1} (Fig. 2a) and $f_1 = 0.55$, $f_2 = 0.45$, $\omega_{r1} = 1.8\times10^{15}$ s^{-1}, $\omega_{r2} = 1.802\times10^{15}$ s^{-1}, $\Delta\omega_1 = 6\times10^{11}$ s^{-1}, $\Delta\omega_2 = 5.5\times10^{11}$ s^{-1}, and $\rho_i N = 10^{-2}$ m^{-1} (Fig. 2b). In the event of a complex gain line, the arrow indicates the operation frequency for which $k'' = 0$ and $D'' < 0$ and the carrier-frequency shift is virtually absent during the propagation of a pulse in a fiber with such a gain line.

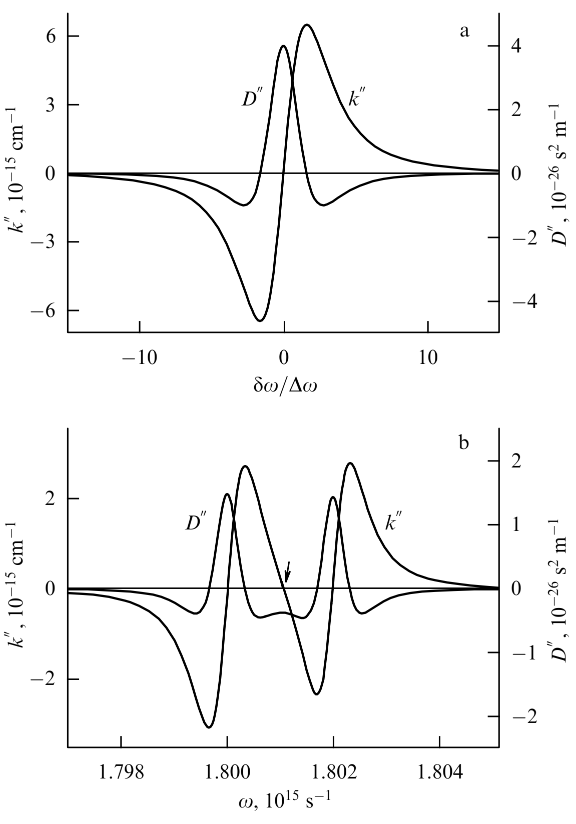

Figure 2. Frequency dependences of dispersion parameters k'' and D'' for a single Lorentzian (a) and complex (b) gain line.

The problem of the carrier-frequency displacement can also be solved by tapping hollow fibers filled with an amplifying gas medium [26] with the inhomogeneous gain line in which a local minimum, the so-called Bennett hole [27], is burnt at the required frequency, or by using microstructured fibers [28]. Notice also that the gain line of a real active medium cannot be described by an ideal Lorentzian and practically always has a sufficiently large number of local extrema.

The analysis performed above gives evidence that the WP carrier-frequency shift can considerably complicate the realization of compression regimes in active optical fibers. However, along with this negative factor, the imaginary first-order dispersive parameter can lead to some specific effects. An important feature of WP propagation in an amplifying medium is the fundamental possibility of the movement of its envelope maximum at a velocity exceeding the speed of light c in vacuum. Such a superluminal propagation of the WP does not mean that energy is transferred at this velocity, but is related to a change in the WP shape due to a stronger amplification at the WP leading edge [29]. Although the possibility of achieving the superluminal velocity of WP traveling in amplifying media was discussed long ago [30–32], this effect is still attracting the close attention of researchers [33–62].

The authors of above-noted papers analyzed WPs with a fairly extended exponential leading edge. However, superluminal propagation regimes for the envelope maximum are also possible for a rapidly decaying Gaussian FM pulse. Taking into account the running time introduced in expressions (7), the propagation velocity of the WP envelope maximum in a medium can be defined by the general expression

$$u_m = \frac{u}{1 + Sk''u} , \qquad (19)$$

where $u = (k')^{-1} = c/n_{ef}$ is a quantity that is usually identified with the WP group velocity, and n_{ef} is the real part of the effective refractive index of the active medium in which the WP is formed. It follows from relationships (19) that the superluminal regime ($u_m > c$) for the WP envelope maximum is possibly attained when the condition $Sk'' < 0$ is fulfilled. This regime can be obtained upon both WP compression and broadening its spectrum for different signs of the quantities $\omega_0 - \omega_r$, α_0, D', and D''. To obtain the superluminal regime at large enough distances, conditions providing the maintenance of the carrier frequency within the gain line ($|\Omega_s| \ll \Delta\omega$) should be fulfilled over the entire WP propagation path. The dynamical features of WPs in media with a considerable dispersion of the gain increment will be discussed in more detail in Section 4.

3. Cascade compression scheme

Let us consider in this section the features of a cascade radiation compression scheme, related to the complex dispersion parameters. We assume that an optical fiber consists of a sequence of active (amplifying) and passive homogeneous segments for which the imaginary and real components of dispersion parameters are deemed constant. This compression technique is a continuation of the amplification technique for FM WPs applied for reducing the influence of nonlinear optical phenomena on the generation, amplification, and transfer of high-power ultrashort light pulses in solid-state laser systems [63–65]. According to this technique [66, 67], a light pulse is transmitted before amplification through a stretcher providing an increase in the pulse duration. The resulting decrease in the peak pulse power considerably reduces the influence of nonlinear effects at the amplification stage. The phase modulation produced during stretching is compensated for after amplification with the aid of a compressor producing an ultrashort laser pulse.

Below, we consider compression conditions for a Gaussian WP propagating in a cascade optical fiber with an input active fiber and an output passive fiber. This scheme solves the problem of the carrier frequency shift and assumes the generation of a long enough WP with a relatively small envelope amplitude and a high FM rate at the first section of the cascade [68, 69]. After the FM rate of the WP becomes as high as possible, the WP is coupled with the second section to obtain compression. In this case, the second section length, at which the WP becomes transform-limited, coincides with the length at which the WP duration becomes minimal. An analysis and comparison of the parameters of a WP propagating in a cascade fiber are performed for forward and backward propagations.

3.1 Two-stage scheme

Let us first consider a two-stage WP compression scheme, when optical radiation propagates in the first amplifying fiber with complex parameters k_1, D_1 and length L_1, and then is coupled into the second fiber with real dispersion parameters k_2, D_2 and length L_2. We assume that a WP coupled with the first fiber has the duration τ_0 and the initial FM rate α_0. According to expression (7), the FM rate of the WP at the output of the amplifying fiber (and at the input to the second passive fiber) is given by

$$\alpha_1 = \frac{\partial^2 \varphi}{\partial \tau^2}\bigg|_{z=L_1} = -\frac{\alpha_0 \tau_0^2 (1-\chi_1 L_1) - \chi_2 L_1}{\tau_0^2 \left[(1-\chi_1 L_1)^2 + (\chi_2 L_1)^2\right]} . \qquad (20)$$

In this case, according to formula (10), the WP duration after propagation through the second fiber is described by the expression

$$\tau_p(L_2) = \tau_1 \left[(1-\alpha_1 D_2' L_2)^2 + \left(\frac{D_2' L_2}{\tau_1^2}\right)^2\right]^{1/2} , \qquad (21)$$

where $\tau_1 = \tau_p(L_1)$ and $\alpha_1 = \alpha(L_1)$ are the WP duration and the FM rate at the output of the first amplifying fiber. It follows from the last formula that the compression regime in the second fiber is possible when the inequality $\alpha_1 D_2' > 0$ is satisfied.

Consider the important case demonstrating the fundamental possibility of the cascade compression of a propagating WP in the absence of its initial FM, i.e., at $\alpha_0 = 0$. After traversing a distance L_1 in the first amplifying part of the fiber, the WP acquires at the output the FM rate

$$\alpha_1 = \alpha(L_1) = \frac{D_1' L_1}{(\tau_0^2 + D_1'' L_1)^2 + (D_1' L_1)^2} . \qquad (22)$$

It is appropriate to determine the optimal length of the first fiber from the condition according to which the WP duration should be minimal at the cascade output. To achieve this, the FM rate $|\alpha(L_1)|$ at the output of the first fiber should be as high as possible. By setting the derivative $\partial\alpha/\partial L_1$ equal to zero, we obtain the optimal length of the first fiber: $L_{10} = \tau_0^2/|D_1|$. As a result, the FM rate for the WP at the output of the first fiber with the optimal length L_{10} reaches the value

$$\alpha_{10} = \alpha(L_{10}) = \frac{D_1'}{2\tau_0^2 \left(|D_1| + D_1''\right)} . \qquad (23)$$

The WP duration after traversing a distance L_1 in the first amplifying fiber is determined by the expression

$$\tau_1 = \tau_p(L_1) = \left[\frac{(\tau_0^2 + D_1'' L_1)^2 + (D_1' L_1)^2}{\tau_0^2 + D_1'' L_1}\right]^{1/2} . \qquad (24)$$

After traversing the optimal length, the WP broadens and its duration becomes equal to $\tau_{10} = \tau_p(L_{10}) = \sqrt{2}\tau_0$.

The values α_1 and τ_1 are the initial (input) for the second, passive fiber. The FM rate and the WP duration after traversing a distance z in the second fiber are defined by the expressions

$$\alpha(L_1 + z) = -\frac{\alpha_1 \tau_1^2 + (\alpha_1^2 \tau_1^2 + \tau_1^{-2})\, D_2' z}{\tau_1^2 \left[(1-\alpha_1 D_2' z)^2 + (D_2' \tau_1^{-2} z)^2\right]} ,$$
$$\tau_p(L_1 + z) = \tau_1 \sqrt{(1+\alpha_1 D_2' z)^2 + (D_2' \tau_1^{-2} z)^2} . \qquad (25)$$

Thus, the FM rate and WP duration at the cascade output are determined by the parameters $\alpha_{12} = \alpha(L_1 + L_2)$ and $\tau_{12} = \tau_p(L_1 + L_2)$. The optimal length L_{20} of the second fiber in the cascade is found from the condition of the WP duration minimum at the cascade output. By solving the equation $\partial\tau_p/\partial L_2 = 0$, we obtain the optimal length

$$L_{20} = \frac{\alpha_1 \tau_1^4}{D_2' (1 + \alpha_1^2 \tau_1^4)} \qquad (26)$$

of the second fiber, on which the maximum compression is reached and where the minimal possible values of α_1 and τ_1 at the cascade output should be chosen as optimal values. In this case, the WP duration at the cascade output reaches the minimal possible value

$$\tau_{12}^{\min} = \tau_p(L_0) = \frac{\tau_{10}}{\sqrt{1 + \alpha_{10}^2 \tau_{10}^4}} = \sqrt{\tau_0^2 + D_1'' L_{10}}$$
$$= \tau_0 \sqrt{1 + \frac{D_1''}{|D_1|}} \qquad (27)$$

for the chirp value α_{10} realized at the output of the first fiber, where the total optimal cascade length L_0 is the sum of the optimal lengths of the first and second fibers, i.e., $L_0 = L_{10} + L_{20}$. It follows from this expression that the minimum WP duration which can be attained in the cascade depends on the sign of the parameter D_1'' and the value of D_1'. Thus, for $D_1' \to 0$ and $D_1'' < 0$, we have $\tau_p(L_0) \to 0$ at the cascade output. According to formula (21) and in view of Eqn (24), the compression condition in the cascade scheme will be expressed by the inequality $D_1' D_2' < 0$. It also follows from Eqn (27) that the value of $\tau_p(L_0)$ is independent of the parameters of the second, passive fiber; however, according to formula (24), the higher degree of compression cannot be achieved without this fiber, i.e., in the active fiber alone.

The analysis of expression (27) shows that, for $|\alpha_1|\tau_1^2 = 1$, the duration of the compressed WP takes the maximum value of $\tau_{\min} = 1/(2|\alpha_1|)^{1/2}$. Therefore, to obtain the WP duration at the cascade output as short as possible, it is necessary to increase the FM rate $|\alpha_1|$ at the first fiber output. In the case of high compression degrees, the inequality $|\alpha_1|\,\tau_1^2 \gg 1$ must be satisfied, and the expressions obtained can be transformed to the form

$$L_{20} \approx \frac{1}{|D_2'\alpha_1|} \approx \tau_0^2 \left| \frac{D_1'}{D_2' D_1''} \right| , \qquad (28)$$

$$\tau_{\min} \approx \frac{1}{|\alpha_1 \tau_1|} = (\tau_0^2 + D_1'' L_1)^{1/2} \left[1 + \left(\frac{\tau_0^2 + D_1'' L_1}{D_1' L_1} \right)^2 \right]^{1/2} . \qquad (29)$$

To achieve efficient cascade compression, the length L_1 of the amplifying fiber should be chosen to provide the validity of the inequality $|\tau_0^2 + D_1'' L_1| \ll |D_1' L_1|$. Then, expression (29) reduces to Eqn (27).

A considerable carrier-frequency shift in the compression scheme constructed here is mainly prevented by the choice of the initial carrier frequency ω_0 determining the value of $k_1''(\omega_0)$. Thus, for the case of $\alpha_0 = 0$, the condition $|\Omega_{\rm s}| \ll \Delta\omega$ determining the restriction on the carrier-frequency shift in the cascade compression scheme can be rewritten in the form

$$\tau_{\min} \gg \sqrt{\frac{|k_1''|\, L_1}{\Delta\omega}} , \qquad (30)$$

where it is taken into account that $\Delta\omega_{\rm s} \approx 1/\tau_{\min}$. The validity of condition (30) is achieved by choosing the parameter $k_1''(\omega_0)$ so that the value of $|k_1''|\, L_1$ is small enough.

Let us turn now to the case of the backward propagation of a WP, when optical radiation first propagates in a passive fiber with parameters k_2, D_2, and L_2 and then is coupled into an active fiber with parameters k_1, D_1, and L_1, i.e., the sections of a cascade fiber are interchanged. We assume that the WP coupled into the cascade, as in the first event, has the duration τ_0 and the initial FM rate $\alpha_0 = 0$. After propagation through the passive fiber, the WP acquires the FM rate and duration determined by the expressions

$$\alpha_2 = \alpha(L_2) = \frac{D_2' L_2}{\tau_0^4 + (D_2' L_2)^2} ,$$
$$\tau_2 = \tau_{\rm p}(L_2) = \sqrt{\tau_0^2 + \left(\frac{D_2' L_2}{\tau_0} \right)^2} . \qquad (31)$$

After propagation through the active section of the fiber, i.e., at the cascade output, the WP duration and the FM rate are described by the expressions

$$\alpha_{21} = - \frac{\alpha_2 \tau_2^2 + (\alpha_2^2 \tau_2^2 + \tau_2^{-2})\, D_1' L_1}{\tau_2^2 \left[(1 - (\alpha_2 D_1' - D_1'' \tau_2^{-2})\, L_1)^2 + (\alpha_2 D_1'' + D_1' \tau_2^{-2})^2 L_1^2 \right]} , \qquad (32)$$

$$\tau_{21} = \tau_2^2 \sqrt{\frac{\left[1 - (\alpha_2 D_1' - D_1'' \tau_2^{-2})\, L_1 \right]^2 + (\alpha_2 D_1'' + D_1' \tau_2^{-2})^2 L_1^2}{\tau_2^2 + D_1''(1 + \alpha_2^2 \tau_2^4)\, L_1}} ,$$

where notations $\alpha_{21} = \alpha(L_2 + L_1)$ and $\tau_{21} = \tau_{\rm p}(L_2 + L_1)$ were introduced.

The dependences of the WP duration on the distance traversed in the fiber for the forward and backward propagations (solid and dashed curves) calculated by these relations are presented in Fig. 3. We assumed that the WP coupled to the fiber input had the initial duration $\tau_0 = 10^{-12}$ s, the FM rate $\alpha_0 = 0$, and the two values of dispersion parameters $D_1' = (2; 0.5) \times 10^{-27}$ s^2 m^{-1}, $D_1'' = (-8; -6) \times 10^{-27}$ s^2 m^{-1}, and $D_2'(-2.2; -2) \times 10^{-26}$ s^2 m^{-1} (curves *1* and *2*, respectively). In the case of arbitrary section lengths $L_1 = (100; 150)$ m and $L_2 = (32.5; 20.3)$ m, the WP at the fiber output exhibits broadening, i.e. its duration at the output enhances that at the input (Fig. 3a). In the case of optimal lengths $L_1 = (121.3; 166.1)$ m and $L_2 = (11.0; 4.2)$ m, the output WP duration is considerably shorter than its initial duration (Fig. 3b). In this event, the minimal pulse time corresponds to the zero FM rate achieved in the second fiber of the cascade. In forward and backward propagations, the WP duration at the cascade output is the same, which demonstrates the manifestation of reciprocity in the linear compression mechanism under consideration (the backward propagation of the WP is equivalent to a change in the order of the sequence of sections in the fiber).

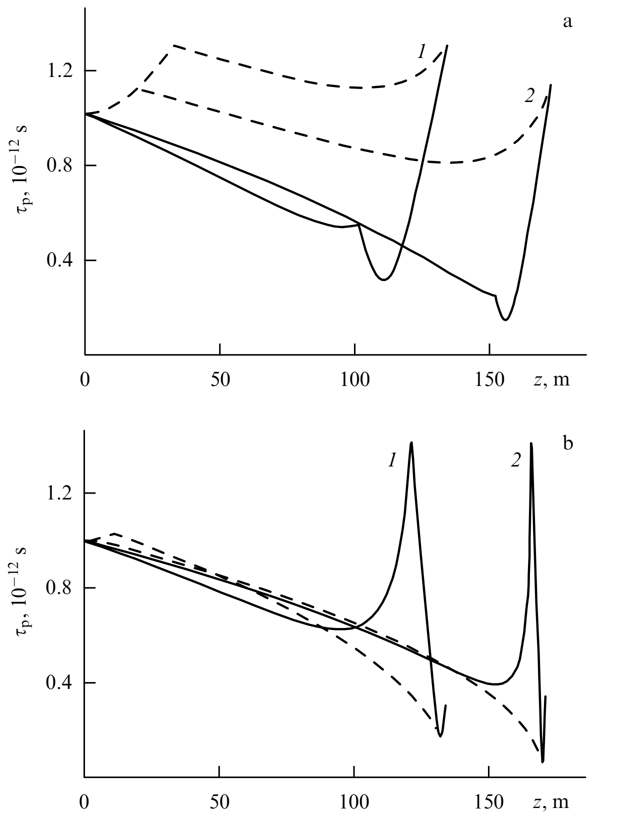

Figure 3. Dependences of the WP duration on the distance traversed in a fiber for arbitrary (a) and optimal (b) lengths of sections, two values of dispersion parameters (curves *1* and *2*), and forward (solid curves) and backward (dashed curves) wave propagations.

The WP dynamics in the cascade are illustrated by a change in the envelope shape during the forward (Fig. 4a) and backward (Fig. 4b) propagations of the wave packet in the cascade with optimal lengths of sections (these dependences were constructed using the values of parameters corresponding to curve *1* in Fig. 3b).

Analysis shows that WP compression is possible for arbitrary lengths of sections as well; however, it is noticeably weaker than that for optimal lengths of sections in the cascade. This is explained by the fact that, for an arbitrary

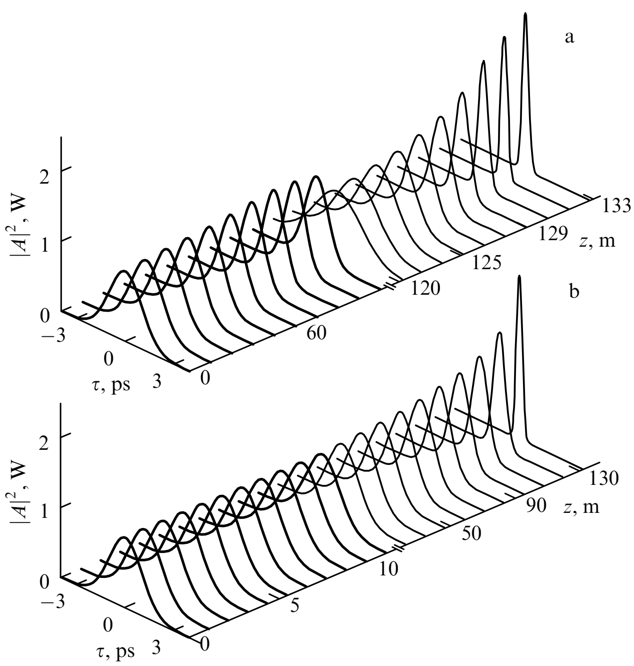

Figure 4. Changes in the shape of the WP envelope in a cascade with optimal lengths of sections for forward (a) and backward (b) propagations.

length L_1, it is impossible to produce the highest possible FM rate of the pulse immediately ahead of the input to the second fiber, where compression is performed. The length of a given fiber on which the pulse becomes transform-limited does not coincide with the length on which the pulse duration becomes minimal. The pulse is also compressed in the active fiber. However, the FM rate of the pulse at the point of its minimal duration is rather high, which is highly undesirable for various technical applications.

3.2 Multistage scheme

A two-stage cascade scheme cannot fully solve the problem of locking the WP carrier frequency in the spectral region corresponding to the negative values of the imaginary components of the second-order dispersion parameters. This problem can be most efficiently solved by designing multisectional fibers consisting of sections supplying radiation to an amplifier, the amplifier itself, and a compensating fiber located behind the amplifier. In so doing, the carrier frequency can be locked in the required frequency region by two radically different techniques.

The first technique is based on the employment of multisectional fibers containing three and more elements, for which the mean value of the parameter $k''(z)$ over the entire propagation path of the pulse is close to zero, while the mean value of the parameter $D''(z)$ is negative, namely

$$\langle k'' \rangle = \sum_{i=1}^{N} \frac{k_i'' L_i}{L} \approx 0 \,, \quad \langle D'' \rangle = \sum_{i=1}^{N} \frac{D_i'' L_i}{L} < 0 \,, \tag{33}$$

where N is the number of sections in the cascade fiber, and L is its total length.

Another technique for carrier-frequency locking is built around a fiber optic line containing a sequence of amplifying elements with resonance frequencies increasing on passing from one element to another ($\omega_{r1} < \omega_{r2} < \ldots < \omega_{rN}$), thereby providing the fulfillment of the condition $\langle D'' \rangle < 0$ over the entire fiber length. In this case, the condition that the WP carrier frequency not escape from the region $D_i'' < 0$ in each of the elements of the fiber can be represented by the inequality

$$|\omega_{si} - \Omega_{si} - \omega_{ri}| > \frac{\Delta\omega_i}{\sqrt{3}} \,, \quad \Omega_{si} = \frac{k_i'' L_i (1 + \alpha_0^2 \tau_0^4)}{\tau_0^2 + (1 + \alpha_0^2 \tau_0^4)\, D_i'' L_i} \,. \tag{34}$$

This model should operate well enough if the carrier-frequency shift in each of the fiber elements is small compared to the detuning of the carrier frequency in the given element from the induced-transition frequency, i.e., $|\omega_{si} - \omega_{ri}| \gg \Omega_{si}$.

Let us consider a cascade consisting of N homogeneous dispersive optical fibers, some of which are characterized by complex dispersion parameters, while the others have real dispersion parameters. We assume that conditions for carrier-frequency locking in the region $D_i'' < 0$ are fulfilled. In this event, the FM compensation condition (for generating a transform-limited WP) at the output of the Nth element of the cascade takes the form

$$\langle D' \rangle \, L = \sum_{i=1}^{N} D_i' L_i = \frac{\alpha_0 \tau_0^4}{1 + \alpha_0^2 \tau_0^4} \,. \tag{35}$$

The transform-limited WP duration at the cascade output is then determined by the expression

$$\tau_{\mathrm{p}} = \left(\langle D'' \rangle \, L + \frac{\tau_0^2}{1 + \alpha_0^2 \tau_0^4} \right)^{1/2} . \tag{36}$$

For $\alpha_0 \neq 0$ and $D_i'' < 0$, the WP duration at the cascade output can be much shorter than τ_0. Thus, for example, for a three-sectional cascade fiber ($N = 3$) and $\alpha_0 \neq 0$, the length of a compensator (the last element of the cascade), in which the compressed WP should be obtained at its output, is given by

$$L_3 = \frac{\alpha_0 \tau_0^4 (1 + \alpha_0^2 \tau_0^4)^{-1} - D_1' L_1 - D_2' L_2}{D_3'} > 0 \,, \tag{37}$$

where L_1 is the length, and D_i' is the real component of the material dispersion of the ith element. The minimal duration of the transform-limited WP at the fiber output, like that for a two-sectional cascade, is determined by the amplifier parameters D_2'', L_2, and k_2'' and the initial FM rate α_0:

$$\tau_{\min}(L_3) = \frac{1}{\Delta\omega_{\mathrm{s}}(L_3)} \approx \left[\frac{1 + \alpha_0^2 \tau_0^4}{\tau_0^2 + D_2'' L_2 (1 + \alpha_0^2 \tau_0^4)} \right]^{-1/2} . \tag{38}$$

Thus, the presence of complex dispersion parameters in cascade optical fibers containing active and passive sections can cause the compression of a WP without the initial FM of low-power input radiation. In this case, the cascade technique allows one to almost completely eliminate negative factors related to the WP carrier-frequency shift and the influence of nonlinear effects accompanying compression in one-sectional amplifying fibers. For active fibers in the cascade compression scheme in the region of the carrier-frequency detuning from the resonance gain line, it is possible to obtain the superluminal propagation regime for the WP envelope maximum in the presence of FM and positive values of the imaginary part of the GVD.

4. Wave-packet dynamics in a nonlinear optical fiber

Nonlinear single-mode optical fibers that are inhomogeneous in length make up efficient tools for controling the parameters of short WPs. Modern technologies make it possible to manufacture optical fibers with the specified type of longitudinal inhomogeneity of a corresponding parameter. Thus, the required GVD profile is formed by changing the fiber diameter or the difference in the refractive indices of the fiber core and cladding [21–23]. The character of longitudinal distribution of material parameters noticeably affects the WP dynamics in a nonlinear fiber [70–80]. For example, in the case of the considerable influence of nonlinear effects, the WP carrier-frequency shift can considerably complicate the realization of compression regimes in active fibers. Moreover, nonlinearity inevitably leads to a strong instability of WPs with the superluminal velocity of the envelope's maximum propagation.

4.1 Solution of a nonlinear equation for the envelope

Let us consider the nonlinear WP dynamics in an amplifying fiber with material parameters depending on the longitudinal coordinate z. The dynamics of WPs of duration $\tau_0 \gg 1$ ns does not virtually depend on the second- and higher-order dispersion effects. Therefore, the WP envelope in a fiber with the Kerr nonlinearity is described by the equation

$$\frac{\partial B}{\partial z} + k(z)\,\frac{\partial B}{\partial t} + \mathrm{i}R(z)|B|^2\,B = -\beta''(z)\,B\,, \tag{39}$$

where R is the Kerr nonlinearity parameter. This equation can be applied in analyzing the radiation dynamics in optical fibers with the inhomogeneous longitudinal distribution of material parameters slowly varying in length. By making the substitution

$$B(z,t) = A(z,t)\,\exp\left(-\int_0^z \beta''(\xi)\,\mathrm{d}\xi\right) \tag{40}$$

into equation (39), we arrive at the equation for the slowly varying amplitude $A(z,t)$:

$$\frac{\partial A}{\partial z} + k(z)\,\frac{\partial A}{\partial t} + \mathrm{i}R_{\mathrm{ef}}(z)|A|^2\,A = 0\,, \tag{41}$$

where the effective nonlinearity parameter is defined in the following way:

$$R_{\mathrm{ef}}(z) = R(z)\,\exp\left(-2\int_0^z \beta''(\xi)\,\mathrm{d}\xi\right). \tag{42}$$

The FM envelope of a Gaussian WP at the fiber input can be described by the expression

$$A(\tau,0) = A_0\exp\left(-\frac{\tau^2}{2\tau_0^2} + \mathrm{i}\,\frac{\alpha_0\tau^2}{2} + \mathrm{i}\varphi_0\right), \tag{43}$$

with the duration constant along the fiber, where $\tau = t - \int_0^z k(\xi)\,\mathrm{d}\xi$ is the running time, and A_0 and φ_0 are the amplitude and phase of the WP envelope at the fiber input. Taking into account that $k(z)$ is a complex quantity, we can write out the expression for the radiation intensity

$$\left|A(\tau,z)\right|^2 = A_0^2\exp\left(-\frac{\tau_{\mathrm{m}}^2}{2\tau_0^2} + \left(\tau_0^{-2} + \alpha^2(z)\,\tau_0^2\right)\times\left(\int_0^z \frac{\partial\beta''(\xi)}{\partial\omega}\,\mathrm{d}\xi\right)^2\right), \tag{44}$$

where the time

$$\tau_{\mathrm{m}}(z) = t - \left(\int_0^z \frac{\partial\beta'(\xi)}{\partial\omega}\,\mathrm{d}\xi + \alpha(z)\,\tau_0^2\int_0^z \frac{\partial\beta''(\xi)}{\partial\omega}\,\mathrm{d}\xi\right) \tag{45}$$

related to the WP envelope maximum was introduced. Then, the instantaneous velocity of the envelope maximum for the FM WP is determined by relation (19), while the delay time of a signal that travelled a distance L is $\Delta\tau_{\mathrm{m}}(L) = t - \tau_{\mathrm{m}}(L)$. Taking these relations into account, we can find the mean velocity of the WP envelope maximum on the fiber length L:

$$\langle u_{\mathrm{m}}\rangle = \frac{L}{\Delta\tau_{\mathrm{m}}(L)} = L\left(\int_0^L \frac{\partial\beta'(z)}{\partial\omega}\,\mathrm{d}z + \alpha(L)\,\tau_0^2\int_0^L \frac{\partial\beta''(z)}{\partial\omega}\,\mathrm{d}z\right)^{-1}. \tag{46}$$

If the parameter β' is assumed constant over the fiber length, expression (46) takes the form

$$\langle u_{\mathrm{m}}\rangle = \left(\frac{1}{u} + \frac{\alpha(L)\,\tau_0^2}{L}\int_0^L \frac{\partial\beta''(z)}{\partial\omega}\,\mathrm{d}z\right)^{-1}, \tag{47}$$

where $u = (\partial\beta'/\partial\omega)_0^{-1}$ is the group velocity of the WP. In the case important in practice, when the carrier-frequency detuning from the resonance frequency of the gain line is relatively small, $|\delta\omega_{\mathrm{s}}(0)| \ll \Delta\omega$, expression (47) can be rewritten in the form

$$\langle u_{\mathrm{m}}\rangle = \left(\frac{1}{u} + \frac{\alpha(L)\,\tau_0^2\rho}{\Delta\omega^2 L}\int_0^L N(z)\,\delta\omega_{\mathrm{s}}(z)\,\mathrm{d}z\right)^{-1}, \tag{48}$$

where the parameter N depends in general on the coordinate. In particular, the pump degeneration effect can lead to a decrease in $N(z)$ with increasing z. On the other hand, tapping distributed fiber amplifiers, it is possible to create active profiles with N increasing with length.

It follows from the relations presented above that the WP dynamics as a whole and the envelope maximum velocity, in particular, considerably depend on the value and sign of the FM rate. To find the dependence of the parameter α on the distance traversed by the WP, we will utilize the equation obtained by the authors in Ref. [74]. For a WP propagating in an amplifying nonlinear medium, by neglecting second-order dispersion effects, we have

$$\frac{\partial\alpha}{\partial z} = \frac{R_{\mathrm{ef}}(z)\,I_0}{\sqrt{2}\tau_0^2}\,, \tag{49}$$

where $I_0 = |A_0|^2$ is the intensity of radiation coupled to the fiber. If the distribution of the effective nonlinearity $R_{\mathrm{ef}}(z)$ along the fiber length is known, the function $\alpha(z)$ can be found from relation (49). By substituting this function into formula (48), we can find the velocity of the WP envelope maximum at any point of the fiber. Notice here that the superluminal velocity of the WP envelope maximum can be obtained even in the case of zero initial FM, because the FM appears during the WP propagation in the fiber due to the Kerr nonlinearity.

4.2 Carrier-frequency shift and a change in the width of the spectrum

As follows from relation (27), the presence of the nonzero imaginary component of the group velocity almost inevitably leads to reshaping the WP and its carrier-frequency shift. The dynamics of carrier-frequency shift in general (including the Lorentzian approximation of the gain line) can be simulated only numerically. In some important particular cases, however, the carrier-frequency shift can be obtained in the analytical form.

If the WP width weakly changes during light propagation (which is the case when the influence of nonlinear effects is insignificant), then the carrier-frequency shift with respect to the maximum of the Lorentzian gain line can be found from the solution of integral equation (17). We will represent the carrier-frequency shift in this equation in the form

$$\Omega_s(z) = \omega_s(z) - \omega_r - \omega_s(0) + \omega_r \tag{50}$$

and introduce the current detuning $\delta\omega_s(z) = \omega_s(z) - \omega_r$ from the resonance frequency. By substituting $\Omega_s(z)$ into Eqn (17), we obtain

$$\delta\omega_s(z) - \delta\omega_s(0) = -\Delta\omega_s^2(z) \int_0^z \rho N(\xi) \frac{\delta\omega_s(\xi)\,\Delta\omega^2}{\left(\Delta\omega^2 + \delta\omega_s^2(\xi)\right)^2}\,d\xi\,. \tag{51}$$

Differentiating this equation with respect to coordinate z yields

$$\frac{\left(\Delta\omega^2 + \delta\omega_s^2(z)\right)^2}{\delta\omega_s(z)} \frac{\partial}{\partial z}\,\delta\omega_s(z) = -\rho N(z)\,\Delta\omega_s^2(z)\,\Delta\omega^2\,. \tag{52}$$

If the WP width remains constant during propagation, i.e., $\Delta\omega_s(z) = \text{const}$ (this is the case for input FM pulses propagating in a medium with a weak cubic nonlinearity), we obtain the functional dependence

$$G(z) - G(0) = -\Delta\omega_s^2 \int_0^z \rho N(z)\,dz\,, \tag{53}$$

determining the dynamics of the detuning $\delta\omega_s(z)$, where

$$G(z) = \frac{\delta\omega_s^4(z)}{4\Delta\omega^2} + \delta\omega_s^2(z) + \Delta\omega^2 \ln\frac{\delta\omega_s(z)}{\delta\omega_s(0)}\,.$$

In the approximation of small detuning of the carrier frequency from the resonance gain line, we have a simple solution to equation (53) describing the carrier-frequency 'pulling' to the region of the gain line maximum:

$$\delta\omega_s(z) = \delta\omega_s(0) \exp\left(-\frac{\Delta\omega_s^2}{\Delta\omega^2} \int_0^z \rho N(z)\,dz\right). \tag{54}$$

In the approximation of narrow WPs with respect to the gain line width, when the carrier-frequency detuning from the resonance frequency can be considered quite accurately constant, i.e., $\delta\omega_s(z) \approx \delta\omega_s(0)$, the velocity of the WP envelope maximum is described by the expression

$$u_m(z) \approx \left(\frac{1}{u(z)} + \frac{\alpha_0 \tau_0^2 \rho N(z)\,\delta\omega_s(0)}{\Delta\omega^2}\right)^{-1}. \tag{55}$$

In the narrowband spectrum and inexhaustible pump approximations, the velocity of the envelope maximum can be assumed constant with a good accuracy. In this case, it follows from relations (50)–(55) that there is a need to use rather long FM pulses (much longer than 1 μs) for observing superluminal waves.

In general, not only the carrier-frequency shift but also a change in the width of the spectrum essentially contribute (and determine to a large extent) to the dynamics of a chirped WP in an active fiber in circumstances where strong nonlinear effects show their worth. For an arbitrarily changing WP linewidth, the analytic solution of equation (51) for the carrier-frequency shift with respect to the resonance frequency (for a small detuning, when $|\delta\omega_s(z)| \ll \Delta\omega$) can be written out in the form

$$|\delta\omega_s(z)| \approx |\omega_0 - \omega_r|\,\kappa(z)\left[1 - 2\int_0^z \kappa^{-1}(\xi)\left(\frac{\partial}{\partial\xi} \ln\frac{\Delta\omega_s(\xi)}{\Delta\omega_s(0)}\right) d\xi\right], \tag{56}$$

where

$$\kappa = \frac{F(z)}{F(0)} \exp\left[-\int_0^z F(\xi)\,d\xi\right], \quad F(z) = \frac{\gamma(\omega_r)\,\Delta\omega_s^2(z)}{\Delta\omega^2}\,,$$

and $\gamma(\omega_r) = \rho N$ is the gain increment at the resonance gain line frequency. In this case, we can assume quite accurately that $F(z)/F(0) \approx \Delta\omega_s^2(z)/\Delta\omega_s^2(0)$ and the current spectral width of the WP is described by the expression

$$\begin{aligned} \Delta\omega_s(z) &\approx \tau_0^{-1}\sqrt{1 + S^2(z)}\,, \\ S(z) &= S_0 + \frac{R I_0}{\sqrt{2}\rho N}\left[\exp(\rho N z) - 1\right], \end{aligned} \tag{57}$$

where $S_0 = \alpha_0 \tau_0^2$. Here, the mean velocity of the envelope maximum on a fiber segment of length L is found from the expression

$$\langle u_m \rangle = \left(\frac{1}{u(z)} + \frac{\alpha(L)\,\tau_p^2(L)\,\rho}{\Delta\omega^2 L} \int_0^L N(z)\,\delta\omega_s(z)\,dz\right)^{-1}. \tag{58}$$

One can see that, for $\alpha(L) < 0$, the superluminal velocity of the envelope maximum for an FM WP can be reached. Because the group velocity is complex, the WP carrier frequency can 'leave' the frequency range corresponding to the superluminal velocity of the envelope maximum. By forming inhomogeneous distributions of fiber parameters, it is possible to maintain the WP envelope maximum in the frequency range corresponding to the superluminal velocity on a certain length of the fiber.

4.3 Velocity and acceleration of the wave-packet envelope maximum

Our analysis showed that the velocity of the WP envelope maximum considerably depends on the initial WP duration and the initial FM rate. This dependence and related features of the dynamics of Gaussian FM WPs open up, under certain conditions, possibilities for achieving the regime of the superluminal propagation of the WP envelope maximum over long distances. It seems that this regime can be most efficiently achieved by using long enough ($\tau_0 > 10^{-3}$ s) and strongly modulated WPs, for which the inequalities $|\alpha_0|\,\tau_0^2 \gg 1$ and $|D''|\,(\tau_0^{-2} + \alpha_0^2\tau_0^2)\,z \ll 1$ are simultaneously valid over distances $z \gg 1$ m. In this case, $S \approx -\alpha_0\tau_0^2$ and, according to formula (19), $u_m \approx u/(1 - \alpha_0\tau_0^2 k'' u)$, where the quantity k'', taking into account the Lorentzian shape of the gain band, is determined by expression (16).

We will illustrate the possibility of achieving the superluminal propagation of the envelope maximum by a numer-

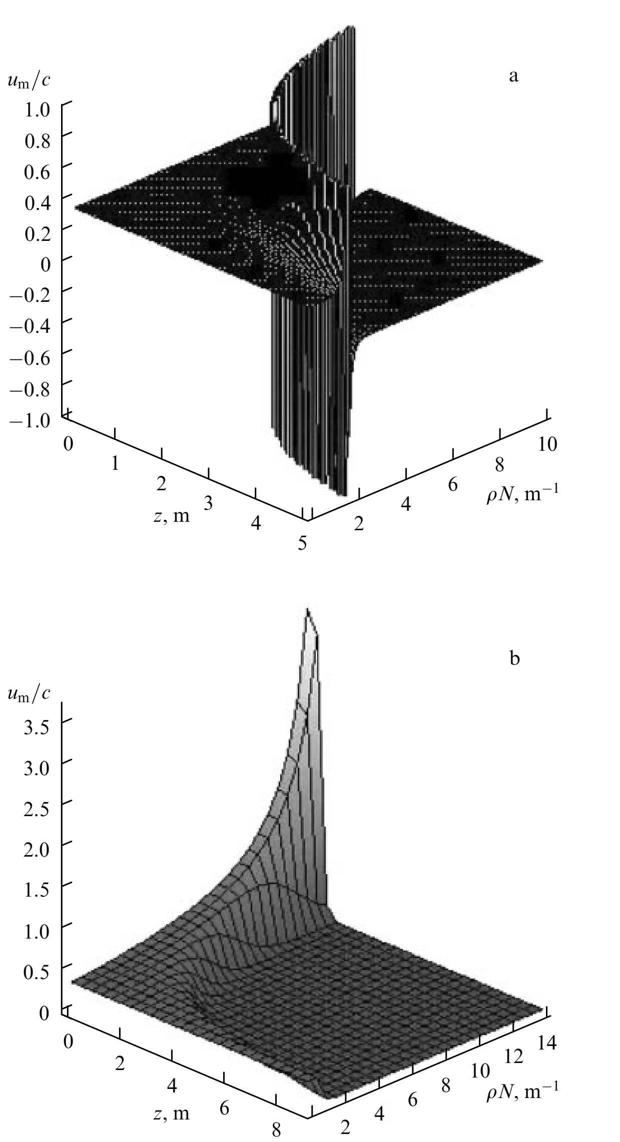

Figure 5. Dependences of the velocity of the WP envelope maximum on the traversed distance and parameter ρN for $\alpha_0 = R_0 = 0$, $(\partial\beta''/\partial\omega) < 0$ (a) and $\alpha_0\tau_0^2 = -2\times10^3$, $RI_0 = 2$ m^{-1} and $(\partial\beta''/\partial\omega) > 0$ (b).

ical example using the parameters typical for erbium-doped fiber amplifiers, which have recently found wide applications [25, 27]. Thus, for a WP with parameters $\tau_0 \approx 10^{-2}$ s, $\alpha_0 \approx 10^8$ s^{-2} in a medium with $|k''| \approx 10^{-12}$ s m^{-1} and $|D_1| \approx |D_2| \approx 10^{-26}$ s^2 m^{-1}, we obtain $u_m > c$. Under these conditions, the carrier-frequency shift is $|\Omega_s| \leqslant 10^3$ s^{-1} for $z \leqslant 10^3$ m. Taking into account that the gain linewidth is typically $\Delta\omega \approx 10^{11} - 10^{14}$ s^{-1}, the carrier-frequency shift by $\approx 10^3$ s^{-1} can be considered extremely small and not affecting the superluminal WP dynamics. It should be noted, however, that in the region of parameters satisfying the relation $\alpha_0\tau_0^2\,\delta\omega_s(z) \approx \Delta\omega^2/\rho N u$, the velocity of the envelope maximum can take arbitrarily large values. Figure 5 shows the dependence of the velocity of the WP envelope maximum on the traversed distance and parameter ρN, obtained for the parameters $\alpha_0 = 0$, $R_0 = 0$, and $(\partial\beta''/\partial\omega) < 0$ (Fig. 5a) and also $\alpha_0\tau_0^2 = -2\times10^3$, $RI_0 = 2$ m^{-1}, and $(\partial\beta''/\partial\omega) > 0$ (Fig. 5b). It is evident that the more accurately the above relation is fulfilled, the greater the velocity of the WP envelope maximum. It is no accident that the velocity of the WP envelope maximum observed in one of the most well-known experimental papers [34] in this field was $u_m \approx 310c$.

The superluminal velocity of the envelope maximum can also be reached with the aid of strongly modulated narrowband WPs, which are produced with the help of gas lasers by introducing light beams into a narrowband amplifying medium (presumably an active gas medium). In doing so, for a long pulse of a duration considerably exceeding 1 ms, a WP with the superluminal velocity of the envelope maximum can be observed over lengths on the order of 1 m.

This result does not contradict the theory of relativity because photons themselves propagate at the speed of light in a corresponding medium. But due to amplification in the medium, the concentration of photons at the leading edge of a WP proves to be much higher than that at the trailing edge. Thus, the WP envelope undergoes deformation, and its maximum begins to propagate at the superluminal velocity.

Notice that, to analyze in more detail the propagation of a Gaussian WP over relatively large distances, it is necessary to separate its leading part (precursor) against the fluctuation (noise) background inevitably appearing in the active medium. Account of fluctuations leads to the deformation of the WP envelope and, therefore, the distortion of information carried by it. The deformation for $k'' \neq 0$ is caused by the difference between velocities of the envelope maximum (amplitude center) u_m and the phase center $u_f = u/(1 - uk''/S)$ of the WP [31] and also by phase distortions related to a change in the group velocity within the WP frequency range.

Of interest is also the possibility of phase conjugation, when $u_m < 0$, which can be caused by the effect of strong amplification and dispersion. As a result, the WP envelope maximum is formed at the WP onset and is shifted to the side opposite to the WP propagation direction. This effect was observed experimentally (see, for example, Refs [38, 81, 86]).

A rapid increase or decrease in the velocity of the WP envelope maximum propagating in an active fiber under nonlinear dynamics conditions should inevitably lead to a strong acceleration as well. It follows from the relations obtained above that the coordinate dependences of the FM rate, the concentration of active particles, the carrier-frequency detuning, and the pulse duration in the region of u_m values close to superluminal velocities can lead to huge accelerations of the WP envelope maximum:

$$a_m = \frac{du_m}{dt} = u_m\,\frac{du_m}{dz}\,. \tag{59}$$

In this case, we can assume that the cross-sectional area ρ of active centers and the gain linewidth $\Delta\omega$ are independent of the coordinate z. Very high accelerations (above 10^{20} m s^{-2}) should take place in the region of parameters satisfying the condition

$$\frac{1}{u(z)} + \frac{\alpha(z)\,\tau_p^2(z)\,\rho}{\Delta\omega^2}\,N(z)\,\delta\omega_s(z) \to 0\,, \tag{60}$$

when, according to formula (55), the velocity $u_m(z) \to \infty$.

It follows from expression (59) that the high accelerations of the envelope maximum are related to the inhomogeneity of the velocity u_m over the coordinate, which can be specified in practice by the z-dependence of the group velocity $u(z)$ and the number $N(z)$ of active centers. The inhomogeneity of the group velocity and concentration of active centers can easily be obtained by the inhomogeneous doping of a fiber amplifier. In this case, the acceleration of the envelope maximum will be described by the expression

$$a_{\mathrm{m}} = u_{\mathrm{m}} \frac{\mathrm{d}u_{\mathrm{m}}}{\mathrm{d}z} \approx u_{\mathrm{m}}^2 \left(\frac{\partial(\ln u)}{\partial z} - \frac{\kappa}{1+\kappa N u} \frac{\partial(Nu)}{\partial z} \right), \qquad (61)$$

where the parameter $\kappa = \alpha_0 \tau_0^2 \rho\, \delta\omega_{\mathrm{s}}(0)/\Delta\omega^2$ is a constant.

It should be noted that the 'superluminal' WPs considered above are, in principle, unstable. It is known that the superluminal propagation of an electromagnetic energy bunch in any medium should produce Vavilov–Cherenkov emission [29, 45], which leads to bunch energy losses and limits the bunch lifetime. Accompanying radiation inevitably causes the deformation and instability of superluminal WPs. Meanwhile, it is specific superluminal optical objects that can be manifested as genuine physical objects which can be called optical tachyons.

Our above-made analysis shows that the velocity of the WP envelope maximum strongly depends on the initial WP duration and the initial FM rate. This dependence and the related features of the dynamics of Gaussian FM WPs open up, under certain conditions, the possibility of anticipating information (with a certain statistical significance) about some observed event. For example, the envelope maximum of the 'second' pulse can arrive from a transmitter at a detector before the 'first' pulse sent earlier with a somewhat different initial chirp (or without it entirely). When the superluminal propagation of the WP envelope maximum takes place, the self-reproduction of a partially transmitted pulse occurs, which is caused by the assumed analyticity of the transmitted signal [31, 33, 35].

5. Conclusions

We have considered the dynamics of an FM pulse in an active (amplifying) medium with the carrier frequency detuned from the gain band maximum. It has been shown that low-power incident transform-limited pulses (not producing noticeable self-phase modulation) can be temporarily compressed in such optical fibers. In this case, under conditions of carrier-frequency detuning from the gain line maximum, the WP carrier frequency inevitably shifts, which negatively affects the temporal compression mechanism under study. In one-sectional optical fibers, the carrier frequency can be maintained by using active media with a complicated gain-line profile having a local extremum in the frequency region with $D'' < 0$. In cascade optical fibers containing active and passive sections, the availability of complex dispersion parameters can produce a strong compression of an optical WP without the initial FM of the coupled low-power radiation. The use of the cascade technique allows one to eliminate almost completely negative factors related to the carrier-frequency shift, accompanying WP compression in one-sectional active optical fibers.

The presence of a first-order dispersive parameter with imaginary components leads, along with the negative factor of carrier-frequency shift, to the fundamental possibility of propagation of the FM WP envelope maximum in a linear amplifying medium at a velocity considerably different from the WP group velocity. Moreover, the velocity of the WP envelope maximum can exceed the speed of light in vacuum.

We have considered within the framework of the first-order dispersion approximation the influence of complex material parameters on the WP dynamics in a length-inhomogeneous optical fiber with Kerr nonlinearity. Our analysis has shown that this model allows the possibility of the superluminal regime of the propagation of the envelope maximum for a Gaussian pulse even in the absence of the initial FM. The achievement of high velocities (in particular, exceeding the speed of light in vacuum) at finite distances can lead to huge accelerations of the envelope maximum of the corresponding WP. In this case, the wave structures under study are fundamentally unstable [33]. Thus, the superluminal movement of an electromagnetic energy bunch in any medium should produce Vavilov–Cherenkov radiation, resulting in bunch energy losses, thereby additionally restricting the bunch lifetime. The presence of specific superluminal radiations gives evidence that the superluminal waves considered in the paper are not caused by kinematic effects but, as pointed out in Refs [29, 45, 82–85], can manifest themselves as real physical objects which can be called tachyons.

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