(2400 Hz) pulsation frequencies of SGR 1806-20 allow one to estimate the capacitance of magnetic loops carrying the current. As a result, we obtain $C_1 \approx 1.5 \times 10^{-2}$ F and $C_2 \approx 8 \times 10^{-7}$ F, whereas the magnitudes of Q factors for minimum and maximum frequencies are $Q_1 \approx 3 \times 10^5$ and $Q_2 \approx 10^7$. Notice that the magnitude of the magnetic field found in this way, $B \approx 10^{13}$ G, is less than the quantum-electrodynamical threshold $B_{\rm QED} = 4.4 \times 10^{13}$ G at which the nonrelativistic Landau energy $\hbar eB/m_{\rm e}c$ is comparable to the electron rest mass $m_{\rm e}c^2$.

8. Conclusion

Natural manifestations of solar and stellar activity — oscillations and waves modulating the emission of the Sun and stars — contain information on coronal parameters, and often it is unique. As a consequence, coronal seismology offers an effective way of diagnosing stellar coronae. The variety of oscillatory and wave processes in solar and stellar coronae is not limited to the cases considered above. In this report, we did not touch on the seismologies of prominences and sunspots, which present separate branches of helioseismology. Further development in methods of coronal seismology is simulated by novel multiwavelength observations of the activity of the Sun and stars. Recent reviews of advances in coronal seismology [29, 38] need to be complemented even now. For instance, fresh SDO observations [39] have revealed manifestations of Kelvin–Helmholtz instability at the boundary of coronal plasma ejection.

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Superfluid neutron stars

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1. Cooling of neutron stars and properties of superdense matter

This talk summarizes the recent interpretation of observations (carried out in 2000–2010 with the NASA's Chandra X-ray orbital observatory) of the young (about 330-year old) neutron star in the Cassiopeia A supernova remnant. The data indicate that the neutron star has a carbon atmosphere and remains warm but shows noticeable cooling, so that its surface temperature has decreased by about 4% in the 10 years of observations. These are the first observations of an isolated neutron star cooling in real time. It is difficult to explain them using the cooling theory for nonsuperfluid neutron stars, but they are naturally explained if the superdense core of the star possesses a strong superfluidity of protons (with a critical temperature higher than 3×10^9 K) and a moderately strong superfluidity of neutrons (with the maximum critical temperature of order $(5-9) \times 10^8$ K over the stellar core). If the observations are correct, these data

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give serious evidence of the presence of superfluidity in neutron star cores.

Typical masses of neutron stars are $M \sim 1.4\,M_\odot$ ($M_\odot$ is the mass of the Sun), and their radii are typically $R \sim 10-14$ km. Therefore, neutron stars are compact and contain superdense matter (see, e.g., Refs [1–3]). The mean density of the matter is a few times ρ_0, and the central density can exceed $(10-15)\,\rho_0$, where $\rho_0 = 2.8 \times 10^{14}$ g cm^{-3} is the saturation density of matter in atomic nuclei. Because of this high density, space–time around neutron stars is noticeably curved, and the notions of mass and radius become ambiguous (e.g., the baryon mass differs from the gravitational one [2]). In what follows, M means the gravitational mass, and R is the circumferential radius of the star. According to current theories, a neutron star has a relatively thin crust ($\lesssim 1$ km in thickness, and $\lesssim 0.01\,M_\odot$ in mass) and a bulky core, which extends from the density of $\rho \approx 0.5\,\rho_0$ to the stellar center and contains superdense nuclear matter. The stellar core is often divided into the outer core [$\rho \lesssim (2-3)\,\rho_0$], and the denser inner core. The outer core consists of close-packed neutrons, with an admixture of protons, electrons and muons, while the inner core may contain other particles, particularly, hyperons and free quarks. All fermions in the core are strongly degenerate; their typical Fermi energies run to a few hundred MeV. The properties of superdense matter are determined by strong interactions of nucleons and other particles (e.g., hyperons). The reliable theory of superdense matter is still absent because of difficulties in describing strong interactions and many-body effects in matter of supranuclear density; the experimental study of such matter in the laboratory is complicated.

One important aspect of the problem consists in superfluidity of superdense matter (see, e.g., Refs [2, 4, 5]). Nucleons, as well as hyperons and quarks, may form Cooper pairs and become superfluid under the action of the attractive component of strong interaction. In the inner crust of a neutron star, where free neutrons appear [1], these neutrons can undergo Cooper pairing in the spin-singlet state. However, near the core–crust interface the singlet-state attraction between neutrons is replaced by repulsion, and this superfluidity disappears. Nevertheless, triplet-state interaction may become attractive instead, so that the neutrons in the stellar core may be superfluid due to triplet-state Cooper pairing. Because the fraction of protons in the neutron star core is relatively small, protons can be superfluid due to singlet-state pairing. Since protons are electrically charged, their superfluidity also means superconductivity. Other strongly interacting particles (hyperons, quarks) can also be superfluid. At densities ρ much higher than ρ_0, the attractive component of interaction between any particles is reduced and any superfluidity disappears.

Microscopically, superfluidity creates a gap in the energy spectrum of particles near the Fermi level. In the case of singlet-state pairing, the gap is isotropic, while for triplet-state pairing it is anisotropic. The gap appears when the temperature falls below some critical temperature T_c; the gap grows with decreasing T and reaches a maximum at $T = 0$. Any superfluidity is specified by the particle pairing type and the critical temperature $T_c(\rho)$ which depends on density ρ. As mentioned above, in the nucleon stellar core one usually considers singlet-state superfluidity of protons and triplet-state superfluidity of neutrons, with the critical temperatures $T_{cp}(\rho)$ and $T_{cn}(\rho)$, respectively. Calculations of the critical temperatures strongly depend on the employed models of nuclear interaction and on methods to account for many-body effects. The theory predicts $T_{cn}, T_{cp} \lesssim 10^{10}$ K (with gaps $\lesssim 1$ MeV). Such superfluidity does not affect the equation of state of matter and neutron star structure but greatly modifies [6] the heat capacities, neutrino processes, kinetics, and hydrodynamics of superdense matter.

The nature of superdense matter remains an important unsolved problem of physics and astrophysics. It is being solved by comparing observations and theory of neutron stars in different ways (as described, e.g., in book [2]). Great progress has been attained in measuring the masses of neutron stars (radio pulsars) in compact binary systems. The recent reliable discovery [7] of the very massive radio pulsar J1614-2230, with a mass of $1.97 \pm 0.04\,M_\odot$, made unrealistic all theoretical models of soft and moderately stiff equations of state of neutron star matter. Such models had predicted that matter is rather soft, so that the maximum mass of stars is lower than the mass of the radio pulsar J1614-2230. Only stiff equations of state, which allow neutron stars to have such a mass, remained realistic. These results are in favor of nucleon models of superdense matter (see, e.g., Ref. [8]); they make models of matter containing other strongly interacting particles (hyperons or quarks) less probable, but do not fully reject these models. The unambigious solution to the problem of superdense matter should be obtained in the future.

Below, we discuss another method to explore superdense matter — by studying cooling of isolated neutron stars (see, e.g., papers [9–13] and references cited therein). Neutron stars are born hot (with an internal temperature of order 10^{11} K) in supernova explosions, but then gradually cool down. In about half a minute after its birth the star becomes fully transparent to neutrinos and cools down via powerful neutrino emission from the hot core. For about 10–200 years, the star is nonisothermal inside, but later it becomes isothermal; a strong temperature gradient remains only in a thin heat blanketing envelope near the surface. In $10^5 - 10^6$ years, the neutrino emission weakens, and the star starts to cool mainly via thermal emission of photons from the surface. The theory gives cooling curves $T_s^\infty(t)$ — the dependence of the effective surface temperature of the star on its age t; the superscript ∞ means that the temperature is redshifted (for a distant observer). The star's cooling depends on the properties of superdense matter in the stellar core. Comparing theoretical curves $T_s^\infty(t)$ with measured surface temperatures of neutron stars of known ages allows one to explore superdense matter. The results have been reviewed in Refs [10–13].

The main cooling regulator of neutron stars with age $t \lesssim 10^5 - 10^6$ years is neutrino emission. For certainty, we restrict ourselves to neutron star models with nucleon cores. References to publications on cooling neutron stars with a more complicated nuclear composition can be found in review papers cited above. The main neutrino processes in stars with nucleon cores are listed in Table 1, where n denotes neutron, p proton, N nucleon (n or p), l electron or muon, and ñ quasineutron (annihilating into a neutrino pair in the presence of superfluidity). Table 1 also presents order-of-magnitude estimates of neutrino luminosity L_ν due to these processes in a neutron star with internal temperature T; the notation $T_8 = T/10^8$ K was used. The three upper estimates are made neglecting superfluidity; the last estimate stands for a star with superfluid neutrons after the splash of neutrino emission which accompanies the onset of superfluidity (see Section 3).

Table 1. Neutrino luminosities L_ν for main reactions in nucleon cores of neutron stars.

Mechanism	Reaction	L_ν, erg s^{-1}
Direct Urca process	$\mathrm{n} \to \mathrm{p l} \bar{\nu}_\mathrm{l}$, $\mathrm{p l} \to \mathrm{n} \nu_\mathrm{l}$	$\sim 10^{40} T_8^6$
Modified Urca process	$\mathrm{nN} \to \mathrm{pNl} \bar{\nu}_\mathrm{l}$, $\mathrm{pNl} \to \mathrm{nN} \nu_\mathrm{l}$	$\sim 10^{32} T_8^8$
Nucleon collisions	$\mathrm{NN} \to \mathrm{NN} \nu \bar{\nu}$	$\sim (10^{30} - 10^{31})\, T_8^8$
Cooper pairing of neutrons	$\tilde{\mathrm{n}}\tilde{\mathrm{n}} \to \nu \bar{\nu}$	$\lesssim (10^{33} - 10^{34})\, T_8^8$

The standard (basic) neutrino mechanism involves the modified Urca process (the chain of two reactions listed in Table 1). In 10^5 years, it cools the star down to the temperature $T_\mathrm{s}^\infty \sim 6 \times 10^5$ K. There are also weaker neutrino processes of neutrino-pair bremsstrahlung in nucleon collisions; they can be of three types — nn, np, and pp. In the inner core of a massive star with a nucleon core that has some specific equations of state, the much more powerful direct Urca process can operate [14] (a sequence of two reactions in Table 1), leading to fast cooling (to $T_\mathrm{s}^\infty \sim 10^5$ K in 10^5 years). If such a process is allowed, massive stars cool down very quickly.

Superfluidity of nucleons strongly affects neutrino emission and heat capacity of the neutron star core, and hence its cooling (see Section 3). The effect of superfluidity has been taken into account for interpretation of the data for a long time but, because the critical temperatures $T_\mathrm{cn}(\rho)$ and $T_\mathrm{cp}(\rho)$ are actually unknown, the observations can be explained in many ways (see, e.g., Refs [10, 13]).

2. Neutron star in Cassiopeia A

Interest in cooling neutron stars noticeably grew in 2009–2010, after the publication of new results of X-ray data processing for observations of the cooling neutron star in the Cassiopeia A supernova remnant. The remnant is the brightest radio source in the Galaxy but weak in optics due to strong interstellar absorption. Its distance is estimated as $3.1^{+0.3}_{-0.1}$ kpc [15], and its age is reliably determined [16] as 330 ± 20 years from observations of the supernova remnant expansion. A compact stellar object (a neutron star or a black hole) created in the supernova explosion had been sought for a long time. However, the collapsar — the neutron star — was discovered in X-rays [17] only after the launch of the Chandra X-ray Orbital Observatory. The object has been observed for a long time [18–20]. The observed spectrum has been fitted by models of the black-body radiation, as well as by hydrogen and iron atmosphere models, but the inferred radius of the emission region has always been small (less than 5 km). These results implied emission from a hot spot on the stellar surface, but the radiation was not pulsating, whereas neutron stars usually spin rapidly producing noticeable pulsations.

The situation changed in 2009, when Ho and Heinke [21] published the results of their own interpretation of the same data with a carbon atmosphere model of the neutron star. They obtained the radius R of radiation region ranging $10-18$ km, compatible with the expected neutron star radius. The inferred stellar mass $M = (1.5-2.4)\, M_\odot$ fell also in the range of expected neutron star masses. According to their interpretation, the magnetic field in the neutron star atmosphere, $B \lesssim 10^{11}$ G, is insufficiently strong to produce noticeable pulsations of the radiation of the spinning star. The inferred surface temperature was $T_\mathrm{s}^\infty \sim 1.5 \times 10^6$ K, in good agreement [22] with the standard cooling of a (nonsuperfluid) neutron star via neutrino emission due to the modified Urca process. This is the youngest neutron star whose surface thermal emission has been detected. Unfortunately, its temperature T_s^∞ was in good agreement with the standard theory, which did not make the results very interesting.

However, the next paper by Heinke and Ho [23] became sensational. The authors reprocessed the same data by ordering them in time and determining the temperature T_s^∞ as a function of time t. It appeared that in 9 years of observations the temperature had dropped by 4% (Fig. 1), and the thermal flux dropped by 21%. The next observations performed in November 2010 [24] confirmed these results. If they are correct, they are the first observations of a cooling isolated neutron star in real time, and this cooling is extremely 'fast' from a theoretical point of view. Formally, it is convenient to introduce the temperature drop rate as

$$s = -\frac{\mathrm{d} \ln T_\mathrm{s}^\infty}{\mathrm{d} \ln t}\,, \qquad (1)$$

which, in our case, is $s \approx 1.35 \pm 0.15$ (here, we present 1σ error bars, while in Fig. 2 we give 2σ error bars). Had the star cooling via neutrino emission been due to the direct or modified Urca process, we would have had $s \sim 0.1$ (see the dashed line in Fig. 2), and the T_s^∞ decline in 10 years would have been unnoticeable. A sharp drop in T_s^∞ occurs in a young neutron star (10–200 years old) at the end of internal thermal relaxation [25–27], but the thermal relaxation of the given star should have been over. In other words, the neutron star is sufficiently warm but cannot be described by the ordinary cooling theory for nonsuperfluid neutron stars.

3. Cooling of superfluid neutron stars

Observations of the neutron star in Cassiopeia A are naturally explained [24, 29] by the cooling theory of superfluid neutron stars. The theory was formulated in its final form in 2004 [28, 30].

We illustrate the cooling calculations following Ref. [24]. The computations have been performed with the equation of state of nucleon matter in the neutron star core suggested by Akmal, Pandharipande, and Ravenhall [31]. Specifically, we have used the parametrization of their results [31] derived in Ref. [32] and denoted as APR I in Ref. [33]. For this equation of state, the maximum mass of stable neutron stars is $M_\mathrm{max} = 1.929\, M_\odot$; the powerful direct Urca process becomes open in stars with $M > 1.829\, M_\odot$. The calculations have been mostly performed for the $1.65\, M_\odot$ star. The central density of such a star is shown by the vertical dotted line in Fig. 1a. The dashed line N in Fig. 1b describes the cooling of this star, neglecting the superfluidity effects.[1] One can see that it cannot provide the required steep slope of the cooling curve. In the inset to Fig. 1b, which is drawn on a smaller scale, one

[1] In the main panel of Fig. 1b, in contrast to the inset, the curve N is slightly raised, in order to fit the figure and demonstrate its insufficiently steep slope.

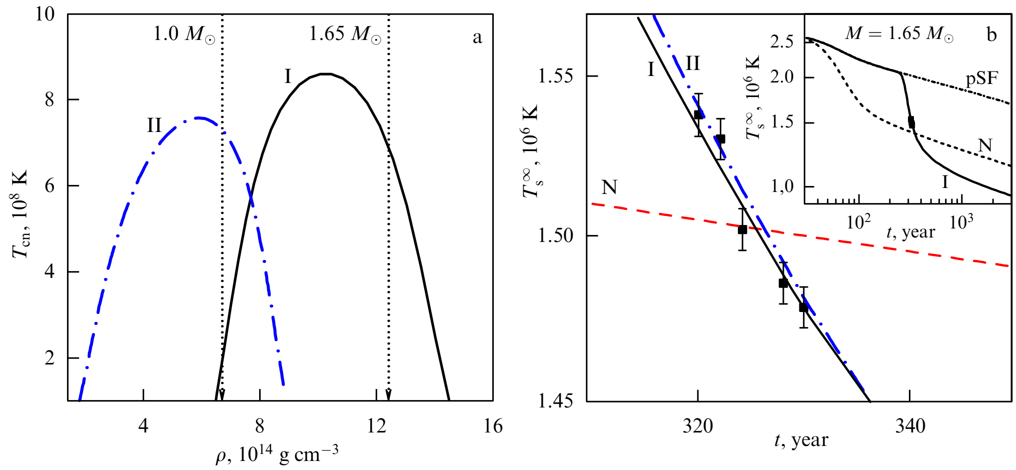

Figure 1. (a) Two models I and II for the critical temperature T_{cn} of triplet-state pairing of neutrons as a function of density ρ in a neutron star core. Vertical dotted straight lines show the central densities of stars of masses $M = 1.0\,M_\odot$ and $1.65\,M_\odot$. (b) Decline in the surface temperature T_s^∞ of the Cassiopeia A neutron star with time t. Dots with error bars are observational data. Curve N depicts a typical cooling curve for a nonsuperfluid star. Curves I and II are theoretical cooling curves for the $1.65\,M_\odot$ star, whose core contains strongly superfluid protons and neutrons with type I and II superfluidity, respectively. The inset shows curves I and N on a smaller scale and also curve pSF for a star with strongly superfluid protons and normal neutrons in the core.

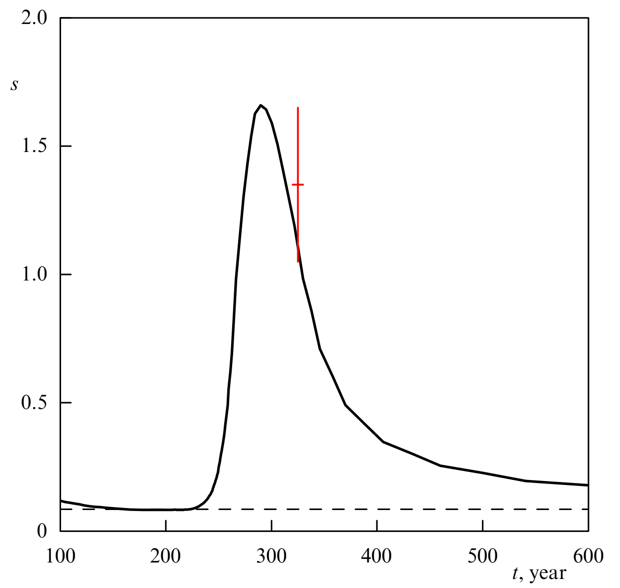

Figure 2. Solid line displays the slope s of the theoretical cooling curve for a $1.65\,M_\odot$ neutron star with strong proton superfluidity and type I neutron superfluidity (see Fig. 1), as compared with the observational data (with 2σ error bars) for the neutron star in Cassiopeia A. Dashed line reproduces $s(t)$ for a nonsuperfluid star (curve N in Fig. 1).

can well observe the desirable steep slope of curve N, but in the earlier epoch ($t \lesssim 100$ yr). It manifests the end of the inner thermal relaxation of the star, which cannot be used to explain the observations because it is difficult to delay the relaxation to the current epoch of $t \approx 330$ years.

More important for us would be the effect of superfluidity on neutrino processes in the stellar core. Because neutrino emission is generated in reactions involving strongly degenerate nucleons, the main contribution to this emission is provided by nucleons with energies near the Fermi level. Superfluidity of nucleons suppresses all the reactions involving these nucleons because of the appearance of a gap in their energy spectrum. When the temperature T falls much below T_c, the suppression becomes exponentially strong (see, e.g., Ref. [5]).

In addition to suppressing standard neutrino processes, superfluidity opens a specific mechanism of neutrino pair emission due to Cooper pairing of nucleons [34]. It can be treated as annihilation of quasiparticles producing neutrino pairs (see Table 1). The process is kinematically allowed owing to a distortion of the nucleon energy spectrum by the gap when T falls below T_c. Its intensity first strongly increases, reaches a maximum at $T \approx 0.8\,T_c$, and then exponentially declines with decreasing T (again, because of a gap occurrence). It is a rare neutrino process whose intensity can increase with decreasing T. The process is strongly affected by many-body (collective) effects, which was first pointed out by Leinson [35] and later studied in a number of papers (see, e.g., Refs [36–41] and references cited therein, as well as a discussion in review [12]). An account for collective effects is complicated (model-dependent), and the results are rather controversial.

In the nucleon core of the neutron star one can expect neutrino emission due to singlet-state Cooper pairing of protons and triplet-state Cooper pairing of neutrons (see Section 1). However, the emission due to pairing of protons is strongly suppressed by the smallness of reaction rate constants (see, e.g., Ref. [6]). It can be additionally reduced by many-body effects. Similar collective effects can strongly suppress neutrino emission due to singlet-state Cooper pairing of neutrons in the inner star's crust, but this emission, even nonsuppressed, is integrally weak due to the small volume of the crust, thus weakly affecting neutron star cooling. The second process of the given type in the stellar core is associated with neutrino emission due to triplet-state pairing of neutrons. Such emission can be sufficiently intense; it can be suppressed by collective effects, but much weaker than suppression due to singlet-state pairing (see, e.g., Ref. [41]). This process is important, and it will be considered below. Its neutrino luminosity is determined by integrating the neutrino emissivity over the region of triplet-state superfluidity of neutrons in the core (see, e.g., Ref. [30]). The latter region is determined by the critical temperature profile $T_{cn}(\rho)$

and by the current core temperature; the region widens when the star cools. The maximum of $T_{cn}(\rho)$ specifies the moment when this neutrino emission starts. The width of the $T_{cn}(\rho)$ profile regulates the intensity of the neutrino outburst and the subsequent efficiency of this neutrino emission in the star. According to computations, the neutrino luminosity due to Cooper pairing of neutrons can be 30–100 times larger than the luminosity due to the modified Urca process in a nonsuperfluid star. This can noticeably accelerate cooling in comparison with the standard one, as reflected in Table 1.

Thus, singlet-state proton superfluidity only suppresses neutrino luminosity of the stellar core, while triplet-state neutron superfluidity can strongly enhance this luminosity.

4. Superfluidity of the neutron star in Cassiopeia A

The theory should explain two apparently contradictory observational facts (see Section 2): the high temperature ($T_s^\infty \approx 1.5 \times 10^6$ K) and fast cooling rate ($s \approx 1.3$) of the star. With this rate, the star should have been much colder than it is. These facts can be naturally explained [24, 29] assuming the presence of strong singlet-state proton superfluidity and moderately strong triplet-state neutron superfluidity in the stellar core. The cooling theory allows one to choose such $T_{cn}(\rho)$ and $T_{cp}(\rho)$ profiles in the core, which lead to the observed values of T_s^∞ and s.

Numerical simulations show that the $T_{cn}(\rho)$ profile should have a wide peak with the maximum $T_{cn}^{max}(\rho) \sim (5-9) \times 10^8$ K. The maximum height guarantees that neutron superfluidity in the core appeared only a few decades ago; the neutrino emission due to Cooper pairing of neutrons gained its full strength, but had no time to greatly cool the star. By way of illustration, Fig. 1a gives two phenomenological $T_{cn}(\rho)$ profiles (curves I and II); their effect on cooling is explained below. The profiles are taken from Ref. [24], where curve I is denoted as (a), and curve II as (c).

To explain the observational data, we also need proton superfluidity. It has to be strong, with the critical temperature $T_{cp}(\rho) \gtrsim 3 \times 10^9$ K in the stellar core. The specific density dependence of T_{cp} is unimportant here. Such superfluidity appears early ($t \lesssim 1$ year) and suppresses all basic neutrino processes involving protons: Urca processes (modified or even direct ones), and neutrino-pair bremsstrahlung due to nucleon–proton collisions. It is only a relatively weak neutrino-pair generation process due to neutron–neutron collisions that survives (before the onset of neutron superfluidity). Hence, the neutrino luminosity of a star with normal neutrons is low, and the star remains noticeably hotter than in the course of the standard cooling due to the modified Urca process. The corresponding cooling curve (curve pSF) is plotted in the inset to Fig. 1b, only in this case the subsequent neutrino outburst due to Cooper pairing of neutrons gives the high observable cooling rate. As seen from Fig. 1, both models I and II for neutron superfluidity in a $1.65\,M_\odot$ star agree with the observations of the neutron star in Cassiopeia A, with model I being slightly better.

Moreover, according to Ref. [24], in the wide range of masses from $1.3\,M_\odot$ to $1.9\,M_\odot$ one can choose such a $T_{cn}(\rho)$ profile in the stellar core, which explains the observations. For $M \gtrsim 1.5\,M_\odot$, these profiles are only slightly different, while the maximum T_{cn}^{max} at lower M should be somewhat higher and shifted towards lower ρ. Theoretical results are also sensitive [24] to poorly studied collective effects on the neutrino emission due to Cooper pairing of neutrons in dense matter (see above).

It is important that the theory predicts a nontrivial behavior of the cooling rate (factor s, see Fig. 2). When the star reaches the state of internal thermal relaxation, the theory gives the standard value of $s \sim 0.1$. However, after the onset of neutron superfluidity, s jumps by a factor of a few tens, reaches a maximum, and then declines to its standard level $s \sim 0.1$. A noticeable rise in s above its standard value is the evidence of neutrino outburst within the star; it does not last very long and indicates a special period in the star's life. One can show that an accurate measurement of $s(t)$ would allow one to infer the most important parameter — the neutrino cooling function (ratio of neutrino luminosity and heat capacity of the star) within that period. It could give very useful information on the internal structure of the neutron star: first and foremost, on the critical temperature profile $T_{cn}(\rho)$ for neutron superfluidity. It seems that the present observations of the neutron star in Cassiopeia A are being made at this very period, but the factor s is measured with large uncertainty. Clearly, to observe the star in this period is a matter of good luck. If the proposed scenario is correct, s should noticeably decline over tens of years, which can be checked in future observations.

5. Cassiopeia A and other cooling neutron stars

At the next stage, it is useful to analyze our results in combination with the data on other cooling neutron stars [24]. It is reasonable to assume that the properties of superdense matter (most importantly, the equation of state) are the same in all neutron stars, but stars have different masses (central densities), rotation periods, magnetic fields, chemical compositions of surface layers, etc. In this case, the $T_{cn}(\rho)$ and $T_{cp}(\rho)$ profiles are the same in all stars (although they extend to different central densities).

For our analysis, we have taken [24] stellar models with the same APR I equation of state. We have calculated the cooling curves for stars of various masses with different $T_{cn}(\rho)$ and $T_{cp}(\rho)$; the results have been compared with the observations of all cooling neutron stars whose T_s^∞ and t have been measured (estimated). Observational data on these stars have been taken from the sources cited in Ref. [24]. The results for strong proton superfluidity and type I or II neutron superfluidity are presented in Fig. 3. Shaded are those ranges of T_s^∞ which are filled by the cooling curves for neutron stars of various masses (from $1\,M_\odot$ to $1.929\,M_\odot$). The upper curves on both panels refer to the $1\,M_\odot$ star; this star shows the slowest cooling in our model. The lower curve refers to the most massive $1.929\,M_\odot$ star representing the fastest cooler. For simplicity, we assume that the powerful direct Urca process of neutrino emission in massive stars is formally allowed but fully suppressed by strong proton superfluidity. If this were not so, massive stars would cool much faster, and the shaded regions in Fig. 3 would drop to much lower temperatures [24]. In addition to the cooling curves for the $1\,M_\odot$ and $1.929\,M_\odot$ stars, we also plotted the intermediate curves for $1.65\,M_\odot$ stars.

As follows from Fig. 1, in model II for neutron superfluidity stars of all masses undergo the stage of neutrino outburst after the onset of neutron superfluidity. It is seen, however, from Fig. 3b that such models for neutron superfluidity contradict observations. These models are good for explaining the coldest neutron stars (for their ages), like the Vela pulsar. However, they cannot explain the warmest middle-aged stars, for instance, PSR 0656+14 — after the neutrino outburst it would have been much colder than

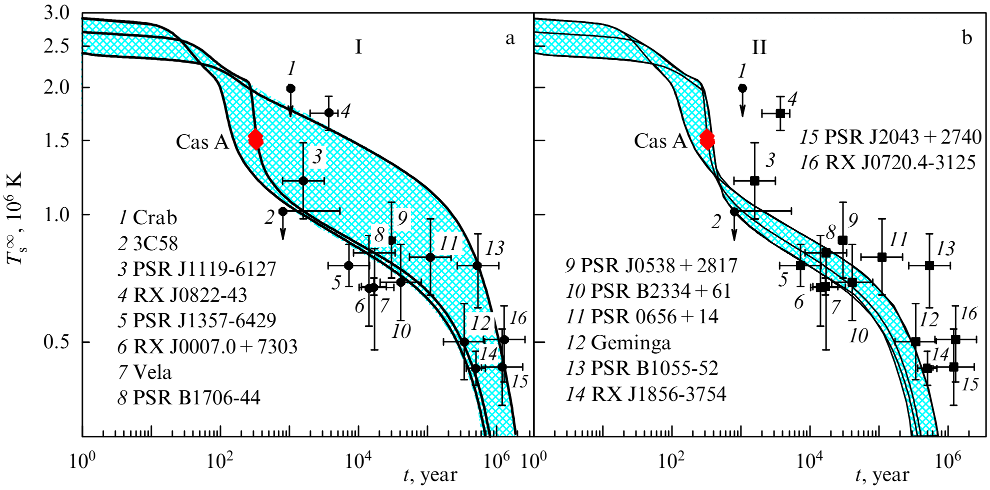

Figure 3. Observational data on cooling neutron stars as compared with theoretical cooling curves for neutron stars of different masses possessing strong superfluidity of protons and type I (a) or II (b) superfluidity of neutrons. Shaded regions are filled by cooling curves of stars of various masses, from $M = M_\odot$ (upper curves) to $\approx 2\,M_\odot$ (lower curves). As an example, the solid lines show cooling of a $1.65\,M_\odot$ star. Superfluidity I can explain the observations of all stars, while superfluidity II cannot.

required. The observations of all stars can be explained only by the bell-shaped $T_{\rm cn}(\rho)$ profile shifted to the inner core of a neutron star, like the type I profile (Fig. 3a). In that case, neutron superfluidity either does not appear at all in a low-mass star or appears too late, when it does not affect the cooling. In more massive stars, it appears earlier and initiates noticeable cooling. A similar $T_{\rm cn}(\rho)$ profile was suggested earlier [30] for explaining the data available at that time. Surprisingly, it also explains the observations of the Cassiopeia A neutron star. Observations of all stars restrict the $T_{\rm cn}(\rho)$ profile rather strongly [30]; the neutron star in Cassiopeia A should not be too light, $M \gtrsim 1.4\,M_\odot$, and the masses of other stars should be ranged. For instance, the Vela pulsar should be massive, and the PSR 0656+14 should have a low mass. Let us stress that the given $T_{\rm cn}(\rho)$ profile does not agree with many microscopic calculations (which indicate that triplet-state pairing occurs at lower densities), but it is not excluded by such numerical simulations (see papers [30, 42] and references cited therein). To avoid this nonstandard $T_{\rm cn}(\rho)$ dependence, we can assume that the $T_{\rm cn}(\rho)$ profile is wide (more standard) and extends to the outer stellar core, but the efficiency of neutrino emission due to Cooper pairing of neutrons at lower densities is strongly suppressed (for instance, by collective effects). Then the neutrino emissivity profile in the core will be similar to that for type I superfluidity.

6. Conclusions

We have discussed recent results obtained by Ho and Heinke [21, 23], who processed 10-year long spectral observations of the young cooling neutron star in the Cassiopeia A supernova remnant using the Chandra X-ray orbital observatory. The results reveal an extraordinarily fast (according to theoretical standards) cooling of the warm star, which is not described by the cooling theory of nonsuperfluid stars. We show that it is easily explained provided the star's core possesses strong singlet-state superfluidity of protons (which suppresses neutrino emission and slows down the cooling just after the star's birth) and moderately strong triplet-state superfluidity of neutrons (which appeared a few decades ago and produced a strong neutrino outburst that is accelerating the cooling in the present epoch). If this explanation is correct, this is the first serious evidence for the presence of superfluidity in neutron star cores that comes from observations of cooling neutron stars. Before these results, the main observational manifestation of superfluidity was thought to be provided by pulsar glitches — sudden changes in pulsar spin periods (see, e.g., Ch. 1 in monograph [2] and references cited therein).

The neutrino outburst triggered by the onset of neutron superfluidity in the Cassiopeia A neutron star should be followed by a rapid decrease in $s(t)$, which can be checked in a few decades of future observations. We stress that the explanation of the data was proposed independently in two publications [24, 29], which differ only in details. It is based on the theory that, in turn, was devised independently in two publications [28, 30] by the same teams.

Two alternative explanations have been suggested for the neutron star phenomenon in Cassiopeia A. The first one [43] assumes that the young neutron star was born rapidly spinning, with its central density very close to the critical density at which the direct Urca process starts (if not suppressed by superfluidity). Then, according to the authors, the star slows down by the pulsar braking mechanism, its central density increases, and the direct Urca process comes into play, triggering powerful neutrino emission and observed rapid cooling. This explanation is, in principle, valid, but its realization is highly improbable (it requires the finest tuning of parameters). The second explanation [44] is based on the nonstandard model of thermal insulation of the neutron star, which allows the authors to delay the thermal relaxation till the present epoch. The observed sharp drop in the surface temperature of the star is explained by the cessation of the thermal relaxation. This model of thermal insulation cannot be justified and contradicts well-known reliable models (see, e.g., Refs [45–47]).

Returning to our primary interpretation, we notice that it allows us to explain observations of cooling of all isolated neutron stars by one and the same model for nucleon superfluidity in the stellar core. Moreover, an analogous model was suggested much earlier to explain the data on cooling neutron stars available at that time [30]. The new data on the neutron star in Cassiopeia A surprisingly support the old results. Sceptics can disagree with the suggested interpretation but may treat it as a hint of how neutrino emissivity within the neutron star should behave to explain the data. In any case, the suggested interpretation of observations of the neutron star in Cassiopeia A is preliminary and has to be confirmed. First, the assumed neutrino outburst is a rare event; it is a surprise that its consequences are being observed. Second, the region of triplet-state pairing of neutrons is shifted too deeply into the star's core, in contrast to the majority of microscopic calculations. Third, the ageing of the X-ray detectors of the Chandra observatory may lead to errors in their calibration and data processing [48]. However, the interest in the neutron star in Cassiopeia A is high, and we hope the situation with observations and theoretical interpretation will be clarified in the near future.

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