Table 3. LISM parameters from the Ulysses measurements of the helium atom flux.

$\lvert V \rvert$, km s^{-1}	λ, deg	β, deg	T, K
26 ± 1	72 ± 2	-2.5 ± 2.7	6700 ± 1500

plane four times and could observe both solar poles. In particular, the latitude asymmetry of the solar wind was measured, which directly confirmed our results obtained from the Prognoz satellites [18].

To register helium atoms that entered the Solar System from the interstellar medium, a group of researchers headed by H Rosenbauer at the Max-Planck Institute for Aeronomy designed a detector based on the interaction of helium atoms having an energy of about 15 eV (which corresponds to the velocity of 25 km s^{-1}) with a thin gold foil. Electrons kicked out from the foil were detected by a channel photomultiplier, which, with the known velocity of the electrons, allowed estimating their local density (in contrast, in our integral method, the radiation on the $\lambda = 584$ Å line was measured along the entire path from the measurement point to infinity). The detector scanned almost the entire sky, enabling the determination of the direction of helium atom trajectories. The measurements were carried out from different points in the Solar System, owing to which a high accuracy was achieved (Table 3). Data obtained in a completely different way are in good agreement with our results obtained by 'optical' means.

6. Conclusion

From the brief consideration of the problem of the motion of the Sun relative to the local interstellar medium given above, we conclude that the basic parameters of the LISM are known quite well. These include the density of atomic hydrogen and helium in the close vicinity of the Solar System (at distances exceeding 20 a.u.) and the direction of the motion of the Sun relative to LISM. The LISM temperature is known with less accuracy. Undoubtedly, the temperature values derived by us from measurements on L_α lines of hydrogen ($\lambda = 1215.7$ Å) and helium ($\lambda = 584$ Å) are different, and this fact calls for explanation. Second, the temperature measurement error on these two lines highly exceeds the relative measurement errors of the solar motion direction relative to the LISM and of the hydrogen and helium atomic density.

It is very plausible that the temperature difference for hydrogen and helium atoms is due to the interstellar atoms crossing the transient zone between the heliosphere and 'pure' interstellar space. Hopefully, new results obtained by the IBEX (Interstellar Boundary Explorer) satellite can be used to improve the parameters of the local interstellar medium.

References

1. Kupperian J E (Jr.) et al. *Planet. Space Sci.* **1** 3 (1959)
2. Morton D C, Purcell J D *Planet. Space Sci.* **9** 455 (1962)
3. Bertaux J L et al. *Kosmich. Issled.* **16** 269 (1978) [*Cosmic Res.* **16** 214 (1978)]
4. Bertaux J L et al. *Astron. Asrophys.* **46** 19 (1976)
5. Kurt V G, Germogenova T A *Astron. Zh.* **44** 352 (1967) [*Sov. Astron.* **11** 278 (1967)]
6. Blum P W, Fahr H J *Astron. Astrophys.* **4** 280 (1970)
7. Blum P W, Pfleiderer J, Wulf-Mathies C *Planet. Space Sci.* **23** 93 (1975)
8. Lallement R et al. *Astron. Astrophys.* **426** 875 (2004)
9. Baranov V B *Space Sci. Rev.* **143** 449 (2009)
10. Katushkina O A, Izmodenov V V *Pis'ma Astron. Zh.* **36** 310 (2010) [*Astron. Lett.* **36** 297 (2010)]
11. Bertaux J L, Lallement R *Astron. Astrophys.* **140** 230 (1984)
12. Bertaux J L, Lallement R, Kurt V G, Mironova E N *Astron. Astrophys.* **150** 1 (1985)
13. Dalaudier F, Bertaux J L, Kurt V G, Mironova E N *Astron. Astrophys.* **134** 171 (1984)
14. Burgin M S et al. *Kosmich. Issled.* **21** 83 (1983) [*Cosmic Res.* **21** 72 (1983)]
15. Kurt V G, Mironova E N, Bertaux I L, Dalaudier F *Kosmich. Issled.* **22** 97 (1984) [*Cosmic Res.* **22** 86 (1984)]
16. Kurt V G, Mironova E N, Bertaux J L, Dalaudier F *Kosmich. Issled.* **22** 225 (1984)
17. Witte M et al. *Adv. Space Res.* **13** (6) 121 (1993)
18. Lallement R, Bertaux J L, Kurt V G *J. Geophys. Res.* **90** 1413 (1985)

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Generation of cosmological flows in general relativity

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1. Introduction

The Copernicus principle is known to cast doubt on the uniqueness of our Universe. Therefore, it seems likely that there is some physical mechanism of gravitational reproduction of cosmological flows of matter expanding from superlarge to small curvatures and densities. We relate the solution of the cosmogenesis problem (the origin of universes) to black holes, in which regions with high space–time curvature are formed in a natural evolutionary manner during gravitational collapse. We only need to continue the singular states that appeared in this way in time and to see what geometrical structures are found beyond them in the future.

An analytic continuation of general relativity (GR) solutions through singular hypersurfaces $r = 0$ is realized in the model class of 'black-white' holes with integrable singularities [1, 2]. In these models, the space–time of a black hole can be connected with a white hole endowed with the metric of a homogeneous cosmological model, allowing an explicit realization of the geometrical concept of a multisheet universe (hyperverse). These topics are discussed in this paper.

2. How the Schwarzschild metric can be continued

A black hole with a positive external mass $M > 0$ without rotation or charge is described in GR by the Schwarzschild metric in the vacuum:

$$\mathrm{d}s^2 = \left(1 - \frac{2GM}{r}\right)\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1 - 2GM/r} - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right), \qquad (1)$$

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where the variable $r > 0$ is defined as the internal curvature radius of a closed homogeneous isotropic 2-space $\mathrm{d}Y^2 = \gamma_{ij}\,\mathrm{d}y^i\,\mathrm{d}y^j$, which does not depend on y^i $(i = 1, 2)$, and G is the gravitational constant. The Y-space is invariant under the group of motions G_3 (two translations that allow moving to an arbitrary point Y, and rotation around a point), and it can be reduced to the form $\mathrm{d}Y^2 = r^2\,\mathrm{d}\Omega$, where $\mathrm{d}\Omega \equiv \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2$ decribes the unit-radius 2-sphere $\mathbb{S}^2$, $y^i = (\theta, \varphi)$. Any of the $\mathbb{S}^2$ points can be taken as the pole $\theta = 0$, and turning around the pole is described by the angle φ.

The space X orthogonal to Y is given in the Eulerian gauge, in which one of the coordinates x^I coincides with r. Additionally, metric (1) is independent of the coordinate $t \in \mathbb{R}^1$. Therefore, the black hole in the vacuum has the group of motions G_4 acting on the hypersurface $\mathbb{C}^3 = \mathbb{R}^1 \times \mathbb{S}^2$ (three translations in $\mathbb{C}^3$ and homogeneous rotations in $\mathbb{S}^2$).

Topologically, geometry (1) represents a 4-cylinder with a homogeneous 3-surface $\mathbb{C}^3$ and the radial coordinate r, which in the region $r > 0$ determines the Schwarzschild sector of a black (or white) hole. Attempts to extend this solution inevitably lead to regions occupied by matter. Therefore, the problem of the analytic continuation of metric (1) must be solved using more general GR metrics with matter, which we restrict by requiring the spherical symmetry G_3 in $\mathbb{S}^2$.

We consider a class of such metrics that in the orthogonal gauge ($g_{Ii} = 0$) have the form

$$\mathrm{d}s^2 = \mathrm{d}X^2 - \mathrm{d}Y^2 = -\mathcal{N}\mathcal{K}\,\mathrm{d}t^2 + \frac{\mathrm{d}\rho^2}{4\rho\mathcal{K}} - \rho(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2)\,, \qquad (2)$$

where the real variable ρ^{-1} is not restricted by the sign and is defined as the Y-space internal curvature scalar that is independent of y^i:

$$R^{(Y)}_{ij} = \rho^{-1}\gamma_{ij}\,. \qquad (3)$$

The Ricci 2-tensor is constructed from the metric γ_{ij}, which by the spherical symmetry can always be reduced to the form $\mathrm{d}Y^2 = \rho\,\mathrm{d}\Omega$. For general reference frames, ρ, $\mathcal{K}$, and $\mathcal{N}$ are 4-scalars depending on the coordinates of the X-space $\mathrm{d}X^2 = n_{IJ}\,\mathrm{d}x^I\,\mathrm{d}x^J$; here, $\mathcal{K}$ is the kinetic term of ρ and $\mathcal{N}$ is restricted to have positive values:

$$\mathcal{K} = \frac{\rho_{,\mu}\,\rho^{,\mu}}{4\rho} \equiv -1 - 2\Phi\,, \qquad \mathcal{N} \equiv N^2 > 0\,, \qquad (4)$$

with Φ being the metric gravitational potential. The energy–momentum tensor has the form $T^{\nu}_{\mu} = \mathrm{diag}\,(T^J_I, -p_\perp, -p_\perp)$, where $T^J_I = (\epsilon + p)\,u_I u^J - p\delta^J_I$. For $\rho > 0$, the functions ϵ, p, $p_\perp$, and $u^\mu = (u^I, 0, 0)$ respectively describe the energy density, longitudinal and transversal tension, and 4-velocities of matter ($u_I u^I = 1$). Metric (2) transforms into solution (1) in the part of the domain $\sqrt{\rho} = r > 0$ that is free of matter. Therefore, by definition, we call the domain $\mathcal{K} > 0$ the T-region of space (2). There, in particular, the component T^t_t describes the pressure[1] and the meaning of the other components depends on the sign of ρ.

The GR equations relate the metric and matter scalars:

$$\Phi' = 2\pi G P\,, \qquad \dot{\Phi} = 2\pi G T^{\rho}_t\,, \qquad (5)$$

$$\frac{N'}{N} = \frac{2\pi G(E + P)}{\mathcal{K}}\,, \qquad (6)$$

$$\frac{(\rho N E)'}{N} + p_\perp - \frac{P}{2} = -\frac{(N^3 T^{t\rho})^{\cdot}}{4N^3\mathcal{K}}\,, \qquad \dot{P} = (T^{\rho}_t)'\,, \qquad (7)$$

where the prime and dot respectively denote partial derivatives with respect to ρ and t, and

$$P \equiv -T^t_t - \frac{\Phi}{4\pi G\rho}\,, \qquad E \equiv T^{\rho}_{\rho} + \frac{\Phi}{4\pi G\rho}\,. \qquad (8)$$

By modeling the state of the effective matter, we can determine P and E from Bianchi identities (7), and integrating (5) and (6) over $\mathrm{d}x^I$ from external solution (1) into the future, we can reconstruct the metric potentials. Similarly, Eqns (5) are transformed for the mass function $m = m(x^I)$:

$$\Phi \equiv -\frac{Gm}{\sqrt{\rho}}\,, \qquad m_{,I} = 4\pi\rho\, e_{IK}\, T^K_J\, \frac{\partial x^J}{N\partial t}\,, \qquad (9)$$

where $(\ldots)_{,I} \equiv \partial/\partial x^I$ and the totally antisymmetric tensor in X is given by

$$e_{IJ} = \left|\det(n_{IJ})\right|^{1/2}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad e^{IJ} = \left|\det(n_{IJ})\right|^{-1/2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \qquad (10)$$

3. Space–time near an integrable singularity

We are interested in solutions (2) generated by metric (1) that are geodesically complete. The sufficient condition for the metric to pass through $\rho = 0$ is the finiteness of the potentials Φ and N and their derivatives. We call such models black-white (or black/white) holes with integrable singularity [1]. They are described by the mean (average over vacuum state (1)) 4-dimensional metric space without punctures, which provides a continuous extension of the affine parameters of world lines of test particles through the singular hypersurface $\rho = 0$. Using reference frames constructed on these particles, we investigate the geometry of black-white holes outside the Schwarzschild sector.

The effective matter generated by a strong gravitational field near the singularity changes the space–time near $\rho = 0$ and is the physical reason for the existence of an integrable singularity. The formation of matter in extremely strong gravitational fields corresponds to the Le Chatelier principle: this is how Nature reacts to a sharp increase in the metric potential amplitudes as $\rho \to 0$ (which would continue to increase in the absence of matter) and thus precludes their divergence. Specific quantum gravitational mechanisms of mutual transformations of matter and gravitational degrees of freedom under extreme conditions need to be considered separately. Here, developing papers [1, 2], we postulate the continuity of gravitational potentials (2) in the presence of effective matter.

In the R-region of space–time (2) with the signature $(+,-,-,-)$ specified by generating metric (1), we have $\mathcal{K} < 0$ and $\rho = r^2 > 0$. Here, the hypersurface $r = 0$ is degenerate and represents a time-like worldline of the center of a spherically symmetric distribution of matter (for example, the center of a star). If there is no matter in the

[1] In particular, under the comoving condition $T^t_t = -p$ in (2).

R-region, it follows from (1) that $r > 2GM$, i.e., the hypersurface $r = 0$ does not lie in the R-region.

In the T-region, we have $\mathcal{K} > 0$, and the variable ρ can have any sign because the physics in this region is determined by nonlinear quantum effects, and *a priori* there are no grounds to believe that the signature remains indefinite [3]. Here, the singular hypersurface $\rho = 0$ splits the 4-space into domains with different signatures $(-,+,-,-)$ for $\rho > 0$ and $(-,-,+,+)$ for $\rho < 0$. The complete geometry depends on the distribution and properties of the effective matter near $\rho = 0$.

We assume that the effective matter distribution maintains the symmetry of the generating field. For example, the region that is evolutionary adjacent to that with the Schwarzschild metric (for example, $T_{\mu\nu} \neq 0$ for $r \leqslant r_0 = \mathrm{const} < 2GM$ and $T_{\mu\nu} = 0$ for $r > r_0$) preserves the Killing t-vector and depends only on ρ. The region inside the star keeps the spherical symmetry and the field homogeneity of the star. We use these constraints in constructing the models in Sections 4–6.

Several properties of geometries (2) should be noted.

• The continuity of the potential Φ implies the integrability of $P(t,\rho)$ along the lines $t = \mathrm{const}$ [see (5)]:

$$\Phi(t,\rho) = \Phi_0 + 2\pi G \int_0 P\,\mathrm{d}\rho\,, \qquad \Phi_0 = \Phi(t,0)\,. \tag{11}$$

• The continuity of Φ and N implies the integrability of $E(t,\rho)$ along the lines $t = \mathrm{const}$ [see (6)]:

$$N(t,\rho) = N_0 \exp\left(2\pi G \int_0 \frac{E+P}{\mathcal{K}}\,\mathrm{d}\rho\right), \quad N_0 = N(t,0)\,. \tag{12}$$

• For $\mathcal{K}_0 \equiv -1 - 2\Phi_0 = K_0^2 = \mathrm{const} > 0$, the space–time at $\rho \to 0$ has the structure

$$\mathrm{d}s^2 = -\,\mathrm{d}\tilde{t}^2 + \,\mathrm{d}U\,\mathrm{d}V - UVK_0^2 \sin^2\theta\,\mathrm{d}\varphi^2\,, \tag{13}$$

$$\tilde{t} = K_0 \int N_0\,\mathrm{d}t\,, \qquad \rho = K_0^2\,UV\,, \qquad \theta = \frac{1}{2K_0}\ln\left|\frac{U}{V}\right|.$$

The direction θ can be taken along any meridian in $\mathbb{S}^2$ and counted from an arbitrarily chosen pole $\theta = 0$.

It follows that the integrable singularity of a black-white hole includes horizon hypersurfaces $U = 0$ and $V = 0$ lying in the T-region and intersecting along the space-like bifurcation line $U = V = 0$. They separate the T-region in cone sectors of the black ($U < 0, V < 0$) and white ($U > 0, V > 0$) holes and the static zone ($UV < 0$). The horizons have the cylindrical symmetry $\mathbb{R}^1 \times \mathbb{S}^2$ with the space-like longitudinal axis $t \in \mathbb{R}^1$ and null geodesics in $\mathbb{S}^2$ ($t = \mathrm{const}$).[2] Photons propagating in these directions are confined gravitationally at $\rho = 0$, where they perform infinite oscillations in Y within a finite interval of the affine parameter. Trajectories of other particles, including photons with a projection in $\mathbb{R}^1$, intersect the singular hypersurface $\rho = 0$ and go away into other metric domains.

4. Geometric maps of an oscillating hole

The structure of a 4-space can be described by 2- and 3-dimensional cross sections and cuts, which can be covered

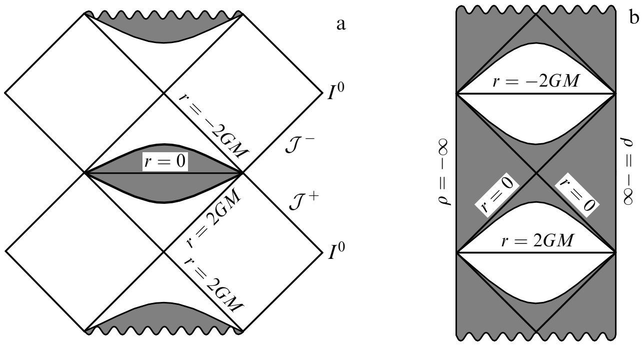

Figure 1. (a) Longitudinal and (b) transversal cross sections of an oscillating black-white hole. $\mathcal{J}^+$ is the light-like infinity of the future for observers in the region $r > 0$, $\mathcal{J}^-$ is the light-like infinity of the past in the region $r < 0$, and I^0 are spatial infinities of the R-regions.

with a net of trajectories of test particles on them. It is convenient to choose coordinates such that the light trajectories have a slope of 45° as in the flat world [4]. By the spherical symmetry, all geodesics in (2) can be represented by three groups of cross sections:

• X-planes: longitudinal cross sections $(\theta,\varphi) = \mathrm{const}$;
• UV-planes: transversal cross sections $(t,\varphi) = \mathrm{const}$;
• tUV-hypersurfaces: longitudinal–transversal cuts $\varphi = \mathrm{const}$.

The X-planes are filled with radial or longitudinal geodesics, the tUV-planes contain particles propagating in both longitudinal and transversal directions, and the UV-planes orthogonal to the bifurcation line are filled with spiral geodesics that do not escape beyond the T-region (see the Appendix) (Fig. 1).

As an example, we consider metric (2), which is independent of t, with matter in the T-region $r \leqslant r_0 = \mathrm{const} < 2GM$. The matter is distributed in the UV-planes along bundles of straight lines $(t,\theta,\varphi) = \mathrm{const}$ intersecting at the points $U = V = 0$. Integrating (9) along the time lines $r = \sqrt{\rho}$ from black-hole region (1) to the future, we obtain continuous potentials

$$\Phi = -\frac{Gm}{r}\,, \quad m = M - 4\pi \int_{r_0} p(r)\,r^2\,\mathrm{d}r = -4\pi \int_0 p\rho\,\mathrm{d}r\,, \tag{14}$$

which directly connect both holes. The black-hole metric is in the sector $r > 0$, while a white hole is obtained from the extension of solution (14) into the region $r < 0$ under the finiteness condition $p\rho = \Phi_0/4\pi G = \mathrm{const}$ at $\rho = 0$. The complete geometry can be restored by integrating from the bifurcation line points along all bundles of lines, including regions $\rho < 0$.

For illustration, we consider the model of an oscillating black-white hole with a triggered matter distribution depending on $\rho \leqslant \rho_0 \equiv r_0^2$ (see [1, 2], case B). In other words, the transversal pressure increases jump-wise: $p_\perp = \lambda_0 \theta(\rho_0 - \rho)$. The longitudinal pressure, which is chosen to be vacuum-like ($p = -\epsilon$), is calculated using Bianchi identities (7):

$$\rho \leqslant \rho_0: \quad P = -E = \frac{2}{3}\,\lambda_0\,, \quad \Phi = \frac{1}{2}\,H_1^2(\rho - 3\rho_0)\,, \tag{15}$$

$$\rho > \rho_0: \quad P = -E = \frac{M}{4\pi|r|^3}\,, \quad \Phi = -\frac{GM}{|r|}\,, \tag{16}$$

[2] A comparison is suggested with horizons $r = 2GM$ of an eternal black (white) hole for the Kruskal metric, which are null in the radial direction and space-like in $\mathbb{S}^2$ and intersect over a space-like bifurcation 2-sphere.

where $H_1^2 \equiv 8\pi G\lambda_0/3 \equiv GM/r_0^3 = \mathrm{const}$. Substituting (15) and (16) in (2), we obtain the metric with matter ($\rho \leqslant \rho_0$):

$$\mathrm{d}s^2 = -\mathcal{K}\,\mathrm{d}t^2 + \frac{\mathrm{d}\rho^2}{4\rho\mathcal{K}} - \rho\,\mathrm{d}\Omega = \frac{-(1-\tilde\rho)^2\,\mathrm{d}\tilde t^2 + H_1^{-2}\tilde\rho^{-1}\,\mathrm{d}\tilde\rho^2 - 4r_1^2\tilde\rho\,\mathrm{d}\Omega}{(1+\tilde\rho)^2} \qquad (17)$$

and the metric in the vacuum ($\rho = r^2 > \rho_0$)

$$\mathrm{d}s^2 = \left(1 - \frac{2GM}{|r|}\right)\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1 - 2GM/|r|} - r^2\,\mathrm{d}\Omega\,, \qquad (18)$$

where $\tilde t = K_0 t$, $\rho = 4r_1^2\tilde\rho(1+\tilde\rho)^{-2}$, $\mathcal{K} = K_0^2 - H_1^2\rho$, $r_1 = K_0/H_1 = r_0(3 - r_0/GM)^{1/2}$, $K_0^2 = -1 - 2\Phi_0$, and $\Phi_0 = \Phi(0) = -3GM/2r_0$. The value of the potential of the integrable singularity is one and a half times lower than that at the matter boundary, and this ratio is independent of the parameter r_0.

We represent (17) using the proper interval on the ρ axis:

$$\rho = r^2 \in [0, \rho_0]: \ \ \mathrm{d}s^2 = \mathrm{d}\tau^2 - \cos^2(H_1\tau)\,\mathrm{d}\tilde t^2 - r_1^2\sin^2(H_1\tau)\,\mathrm{d}\Omega\,, \qquad (19)$$

$$\rho \leqslant 0: \ \ \mathrm{d}s^2 = -\,\mathrm{d}x^2 - \cosh^2(H_1 x)\,\mathrm{d}\tilde t^2 + r_1^2\sinh^2(H_1 x)\,\mathrm{d}\Omega\,, \qquad (20)$$

where $r = -r_1\sin(H_1\tau) \in [-r_0, r_0]$ and $\rho = -r_1^2\sinh^2(H_1 x) \leqslant 0$. The space–time of static domains is asymptotically anti-de Sitter:

$$\rho \ll -r_0^2: \quad T_\nu^\mu = -\lambda_0\delta_\nu^\mu\,, \quad R_\nu^\mu = 3H_1^2\delta_\nu^\mu\,. \qquad (21)$$

For this reason, the regions $\rho < 0$ are also referred to as anti-de Sitter (AdS) zones.

5. Source of an eternal black (white) hole

Eternal holes that are almost everywhere free of matter can be obtained in the limit $r_0 \to 0$ in Eqns (15)–(18):

$$r \in \mathbb{R}^1: \qquad \epsilon = -p = 2p_\perp = M\,\frac{\delta(r)}{2\pi r^2}\,, \qquad (22)$$

$$\rho < 0: \quad p = -\epsilon = p_\perp = \lambda_0 \equiv \frac{3M}{8\pi r_0^3}\,. \qquad (23)$$

The AdS zones are strongly curved by the dense vacuum λ_0 [see (20)]:

$$\rho \ll -r_0^2: \ \ \mathrm{d}s^2 = -\,\mathrm{d}x^2 + \frac{3}{4}\,r_0^2\,\exp(2H_1 x) \times (-H_1^2\,\mathrm{d}t^2 + \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2)\,, \qquad (24)$$

which produces a δ-like material source with the geometry of eternal black hole (18) in the region $r \in \mathbb{R}^1$. This source, localized at $|r| \leqslant r_0$, has the density $\sim \lambda_0$ and the total mass M. The value of r_0 can be calculated in quantum field theory. In our classical treatment, r_0 is a free parameter of the problem. We also note that the relation between the longitudinal and transversal tensions in (22) is not universal: it depends on the chosen condition of the model $p = -\epsilon$.

Solutions (22)–(24) show that the polarized vacuum in static AdS zones is the source of eternal black holes. The gravitational mass of the effective matter of each of the AdS zones, via its light hypersurface of the future $\rho = 0$, generates the causally connected Schwarzschild metric of a black-white hole in the vacuum, which extends in time to the next integrable singularity $\rho = 0$, where the process repeats (see Fig. 1). The full geometry is invariant under the reversal $r \to -r$ of the spherical coordinate system relative to the bifurcation lines. In this sense, phase transitions between gravitational and material degrees of freedom in this model are reversible.

In Section 6, we construct one more example of a reversible geometry, in which the parent star, collapsed from the R-region with the formation of a singularity in the T-region, is the source of a black hole. During the evolution, this model passes the stage of the effective matter and transforms into a white hole with the metric of a homogeneous cosmological model.

6. Astrophysical black-white hole

We consider the model of a black-white hole with an integrable singularity, assuming that the black hole was formed during the collapse of a star from the parent universe. The star is modeled as a homogeneous sphere with the radius $r(T, R = 1) = a(T)$, which was at rest in a flat space–time and had a mass M ($a \gg 2GM$ at $T \to -\infty$), and then started collapsing due to self-gravitation, with the initial pressure being negligible. Here, T and R are the Lagrange coordinates comoving with the star material, and the proper time $T \in \mathbb{R}^1$ and radial markers of spherical shells $R \geqslant 0$ are normalized such that $R = 1$ on the star surface.

The symmetry of the field is determined by the initial and boundary conditions of the problem. Inside the star ($R \leqslant 1$), the symmetry of $\mathbb{R}^1 \times \mathbb{E}^3$ is the 6-parameter group G_6 on $\mathbb{E}^3$ and the potentials a, H, ϵ, p are functions of T. Outside the star ($R > 1$), the group of motions G_4 on $\mathbb{C}^3$ and all potentials depend on r. To avoid crossing matter flows, we assume that the longitudinal tension, which is produced outside the star at large curvatures, is vacuum-like. Then, at $R > 1$, $N = 1$ and the energy–momentum tensor is invariant under motions in X: $T_\nu^\mu = -\mathrm{diag}\,(p, p, p_\perp, p_\perp)$. Inside $R \leqslant 1$, the matter is Pascalian, $T_\nu^\mu = (\epsilon + p)\,u_\nu u^\mu - p\delta_\nu^\mu$ (with the 4-velocity $u_\nu = T_{,\nu}$), and its state is calculated from the boundary conditions. The homogeneity of the star suggests the continuity of Φ and p at the boundary $R = 1$ ($r = a$), whereas the density and transversal pressure can change discontinuously.

Inside the star, the metric has the form [5]

$$\mathrm{d}s^2 = \mathrm{d}T^2 - a^2(\mathrm{d}R^2 + R^2\,\mathrm{d}\Omega) = \frac{a^4H^2\,\mathrm{d}t^2 - \mathrm{d}r^2}{1 + 2\Phi} - r^2\,\mathrm{d}\Omega\,, \qquad (25)$$

where the Euler and Lagrange coordinates are related as

$$R \leqslant 1: \quad r = aR\,, \quad t = -\int\frac{\mathrm{d}T}{a^2H} - \frac{R^2}{2}\,, \qquad (26)$$

and the Hubble function $H = \mathrm{d}a/a\,\mathrm{d}T$ can be found from the Friedman equations:

$$H^2 = \frac{8\pi G}{3}\,\epsilon = \frac{2Gm}{r^3} = -\frac{2\Phi}{r^2}\,, \quad \frac{\mathrm{d}\epsilon}{\mathrm{d}T} + 3H(\epsilon + p) = 0\,. \qquad (27)$$

Initially, the pressure is absent and the star collapses freely ($H < 0$, $a^3H^2 = 2GM$). The Newtonian potential in this limit is $\Phi_{\rm N} = GM(R^2 - 3)/2a$. The internal tension in the star appears at $a \leqslant r_0 < 2GM$, and it can be calculated from the matching conditions with the effective matter at the boundary $R = 1$.

Outside the star, the metric has the form

$$\begin{aligned} {\rm d}s^2 &= (1 + 2\Phi)\,{\rm d}t^2 - \frac{{\rm d}r^2}{1 + 2\Phi} - r^2\,{\rm d}\Omega \\ &= {\rm d}T^2 + 2\Phi\,{\rm d}R^2 - r^2\,{\rm d}\Omega\,, \end{aligned} \tag{28}$$

where the Lagrange reference frame comoves with the shells of free dust particles following the collapse of the star surface:

$$R > 1: \quad R - T = \int \frac{{\rm d}r}{\sqrt{-2\Phi}}\,, \quad t = T - \int \frac{\sqrt{-2\Phi}\,{\rm d}r}{1 + 2\Phi}\,, \tag{29}$$

$$\Phi = -\frac{Gm}{r} \equiv -\frac{1}{2}\,H^2 r^2\,, \quad m = M - 4\pi \int_{r_0} p(r)\,r^2\,{\rm d}r\,. \tag{30}$$

The source of the Schwarzschild metric is the mass M of the star with the continuous potential $\Phi = -GM/r$ at the star boundary. The discontinuity of the functions N, t, and g_{RR} is due to the density jump at $R = 1$. The effective matter outside the star emerges at $r \leqslant r_0$ and preserves the symmetry of the parent metric. The continuity of Φ suggests that $m(0) = 0$ and leads to the formulas in Section 4.

By extending the reference frame of free particles to the entire cone $\rho = r^2 \geqslant 0$, we continue the metric from the black-hole region ($r > 0$) to the white-hole region ($r < 0$), bypassing the body of the star itself ($R > 1$) [see (14)–(18)]. Closing the external metric, we then restore the complete solution on the entire manifold $R \geqslant 0$ by matching p at the star boundary (Fig. 2). As a result, everywhere in the effective matter zone, we obtain $P = 2\lambda_0/3$ and

$$\begin{aligned} |r| \leqslant r_0 \tilde{R}: \quad & r = -\sqrt{3} r_0 \tilde{R} \sin(H_1 \tilde{T})\,, \\ & \Phi = \frac{3}{2}\,\Phi_0 \tilde{R}^2 \cos^2(H_1 \tilde{T})\,, \end{aligned} \tag{31}$$

where $\tilde{R} \equiv \min(R, 1)$ and $\tilde{T} \equiv T + (1 - R)\,\theta(R - 1)$. The state of effective matter inside the star is described by the equation $\epsilon + 3p = 2\lambda_0$.

Freely falling shells of the star intersect simultaneously at one point $r = 0$ separating the time-like and space-like parts of the axis $r = 0$ (see Fig. 2). At this bifurcation point, the equation of state of the effective matter is $p = -\epsilon/3$ and the motion is rectilinear: $a \propto T$. Gravity is 'switched off' due to the mass ar $r = 0$, which provides the finite amplitude of tidal forces and the continuation of matter world lines into the future.

7. Physical nature of integrable singularities

The concept of integrable singularities allowed us to radically advance in solving the cosmogenesis problem using the new class of black-white-hole models in GR. Are these solutions only research tools or do they really exist? If they do, then why and how do they form? These questions require separate studies. Here, we discuss some ideas on the physical nature of integrable singularities.

Reasons for the formation of singular structures with a finite gravitational potential are related to the redistribution of matter and space–time degrees of freedom in strong gravitational fields of the singularities themselves. (At high intensities and small scales, quantum effects apparently play the key role.) That is why we talk about the effective matter, which includes both material and gravitational degrees of freedom that remain after averaging the metric over the quantum state. However, we keep notions of energy–momentum and laws of motion in the form of Bianchi identities. This allows describing the back reaction using the Einstein equations in which the left-hand side is defined using the mean metric $g_{\mu\nu}$ and the right-hand side contains the energy–momentum tensor of the effective matter T_ν^μ, including all polarization, gravitationally modified, and other terms of quantum theory that need to be calculated.

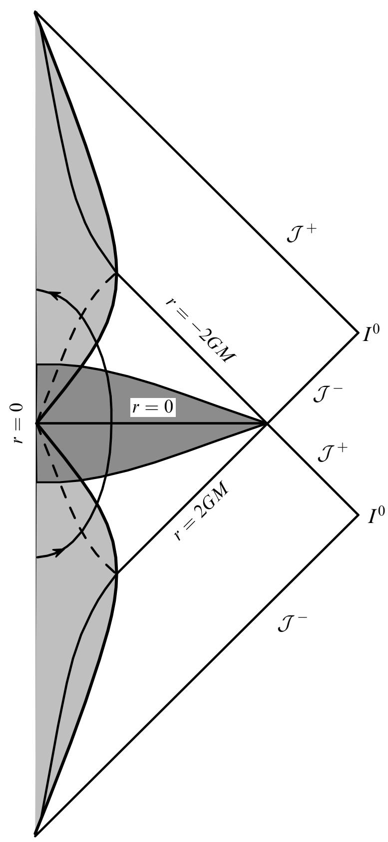

Figure 2. Penrose diagram of an astrophysical black-white hole. The light gray region is the body of the star, the dark gray region is the effective matter, the dashed line separates the R- and T-regions inside the star, and the line with arrows is a contour $t = $ const.

We illustrate the method by considering the development of one degree of freedom in the given physical symmetry of a collapsing star. Let the physical variable be described by a massless field φ minimally coupled to metric (25). Secondary quantization leads to the following equation for the amplitudes of Fourier harmonics $\varphi_k = v_k(\eta)/a$ [5]:

$$v_k'' + (k^2 - U)\,v = 0\,, \tag{32}$$

$$U = U(\eta) = \frac{a''}{a} = a^2 H^2 + (aH)' = -2\Phi + \frac{\Phi'}{\sqrt{-2\Phi}}\,, \tag{33}$$

where the prime denotes the derivative with respect to the conformal time $\eta = \int {\rm d}T/a$, k is the wave number, and Φ is the gravitational potential on the stellar surface.

As long as the pressure in the star is low, the tidal potential increases with decreasing the radius: $U = GM/a = a^2 H^2/2$.

At large a, the state of the field is vacuum-like: it oscillates near an equilibrium point in the adiabatic zone $U < k^2$.

During the collapse, the field enters the parametric zone, and its amplitude unlimitedly increases with increasing $U > k^2$:

$$\varphi_k = \frac{\exp(-\mathrm{i}k\eta)}{a\sqrt{2k}} \to \frac{H}{k^{3/2}}\,, \quad \varphi_{,\mu}\varphi^{,\mu} \simeq H^4 \ln U\,. \tag{34}$$

At this stage, the pressure cannot be neglected any more; the vacuum is polarized and in turn affects the metric, which changes the gravitational potential and the rate of collapse near $a = 0$ (for the solar mass and the number of degrees of freedom ~ 100, this occurs at $H \simeq 0.1\,M_{\mathrm{P}}(\ln U_0)^{-1/2} \sim 10^{17}$ GeV).

We can suppose that the back reaction described by the Friedman equations restructures the solution such that tidal potential (33) stops increasing infinitely and saturates: $U(r \to 0) = U_0 = -2\Phi_0 = \mathrm{const}$. This precludes an ultraviolet catastrophe and saves oscillators from being destroyed, because high-frequency modes with $k^2 > U_0$ remain in the adiabatic zone and are not polarized. Thus, the notion of the Heisenberg state vector stays valid, which corresponds to the Minkowski vacuum in the R-region, and the main requirement of the metric theory on the finiteness of the gravitational potential on a singular hypersurface is secured. We call such a structure an integrable singularity.

This example illustrates the difference between our theory and models with bounces (see, e.g., [6]), which have no singularity, $a > 0$, and $H = \Phi = 0$ at the bounce. The last requirement, in our opinion, is redundant and is not a necessary condition. In our models, the potential Φ reaches an extremum at the singularity $r = 0$, which weakens the singularity but does not fully eliminate it. For $a \propto T$, the situation resembles the explosive model of nongravitating particles by Milne. At the moment of crossing, particles move by inertia and do not feel the central mass attraction. Gravitation is switched off at this moment ($m(0) = 0$); therefore, the space does not curve, although the energy density diverges.

8. The arrow of time and natural selection

Gravitational tidal models of black-white holes containing expanding flows of matter elucidate many fundamental unanswered questions in modern physics. One of them is the causality principle, which puts the causal relationships in correspondence with the arrow of time. In the context of our work, we can discuss the origin of the cosmological arrow of time, which we regard as the orientation of the future light cone in the direction of the volume expansion of a large-scale flow of matter.

As is well known, dynamical equations describing microscopic processes are invariant under the change in the sign of time. However, the local dynamics have to be completed with an external arrow of time, because the invariance under time reversal is lost in the limit transition to the global geometry.

Geodesically full geometries with integrable singularities suggest the origin of the arrow of time. They include different space–time domains separated by event horizons $r = 0$ and $r = 2GM$: nonstationary regions of black and white holes (lying between the Schwarzschild and singular horizons), alternating static R-zones (connecting the Schwarzschild horizons of white and black holes), and AdS zones (connecting singular horizons of black and white holes). Here, all possibilities are realized. Each collapsing and anticollapsing (cosmological) region has its own time. The static regions are time-independent, but are not spatially homogeneous.

We are dealing with a unique geometry that is split into sectors with time and space parity. When crossing any of the horizons, the meaning of the coordinate r on which the metric depends changes [7]. In some domains, r is the time coordinate (and we then obtain black holes and/or cosmological models), while in other domains, it is a spatial coordinate (static zones). The complete geometry can then remain invariant under the change $r \to -r$.

Therefore, the origin of the cosmological arrow of time is related to the initial conditions. We (observers) belong to a cosmological flow of matter and live in its proper time, which started 14 bln years ago at the moment $r = 0$. The time coordinate can be extrapolated into the past, to the precosmological epoch, but there it described the time in the T-zone of the parent black hole. In the even more remote past, this coordinate represented the radial coordinate of an asymptotically flat space of the parent universe in which the star had lived before it collapsed into a black hole. The integrable singularity $r = 0$ that emerged during the collapse 'kindled' our Universe, and after several billion years, the nonlinear large-scale structure began developing to initiate the process of star formation. As a result, new black holes appeared; they can be entrances to new universes.

This process of evolution of a multisheet space–time resembles the growth of a tree (a genealogical tree, so to speak). Such a tree can flourish, or can wither if no new black holes form in the daughter universes. The critical situation appears when the development conditions do not provide the production of seed density fluctuations to form gravitationally bound matter clumps and their collapse into black holes. But another scenario can be realized: one collapse under favorable conditions, which gave rise to inflationary parameters and phase transitions in a white hole, can lead to the flourishing of a whole tree with nondecaying chains of new universes. Because of these processes, cosmological natural selection works [8]: only universes where black holes can be formed survive and develop, and this is possible for a certain set of parameters, world constants, etc.

This concept of a multisheet universe is based on the gravitational instability processes, which resemble oscillating tides. Anticollapsing space–time regions (white holes) stem from collapsing black holes, and, conversely, an expanding quasihomogeneous flow of matter of a white hole disintegrates into clumps collapsing into black holes. The former process is related to the $r = 0$ horizon and the latter is related to the $r = 2GM$ horizon. At both horizons, the gravitational potential is relativistic, and quantum gravitational processes of vacuum polarization and pair creation should be taken into account [9]. However, while these effects are suppressed by the mass parameter at the Schwarzschild horizon (the Hawking evaporation), they dominate at $r = 0$ and form structures of integrable singularities.

9. Conclusions

In the framework of our concept of integrable singularities, a new class of black-white holes in GR is obtained, which can be the key to solving the cosmogenesis problem. An integrable singularity at $r = 0$ can be compared with a classical cusp, in which the energy density or the longitudinal tension of matter diverges, but the mass is zero,[3] $m(0) = 0$, and the gravitational potential $r = 0$ is bounded. Because of this property, the tidal forces are finite and any geodesics freely extend from a black hole into a white hole, where the geometry is equivalent to that of an expanding cosmological model.

The mass of matter in a multisheet universe can be arbitrarily large, because it is compensated by the negative gravitational binding energy. Hence, the total energy of holes, measured in static zones, is constant in time. The integrable singularities are reminiscent of machines for reprocessing gravitational degrees of freedom into material ones; however, the quantitative characteristics of this process can be determined only in the self-consistent quantum theory.

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10. Appendix. Motion of test particles in a black-white hole

We consider the motion of test particles in metric (2), which is independent of the time t. Let $k^\mu = dx^\mu/d\lambda$ and λ be the tangent vector, and the affine parameter along the trajectory be $x^\mu = x^\mu(\lambda)$. From the geodesic equation $k^\mu k_{\nu;\mu} = 0$, we obtain

$$k \equiv k_t = \text{const}\,, \quad k_\perp \equiv k_\theta + \frac{k_\varphi}{\sin^2\theta} = \text{const}\,, \tag{35}$$

where $k_\varphi = \text{const}$ is the azimuthal angular momentum. Unlike invariants of motion for the longitudinal and transversal momenta, the value of the azimuthal number depends on the orientation of the polar coordinate system. By adjusting the pole $\theta = 0$ with one of the points on the trajectory, we have $\varphi = \text{const}$ and $k_\varphi = 0$ everywhere on the world line of a particle. In the projection on the 2-sphere, the particle moves along a meridian with the angle θ monotonically increasing within the interval 2π by the number of turns in $\mathbb{S}^2$:

$$\frac{d\theta}{d\lambda} = -\frac{k_\perp}{\rho}\,, \quad \left(\frac{d\rho}{2\,d\lambda}\right)^2 - (k_\perp^2 + n\rho)\,\mathcal{K} = \frac{k^2\rho}{N^2}\,. \tag{36}$$

The equation for $\lambda(\rho)$ follows from the normalization integral $k_\mu k^\mu \equiv n = \text{const}$, where $n = 0$ for the null and $n = 1$ for the time-like geodesics. Inside the cone $\rho = r^2 \geqslant 0$ of a black-white hole, Eqn (36) has the form

$$\left(\frac{dr}{d\lambda}\right)^2 + \phi = \frac{k^2}{N^2} - n\,, \quad \phi = \frac{k_\perp^2}{r^2}\,(1 + 2\Phi) + 2n\Phi\,, \tag{37}$$

where the potential $\phi = \phi(r)$ tends to zero as $r \to \infty$ (Fig. 3).

The longitudinal light geodesics ($n = k_\perp = 0$) propagate directly from the black hole into the white hole with the continuous affine time $\lambda = -k^{-1}\int N\,dr$. The spiral light

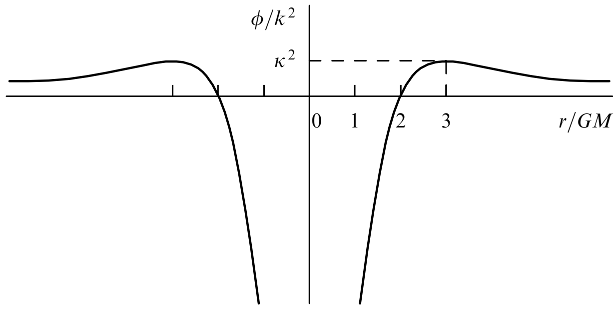

Figure 3. The potential $\phi(r)$ of light geodesics in a black-white hole.

geodesics on the cross sections ($n = k = 0$) lie in the T-region of the black or white hole, $\mathcal{K} = K^2 \geqslant 0$, coming from AdS zones and going back to AdS zones: $\theta = -k_\perp \int d\lambda/\rho = \pm\int d\rho/(2\rho K)$. Of special interest are photons with $n = k = k_\perp = 0$ living on the horizons $\rho = 0$:

$$\theta = \pm\frac{\ln|\lambda - \lambda_0|}{2K_0}\,, \quad \rho = \mp 2K_0(\lambda - \lambda_0)\,k_\perp \to 0\,, \tag{38}$$

where λ_0 is the value of the affine parameter on the bifurcation line.

Trajectories of photons with the impact parameter $\kappa \equiv k_\perp(3\sqrt{3}GMk)^{-1} < 1$ connect the R- and T-regions. Photons with $\kappa = 1$ return to R-regions at the radius $r = 3GM$. For $\kappa > 1$, there are photons of two kinds: in the R-region with $r > 3GM$, and in the zone $r < 3GM$ that unites the T-region and the inner part of the R-region adjacent to it.

References

1. Lukash V N, Strokov V N, arXiv:1109.2796
2. Lukash V N, Mikheeva E V, Strokov V N *Usp. Fiz. Nauk* **182** 216 (2012) [*Phys. Usp.* **55** 204 (2012)]
3. Sakharov A D *Zh. Eksp. Teor. Fiz.* **87** 375 (1984) [*Sov. Phys. JETP* **60** 214 (1984)]
4. Hawking S, Penrose R *The Nature of Space and Time* (Princeton, N.J.: Princeton Univ. Press, 1996)
5. Lukash V N, Mikheeva E V *Fizicheskaya Kosmologiya* (Physical Cosmology) (Moscow: Fizmatlit, 2010)
6. Novikov I D *Pis'ma Zh. Eksp. Teor. Fiz.* **3** 223 (1966) [*JETP Lett.* **3** 142 (1966)]
7. Novikov I D *Astron. Zh.* **38** 564 (1961) [*Sov. Astron.* **5** 423 (1961)]
8. Smolin L *The Life of the Cosmos* (Oxford: Oxford Univ. Press, 1999)
9. Frolov V P, Shapiro I L *Phys. Rev. D* **80** 044034 (2009)

[3] The time-like axis of the zero mass (the line of the star center) lies in the R-region, and the mass function can be found by integrating the density over radial shells [see (9)]: $m = 4\pi\int_0 \epsilon(r)\,r^2\,dr$. The space-like axis of the zero mass (bifurcation lines [see (13)]) lies in the T-region, and the mass function of a black-white hole can be obtained by integrating the longitudinal tension over time: $m = -4\pi\int_0 p(r)\,r^2\,dr$.