METHODOLOGICAL NOTES

Zero point energy and zero point oscillations: how they are detected experimentally

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Contents

Abstract. The zero point energy of a system in a potential well is reviewed as a concept, with some history of the development behind it, and a discussion is given of how it can be detected experimentally from the electronic-vibrational spectrum of molecules with different isotopes (isotope effect). Also discussed is how the zero point oscillations of crystal lattice atoms show up in the diffraction of X-rays and neutrons from crystals and in the temperature dependence of the Mössbauer effect probability. Other topics include measuring zero point oscillations of water molecules in a nanotube to determine the form of the potential energy of the system; the role of zero point oscillations in the dynamics of electrons in semiconductors, and experiments on the optical cooling and quantum behavior of mechanical oscillators.

1. Introduction

In classical physics, the total energy of a particle undergoing the action of a certain conservative force with potential U is expressed as

$$E = \frac{mv^2}{2} + U(r). \tag{1}$$

The particle's ground state can be found by 'reducing' its velocity to zero and placing the particle at a point where its

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potential energy U is minimal. In this case, however, the result is none other than the classical approximation to the real world. The fact is that in classical physics the particle's position in space and its momentum (velocity) are determined unambiguously at variance with the fundamental principles of quantum physics, where the particle's position in the phase space is specified by a certain region of size $(2\pi\hbar)^3$ rather than a point, in agreement with the uncertainty principle. This means that the particle possesses the so-called zero point energy E_0, i.e. its total energy is higher than the minimal classical energy by E_0. As shown for the first time by Werner Heisenberg in 1925 [1], the zero point energy of a particle in a one-dimensional parabolic potential (harmonic oscillator) equals $\hbar\omega/2$, where ω is the classical circular frequency of the particle's oscillations (in a three-dimensional harmonic potential one has $E_0 = 3\hbar\omega/2$). The zero point energy playing the role of an additive constant can be regarded as the energy counting level.

It is natural to relate zero point energy to a certain motion; therefore, the particle is said to undergo zero point oscillations. Certainly, this assertion stems from a mere analogy with classical physics; there are actually no 'classical' movements of the particle, its momentum in the stationary state having no definite value (i.e. the particle lacks a definite trajectory). According to the quantum-mechanical description, the particle's state is determined by the wave function $\psi(r)$, the absolute value squared of which gives the probability distribution of coordinate values: $|\psi|^2 \, \mathrm{d}q$ is the probability that measurement of the system will reveal the particle coordinate values in element $\mathrm{d}q$ of the configuration space.

A natural question that arises is whether zero point energy can be detected and zero point oscillations observed in experiment.

Although zero point oscillations are considered in many monographs and textbooks, the problem of their experimental validation is rarely discussed, with only some equivocal phrases to the effect that light scattering experiments give

direct evidence of this phenomenon. By way of example, D I Blokhintsev writes in his book, *Fundamentals of Quantum Mechanics* [2]:

"It is possible to prove experimentally the existence of zero point energy and zero point oscillations of the atoms by observation of light scattering from crystals. Light scattering is due to atomic oscillations. According to the classical theory, their amplitude must decrease infinitely with decreasing temperature; in parallel, light scattering must disappear. It is known from experience, however, that the intensity of light scattering tends toward a certain limiting value as temperature drops. This suggests that atoms continue to oscillate even at absolute zero. This fact confirms the existence of zero point oscillations."

Many researchers supported practically the same statement, viz. Zhdanov [3], Migdal [4], Feinberg [5], Levich et al. [6], Savel'ev [7], Irodov [8], Martinson and Smirnov [9], Mostepanenko and Trunov [10], and El'yashevich and Barsukov [11]; it even found its way to the Internet [12]. Taken together, these observations encouraged the author to look into the problem in more detail. In what follows, it will be shown that the assertion about the matter under consideration in the publications listed above is not quite correct.

2. Development of the zero-point energy concept

As mentioned in the Introduction, Heisenberg was the first to propose the quantum-mechanical explanation of zero point energy in 1925. However, the term itself had been coined even earlier, in 1911, i.e. before the advent of quantum mechanics, in the so-called second article by Max Planck [13]. The reaction of the scientific community to the appearance of the term 'zero point energy' is described in the book by Frenkel and Yavelov [14]. In the foreign literature, the same issue is dealt with in the work by Milonni and Shih [15], and Milonni [16, 17].

In the first article devoted to the equilibrium thermal radiation theory, Planck considered a system of a large number of resonators and radiation, assuming that the energy of an oscillator absorbing and emitting thermal radiation of frequency v is always the product of a quantum of energy hv and an integer. As the reader knows, Planck did not like that he had to 'make' the oscillator absorb and emit the energy in separate portions, hv, to explain the experimentally observed radiation spectrum of a black body. In an attempt to avoid or at least smooth over this striking contradiction with classical electrodynamics, Planck created in 1911 his second quantum theory, in which only emission of energy was regarded as discrete, while its absorption was assumed to be continuous. This approach led Planck to derive the experimentally verified formula for the emission spectrum of a black body, but he found himself facing a new puzzle. Because the oscillator absorbs energy smaller than a quantum, which it cannot emit (the energy continues to be emitted in whole quanta), this fraction of the quantum energy will never leave the oscillator, even at absolute zero temperature, when its vibrations must totally stop (in agreement with the classical theory) and energy must fall to zero. It followed from Planck's second theory that the mean oscillator energy at absolute zero temperature is $hv/2$ rather than 0.

Let us consider in more detail how Planck arrived at the conclusion about the existence of zero point energy. He suggested that radiation is absorbed in conformity with the classical theory, whereas the energy is emitted in discrete portions (quanta). Let the oscillator emit only after it has continuously absorbed energy hv, and let P_n be the probability that the oscillator energy lies in the range between $(n-1)\,hv$ and nhv. When the energy becomes nhv after the oscillator has absorbed the radiation, it will emit this energy with probability p and will continue to absorb the incident radiation with probability $1-p$. Thus, the subsequent probabilities are as follows:

$$P_2 = P_1(1-p), \quad P_3 = P_2(1-p) = P_1(1-p)^2, \ldots,$$
$$P_n = P_1(1-p)^{n-1}, \tag{2}$$

while

$$\sum_{n=1}^{\infty} P_n = \sum_{n=1}^{\infty} P_1(1-p)^{n-1} = \frac{P_1}{p}, \tag{3}$$

but sum (3) must equal 1. Therefore, $P_1 = p$ is the probability that the oscillator is in equilibrium with the radiation and its energy lies between 0 and hv; $P_2 = p(1-p)$ is the probability that its energy falls between hv and $2hv$. Accordingly, $P_n = p(1-p)^{n-1}$ is the probability that the oscillator energy falls within the range between $(n-1)\,hv$ and nhv.

It follows from the above that oscillator entropy is expressed in the form

$$S = -k_B \sum_{n=1}^{\infty} P_n \ln P_n = -k_B \sum_{n=1}^{\infty} p(1-p)^{n-1} \ln\left[p(1-p)^{n-1}\right]$$
$$= -k_B\left[\frac{1}{p}\ln p + \left(\frac{1}{p}-1\right)\ln\left(\frac{1}{p}-1\right)\right], \tag{4}$$

k_B is the Boltzmann constant. Furthermore, Planck suggested that with equal probability an oscillator has an energy in the interval from $(n-1)\,hv$ to nhv, which accounts for its average energy within this range:

$$\overline{U}_n = \frac{1}{2}\,(n+n-1)\,hv = \left(n-\frac{1}{2}\right)hv. \tag{5}$$

Thus, the average oscillator energy takes the form

$$\overline{U} = \sum_{n=1}^{\infty}\left(n-\frac{1}{2}\right)hvP_n = hv\sum_{n=1}^{\infty}\left(n-\frac{1}{2}\right)p(1-p)^{n-1}$$
$$= \left(\frac{1}{p}-\frac{1}{2}\right)hv, \tag{6}$$

i.e. $1/p = \overline{U}/hv + 1/2$, and the following expression holds for entropy:

$$S = k_B\left[\left(\frac{\overline{U}}{hv}+\frac{1}{2}\right)\ln\left(\frac{\overline{U}}{hv}+\frac{1}{2}\right) - \left(\frac{\overline{U}}{hv}-\frac{1}{2}\right)\ln\left(\frac{\overline{U}}{hv}-\frac{1}{2}\right)\right]. \tag{7}$$

In the end, Planck made use of the thermodynamic relation $\partial S/\partial U = 1/T$ and arrived at the following formula for the equilibrium radiation energy:

$$\overline{U} = \frac{1}{2}\,hv\,\frac{\exp\left(hv/k_BT\right)+1}{\exp\left(hv/k_BT\right)-1} = \frac{1}{2}\,hv\,\frac{\exp\left(hv/k_BT\right)-1+2}{\exp\left(hv/k_BT\right)-1}$$
$$= \frac{hv}{\exp\left(hv/k_BT\right)-1} + \frac{1}{2}\,hv. \tag{8}$$

It follows from the last formula that $U \neq 0$ as $T \to 0$, i.e. the energy $\bar{U} \to h\nu/2$ as $T \to 0$.

It seems that substitution of the finite zero point energy for quantum absorption gives no additional advantage as regards reconciliation with the classical picture. However, Planck was more content with his second theory just because it gave him an opportunity to get rid of Einstein's light quanta that he disliked so much, even if they allowed the photo-electric effect to be successfully explained. According to Max Born, this strange theory initially received a rather tepid reception, although Planck himself considered it very valuable. From the standpoint of modern quantum mechanics, the second Planck theory cannot be considered valid; however, the conclusion about the existence of energy at absolute zero temperature (what one might call zero point energy) proved correct.

This idea was soon taken up by Einstein and Stern [18], who showed that in this case the Planck spectrum can actually be derived from the purely classical consideration. These authors demonstrated that the Planck spectrum of thermal radiation can be easily derived from the postulate of zero point energy of the oscillator, i.e. on the assumption that its average energy is $U + \hbar\omega$. True, the zero point energy postulated by Einstein and Stern was twice that of Planck. It therefore looks strange that they arrived at the right result. However, Einstein and Stern ascribed the total zero point energy to the material dipole oscillator instead of taking into consideration the zero point energy of the field. Milonni and Shih [15] showed that the addition of $h\nu/2$ to the energy of each mode of the electromagnetic field really leads to the correct Planck formula

$$\bar{U} = \frac{h\nu}{\exp\left(h\nu/k_\mathrm{B}T\right) - 1} \,. \tag{9}$$

Einstein and Stern noticed an odd circumstance concerning formula (9). Let us consider the classical limit of formula (9), i.e. the case of $k_\mathrm{B}T \gg \hbar\omega$:

$$U = \frac{h\nu}{\exp\left(h\nu/k_\mathrm{B}T\right) - 1}$$

$$\approx \frac{h\nu}{1 + h\nu/k_\mathrm{B}T + (1/2)\left(h\nu/k_\mathrm{B}T\right)^2 - 1} \approx k_\mathrm{B}T - \frac{1}{2}\,h\nu\,, \tag{10}$$

In this case, the quantity $U + h\nu/2$ rather than U tends toward the classical limit $k_\mathrm{B}T$, and zero point energy needs to be added to the mean energy of the oscillator, as Planck did. Of course, there is no reasonable cause to take this conclusion seriously.

Nevertheless, the idea of zero point energy soon became popular. The paper by Einstein and Stern [18] was followed by numerous theoretical calculations making use of the notion of zero point energy, as well as of movements, and oscillations at zero temperature for the explanation of various uncompre-hended experimental dependences; however, most of this work was speculative and poorly substantiated.

The discussion of zero point energy at the 2nd Solvay Congress (autumn 1913) was provoked by Laue's lecture on X-ray diffraction from the crystal lattice [19]. Nernst addressed in his report the question of the reality of zero point oscillations of the atoms in the crystal lattice, corre-sponding to zero point energy. Given that these oscillations are as real as usual elastic oscillations responsible for the heat capacity of a solid body, they must somehow influence the X-ray diffraction pattern considered below. Einstein strongly opposed such an interpretation of zero point energy in the discussion on this issue. He said: "There are serious argu-ments against the hypothesis that zero point energy is related to elastic oscillations. Indeed, if the energy of elastic oscillations tends, as it decreases, toward a finite positive value rather than zero, then all temperature-dependent properties of solid bodies should be expected to exhibit a similar dependence, i.e. a tendency toward constant and finite values at low temperatures."

During further conversational exchange over the issue, Einstein noted that, in his opinion, the zero point energy of the Planck oscillator is not equivalent to zero point elastic oscillations of the crystal lattice, which, if any, must inevitably manifest themselves in the X-ray diffraction pattern in the crystal, as postulated by Nernst. It will be shown below that it is the influence of zero point oscillations on the X-ray diffraction pattern that was the first experi-mental evidence of their existence.

That an electromagnetic field must also have zero point energy was first suggested in 1916 by Nernst [20], who analyzed the conclusion about the existence of zero point energy from the second Planck article on thermal radiation and the results of Einstein and Stern. He arrived at the conclusion that the Universe must be filled with a huge zero point energy. Lamb noted that absolute vacuum in the classical sense is nonexistent.

3. Isotope effect in electronic-vibrational spectra of molecules

R S Mulliken [21] was the first to point out the reality of zero point energy in 1924. He analyzed the emission spectra of two BO molecules containing light (^{10}B) and heavy (^{11}B) boron isotopes. His conclusion was later confirmed by Jenkins and de Laszlo [22].

The optical spectra of atoms are called line spectra, since they consist of separate lines that reproduce the shape of the spectrometer entrance slit. The spectra of molecules are referred to as banded, since they contain spectral bands. Let us consider the vibrational spectra of two diatomic molecules differing only in the isotopic composition of their constituent atoms. The reduced masses μ of these two oscillatory systems being different, their vibrational frequencies $\omega = \sqrt{\gamma/\mu}$ are different, too (γ is the vibrational rigidity of the molecule). This difference is small but detectable. The vibrational spectrum of each of the two molecules is described, taking account of anharmonicity, by the expression

$$E_v = \hbar\omega\left[\left(v + \frac{1}{2}\right) - x_\mathrm{e}\left(v + \frac{1}{2}\right)^2 + y_\mathrm{e}\left(v + \frac{1}{2}\right)^3 + \dots\right], \tag{11}$$

where constants ω, x_e, $y_\mathrm{e}, \dots$ are determined by concrete properties of the molecule, and v is the vibrational quantum number: $v = 0, 1, 2, \dots$. The energy difference between the vibrational states corresponding to vibrational numbers v and v' is given by the expression

$$\hbar\omega_{v,v'} = \left|E_v - E_{v'}\right|. \tag{12}$$

When analyzing the vibrational structure of the electronic spectra of the two molecules, ^{10}B ^{16}O and ^{11}B ^{16}O, Mulliken

found out that the best description of emission spectra is possible only by taking into account the term 1/2 (i.e. the zero point energy of the molecules) in formula (11). It should be specially emphasized that the energy of excited vibrational states sets the position of vibrational levels with respect to the state with $v = 0$, rather than to the minimum of the potential energy curve. However, comparison of the band systems of the two different isotopic molecules implies knowledge of the absolute positions of the vibrational levels of the two electronic states, i.e. the distance from these levels to the minima of the respective potential curves. Therefore, the difference $\Delta(\hbar\omega)$ between the energies of states v' and v'' should be written out in the form

$$\Delta(\hbar\omega) = \hbar\omega'_{\mathrm{e}} + \hbar\omega'_{\mathrm{e}}\left(v' + \frac{1}{2}\right) + \hbar\omega'_{\mathrm{e}}x'_{\mathrm{e}}\left(v' + \frac{1}{2}\right)^2 + \dots$$

$$- \left[\hbar\omega''_{\mathrm{e}}\left(v'' + \frac{1}{2}\right) + \hbar\omega''_{\mathrm{e}}x''_{\mathrm{e}}\left(v'' + \frac{1}{2}\right)^2 + \dots\right] \quad (13)$$

with its half-integer vibrational quanta. Here, one prime labels quantities related to the excited state, and the other denotes those related to the ground state.

It is natural to think that electronic energies and potential curves for each electronic state of the two isotope molecules, such as ^{10}BO and ^{11}BO, are practically identical, the number of electrons and the field in which they move being equal in both cases. Therefore, the energies of purely electronic transitions ($\hbar\omega$) for the two isotopic molecules are equal, as well. However, the vibrational band system of the heavier molecule is compressed compared with that of the lighter molecule (Fig. 1).

Table 1 shows isotope shifts (in cm^{-1}) in the v' series at $v'' = 0$ for α-bands of the BO molecule, which were measured by Jenkins and McKellar [23].

Of special importance is the directly observable difference $E^l_{00} - E^h_{00}$ between the energies of the transition from the vibrational state $v' = 0$ to $v'' = 0$ (0–0 band displacement). This fact gives direct evidence of the existence of zero point energy. Moreover, isotope shifts calculated on the assumption of half-integer vibrational quanta are also in excellent agreement with experiment.
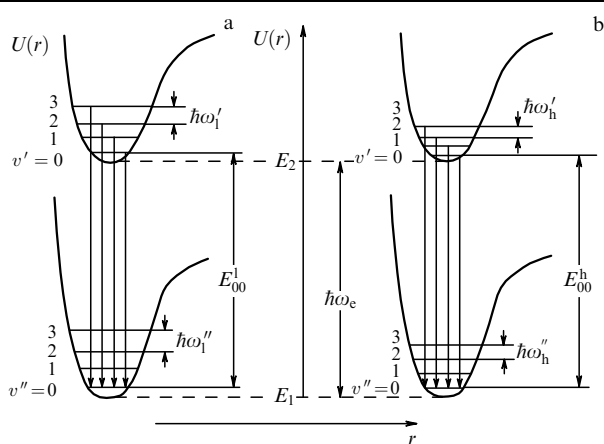
Figure 1. Qualitative picture of electronic–vibrational transitions: vibration spectrum of a molecule with a light (a) and heavy (b) isotope (the scale is grossly distorted for illustrative purposes). Arrows show transitions from states v' to states $v'' = 0$ observed in the emission spectra. Indices l and h label the light and heavy boron isotopes, respectively.

Table 1. Isotope shifts [cm^{-1}] in the v' series at $v'' = 0$ for the α-bands of a BO molecule [23].

Band	Observed energy difference ($^{10}BO - {}^{11}BO$)	Calculated results	
		Taking account of zero point energy	Disregarding zero point energy
0–0	−8.6	−9.08	0
1–0	+26.7	+26.29	+35.69
2–0	+60.8	+60.36	+70.09
3–0	+93.6	+93.14	+103.20

The nonzero isotope shift for the 0–0 band is due to the difference between zero point energies in the upper and lower states for the two isotopomeric molecules. The reason is that the potential energy curves for the ground and excited electronic states are different, i.e. they correspond to different vibration stiffness and therefore to different vibrational frequencies of the molecule. Let us consider, by way of example, a hydrogen molecule involving only two electrons. If one of them (having a large quantum number n) travels far from the nucleus, the attraction between the two atoms is quite different from that in the case when both electrons are very close to the nucleus.

Thus, suppose there are two diatomic molecules in which one of the molecules has identical atoms (μ_1), while the other is composed from atoms with different masses ($M_1 < M_2$), i.e. the reduced mass of the molecule with the light isotope (μ_1) is smaller than that of the molecule with the heavy isotope (μ_2). Let the vibrational frequency of the atoms in the excited electronic states be $\beta < 1$ times lower due to the difference between the potential curves of the molecules (for both isotopic molecules), i.e., $\omega' = \beta\omega''$. Furthermore, let the vibrational frequency $\omega = \sqrt{k/\mu}$ be α times higher ($\alpha > 1$) in the molecule with a smaller mass of one of the atoms by virtue of the different reduced molecular masses. Then, the difference between the energies of 0–0 transitions from the state $v' = 0$ to the state $v'' = 0$ in these molecules (without regard to anharmonicity) is equal to

$$\Delta E_{00} = E^l_{00} - E^h_{00} = \hbar\omega_{\mathrm{e}} + \frac{\hbar}{2}\,\omega''_l(1 - \beta) - \hbar\omega_{\mathrm{e}}$$

$$- \frac{\hbar}{2}\,\omega''_h(1 - \beta) = \frac{\hbar}{2}\,(1 - \beta)(\omega''_l - \omega''_h)$$

$$= \frac{\hbar}{2}\,\omega''(1 - \beta)(\alpha - 1)\,, \quad (14)$$

where $\hbar\omega_{\mathrm{e}}$ is the energy difference between excited and ground electronic states (see Fig. 1). Clearly, $\Delta E_{00} = 0$ both at $\alpha = 1$ (identical molecules) and at $\beta = 1$ (equal potential in the ground and excited states).

4. Diffraction of X-rays and neutrons from crystals

To begin with, we shall consider X-ray scattering. It should be emphasized that the cross section of X-ray scattering from the electrons is $\approx 10^7$ times that from the nuclei; therefore, we may take into account only the interaction between the radiation and the electrons. For the purpose of describing multielectron systems in the ground state, we introduce a single-particle function of electron density ρ. Electron density ρ is the density function of N electrons in the ground state averaged over electron–electron interactions and dependent

only on the coordinates of the points of physical space being considered. Quantity F introduced to describe X-ray scattering is called *the atomic scattering factor or formfactor*:

$$F = \int_0^\infty 4\pi r^2 \rho(r) \frac{\sin(4\pi r \sin\theta/\lambda)}{4\pi r \sin\theta/\lambda}\, dr = F\left(\frac{\sin\theta}{\lambda}\right), \quad (15)$$

where $\rho(r)$ is the electron density distribution function of an atom, and θ is the scattering angle. The atomic formfactor shows how much the scattering amplitude is greater than the amplitude of a wave scattered by the electron under the same conditions. Relative intensities of different reflections from crystal planes characterized by the Miller indices (hkl) are found by multiplying the atomic formfactor by the module squared of structure amplitude, $|F_{hkl}|^2$, which is a dimensionless quantity characterizing only the crystal structure.

Laue [24] was the first to derive formulas for X-ray scattering from crystals in 1912 on the assumption that the atoms are point-like and immobile. The influence of temperature on the intensity of X-ray reflection from the crystals was first considered theoretically by Debye in 1913 [25, 26]; it was later modified by Waller in 1925 [27]. Heating of the crystal lattice induces irregular deflections of atoms from equilibrium. In order to take thermal motion of the atoms into account, the crystal is considered to be an equilibrium thermodynamic ensemble whose members pass through all possible vibrational states during measurement. The measured intensity constitutes the average taken over all nuclear configurations. For this reason, X-ray scattering by such oscillating atoms is out of phase with that by an ideal lattice, and its intensity is lower. Evidently, the higher the temperature, the lower the intensity of the observed peaks corresponding to Bragg reflection. Debye showed that the intensities of interference maxima should be multiplied by factor $\exp(-W)$, where

$$W = \frac{6\hbar^2}{Mk\Theta}\frac{\phi(x)}{x}\frac{\sin^2\theta}{\lambda^2}. \quad (16)$$

In this formula derived for a simple cubic lattice composed of identical atoms with mass M, Θ is the Debye temperature, θ is the sliding angle of X-rays with wavelength λ, and parameter $x = \Theta/T$ is the relative inverse temperature. Function $\phi(x)$ was tabulated by Debye. Given zero point oscillations of the atoms are taken into account, $\phi(x)/x$ must be replaced by the function $\phi(x)/(x + 1/4)$ (Debye introduced zero point oscillations into his formula as early as 1913!). Waller pointed out in 1925 that the exponential factor for a simple lattice must have the form $\exp(-2W)$. The thermal motion disappears in the low-temperature limit, and the Debye–Waller factor is determined by zero point oscillations alone.

In the late 1920s, James and Firth [28] measured X-ray scattering by rock salt (NaCl) crystals at different temperatures (down to a liquid-nitrogen temperature). Based on the results thus obtained, James, Waller, and Hartree published a paper [29] that turned out to be the first experimental confirmation of zero point oscillations of atoms in a crystal lattice. Each author made an appreciable contribution to the joint work. James and his colleagues provided reliable data on X-ray scattering from NaCl crystals. Waller calculated X-ray scattering by different atoms of the crystal lattice, and Hartree computed electron density distributions in Na and Cl atoms based on the Schrödinger equation. Evidently, if the state of an electron is described by the wave function ψ_n, the
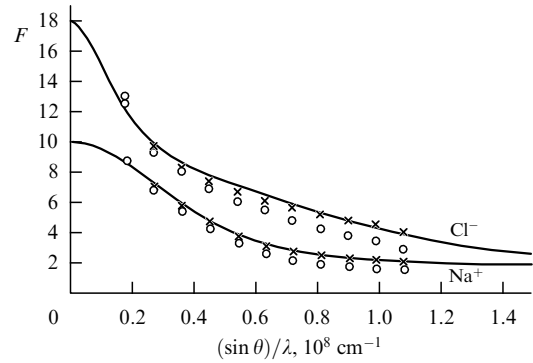
Figure 2. Comparison of the value of formfactor F obtained in experiment for Na and Cl atoms with the results of calculations (solid lines); ○ — experimental data without regard to zero point oscillations, and × — data taking account of zero point energy.

charge density distribution is $e|\psi_n|^2$. Charge density in an atom can be found by summing up numerically the contributions from all atomic electrons. It was assumed in the computations that the elementary volume of charge density in an atom scatters radiation as a free electron does, in accordance with the classical laws derived by J J Thomson. Figure 2 presents the original data taken from Ref. [29].

It shows that the experimental findings agree with the computations only if zero point oscillations of atoms are taken into account, as is especially clear at large scattering angles. This seems natural because the greatest relative contribution of zero point oscillations occurs at high-order reflections, i.e. at large values of parameter $\sin\theta/\lambda$.

Notice that it proved possible to reveal the influence of zero point oscillations on Bragg peak intensity only due to the fact that the root-mean-square amplitude of zero point oscillations is of the same order as the radius of the K-electron orbit. The authors of Ref. [29] reported this amplitude to be 0.12 Å for Na and 0.11 Å for Cl, whereas the electron density maximum of the K-electron in the Na and Cl atoms is located at a distance of 0.052 Å and 0.035 Å from their centers, respectively.

In principle, the values of the root-mean-square amplitudes $\overline{u^2}$ of zero point oscillations in a crystal is easy to estimate in the Debye approximation. Zero point oscillations occur at any frequency within the $(0, \omega_D)$ range. The following relation holds at an arbitrary frequency ω:

$$2\frac{M\omega^2\overline{u^2}}{2} = \frac{3}{2}\hbar\omega. \quad (17)$$

In the classical model, $\overline{u^2} = A^2/2$, where A is the classical oscillation amplitude (this result also holds true in quantum mechanics). Whence it follows that $A^2(\omega) = 3\hbar/M\omega$. In the Debye model, the frequency distribution function (vibrational density) $g(\omega) = 3V\omega^2/(2\pi^2 s^3)$. Therefore, one has

$$\langle A^2 \rangle = \frac{\int_0^{\omega_D} g(\omega)A^2(\omega)\, d\omega}{\int_0^{\omega_D} g(\omega)\, d\omega} = \frac{9V\hbar\omega_D^2}{4M\pi^2 s^3}\frac{1}{3N} = \frac{3(k_B\Theta)^2}{4\hbar\rho s^3}. \quad (18)$$

For an NaCl crystal, density $\rho = 2.17$ g cm^{-3}, the averaged speed of sound $s = 4.5 \times 10^5$ cm s^{-1}, and the Debye temperature $\Theta = 275$ K. Substituting these values into formula (18) yields

$$\sqrt{\langle A^2 \rangle} \simeq 0.1 \text{ Å}.$$
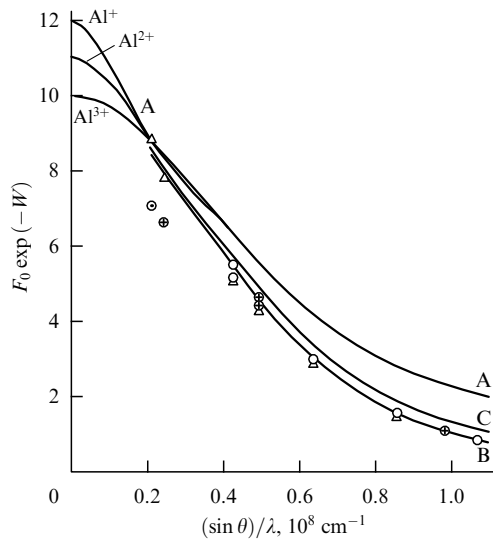
Figure 3. Angular dependence of formfactor F for aluminium at 290 K. Different symbols indicate the results of measurements of reflection from different crystal planes. Curves A correspond to calculations for immobile Al^{1+}, Al^{2+}, and Al^{3+} ions, while curves B and C to computations for Al^{1+}. Curve B was calculated taking account of zero point oscillations, and curve C without regard to their existence.

Although this estimate was obtained for a monoatomic cubic crystal, it is close to that reported in Ref. [29].

In the next year of 1929, James and co-workers carried out similar experiments with aluminium [30] and confirmed the existence of zero point oscillations of crystal lattice atoms (Fig. 3). (When discussing zero point oscillations in his textbook [2], Blokhintsev referred just to Ref. [30]).

Calculations have been made for different Al valences, since its true valence in a crystal was unknown. As shown, F values for all ions are practically identical everywhere, barring the small angle region. Curves B and C correspond to the computations for Al^{1+} at 290 K. Curve B was calculated on the assumption that zero point oscillations do exist, and curve C without regard to their existence. Different symbols indicate the values of the F factor obtained for reflection from different crystal planes.

However, these both experiments are not so-called *crucial experiments*, the authors making conclusions based on comparisons of the observed and theoretical data. In this respect, the situation with neutron diffraction by crystals is somewhat better, because neutrons are scattered from nuclei and measurements of the temperature dependence of the Debye–Waller factor would give direct confirmation of zero point oscillations, since this factor cannot be unity even at absolute zero. However, absolute measurements of the Debye–Waller factor in the case of neutrons (as well as X-rays) are hampered by weakening of the incident and diffracted beams (e.g., due to extinction) and the presence of the diffuse background. Moreover, a detecting system always possesses a finite resolving power that accounts for the broadening of the observed diffraction peaks and their decreased height. The temperature dependence of the Debye–Waller factor was measured by many authors, but most of them were focused on studying the anharmonicity of atomic vibrations, i.e. the temperature dependence of the Debye temperature. The author of the present paper is unaware of such measurements at low temperatures. The discovery of the Mössbauer effect in the early 1960s opened

up the possibility of directly observing zero point vibrations of lattice atoms (see Section 5 below).

Zero point oscillations of the atoms in crystals are interesting to discuss in conjunction with the role of zero point oscillations in the dynamics of electrons in semiconductors. Scattering of electrons from zero point oscillations of the lattice is impossible. Lattice oscillations near absolute zero are in the lowest energy state and consequently cannot transfer even a single phonon to an electron. On the other hand, the eigenenergy of the electron is too low to enable phonon production; for this reason, the electron cannot scatter, i.e. change the value of its wave vector.

Under near-equilibrium conditions, e.g. in studies of electron mobility in weak fields, the contribution from scattering by zero point oscillations associated with induced phonon emission is also small, even at low temperatures. This can be accounted for by electron gas degeneracy which renders processes with phonon emission impossible.

However, the assumption of equidistant energy distribution is invalid for hot electrons at low temperatures. Instead, the condition of $n \ll 1$ is fulfilled for the number of phonons involved in scattering, meaning that electron scattering with phonon absorption or induced emission is negligibly small; only scattering due to zero point oscillations matters [31].

Since the value of the Debye–Waller factor is practically determined at low temperatures by zero point oscillations alone, it opens up ample opportunities for studying the potential energy of the system of interest. In this respect, the experiment on the behavior of water in nanotubes described by Kolesnikov et al. [32] is very illustrative. Water in one-dimensional channels is of great interest for biology, geology, and materials science. Carbon nanotubes provide an excellent model for the investigation of such systems, because the interaction between carbon and water molecules is very weak, and this concerns practically quasi-one-dimensional water. The authors of Ref. [32] measured elastic scattering of neutrons in such a system and compared the results with similar measurements in ice. Figure 4 illustrates temperature dependences of root-mean-square vibrations of water molecules. Evidently, their amplitude in a nanotube is much higher
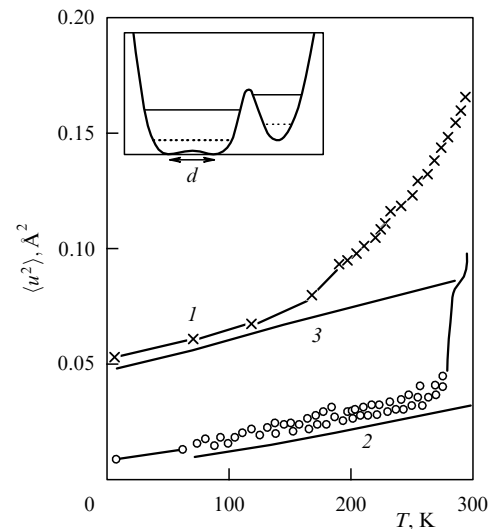
Figure 4. Temperature dependence of the root-mean-square amplitude $\langle u^2 \rangle$ of water molecule vibrations in a nanotube (curve *1*) and ice (curve *2*); curve *3* — $\langle u^2 \rangle + d^2$ (data taken from Ref. [31]).

than for ice. The authors concluded that water is present in the nanotube in a wide double-well potential (shown in the inset to Fig. 4), so that they estimated delocalization of water molecules at roughly 2 Å.

5. Manifestation of zero point oscillations in the Mössbauer effect probability

Unlike nonresonant elastic scattering of X-rays by crystals considered in the preceding paragraphs, the Mössbauer effect comprises the resonant scattering of gamma rays by the nuclei of a crystal lattice. Because emission (scattering) is exercised by the lattice nuclei, phonons are also involved in the process. Therefore, the temperature dependence of the Mössbauer effect is directly related to atomic vibrations, including both thermal and zero point vibrations. It will be shown that this fact allows the existence of zero point oscillations of atoms to be directly confirmed in experiment.

It seems appropriate to recall certain theoretical aspects of the Mössbauer effect before proceeding with the discussion of the data of interest.

For a nucleus in a crystal lattice, the energy is consumed only for lattice excitations (phonon production or absorption). The lower the lattice temperature, the fewer phonons it contains and the higher the probability of phonons being uninvolved in the emission and absorption of a gamma quantum by the nucleus [phonons comprise Bose particles for which the probability of absorption (emission) is proportional to the number of particles present in the system]. The Mössbauer effect actually exhibits the phononless emission (absorption) of gamma quanta.

In one of his early studies [33], R L Mössbauer noted that the theoretical description of the effect in question practically coincides with the theory of resonant capture of slow neutrons in crystals, developed by Lamb [34]. In the Debye model of the spectrum of crystal vibration excitations, the relative intensity f of the Mössbauer line, i.e. the fraction of the emission (absorption) processes proceeding without alteration of the vibrational state of the crystal, is actually determined by the Debye–Waller temperature factor

$$f = \exp\left(-2W\right), \tag{19}$$

where

$$2W = \frac{6R}{k_B \Theta}\left[\frac{1}{4} + \left(\frac{T}{\Theta}\right)^2 \int_0^{\Theta/T} \frac{x\,dx}{\exp\left(x\right) - 1}\right], \tag{20}$$

and Θ is the Debye temperature of the crystal. The term $1/4$ on the right-hand side of expression (20) reflects zero point oscillations of crystal atoms; evidently, the relative intensity of the Mössbauer line is never equal to unity, even at zero temperature.

The latter inference also ensues from the possibility of writing down the intensity of the Mössbauer line (as shown by Lipkin [35, 36]) in the form

$$f = \exp\left(-k_q^2 \langle x^2 \rangle\right), \tag{21}$$

where k_q is the wave number of the emitted quantum, and $\langle x^2 \rangle$ is the mean square deflection of an oscillating lattice atom from equilibrium. Thermal vibrations disappear at low temperatures and $\langle x^2 \rangle$ is determined by zero point oscillations, i.e. expression (20) never equals zero.
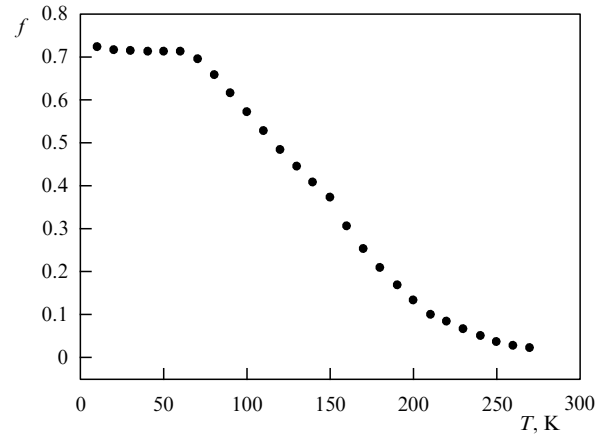

Figure 5. Temperature dependence of the Debye–Waller factor for tin (data taken from Ref. [38]).

As noted in Section 4, the data of X-ray experiments can be compared only with the calculated results. In this context, the Mössbauer effect provides more direct experimental evidence of zero point oscillations of the atoms in a crystal lattice. The physical cause behind the difference between the two experiments consists in the fact that although the γ-quantum wavelength (the resonance energy for tin is 14 keV) is practically identical in both cases but it is on the order of the atomic size in the X-ray experiment where the electron density distribution (structural factor) depends on temperature, whereas in the Mössbauer effect an X-ray quantum interacts with a nucleus, the size of which is four orders of magnitude smaller, i.e. the nucleus can be regarded with a very high degree of accuracy as a point-like scatterer.

The first experiments on measuring the temperature dependence of the probability of the Mössbauer effect were carried out by Boyle et al. [37] in 1960. These authors measured the probability of resonant absorption by tin (^{119}Sn) upon increasing temperature from 120 K to the melting point, but their primary objective was to study the anharmonicity of oscillations of lattice atoms, resulting in a temperature-dependent variation of the Debye temperature.

In 1965, Hohenemser [38] performed similar measurements at a temperature decreasing to 1.3 K. The interest in these experiments was due to the possibility of verifying calculations of the phonon spectrum, the value of $2W$ in the Mössbauer effect being determined by the integral taken over the phonon spectrum of the crystal. However, we are interested now in the behavior of the temperature factor in the low-temperature region. Figure 5 presents Hohenemser's results which show unambiguously that factor $2W$ at low temperatures depends on zero point oscillations rather than thermal ones; in other words, the Debye–Waller factor always differs from unity.

6. Observation of quantum effects in macroscopic oscillators

Quantum behavior has, until recently, been observed only in atomic and subatomic structures or in molecules. The most massive objects showing quantum behavior are fullerenes, i.e. carbon molecules containing 60 atoms (C_{60}) each. In 2002, a group of researchers at the University of Vienna examined classical diffraction of fullerenes by a diffraction grating [39]. The progress in nanotechnologies achieved in recent decades
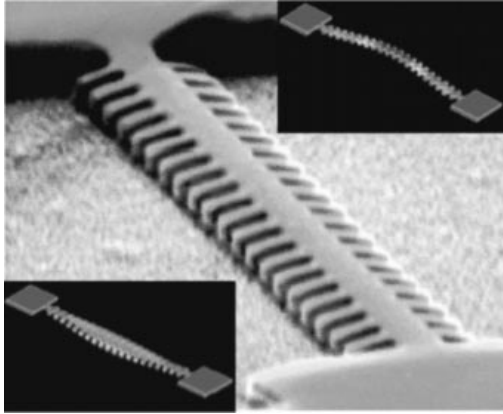
Figure 6. An image of a mechanical nanooscillator produced by scanning electron microscopy. The top inset shows schematically low-frequency vibrations of the oscillator. The inset at the bottom displays a schematic of the collective vibrational mode at higher frequencies.

has lastly provided a basis for the production and investigation of macroscopic oscillators. Realization of a quantum-mechanical oscillator is of interest for many disciplines, viz. quantum measurements, quantum computers, quantum optics, condensed medium physics, and the detectors of gravitational waves. In the current situation, the mean number $\langle n_r \rangle$ of quanta (photons) in the oscillator come to unity. In fact, the zero quantum state of the mechanical oscillator has been reached and the possibility of checking the applicability of the quantum theory to macroscopic objects, i.e. verifying the basic principles of physics, opened up.

In order to transfer a macroscopic oscillator to a state with the mean number of thermal quanta $\langle n \rangle = [\exp{(\hbar\Omega/k_B T)} - 1]^{-1} \simeq 1$ (where Ω is the resonance frequency of the oscillator), i.e., to a state close to the ground state, the vibrational quantum energy $\hbar\Omega$ must satisfy the condition $\hbar\Omega > k_B T$. This means, for example, that for a pendulum swinging with an acoustic frequency of roughly 1 kHz, temperatures on the order of 50 nK are needed that are impossible to reach by standard cooling techniques. The oscillator frequency must be around 1 GHz at real temperatures on the order of 50 mK, attainable with a refrigerator.

Two studies [40] and [41] published in 2005 and 2010, respectively, contributed to considerable progress in the creation and investigation of macroscopic oscillators.

In the former, researchers from Boston University designed an intricately shaped microscopic mechanical oscillator with a resonance frequency of several gigahertz [40]. They used electron beam lithography to fabricate a structure resembling a TV aerial or a double sided hair comb (Fig. 6). It consisted of a 10.7-μm long and 400-nm wide silicon base plate with the toothplates 500 nm in length and 200 nm in width. The total thickness of the structure was 245 nm, the silicon base plate was 185 nm thick, and the thickness of a gold electrode sprayed over the silicon was 60 nm.

Such an oscillator shape results in the appearance of two types of vibrational modes: low-frequency ones corresponding to vibrations of the construction as a whole (see the inset at the top of Fig. 6), and collective (high-frequency) modes whose eigenfrequency exceeds 1 GHz when in-phase 'tooth' vibrations cause the entire structure to vibrate (see the inset at the bottom of Fig. 6). The construction was placed in a cryostat with a superconducting magnet supplying a magnetic field of up to 16 T. Vibrations were induced by an alternating current passed through the 'comb'; it resulted in the Lorentz force acting on the structure. The researchers could observe vibrations of the mechanical oscillator by measuring a voltage drop at the gold electrode, which under such conditions was proportional to the displacement of the structure. When the frequency of the driving force coincides with one of the eigenfrequencies of system's vibrations, resonance must take place, i.e. the amplitude of oscillator vibrations must grow, as appears from a change in maximum voltage.

When working in a low-frequency region (21 MHz) at a temperature of 60 mK, the researchers could see a purely classical picture, i.e. the amplitude of oscillator vibrations gradually increasing with the magnetic field and, accordingly, the increasing force that acted on the structure. They observed just the same picture at a frequency coincident with the eigenfrequency of the high-frequency collective mode (about 1.5 GHz) at a temperature of 1 K (Fig. 7).

However, the picture was qualitatively different as the temperature fell to 110 mK, at which the value of $k_B T$ became comparable to $h\nu$, and the quantum character of vibrations could be expected to manifest itself. Specifically, the vibration amplitude did not undergo continuous alteration under changes of the magnetic field. Instead, the researchers observed voltage jumps between two specified values (Fig. 8). Such jumps can be interpreted as transitions of the quantum oscillator between the ground and first excited states.
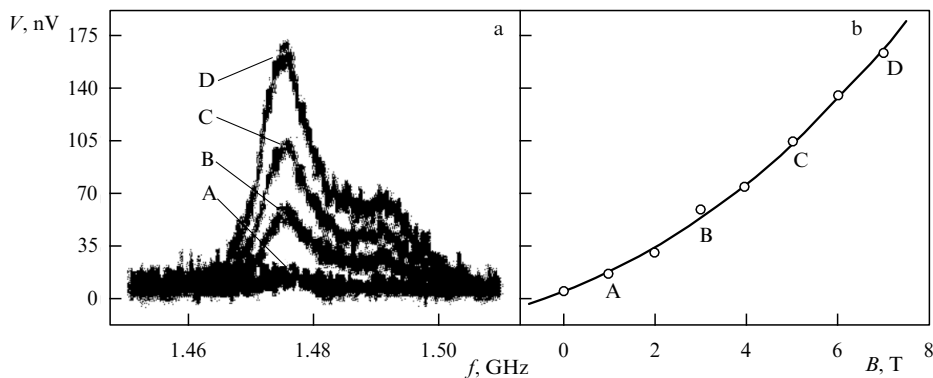
Figure 7. The classical response of the collective vibrational mode of a resonant structure at 1 K. (a) The dependence of maximum voltage and, accordingly, vibration amplitude on the driving force frequency near resonance. (b) The dependence of the maximum voltage at resonance on the magnetic field.
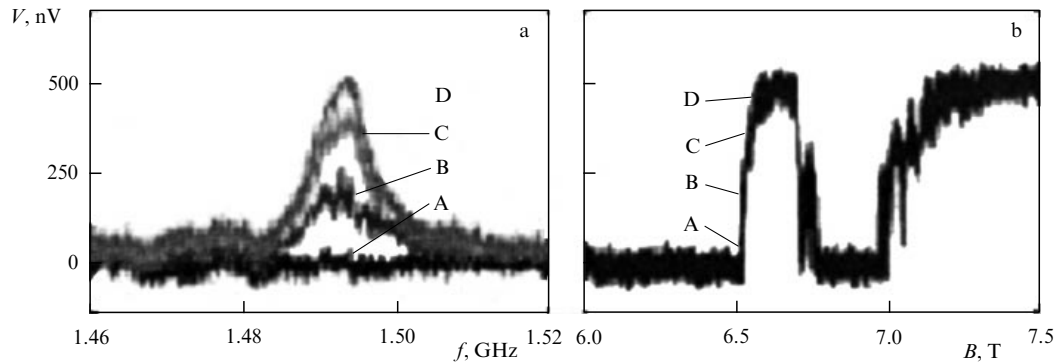
Figure 8. Nonclassical behavior of the oscillator at 110 mK. (a) The dependence of maximum voltage and, accordingly, vibration amplitude on the driving force frequency near the resonance. (b) The dependence of maximum voltage at resonance on the magnetic field.

Because the authors of Ref. [40] measured the absolute value of oscillator displacement, $|x|$, they naturally supposed that the discreteness of the oscillator excitation reflects the discreteness of its energy levels, since $|x| \propto \sqrt{E}$ (E is the oscillator energy).

The following comment is in order as regards Ref. [40]. The authors of Ref. [42], published in the same year as paper [40], questioned the correctness of the measurement of the oscillator vibrational energy and amplitude, i.e., the correctness of the interpretation of the data reported in paper [40]. It is worth noting in light of this criticism that the appearance of voltage jumps in response to magnetic field variation is in itself unambiguous evidence of the discreteness of oscillator energy states at low temperatures, observed in Ref. [40].

In the second aforementioned work [41], published in 2010 by physicists from the University of California, Santa Barbara, a mechanical system resembling a miniature ($\sim 30~\mu m$ in size) springboard was adopted; it was composed of a core layer of a piezoelectric material (aluminium nitride) flanked by aluminium layers. When operated, the structure did not oscillate from side to side like a pendulum, but alternately contracted and dilated, giving rise to an oscillating electric field (Fig. 9).

Such a mechanical system had a resonance frequency of 6 GHz, which enabled measurements at relatively high temperatures. It was cooled to 25 mK with a dilution cryostat. At this temperature, the mean number $\langle n_r \rangle$ of phonons in the oscillator was < 0.07, i.e. the oscillator resided in the ground state with a probability higher than 93%.

Mechanical vibrations of the piezoelectric were generated by electrical signals and, vice versa, resonator oscillations were recorded by the signal from the piezoelectric.

In order to observe the effects of interest in the mechanical resonator, the authors connected it to a quantum-mechanical system — a Josephson junction-based superconducting qubit consisting of superconductors insulated with a thin dielectric layer. The qubit constitutes a two-level quantum system with an energy level spacing of ΔE. The qubit frequency $\omega = \Delta E/\hbar$ is controlled by the bias voltage applied across the junction.

The mechanical oscillator and the qubit were mounted on a single plate (Fig. 10a). Figure 10b depicts the equivalent electrical diagram of the qubit and oscillator.

The qubit was excited by a high-frequency pulse, and the authors observed how excitation was transmitted to the resonator. Thus, the experiment being described demonstrated the possibility of controling vibrational motion quanta, i.e. phonons. By adjusting the frequencies of the
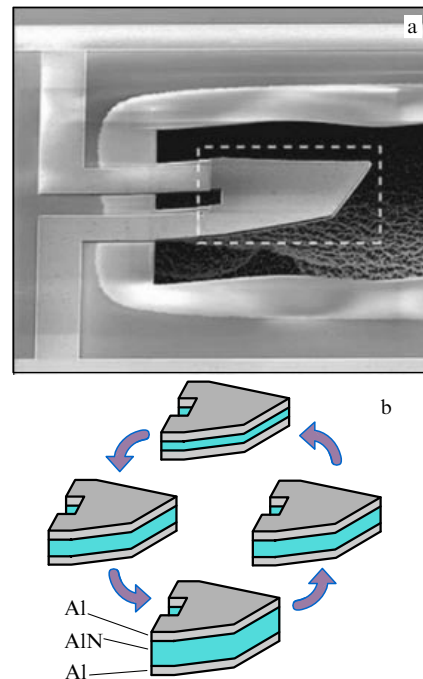

Figure 9. (a) An image of a mechanical nanooscillator produced by scanning electron microscopy (the vibrating part is enclosed by the dashed line). (b) Schematic representation of dilatometric 'springboard' oscillations.

qubit and oscillator, the authors ensured a two-way 'transfer' of quanta.

7. Optical cooling of nanomechanical oscillators

As noted in Section 6, macroscopic oscillators need to be cooled to as low a temperature as possible to enable observation of their quantum behavior. Low-temperature refrigerators allow reaching only temperatures of a few dozen millikelvins. In recent years, experimentalists have exerted considerable effort to achieve stronger cooling by optical methods that brought about very promising results in 2011 (see below). A detailed description of the theory of optical cooling of mechanical oscillators and the relevant experimental data obtained before 2009 can be found in the work of Gorodetskii et al. [43].

The idea of optical cooling stems from the phenomenon of anti-Stokes luminescence, i.e. photoluminescence, with a
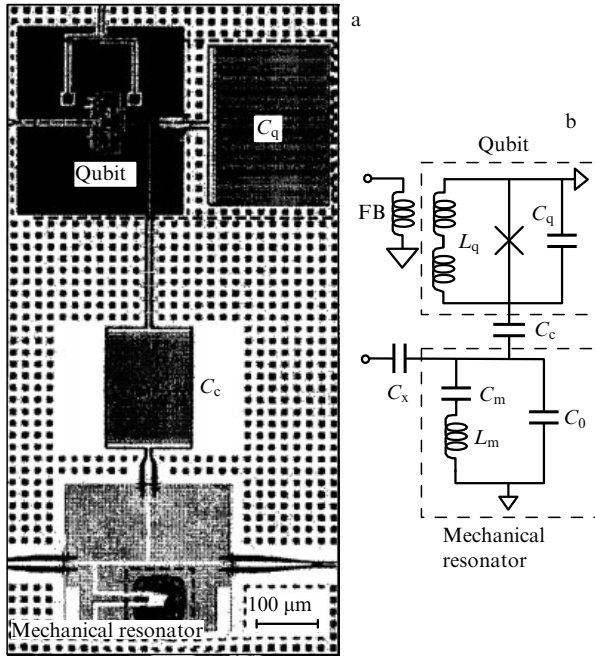
Figure 10. (a) Optical micrograph of a mechanical resonator connected to a qubit through capacitor C_c. (b) Equivalent electrical diagram of the qubit and resonator. The symbol $\times$ denotes the Josephson junction. Capacitor C_q is connected in parallel to junction with inductor L_q through which high-frequency current is transmitted to the qubit over an FB fiber-optic cable to change the qubit state. Capacitor C_c connects the qubit and the resonator. Capacitor $C_x = 0.5$ fF is used to transmit microwave signals to the resonator.

Figure 11. Stokes and anti-Stokes side frequencies resulting from amplitude modulation of a high-frequency signal with thermal vibrations of the mechanical oscillator. The width of the anti-Stokes component increases with the pumping power.

Figure 12. (a) Photograph of a microwave resonator formed by a spiral inductor and a capacitor consisting of two parallel disks. (b) Photograph of the moving membrane of the capacitor.
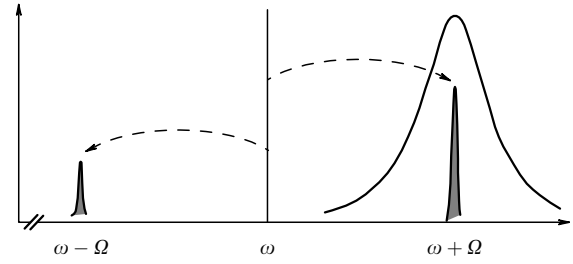
shorter wavelength compared to that of the exciting light. Substance cools because the energy of quanta increases at the expense of the thermal motion energy. This effect is called optical cooling. Laser cooling of atoms is well known and has been applied in many experiments on Bose condensation of alkali atom vapors [44]. In the papers discussed below, this method was employed to cool macroscopic systems, such as nanomechanical oscillators.

If an electromechanical resonant system has a moving part, its motion results in a change in the resonance frequency ω_0 of the radio-frequency circuit. In the case of the pumping frequency ω being lower than the resonance frequency of the contour, thermal vibrations cause amplitude modulation of high-frequency pumping. In turn this gives rise to frequency modulation, i.e. to appearing two side frequencies $\omega \pm \Omega$, as shown in Fig. 11.

The amplitude of the anti-Stokes component is greater compared with that of the Stokes one. Due to this, the former carries away the energy of the mechanical oscillator (one phonon) when it leaves the resonator; therefore, oscillator vibrations decay and set up parametric cooling of a mechanical oscillator.

Since the mechanical motion is 'encoded' in emitted photons, the measurement of emission intensity yields the temperature of the mechanical oscillator. In other words, the measurement of emission intensity of the anti-Stokes component provides a method for the direct estimation of the thermal population of the mechanical oscillator. This idea was successfully realized in the work discussed below.

Let us consider first the data reported in Ref. [45]. The mechanical oscillator was a 100-nm thick aluminium membrane 15 μm in diameter placed 50 nm above the aluminium
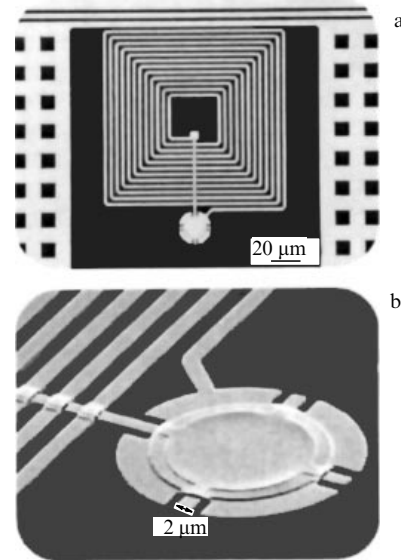
layer coating the sapphire substrate. The mass of the oscillating membrane $m = 48$ pg, and the resonance frequency $\Omega = 10.56$ MHz. These two parallel metallic layers formed a capacitor shunted by a 12-nH spiral inductor. The resulting structure operated as a quasistationary resonator in which the electric field was largely concentrated between the capacitor plates (Fig. 12). Such a resonant system provided a strong electromechanical coupling due to the fact that almost the entire electric field was concentrated near the capacitive mechanical part of the resonator.

The resonance frequency of the radio-frequency (RF) contour was $\omega_0 = 2\pi \times 7.54$ GHz. The system was cooled in a dilution refrigerator to 15 mK, at which the aluminium became superconducting and the quality factor of the RF contour was rather high. The mean number of phonons at such a temperature was $n_m = [\exp{(\hbar\Omega/k_B T)} - 1]^{-1} \approx 30$. RF pumping of the resonance circuit resulted in the cooling of the mechanical oscillator practically to the temperature of the ground quantum state, with the number of photons in the resonator being $n_c = 4000$ (Fig. 13).

The thermal motion of the membrane gives rise to a well-resolvable peak against the background of a noise microwave signal.

The area under the resonance curve of the spectral displacement density S_n is proportional to the effective
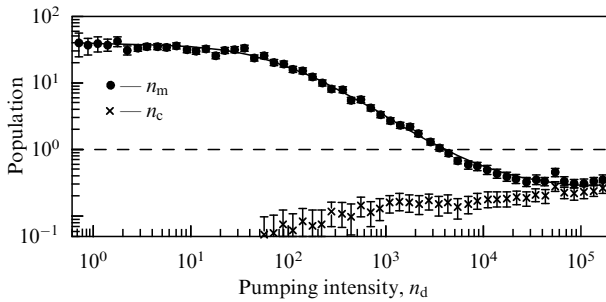
Figure 13. Dependence of the mean number of mechanical quanta (phonons) in the oscillator on the number of photons in the resonator (pumping power). At the initial temperature $T = 20$ mK, when the mean number of phonons $n_m = [\exp(\hbar\Omega_m/k_B T) - 1]^{-1} = 40$, anti-Stokes radiation cools the mechanical oscillator, due to a strong photon–phonon interaction, to the temperature of the ground quantum state, at which $n_m = 0.3 \pm 0.05$. The number of thermal photons n_c in the resonator equals 4000.

Figure 14. A silicon strip that simultaneously serves as a mechanical vibrator and a waveguide type optical cavity.

Figure 15. The dependence of the oscillator population on the pumping strength. The insets show results of the measurement of power spectral density (PSD) at the low and high pumping intensities.

temperature of the mechanical mode, which means that the measurement of the noise power makes it possible to unambiguously relate this area to the temperature of the mechanical oscillator.

As the intensity of optical pumping of the resonator increases, the 'curve base' diminishes, the resonance becomes wider, and the area under the peak decreases; in other words, the oscillator Q-factor decreases significantly as the temperature goes down (see the insets to Fig. 15). It can be accounted for by the fact that the radiation delivered to the resonator not only cools the mechanical oscillator but also damps its motion. As shown in Ref. [46], additional friction in a resonator with a movable mirror arises because the pressure exerted on the mirror upon its displacement develops not instantaneously but after a time needed for vibrations to settle down in the resonator. Then, the negative rigidity imparted by radiation pressure gives rise to additional dissipation (dissipative ponderomotive effect of electromagnetic radiation).

Figure 13 displays how the spectral density of membrane displacements varies with a rise in the pumping strength, and the oscillator practically passes to the ground quantum state as a result of optical cooling of the system.

Another experiment [47] with the so-called optomechanical resonator [48] was carried out jointly by researchers from the California Institute of Technology and the University of Vienna. As illustrated in Fig. 14, the mechanical oscillator was a thin silicon strip with holes arrayed along the axis; it simultaneously served as a waveguide type optical cavity (the resonance frequency of the vibrator amounted to 3.68 GHz, and laser pumping frequency was 195 THz).

Light modulation occurred due to thermal vibrations of the strip that caused it to lengthen and thereby altered the light cavity frequency, i.e. induced frequency modulation of the pumping signal.

Figure 15 presents the experimentally found dependence of the oscillator population on the pumping intensity. These values were determined, as in the previously considered experiment, from the Lorentz noise spectrum area (in units of $m^2\,Hz^{-1}$).

These results indicate that the authors managed to reach the state of a macroscopic oscillator with the mean number of phonons $\langle n \rangle = 0.85 \pm 0.04$ starting from a rather high temperature.
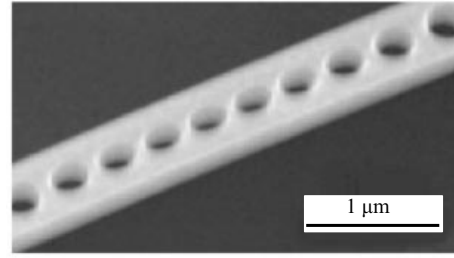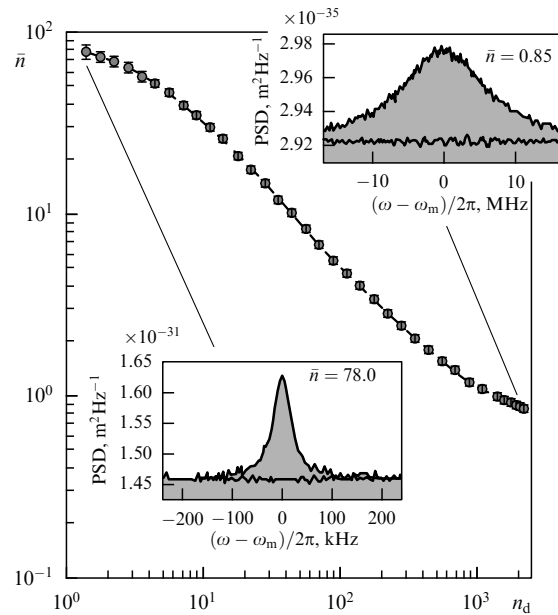
Calculations show that the ground state can be achieved starting from temperature T only if the quality factor of the mechanical oscillator, Q_m, and the width of the RF resonator, $\Delta\omega$, satisfy the relation $\hbar\Delta\omega \geq k_B T/Q_m$. Parameters of the electrooptical resonator obtained in Ref. [47] make it possible to reach the quantum state of the mechanical oscillator starting from room temperature. Naturally, this opens up wide opportunities for extensive experimentation with macroscopic oscillators.

8. Conclusion

Zero point oscillations are a fundamental consequence of the quantum-mechanical description of the laws of Nature totally contradicting classical physics. As noted above, zero point oscillations are inherent in any particle placed in a potential well; therefore, their experimental detection is of paramount importance. The most conspicuous manifestation of zero point oscillations is the experimentally observed existence of helium in the liquid phase under normal conditions at temperatures down to absolute zero.

Another surprising consequence of the recognition of zero point oscillations is the assertion of nonexistence of strictly one-dimensional crystals (R Peierls), the zero point vibrations

of atoms being so strong even at absolute zero that they destroy long-range order in the atomic arrangement. Of course, this does not imply prohibition of the existence of quasione-dimensional crystals in which the interaction between periodic chains of atoms is weak compared with that between the atoms within the chain.

The above experimental results show that the reality of zero point oscillations is unambiguously confirmed by studies of temperature dependence of the Debye–Waller factor in experiments on X-ray radiation, neutron, and gamma quanta scatterings by crystals. Of special interest are recent experiments on observing quantum effects in macroscopic mechanical oscillators. Progress in nanotechnologies made it possible to prepare mechanical resonant systems with an eigenfrequency lying in the gigahertz region, so that at millikelvin temperatures they reside in the zero state, and quantum transitions between the ground and the first excited states are well-apparent. The above experiments provided the first observation of the quantum behavior of a micrometer-sized mechanical oscillator.

Although zero point oscillations of quantum-mechanical systems were predicted in the very first years after the advent of quantum mechanics in the 1920s, they remain a subject of extensive research (see, for instance, Refs [49–52]) and a tool for the investigation of the dynamics of various systems.

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