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Laser radiation enhancement of forbidden orbital electron captures and of neutrinoless double electron captures by nuclei

M Yu Romanovskii

1. Introduction

The acceleration of the beta decay of particles (a process inverse to electron capture) under the action of an external electromagnetic field has been studied for more than four decades since the pioneering studies [1, 2]. The acceleration of nuclear beta decay was explained by the transformation of the wave function (WF) of a free emitted electron in a (high) electromagnetic field [3–7].[1] In this case, the intensity of an external electric field required for the several-fold acceleration of the process rate was estimated as $E_{\rm crit} = m_{\rm e}^2 c^3/e\hbar \sim 1.3 \times 10^{16}$ V cm^{-1} (where $m_{\rm e}$ is the electron mass, e is the elementary charge, c is the speed of light in vacuum, and $\hbar$ is the Planck constant).

The acceleration of the capture of orbital electrons (a process inverse to beta decay) has not yet drawn significant attention, partly because this effect has been known for a long time (e.g., the rate of the ^{7}Be $\rightarrow$ ^{7}Li process depends on the chemical bond involving a beryllium atom (see Ref. [8]). Furthermore, the theoretically and experimentally studied range of actions on the wave functions of orbital electrons was very wide. It covered the effects of the same chemical bonds and high pressure [9], the thermal effects (including superconductivity), the action of internal electric and magnetic fields of the medium, and plasma effects (see review [10]). K-electron capture was primarily considered, where the possible degree of rate acceleration did not exceed 10^{-2}. The known work (see, e.g., Refs [11–14]) on nuclear excitation accompanying electron transitions in the respective atoms was along the same lines.

The capture of electrons from atomic shells higher than the K-shell is also well known [15–17], including the capture of electrons with a nonzero orbital quantum number l [18, 19]. Just the latter process can be accelerated by an external

[1] Numerous incompletely accurate works predicting the acceleration of beta decay by comparatively moderate external fields are not mentioned here.

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electric field. In the case of a forbidden electron capture, a bound electron–free neutrino pair should compensate for the change in the total angular momentum of the nucleus, i.e., it should be either altered due to the captured orbital electron or 'carried away' by the neutrino. The probability of the transfer of the orbital angular momentum of the electron to the nucleus or carrying away this momentum by the neutrino is determined by two factors. One is the relationship among the nuclear radius, the characteristic radius of the electron wave function, and the de Broglie wavelength of the neutrino. The probability that the orbital angular momentum is carried away by the free neutrino is determined by the ratio of the nuclear radius r_n to the de Broglie wavelength of the neutrino raised to the power of $2l$ (l is the carried away orbital angular momentum). At the same time, the probability of the orbital angular momentum transfer from the orbital electron to the nucleus is determined by the ratio of the nuclear radius to the characteristic radius of the wave function of the orbital electron to the same power $2l$. The second factor is that the orbital electron already carries the angular momentum ready to be transferred to the nucleus, whereas the escaping neutrino has to acquire it during the decay. Then, it follows from the uncertainty relation that this process is rather fast, given the high energies E_ν of the emitted neutrino. Taken together, the above two factors account for the described deceleration of the forbidden capture of orbital electrons. Thus, for $E_\nu < 1$ MeV, the forbidden capture largely involves the corresponding electrons in the p-, d-,... states, i.e., electrons from L_{III}, M_V, N_{VII}, and more deeper-lying shells. A characteristic example is the electron capture by the nucleus of ^{205}Pb: the ratio of K- and L-capture rates is estimated at 10^{-4} [18].

The theory of orbital electron capture by nuclei (see the consistent exposition in monograph [20]) states that probabilities of allowed and unique first-, second-forbidden, etc. captures are proportional to the matrix element squared of the corresponding transition between parent and daughter nuclei, the wave function squared of the electron being captured at the nucleus, the energy of the neutrino raised to the power of 2 for allowed captures, to the power of 4 for the first-forbidden capture, to the power of 6 for the second-forbidden capture, etc., and to the Fermi constant squared of weak interactions. In what follows we will be interested first and foremost in unique first- and second-forbidden transitions expressed through a single nuclear matrix element.[2] Such electron captures are known fairly well. Suffice it to point to the mentioned first-forbidden process in ^{205}Pb. Other quite interesting examples are considered below.

2. The action of an external electric field on electron capture

Among the above terms entering the probability of the electron capture process, only the wave function of the nucleus-bound electron can be affected by an external electric field. Indeed, such a field polarizes the atom, with wave functions of all electrons becoming 'displaced' with respect to the nucleus. Clearly, for electrons in the s-state, this leads only to a decrease in the wave function amplitude at the nucleus where the WFs of such electrons have a maximum. At the same time, there is a point in the nucleus at which wave functions of electrons in the p-, d-,... states vanish. It occurs both in the Dirac description of the single-electron wave functions in an atom and in the description guided by the simple Slater approximation [21, 22]. A polarization shift can only increase densities of electron states of these WF; it is this increase in electron density with a needed orbital momentum at the nucleus that makes possible the acceleration of electron capture rate.

At present, the theory of beta processes [20] is adopting the description of the single-electron state by the Dirac relativistic equation (see, for instance, books [23, 24]) with the self-consistent Hartree–Fock potential. Earlier researchers (up to the 1960s inclusive) made use of electron wave functions in the above-mentioned Slater approximation [25], its main inaccuracy laying just with the failure to take into account relativistic effects. However, it is quite sufficient for our purpose to demonstrate the possibility of acceleration of electron capture by a laser field.

The wave functions in the Slater approximation are written for the purely Coulomb electron–nucleus interaction in the Schrödinger equation. However, this hyperfine interaction causes a certain constant addition to the electron WF to appear at the nucleus. It can be easily shown that at $l = 1$ and 2, i.e., for the first- and second-forbidden electron captures, this addition does not affect the laser radiation-induced acceleration of the above electron captures, while the hyperfine interaction weakens the effect of acceleration starting from the third-forbidden capture.

The correct calculation of the magnitude of electron capture effect from the p-, d-,... states has to include computation of the WFs of these electrons near the nucleus, taking account of the influence of the quasistationary electric field. The perturbation theory may be applied here because the external laser field under such conditions is assumed to be low for $I < Z_{\rm eff}^6 I_0 \sim (10^8 - 10^{10}) I_0$, i.e., in the nonhyperrelativistic case; here, $Z_{\rm eff}$ is the effective shielded charge, $I_0 = cE_{\rm at}^2/8\pi$, $E_{\rm at} = e/r_{\rm B}^2$, and $r_{\rm B}$ is the Bohr radius.

3. Wave functions of electrons at the nucleus in an external electric field

Let us apply the Heisenberg–Schrödinger perturbation theory to elucidate how an external electric field affects the single-particle hydrogen-like Slater wave functions of electrons. Only WF at the nucleus will be needed for the purpose, i.e., at r on the order of several r_n. We shall write out the Schrödinger equation with the effective Coulomb charge and external electric field:

$$\Delta\psi + \frac{2m_e}{\hbar^2}\left(E + \frac{Z_{\rm eff}\, e^2}{r} - eAr\right)\psi = 0\,. \qquad (1)$$

Here, ψ and E are the wave function and the energy of the corresponding electron state, respectively. As usual for problems with an external electric field, we shall move in Eqn (1) to the parabolic coordinates ξ, η, φ (see the best description in Ref. [26]). In these coordinates, the variables are separated, the normalized WF depending on parabolic quantum numbers n_1, n_2, and magnetic number m is expressed in the form

$$\psi_{n_1 n_2 m} = \sqrt{2}\,\varepsilon^{2/3} f_{n_1 m}(\varepsilon\xi) f_{n_2 m}(\varepsilon\eta)\,\frac{\exp(im\varphi)}{\sqrt{2\pi}}\,. \qquad (2)$$

Here, ε, unlike the same quantity in Ref. [20], has the sense of a dimensional constant (inverse effective Bohr radius $r_{\rm B}$): $\varepsilon = Z_{\rm eff}/nr_{\rm B}$, $n = n_1 + n_2 + |m| + 1$ is the principal quantum

[2] For simplicity, we shall consider the action of the electric field on electron capture without regard for exchange and overlap effects. Their influence is pronounced but not decisive [20].

number, functions $f_{n_1 m}$ and $f_{n_2 m}$ satisfy equations

$$\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi\,\frac{\mathrm{d}}{\mathrm{d}\xi}\right) f_{n_1 m} + \left(\frac{m_\mathrm{e} E}{2\hbar^2}\,\xi - \frac{m_\mathrm{e} e A}{4\hbar^2}\,\xi^2 - \frac{m^2}{4\xi}\right) f_{n_1 m} = -\beta_1 f_{n_1 m}\,,$$

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\,\frac{\mathrm{d}}{\mathrm{d}\eta}\right) f_{n_2 m} + \left(\frac{m_\mathrm{e} E}{2\hbar^2}\,\eta - \frac{m_\mathrm{e} e A}{4\hbar^2}\,\eta^2 - \frac{m^2}{4\eta}\right) f_{n_2 m} = -\beta_2 f_{n_2 m}\,, \tag{3}$$

$$\beta_1 + \beta_2 = \frac{Z_\mathrm{eff} e^2 m_\mathrm{e}}{2\hbar^2}\,, \qquad \beta_1 = \left(n_1 + \frac{|m|+1}{2}\right)\frac{Z_\mathrm{eff} e^2 m_\mathrm{e}}{2\hbar^2}\,,$$

$$\beta_2 = \left(n_2 + \frac{|m|+1}{2}\right)\frac{Z_\mathrm{eff} e^2 m_\mathrm{e}}{2\hbar^2}\,.$$

As mentioned above, the term with the electric field in Eqn (3) can be regarded as a small perturbation up to the amplitudes as high as the laser field amplitudes: $A < Z_\mathrm{eff}^3 E_\mathrm{at}$, i.e., for all the existing laser systems. Then, the first-order correction to the wave function $f_{n_i m}^{(1)}(x)$ is written as [27]

$$f_{n_i m}^{(1)}(x) = \frac{A}{4Ze\varepsilon^2}\sum_{n_i \neq n_j}\frac{\langle x\rangle_{n_i n_j}^2}{n_i - n_j}\,f_{n_j m}^{(0)}(x)\,, \tag{4}$$

where $f_{n_i m}^{(0)}(x)$ is the solution of equations (3) with $A = 0$, and the matrix element $\langle x\rangle_{n_i n_j}^2$ is the mean of the square of ξ or η over the corresponding unperturbed wave function $f_{n_i m}^{(0)}(x)$ (see Ref. [26] for the values of these matrix elements). The second-order correction $f_{n_i m}^{(2)}(x)$ has a similar form [27]:

$$f_{n_i m}^{(2)}(x) = \frac{A^2}{16Z^2 e^2 \varepsilon^4}\left[\sum_{n_i \neq n_k}\sum_{n_i \neq n_l}\frac{\langle x\rangle_{n_i n_l}^2}{n_k - n_i}\,\frac{\langle x\rangle_{n_l n_k}^2}{n_l - n_i}\,f_{n_k m}^{(0)}(x) - \sum_{n_i \neq n_s}\frac{\langle x\rangle_{n_i n_i}^2\langle x\rangle_{n_i n_s}^2}{(n_s - n_i)^2}\,f_{n_s m}^{(0)}(x) - \frac{1}{2}\sum_{n_i \neq n_s}\frac{\left(\langle x\rangle_{n_i n_s}^2\right)^2}{(n_s - n_i)^2}\,f_{n_i m}^{(0)}(x)\right]. \tag{5}$$

Evidently, the first-order correction to the total wave function (2) is expressed in the form

$$\psi_{n_1 n_2 m}^{(1)} = \sqrt{2}\,\varepsilon^{3/2}\left[f_{n_1 m}^{(1)}(\varepsilon\xi) f_{n_2 m}^{(0)}(\varepsilon\eta) + f_{n_1 m}^{(0)}(\varepsilon\xi) f_{n_2 m}^{(1)}(\varepsilon\eta)\right] \times \frac{\exp(\mathrm{i}m\varphi)}{\sqrt{2\pi}}\,, \tag{6}$$

and the second-order correction is given by

$$\psi_{n_1 n_2 m}^{(2)} = \sqrt{2}\,\varepsilon^{3/2}\left[f_{n_1 m}^{(2)}(\varepsilon\xi) f_{n_2 m}^{(0)}(\varepsilon\eta) + f_{n_1 m}^{(1)}(\varepsilon\xi) f_{n_2 m}^{(1)}(\varepsilon\eta) + f_{n_1 m}^{(0)}(\varepsilon\xi) f_{n_2 m}^{(2)}(\varepsilon\eta)\right]\frac{\exp(\mathrm{i}m\varphi)}{\sqrt{2\pi}}\,. \tag{7}$$

We are actually interested in the behavior of the wave functions perturbed by the laser field in the spherical coordinates near zero. Small r correspond to small ξ and η, so that functions $f_{n_1 m}$ and $f_{n_2 m}$ can be expanded into a Taylor series. Pronounced WF polarization by the laser field takes place only for the states with $|m| = 0$.[3] Then, one has

$$\psi_{n_1 n_2 0}^{(1)}(\xi, \eta \sim 0) = \frac{3(n_1 - n_2) n^{1/2} A r_\mathrm{B}^{1/2}}{8\sqrt{\pi}\,Z_\mathrm{eff}^{3/2} e}\,, \tag{8}$$

[3] The action of a uniform constant electric field on single-electron WFs with $m \neq 0$ is different. Because these WFs change signs, besides being nonzero in axially symmetric regions, the zero WF value remains immobile at point $r = 0$. Only the electro-induced change in the coefficient of r in equation (1) takes place for WF with $l = 1$, in the coefficient of r^2 for WF with $l = 2$, etc.

i.e., the first-order correction is proportional to the difference between parabolic quantum numbers; it vanishes when these numbers become equal [26, 27]. Accordingly, the second-order correction to wave function (2), namely

$$\psi_{n_1 n_2 0}^{(2)}(\xi, \eta \sim 0) = \frac{n^{5/2} A^2 r_\mathrm{B}^{5/2}}{8\sqrt{\pi}\,Z_\mathrm{eff}^{9/2} e^2}\left(n_1^3 + n_2^3 - n_1^2 - n_2^2 + 2n_1 + 2n_2 - \frac{3}{2}\right), \tag{9}$$

does not vanish even at equal parabolic quantum numbers.

Let us turn now to the calculation of the WF in a laser field. Because the exact hydrogen-like WFs in the spherical coordinates are the linear combinations of the WFs in the parabolic coordinates [26], then it follows:

$$\psi_{2,1,0}(r, \theta, 0) = \frac{1}{2\sqrt{2}}\left[\psi_{1,0,0}(\xi, \eta) - \psi_{0,1,0}(\xi, \eta)\right], \tag{10}$$

$$\psi_{3,2,0}(r, \theta, 0) = \frac{1}{2\sqrt{3}}\left[\psi_{2,0,0}(\xi, \eta) + \psi_{0,2,0}(\xi, \eta) - 2\psi_{1,1,0}(\xi, \eta)\right]. \tag{11}$$

The substitution of formula (8) into formula (10) shows that even the first-order correction to the first WF at zero point does not equal zero, and

$$\psi_{2,1,0}(r \to 0, \theta, \varphi) \simeq \frac{Z_\mathrm{eff}^{5/2}}{4\sqrt{2\pi}\,r_\mathrm{B}^{5/2}}\,r\cos\theta + \frac{3A\sqrt{3r_\mathrm{B}}}{8\sqrt{\pi}\,Z_\mathrm{eff}^{3/2} e}\cos(\theta - \gamma)\,, \tag{12}$$

where γ is the angle between the direction of the laser electric field and the WF polar angle, and $Z_\mathrm{eff} = Z_{\mathrm{L_{III}}}$. The analogous calculation of the first-order correction to the second WF at zero point yields its zero value and necessitates taking account now of the second-order correction. Thus, one obtains

$$\psi_{3,2,0}(r \to 0, \theta, \varphi) \simeq \frac{\sqrt{3}\,Z_\mathrm{eff}^{7/2}}{324\sqrt{2\pi}\,r_\mathrm{B}^{7/2}}\,r^2\left(\cos^2\theta - \frac{1}{3}\right) + \frac{9\sqrt{3}\,A^2 r_\mathrm{B}^{5/2}}{2\sqrt{\pi}\,Z_\mathrm{eff}^{9/2} e^2}\left[\cos^2(\theta - \gamma) - \frac{1}{3}\right]^2. \tag{13}$$

Here, $Z_\mathrm{eff} = Z_{\mathrm{M_V}}$. For the third-order forbidden capture of electrons, the third-order correction to the wave function $\psi_{4,3,0}(r \to 0, \theta, \varphi)$ needs to be calculated.

As mentioned above, the electron capture acceleration factor α can be found from the absolute values squared of wave functions (12), (13) averaged over time and the orientation of the laser electric field vector and integrated over the nucleus volume. The quantity thus obtained must then be carried over to the integral of unperturbed WFs over the nucleus volume. Of special interest are large α, at which the WF polarization shift markedly exceeds r_n. Thus, for the first-forbidden electron capture, one finds

$$\alpha_1 = \alpha_{\mathrm{L_{III}}} \approx \frac{25}{4Z_{\mathrm{L_{III}}}^8}\left(\frac{r_\mathrm{B}}{r_\mathrm{n}}\right)^2\frac{I_{l0}}{I_0}\,. \tag{14}$$

The acceleration factor ($I_{l0} = cA^2/8\pi$) of the second-forbidden electron capture under analogous conditions is given by

$$\alpha_2 = \alpha_{\mathrm{M_V}} \approx \frac{3^{15}\pi}{8Z_{\mathrm{L_{III}}}^{16}}\left(\frac{r_\mathrm{B}}{r_\mathrm{n}}\right)^4\frac{I_{l0}^2}{I_0^2}\,. \tag{15}$$

Let us consider concrete examples. The decay of a long-lived lead isotope (with a half-life of 1.53×10^7 years) $^{205}\mathrm{Pb} \to {}^{205}\mathrm{Tl}$ proceeds with a change in the total nuclear momentum by 2 and a change in parity. This is the unique first-order forbidden capture of electrons. The acceleration $\alpha_1 \simeq 1.7 \times 10^{-6}\, I_{l0}/I_0$. A laser radiation fluence of $\sim 6 \times 10^6\, I_0 \sim 10^{23}$ W cm^{-2} is needed for the appreciable ($\alpha_1 \sim 10$) acceleration of this capture. Such fluences remain to be reached. Another example of a similar process is the capture of electrons in a long-lived $^{81}\mathrm{Kr}$ isotope (with a half-life of 2.29×10^5 years [17]). For $^{81}\mathrm{Kr} \to {}^{81}\mathrm{Br}$ decay, $\alpha_1 \simeq 0.35\, I_{l0}/I_0$. A tenfold acceleration is already achieved at fluences $I_{l0} \sim 10^{18}$ W cm^{-2}.

The possibility of the experimental observation of acceleration of the second-order forbidden capture in the laser field is much more obvious: the effective charge is lower, and polarizability higher. The second-order forbidden electron capture, e.g., in a $^{133}\mathrm{Ba}$ isotope (with a half-life of 10.51 years), $^{133}\mathrm{Ba} \to {}^{133}\mathrm{Cs}$, is characterized by $\alpha_2 \simeq 300\,(I_{l0}/I_0)^2$, i.e., $\alpha_2 \sim 10$ at $I_{l0} \sim 10^{16}$ W cm^{-2}.

It is especially interesting to consider processes involving stable nuclei. For the second-order forbidden electron capture $^{123}\mathrm{Te} \to {}^{123}\mathrm{Sb}$, $\alpha_2 \simeq 1000(I_{l0}/I_0)^2$, i.e., $\alpha_2 \sim 10$ at $I_{l0} \sim 10^{15}$ W cm^{-2}. This process has until recently been interpreted as a K-capture, in conformity with the emission of quantum with the energy in the 28-keV region — the K_α-line of $^{123}\mathrm{Sb}$ with a lifetime exceeding 10^{13} years [28]. The same value was collated in tables [17]. However, later (more accurate) studies demonstrated that there is no such emission, and the $^{123}\mathrm{Te} \to {}^{123}\mathrm{Sb}$ decay cannot be interpreted as K-capture [29]. The half-life was shown to be constrained by the value of $t_{1/2} > 9.2 \times 10^{16}$ years [29].

4. Possibilities of experimental realization

The intensities of laser radiation needed to accelerate the first-forbidden electron capture are rather high: at present, they are realized in short (less than 1 ps) superpower laser pulses with a low repetition rate. Nevertheless, amplification of characteristic X-ray radiation with transitions from the L_{III}-shell is possible to observe even if in the X-ray photon counting mode. The process of electron capture acceleration is likely to be accompanied by marked ionization of the atoms being studied.

The acceleration of the second-forbidden electron capture is easier to realize. The necessary radiation intensities can be reached even by focusing powerful radiation of continuous wave lasers. In order to prevent the formation of highly ionized plasma, the experiment may consist in focusing laser radiation into strongly rarefied atomic vapor 'clouds'. The rather small total number of 'working' nuclei is compensated for by the continuity of laser irradiation. We note that many interesting nuclei, first and foremost light ones, remain beyond the framework of the approximation. By way of example, it can be expected that a 10-fold acceleration of electron capture in $^{54}\mathrm{Mn} \to {}^{54}\mathrm{Cr}$ ($t_{1/2} = 312$ days)[4] will be possible to achieve at a laser radiation intensity of $\sim 10^{11} - 10^{12}$ W cm^{-2}. The experiment can be conducted by the irradiation of a solid state target with a large number of decaying nuclei. The conditions for the third-forbidden electron capture are even more readily available [30].

5. Acceleration of neutrinoless double capture of orbital electrons

Here are some physical considerations as regards the possibility of achieving acceleration of significant (thus far hypothetical) neutrinoless double capture of orbital electrons by stable nuclei through the application of a strong electric field (including a laser field) to the atoms. Such a process, hypothesized as early as 1955 [31], can be realized if a neutrino coincides with its antiparticle (E Majorana's hypothesis). In such a case, two consecutive electron capture processes need to be summarized for a nucleus of mass number $A + 2$:

$$\begin{aligned}&\left[(A+2) + \mathrm{e} \to (A+1) + \nu_\mathrm{e}\right] \\ &\quad + \left[(A+1) + \mathrm{e} + \bar{\nu}_\mathrm{e} \to A + \gamma\right] \\ &\quad = (A+2) + 2\mathrm{e} \to A + \gamma\,. \end{aligned} \tag{16}$$

Formally, in this case the neutrino of the first process and the antineutrino of the second one cancel each other out (physically, the neutrino is emitted in the former process, and absorbed in the latter). The two events are integrated into the resultant process with the emission of a gamma quantum. Such transitions occur between nuclei $0^+ \to 0^+$; there are a total of 12 such pairs with pure electron capture without positron emission [17].

Evidently, the time between the first process and the second one is not too large: it is at least shorter than the neutrino time of flight in the nucleus of interest, and the distance travelled by the neutrino is shorter than the nuclear radius. This fact accounts for the magnitude of the nuclear transition matrix element between the parent $(A + 2)$ and daughter (A) nuclei: it appears to be lower for transitions with a smaller intermediate nuclear momentum. Indeed, the uncertainty relation for the angle φ and angular momentum L_z is written as [23, 24]

$$\left\langle (\Delta L_z)^2 \right\rangle \left\langle (\Delta\varphi)^2 \right\rangle \geqslant \frac{\hbar^2}{4}\,,$$

and at small angular displacements $\Delta\varphi$ (the spatial displacements of protons that capture an electron inside the nucleus being small, too) it determines the rather large values of ΔL_z associated with the first capture, i.e., in the intermediate $(A + 1)$ nucleus. For this reason, the above-mentioned nuclear matrix element must be larger (much larger) for such neutrinoless double captures in which electron capture by the intermediate nucleus is forbidden. It is clear, however, that the wave function of the electron being captured at the nucleus is smaller (by a factor of $Z_{\mathrm{eff}} r_\mathrm{n}/r_\mathrm{B}$ to the appropriate power), and the result for the double forbidden and allowed capture rates is roughly equal (in any case, the difference will not be as great as for single-electron captures [20, 25]). When the wave function of the captured electron with the nonzero orbital quantum number is shifted by an electric field with respect to the nucleus, the amplitude of this function at the nucleus increases, which leads to the acceleration of the double electron capture. In transitions between nuclei $0^+ \to 0^+$, the acceleration factor is simply the corresponding acceleration factor squared of the forbidden single-electron capture (14), (15).

[4] Breakdown voltage of pure Mn salts amounting to 10^9 W cm^{-1} (such a field amplitude corresponds to a radiation intensity of 10^{15} W cm^{-2}), the experiment can be conducted by placing a sample of such salt in a constant electric field of ~ 10 MV cm^{-1}.

6. Neutrinoless double capture to the resonance states of daughter nuclei and possibilities of its experimental realization

The 'surplus' energy in neutrinoless double electron capture must be eliminated by a γ-quantum with the appropriate energy. If the daughter nucleus lacks resonance levels in close proximity to the ground state (minus the binding energy of the electrons being captured), the main process by which such γ-quanta are generated in the parent nucleus is bremsstrahlung [32]. The presence of such closely spaced excited levels would sharply increase the probability of double capture, because it might occur to the resonance state with a subsequent resonant release of the corresponding γ-quantum. The calculation of such resonant processes for certain nuclei is reported in Refs [33–36]; it is proposed to employ synchrotron radiation to realize induced transitions to the excited state of the daughter nucleus [37]. The resonance parameter F in the probability of double capture to the excited state has the form [38]

$$F = \frac{\Gamma_{2h}}{\Delta^2 + \Gamma_{2h}^2/4}\,, \tag{17}$$

where $\Delta = Q - B_{2h} - E_\gamma$, B_{2h} and Γ_{2h} are the energy and the width, respectively, of a double electron hole in the atomic electron shell of the daughter nucleus, Q is the difference between the binding energies of the parent and daughter nuclei, and E_γ is the energy of the excited level in the daughter nucleus.

Isotope ^{74}Se possesses the lowest Δ among known nuclei: for electron capture from L_{III} and L_I shells, $\Delta_{Se} = 2.6$ keV (the data of Ref. [33]; the value of Q for a $^{74}\text{Se} \to {}^{74}\text{Ge}$ pair was measured to an accuracy of 2.3 keV [33]). Initially, the first-forbidden capture to the virtual ^{74}As nucleus ground state 2^- occurs; it is followed by the allowed capture to the excited state 2^+ of ^{74}Se. According to Ref. [33], the upper estimate of the half-life at such a mismatch Δ_{Se} (in the first capture from the L_{II} but not the L_{III} shell) amounts to 0.55×10^{19} years. The above reasoning leads to the conclusion that the double electron capture from the L_{III} and L_I shells must be faster.

In the course of atomic ionization, the absolute value of the electron binding energy in the remaining shells becomes higher, because nucleus shielding weakens. Moreover, the ionization energy during transition between electron shells undergoes a strong jump (the ionization energy of neonlike Se^{24+} is 2542 eV compared with 1036.3 eV in the preceding Se^{23+} ion). This fact was utilized, for instance, in designing plasma X-ray lasers [39, 40]. The electron binding energy in the L_{III} shell of neonlike $^{74}Se^{24+}$ roughly equals the ionization energy, 2542 eV; it is 15% higher or 2923 eV (as in the initial atom [41]) in the L_I shell. Thus, $B_{2h}^i \approx 5.5$ keV in an $^{74}Se^{24+}$ ion, i.e., $\Delta_{Se}^i \approx 0$! (see Fig. 1), which means that the value of Δ_{Se}^i for the $^{74}Se^{24+}$ ion is on the order of several dozen electron-volts. The value of Γ_{2h}^i in the $^{74}Se^{24+}$ ion being also different (1.5 times higher) from the atomic value of Γ_{2h}, this fact should be taken into account in the acceleration factor.

Thus, we have to deal with two acceleration factors of neutrinoless double capture of electrons from the L_{III} and L_I shells of ^{74}Se in a strong laser field. The strongest of them is bringing quantity (17) into resonance as a result of the Stark shift of the inner electron energy levels in the ion as compared with those in the atom. In this case, the acceleration of the first-forbidden capture from the L_{III} shell to a virtual nucleus [in the absence of acceleration of the capture from the L_I shell (see above)] is not too high. The resultant acceleration factor α_1^{res} can be obtained by multiplying the resonance factor ratio (17) for the ion and atom by α_1. In this case, if

$$\frac{25}{4Z_{L_{III}}^8}\left(\frac{r_B}{r_n}\right)^2 \frac{I_{l0}}{I_0} > 1\,,$$

then

$$\alpha_1^{res} \approx \frac{25}{4Z_{L_{III}}^8}\left(\frac{r_B}{r_n}\right)^2 \frac{I_{l0}}{I_0}\,\frac{\Delta^2\Gamma_{2h}^i}{(\Delta^i)^2\Gamma_{2h}}\,; \tag{18a}$$

in the opposite case, one finds

$$\alpha_1^{res} \approx \frac{\Delta^2\Gamma_{2h}^i}{(\Delta^i)^2\Gamma_{2h}}\,. \tag{18b}$$

Substituting parameters of the ^{74}Se L_{III} and L_I shells and $\Gamma_{2h} \sim 1$ eV, and taking for once into consideration the absence of acceleration of electron capture from the L_{III} shell (the intensity of the laser field is sufficient to maintain only a plasma with ions of the desired degree of ionization and case (18b) is realized), we arrive at $\alpha_1^{res} \sim 10^4$ and, therefore, the time of such a process in the laser field of the specified intensity covers $\sim 5 \times 10^{14}$ years).

The maintenance of dense, hot $^{74}Se^{24+}$ plasma can be achieved in different ways [39, 40]. One can confine it in a trap created by pulsed beams of one or several CO_2 lasers (see Refs [42, 43] for the description of traps). When a system of laser beams [42, 43] keeps the number of ions relevant to 1 g of ^{74}Se, the minimum time of an experiment needed to detect γ-quanta with energies 608.35 and 595.85 keV from an emitting ^{74}Ge nucleus may be as short as a few seconds.

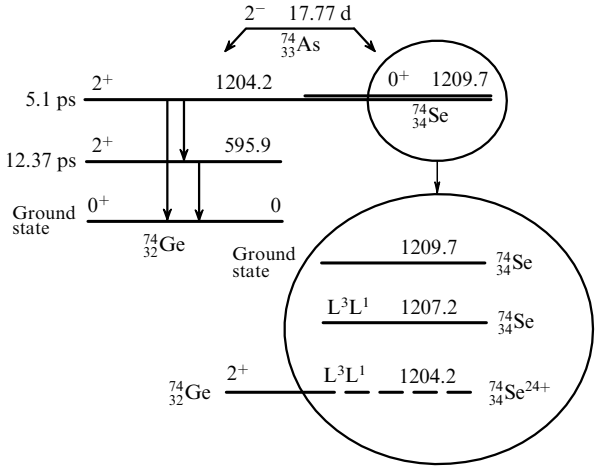

Figure 1. Schematic of energy layers in the neutrinoless electron capture $^{74}\text{Se} \to {}^{74}\text{Ge}$. Left: lower excited levels of ^{74}Ge and the corresponding lifetimes. The ellipse encompasses energy levels of the ^{74}Se ground state, double holes in the L_I and L_{III} electron shells of a ^{74}Se atom, and a neonlike $^{74}Se^{24+}$ ion. The $1204.2 \to$ ground state transition in ^{74}Ge is roughly half as probable as the $1204.2 \to 595.9$ transition [13] (all energies are expressed in kiloelectron-volts).

7. Conclusion

The intensities of laser radiation needed to accelerate the first-forbidden electron capture by nuclei are relatively high: at present, they are realized in short superpower laser pulses less than 1 ps in duration at a low repetition rate. Nevertheless,

observation of the acceleration of characteristic X-ray radiation with transitions from the L_{III} electron shell appears possible even if in the X-ray photon counting mode.

Acceleration of the second-forbidden capture is easier to realize. The necessary intensities of laser radiation can be achieved even by focusing radiation from powerful continuous wave lasers. Equally possible are experiments in a constant electric field of 10–100 MV cm^{-1}, i.e., in a field lower than the breakdown field voltage of many pure dielectrics containing nuclei of interest.

Finally, considerable acceleration of the neutrinoless double capture of electrons, as well as verification of Majorana's hypothesis for the nature of neutrino, is possible in an experiment with a plasma consisting of electrons and neonlike $^{74}Se^{24+}$ ions. Also, the use of heavy ^{168}Yb ions offers great prospects.

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Computational physics and testing theoretical predictions

L N Shchur

1. Introduction

Computational physics was born simultaneously with the creation of the first electronic computers.[1] Physicists used computers to achieve a practical goal important at that moment: to develop thermonuclear weapons. One of the first computational tasks needed for peaceful applications was the work of Fermi, Pasta, and Ulam [1] on simulating the dynamics of the one-dimensional nonlinear chain; the work was done on the MANIAC 1 mainframe computer in Los Alamos [2]. The opinion prevalent at the beginning of the 1950s was that nonlinearity should lead to equipartition of energy over degrees of freedom, i.e., to stochastization. Contrary to expectations, numerical experiments revealed quasiperiodic behavior. This phenomenon was explained in 1965 by Zabusky and Kruskal [3], who numerically identified solitons (and introduced the very term 'soliton') and found their inelastic scattering. This result led to the discovery of the inverse scattering problem method [4] which, in turn, became the key to obtaining exact solutions of nonlinear problems (see, e.g., monograph [5]). This is an impressive example of the

[1] The author uses the Russian term 'electronic computing machine' (EVM in *Russ. abbr.*) interchangeably with the currently widespread term 'computer' in those section of the text where it was historically justifiable. The English translation uses the term 'computer' throughout.

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