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LETTERS TO THE EDITORS

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Is singularity-free cloaking possible? New cloaking ideas
(a reply to comments on the paper “Invisible cloaking of material bodies using the wave flow method” [*Phys. Usp.* **53** 455 (2010)])

A E Dubinov, L A Mytareva

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Abstract. Replies are made to comments in N N Rozanov's letter to the editors, “Can even monochromatic radiation ensure ideal invisibility?” (see *Usp. Fiz. Nauk* 181 787 (2011) [*Phys. Usp.* 54 763 (2011)]) concerning our paper “Invisible cloaking of material bodies using the wave flow method” (*Usp. Fiz. Nauk* 180 475 (2010) [*Phys. Usp.* 53 455 (2010)]). Examples are given of spatial configurations that enable the creation of singularity-free cloaking materials. Some emerging cloaking ideas are discussed.

1. Genesis

First, a word on priority: unfortunately, we had no previous knowledge of L S Dolin's work [1] referred to in the letter [2] to the editors of *Physics–Uspekhi*. It so happened, however, that just a few days after our paper [3] came online, L S Dolin himself kindly sent us a reprint of his work. The work [1] does indeed have direct relevance to wave cloaking of material bodies, which is the research subject of Ref. [3], and derives expressions for the $\hat{\varepsilon}$ and $\hat{\mu}$ tensor components of a spherical cloaking shell, something which was done forty five years later by J Pendry et al. [4]. While presenting the cloaking idea in an admittedly less colorful way compared to subsequent foreign work, Ref. [1] points quite definitely to coordinate transformations as a tool “to design nonreflecting inhomogeneities.” In what follows, it will be taken for granted that the credit for priority goes to Ref. [1].

A E Dubinov, L A Mytareva Sarov State Institute of Physics and Technology,
ul. Dukhova 6, 607186 Sarov, Nizhny Novgorod region, Russian Federation
Tel. + 7 (83130) 4 51 44, + 7 (83130) 5 12 35
E-mail: dubinov-ae@yandex.ru, mytareva-la@yandex.ru

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2. Singularity-free cloaking

We now turn to the essence of the problem. As pointed out in letter [2], what mainly prevents ideal wave flow cloaking is a singularity that arises in the coordinate transformation used in calculating a cloaking shell. As a result, some components of the $\hat{\varepsilon}$ and $\hat{\mu}$ tensors inevitably become infinite on a certain set of points within the shell. This problem, it is argued, is insurmountable, and hence ideal cloaking is fundamentally impossible.

It is indeed known that there is no one-to-one mapping between a simply connected and a doubly connected domain. For example, when mapping a circular cylinder to a cylinder with a coaxial cavity (or a circle to a ring) (Fig. 1), each point on the axis is put into correspondence with the infinite set of points on the inner surface (circle). Then, the linear transformation

$$r' = \frac{b-a}{b}\, r + a\,, \qquad \phi' = \phi\,, \qquad z' = z \tag{1}$$

yields the following expressions for the permittivity tensor components:

$$\begin{gathered} \varepsilon_{rr} = \mu_{rr} = \frac{r-a}{r}\,, \qquad \varepsilon_{\phi\phi} = \mu_{\phi\phi} = \frac{r}{r-a}\,, \\ \varepsilon_{zz} = \mu_{zz} = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r}\,, \end{gathered} \tag{2}$$

with a and b being the respective inner and outer radii of the shell. It is seen that the ϕ components at the inner boundary tend to infinity.

Reference [2] is, as we understood it, a critique of our paper [3] in which, it is claimed, we overlook the fact that some $\hat{\varepsilon}$ and $\hat{\mu}$ tensor components inevitably diverge, and we repeatedly use the concept of ‘ideal cloaking’, something which these divergences (singularities) totally rule out (see the title of Ref. [2]).

Let us object. First, the concept of ‘ideal cloaking’ is valid to use even if cloaking can never be ideal. After all, knowing

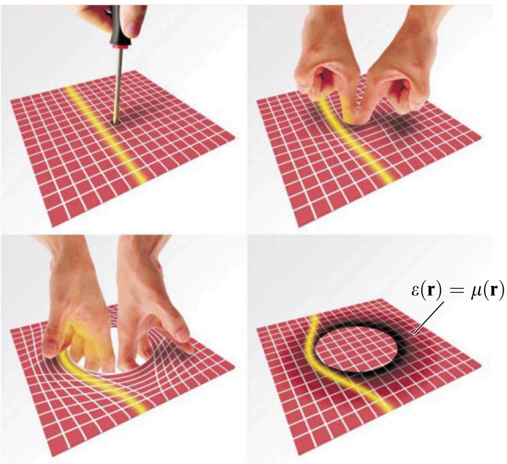

Figure 1. Coordinate transformation (1). (Taken from Ref. [5].)

that all real gases have the equations of state differing from that of an ideal gas does not prevent us from speaking of an 'ideal gas'. Just try to declare publicly that all the literature on ideal gases is erroneous and useless!

Second, the problem of singularity was by no means ignored in our work (see, for example, Section 7.3 of Ref. [3]). We, too, are of the opinion that this is the key (if not the only) problem in designing ideal cloaking shells.

Let us take a look at how the problem of singularity has been approached in the literature. It should be understood from the outset that there are at least two ways around this problem: to learn to create shells with a specified singularity on a specified set of points, or to find such coordinate transformations whose resulting shell tensor components are free of singularities.

The former approach has met with little or no success. Reference [6], the only noteworthy one in this context, analyzed a nonideal cylindrical shell with parameters (2) and without a thin inner layer δ (Fig. 2). The goal was to see the extent to which the cloaking effect is influenced by removing the thin inner layer containing points where the ϕ-components of the ε and μ tensors diverge.

The authors of Ref. [6] obtained an expression for the amplitude of radiation scattered from a shell with an inserted perturbation δ (see Fig. 2). Letting δ tend to zero, they observed that decreasing δ by three orders of magnitude decreased scattering by at least an order of magnitude! What this slow convergence implies is that even the slightest deviation of the shell from the ideal shape leads to the appearance of significant scattering.

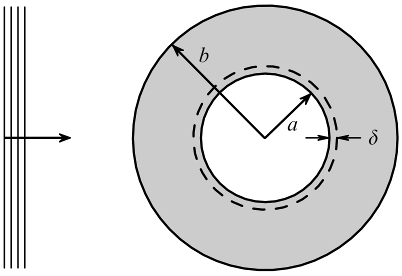

Figure 2. Cylindrical cloaking shell with perturbation δ. (Taken from Ref. [6].)

Indeed, it is these findings that stimulated much research into solving the singularity problem by finding cloaking shells with shapes and profiles free of singularity points. Importantly, the second approach turned out to be more efficient than the first.

The reason is that coordinate transformation (1) and its resulting shell configuration (2) are not the only ones possible for this geometry (a point which is strongly emphasized and illustrated with examples in Ref. [3]), and this makes it possible to find the coordinate transformation required.

The idea of how to find such a transformation came from Ref. [7], which argues that whether or not the shell has singular points depends on the measure of the set of points from which the cavity 'blows up' under the coordinate transformation. There will be no such points if the original set of points and its image have identical measures.

This requirement is not met for the cylindrical shell obtained by linear transformation (1), the reason being that its cavity 'grows' from the axis of the cylinder (a set of measure 1). This is where the singularity comes from.

Because the image is always a surface (the inner surface of the shell), it follows that the original must also be a set of measure 2. But how can this be achieved?

In Ref. [7], for example, the solution for an elliptic cylinder-shaped shell is obtained by linearly transforming the conventional elliptic cylinder coordinates in which the coordinate lines in the cross sectional plane are confocal ellipses (Fig. 3). Then, the inner surface of the elliptic shell is obtained from a portion of the plane (the segment $[-p, p]$ in the cross section shown in Fig. 3), and the dielectric permittivity and magnetic permeability components of this shell are nowhere divergent. This kind of a singularity-free shell was pointed to in Ref. [3].

One further singularity-free cloaking proposal, also discussed in Ref. [3], was suggested in paper [8] titled, "Hiding under the carpet: a new strategy for cloaking" (this vivid image is used to describe the idea of cloaking by a flat layer (against the background of a smooth surface) whose lower surface has a small indentation where the object to be hidden is placed (Fig. 4).

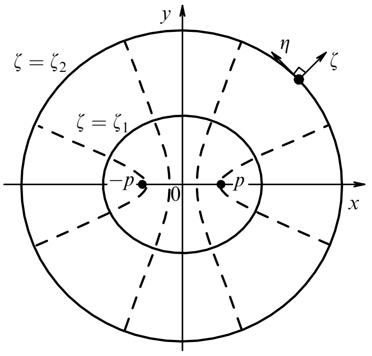

Figure 3. Elliptic cylinder coordinate system. (Taken from Ref. [7].)

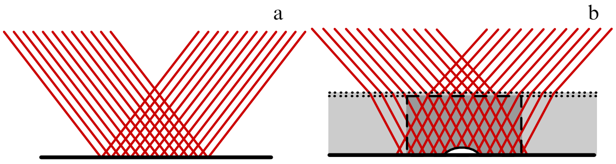

Figure 4. Ray trajectories on (a) a background surface, and (b) a carpet-like cloaking layer. (Taken from Ref. [8].)

In this case, the transformation used is one-to-one, which rules out any singularities and likely allows ideal cloaking (at least from the point of view of the absence of divergences!). Notably, it is various types of such 'carpets' that are currently a major focus of foreign wave cloaking research [9–14]. We have already reported on an experimental realization of an ultrahigh frequency carpet [3], a development which demonstrated the possibility not only of avoiding singularities but also of getting rid of shell anisotropy and of the monochromaticity of the effect (the shell exhibited the cloaking effect at four different frequencies: 13, 14, 15, and 16 GHz). Reference [15] used a nanostructured carpet to experimentally demonstrate the cloaking effect in the optical range ($\lambda = 1520$ nm), and in a recent study [16] a layered plastic carpet produced acoustic cloaking at 3 kHz.

Added to the above, calculations have recently been made of specifically shaped and arbitrarily shaped shells assumed to have finite material parameters (see, for example, Refs [17–19]).

Thus, our paper [3] indisputably did pay special attention to the divergence problem and indeed presented two kinds of singularity-free shells. And even though we totally agree with the author of Ref. [2] that an ideal shell containing singularities cannot be created, our feeling is that the problem is likely solvable.

One more remark is in order here. Our extensive monitoring of cloaking literature over the last four years has revealed the occurrence during this period of a number of seemingly insurmountable problems on the way to creating and applying a cloaking shell with more or less ideal performance properties. But, insurmountable though a problem appeared, each time, invariably, a new and nontrivial way out was found. So, let us be optimistic!

3. Cloaking news

The field of cloaking technology is currently experiencing dynamic and large-scale growth as witnessed, in particular, by the fact that within a mere year after our paper [3] was published, a number of unusual and indeed strikingly new cloaking ideas took shape — including outer, anticloaking, mirage, and illusion technologies — and we cannot resist this opportunity to describe them, if only briefly.

A major problem mentioned in Ref. [3] is that an object under a cloak is blind in the sense that it has no channel to communicate with the world around it. Reference [3] also pointed to a way to overcome this blindness by simply making a small window in the cloaking shell. As an extension of this idea, a multiwindow cloak was suggested and studied [20].

An interesting geometric approach, now referred to as *outer cloaking*, was suggested in Ref. [21]. If a number of cloaking elements are arranged in a cluster-like spatial pattern, an object placed inside the cluster will be invisible, and windows that exist between the elements will serve as communication channels between the cloaked object and the outer world. Figure 5 illustrates the concept.

Three successive time snapshots illustrating the scattering of a spherical wave pulse from a metal body of complex shape are shown in the upper part of Fig. 5. The reflected wave and the shadow region are clearly distinguishable, making it an easy matter to detect the scatterer, but if the body is placed inside the cluster of cloaking elements, the reflected wave and the shadow both disappear. Further work along these lines was pursued in Refs [22, 23].

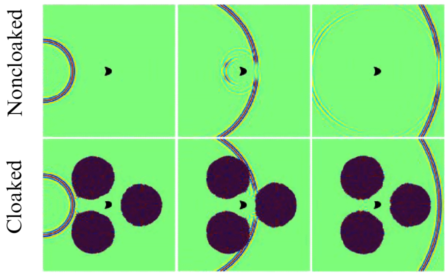

Figure 5. Outer cloaking modeling results for three successive times. Upper panel: scattering of a spherical electromagnetic pulse by a metal body, resulting in a reflected wave and a shadow. Lower panel: outer cloaking by a cluster of three cloaking elements. (Taken from Ref. [21].)

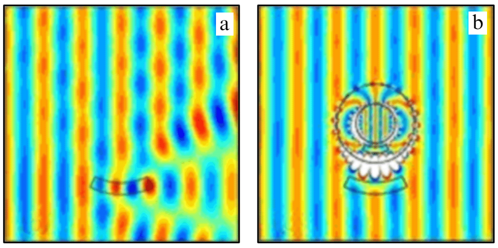

Figure 6. Cloaking using the Chinese umbrella technique: (a) opened scatterer; (b) scatterer cloaked by a Chinese umbrella. (Taken from Ref. [24].)

In another example of outer cloaking — the so-called Chinese umbrella suggested by Chinese researchers [24] (see Fig. 6) — a U-shaped dielectric scattering object makes a shadow for a plane wave. A specially designed circular cloaking element placed next to the scatterer serves as an umbrella to hide it (the complex wave field structure near the umbrella indicates that the cloaking approach used relied on a wave type transformation, with an electromagnetic wave transformed to and back from surface plasmons).

Yet another counter-blindness approach, which its authors [25] called anticloaking, consists in simply surrounding the object for a short time by an inner shell which neutralizes the effect of the outer cloaking shell (Fig. 7). The important result of Ref. [25] is that the structure of the inner anticloaking shell can be calculated for arbitrary geometries of the object and of the outer cloaking shell. (For more on this see Refs [26, 27].)

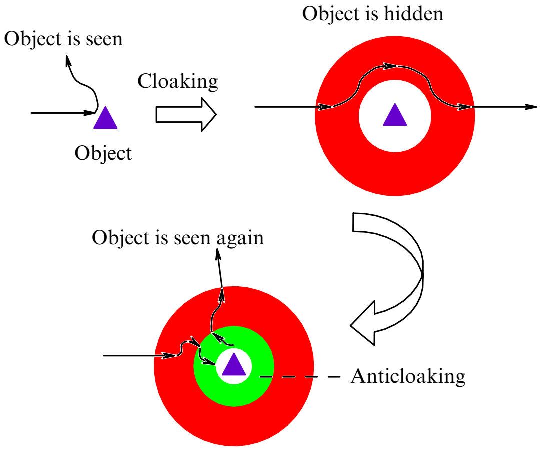

Figure 7. Anticloaking diagram [24].

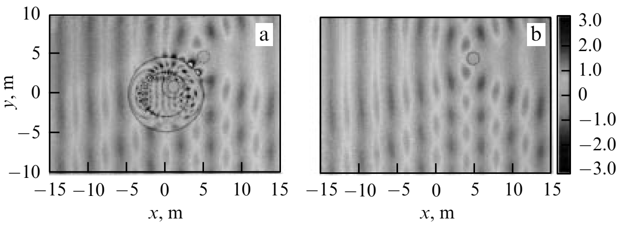

Figure 8. Wave patterns explaining mirage [29]. Identical scattering fields form for a plane wave incident on a metal cylinder of radius 0.8 m (a) surrounded by a special shell and centered at point $(1, 1)$, and (b) surrounded by no shell and centered at point $(5, 5)$.

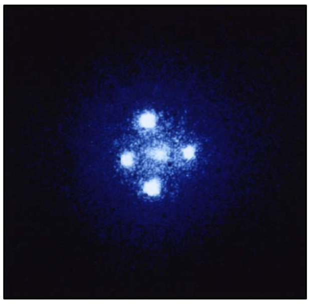

Figure 9. Hubble image of the Einstein Cross. (From http://upload.wikimedia.org/wikipedia/commons/7/7e/EinsteinCross.jpg.)

It is to be stressed that the key task of wave flow cloaking is to mislead the outer observer as to the location and indeed the very existence of the object, and as to its main properties (size, shape, material composition, etc.), by endowing it with invisibility.

Invisibility, though, is not the only answer to this problem. We can, for example, mislead the observer by making him or her see the object where it is not resided. This technology has come to be known as *mirage*. Notable work on this topic includes Refs [28] and [29]. While the shell calculated in the former produces a mirage at a distance of only one third of the wavelength from the object, the setup offered in the latter considerably increases this distance (Fig. 8).

It also proves possible to create a large number of similar images of one object at different places. This *multiple mirage* phenomenon has long been known to astronomers: as a result of rays being gravitationally lensed by a massive body, two or more images of one and the same compact object can be observed in the sky. An example is the so-called Einstein Cross, a quadruple image of the quasar Q2237 + 030 (or QSO 2237 + 0305) lensed by the ZW 2237 + 030 galaxy (Fig. 9).

In yet another cloaking approach, known as *illusion*, the observer is misled as to the shape, size, and material of the object to be cloaked. The results of Ref. [30] illustrate this idea.

Suppose that we need to conceal information about a dielectric spoon in such a way as to make the outer observer see it as a metallic handled cup, i.e., the observer has to be misled as to the material, shape, and topology of the body. Surprising though this may seem, the problem does have a solution! The solution is a shell chosen and calculated in a special way which, when wrapped around the spoon, scatters the wave in such a way as to produce the required illusion. The results of the calculations are given in Fig. 10. The reader is referred to Refs [31–33] for subsequent work on the analysis and development of the illusion idea.

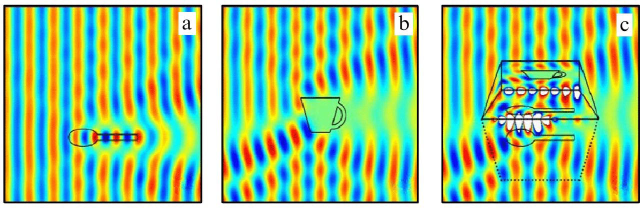

Figure 10. Wave patterns explaining illusion: (a) scattering of a plane wave by a spoon with permittivity $\varepsilon = 2$; (b) scattering of a plane wave by a metal cup with $\varepsilon = -1$; (c) scattering of a plane wave by a spoon in a shell producing the same scattering as the cup, i.e., the wave pattern of the illusion. (Taken from Ref. [30].)

Notice that the calculation of a mirage- (or illusion-) producing shell or of a cloaking shell proper is a problem in transformation optics. The basic aspects of this relatively new field are discussed, for example, in the review paper [34].

4. Conclusion

In summary, the material presented above complements our review [3] and convincingly demonstrates that considerable advances are being made in the understanding and development of wave cloaking and that the problems that arise in the field — such as singularities in the material parameters of cloaking shells or the blindness of a cloaked object — are being steadily solved.

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